Chapter III

THEORETICAL CONSIDERATIONS

Evaporation on an Inclined Heated Plate

A liquid solution is being fed on an inclined heated plate at its boiling temperature. The plate makes an angle of θ° with the vertical line. As the solution is flowing downward in a form of thin film, under the gravitational force, it gains heat from the heated plate, water in the solution vaporized, the concentration of the solution increases, and the thickness of the film might change.

Let the coordinates of the system be assigned as shown in Figure 2. It is assumed that the width of the plate in z-direction is infinite as compared to the thickness of the liquid film. There are no momentum, heat, and mass transferred in z-direction. It is also assumed that the temperature gradient in y-direction is negligible. Therefore, all the heat used for vaporization of water is being transferred in x-direction from the plate surface to the vapor-liquid interface. If the liquid film is very thin, the flow is laminar, consequently, the heat is conducted in x-direction.

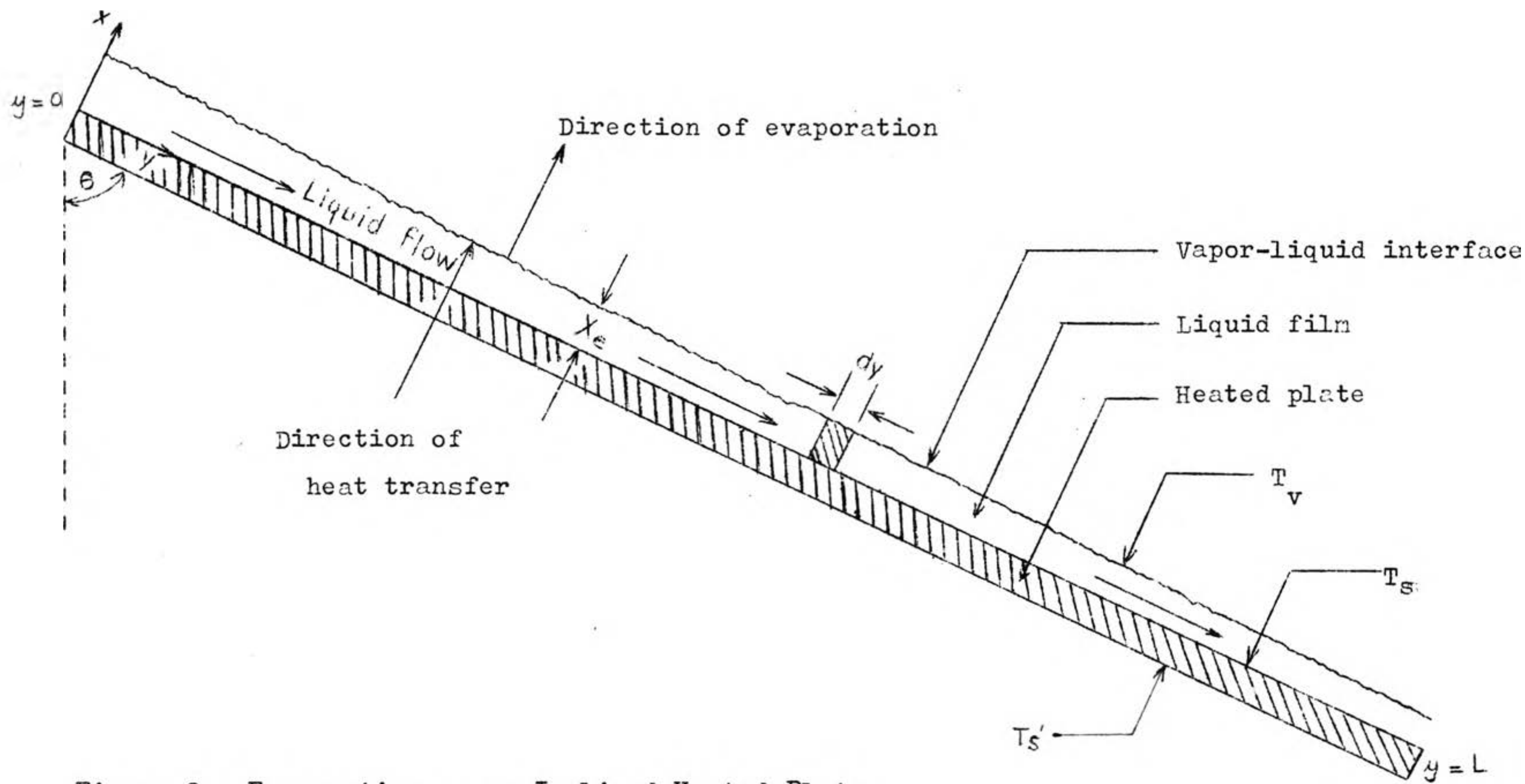

Figure 2. Evaporation on an Inclined Heated Plate

The energy balance of a differential element, is

$$dq = kBdy \frac{Ts - Tv}{Xe} \quad (1)$$

where q is the rate of heat transfer

k is the thermal conductivity of the liquid

B is the width of the plate

Ts is the surface temperature of the plate

Tv is the vaporization temperature of the liquid

Xe is the liquid film thickness at y = y

However, from the conventional definition of heat transfer coefficient, h,

$$dq = hBdy\,(Ts - Tv) \quad (2)$$

Therefore $h = k/Xe$ (3)

The amount of heat transferred may be determined from the change in liquid flow rate.

$$dq = -\lambda\, dw \quad (4)$$

where w is the liquid flow rate at y = y

λ is the heat of vaporization per unit mass

Then equation (2) becomes

$$-\lambda dw = hBdy\,(Ts - Tv)$$

$$h = \frac{-\lambda}{Ts-Tv} \cdot \frac{1}{B} \cdot \frac{dw}{dy}$$

$$\frac{k}{Xe} = h = \frac{-\lambda}{Ts-Tv} \frac{d\Gamma}{dy} \quad (5)$$

there Γ is the flow rate of liquid per unit width.

$$(Ts - Tv) = -\frac{Xe\lambda}{k} \cdot \frac{d\Gamma}{dy} \quad (6)$$

For the entire length L of the plate, let a mean heat transfer coefficient, h_m, be defined as

$$q = h_m BL\,(Ts - Tv) = \lambda(w_0 - w_L)$$
$$= \lambda \Delta w \quad (7)$$

where w_0 is the flow rate of the feed solution

w_L is the flow rate of the concentrated solution

$$Ts - Tv = \frac{\lambda \Delta w}{h_m BL} = \frac{\lambda \Delta \Gamma}{h_m\ L} \quad (8)$$

Combining equations (6) & (8) gives

$$dy = \frac{-Xe \cdot h_m L}{k \Delta \Gamma} \cdot d\Gamma \quad (9)$$

According to the continuity equation[1]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \quad (10)$$

For imcompressible fluid and one dimensional flow,

ρ is constant

$$\frac{\partial \rho}{\partial t} = 0 \ , \quad \text{and} \quad v_x = v_z = 0$$

Therefore
$$\frac{\partial v_y}{\partial y} = 0 \quad (11)$$

According to the equation of motion (Navier-Stroke's equation)[2] in y - direction:

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[1] R. Byron Bird, Warren E. Stewart, and Edwin N. Lightfoot, Transport Phonomona (Tokyo: Toppan Company, Ltd., 1960), p. 75

[2] Ibid., p. 84

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_z}{\partial z} =$$

$$\frac{g_c}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + g_y \qquad (12)$$

For steady-state one-dimensional flow without any external forces except the gravity, equation (12) becomes

$$\frac{d^2 v_y}{dx^2} = -\frac{g_y}{\nu}$$

$$\frac{d^2 v_y}{dx^2} = -\frac{g \cos\theta}{\nu} \qquad (13)$$

To solve this equation, two boundary conditions are required. They are:

B.C 1: at $x = X_o$, $\frac{dv_y}{dx} = 0$

B.C 2: at $x = 0$, $v_y = 0$

Integrating equation (13) gives

$$\frac{dv_y}{dx} = -g\frac{\cos\theta}{\nu}x + C_1 \qquad (14)$$

Applying B.C 1 yields

$$C_1 = g\frac{\cos\theta}{\nu} Xo \qquad (15)$$

Then

$$\frac{dv_y}{dx} = g\frac{\cos\theta}{\nu}(Xo - X) \qquad (16)$$

Integrating equation (16) gives

$$v_y = g\frac{\cos\theta}{\nu}\left(Xo.X - \frac{X^2}{2}\right) + C_2 \qquad (17)$$

Applying B.C 2 yields

$$v_y = g \frac{\cos\theta}{\nu}(Xe.X - X^2/2) \quad (18)$$

The mass flow rate of the solution at any value of y is

$$w = \int_0^{X_e} v_y \rho B dx$$

$$= B \rho g \frac{\cos\theta}{\nu} \int_0^{X_e} (Xe.X - X^2/2) dx$$

$$= B \rho g \frac{\cos\theta}{\nu} \left(\frac{Xe^3}{2} - \frac{Xe^3}{6} \right)$$

$$= B \rho g \frac{\cos\theta}{\nu} \frac{Xe^3}{3} \quad (19)$$

Therefore $$\Gamma = \frac{w}{B} = \rho g \frac{\cos\theta \, Xe^3}{3} \quad (20)$$

$$Xe = \left(\frac{3 \nu \Gamma}{g \cos\theta} \right)^{1/3} \quad (21)$$

Substituting Xe into equation (9) yields

$$dy = - \frac{h_m L}{k \Delta \Gamma} \left(\frac{3 \nu \Gamma}{g \cos\theta} \right)^{1/3} d\Gamma \quad (22)$$

Integrating gives

$$\int_0^L dy = - \frac{h_m L}{k \Delta \Gamma} \left(\frac{3 \nu}{\rho g \cos\theta} \right)^{1/3} \int_{\Gamma_o}^{\Gamma_L} \Gamma^{1/3} d\Gamma$$

$$L = \frac{h_m L}{k \Delta \Gamma} \left(\frac{3 \nu}{\rho g \cos\theta} \right)^{1/3} \cdot \frac{3}{4} \left(\Gamma_o^{4/3} - \Gamma_L^{4/3} \right)$$

$$h_m = \frac{4}{3} \frac{k \Delta \Gamma}{\Gamma_o^{4/3} - \Gamma_L^{4/3}} \left(\frac{\rho g \cos\theta}{3 \nu} \right)^{1/3}$$

$$= 0.925 \frac{\Delta \Gamma}{\Gamma_o^{4/3} - \Gamma_L^{4/3}} \left(\frac{\rho g k^3 \cos\theta}{\nu} \right)^{1/3} \quad (23)$$

From equation (8)

$$h_m = \frac{\lambda \Delta \Gamma}{L(Ts - Tv)} \qquad (24)$$

Combining equations (23) and (24) gives

$$\left(\Gamma_o^{4/3} - \Gamma_L^{4/3} \right) = 0.925 \frac{L(Ts - Tv)}{\lambda} \left(\rho k^3 g \frac{\cos\theta}{\nu} \right)^{1/3}.$$

$$= 0.925 L \frac{(Ts - Tv)}{\lambda} \left(\rho^2 k^3 g \frac{\cos\theta}{\mu} \right)^{1/3} \qquad (25)$$

Equation (25) is derived based on the assumptions that the physical properties of liquid, density, viscosity, thermal conductivity, and heat of vaporization, temperature drop Ts - Tv are constant. In fact, these values are not constant but vary to a certain extent. However, if their mean values are used the result might be agreeable with the equation.