การเปรียบเทียบแบบจำลองควินเทสเซนซ์แบบต่าง ๆ

นาย คัมภีร์ ค้าแหวน

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต
สาขาวิชาฟิสิกส์ ภาควิชาฟิสิกส์
คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
ปีการศึกษา 2544

ISBN 974-03-0211-4

COMPARISON OF QUINTESSENCE MODELS

Mr. Khamphee Karwan

A Thesis Submitted in Partial Fulfillment of the Requirements

for the Degree of Master of Science in Physics

Department of Physics

Faculty of Science

Chulalongkorn University

Academic Year 2001

ISBN 974-03-0211-4

Thesis Title	Comparison of Quintessence Models
By	Mr. Khamphee Karwan
Field of Study	Physics
Thesis Advisor	Dr. Ahpisit Ungkitchanukit

Accepted by the Faculty of Science, Chulalongkorn University in Partial Fulfillment of the Requirements for the Master's Degree.

....Dean of Faculty of Science
(Associate Professor Wanchai Phothiphichitr, Ph.D.)

THESIS COMMITTEE

....Chairman
(Rujikorn Dhanawittayapol, Ph.D.)

....Thesis Advisor
(Ahpisit Ungkitchanukit, Ph.D.)

....Member
(Assistant Professor Pisistha Ratanavararak, Ph.D.)

....Member
(Assistant Professor Pornchai Pacharintanakul, Ph.D.)

คัมภีร์ ค้าแหวน : การเปรียบเทียบแบบจำลองควินเทสเซนซ์แบบต่างๆ
(COMPARISON OF QUINTESSENCE MODELS)
อาจารย์ที่ปรึกษา : อ.ดร. อภิสิทธิ์ อึ้งกิจจานุกิจ 74 หน้า ISBN 974-03-0211-4

จุดมุ่งหมายของวิทยานิพนธ์คือการเปรียบเทียบแบบจำลองควินเทสเซนซ์ ได้มีการรวบรวมและนำเสนอแนวความคิดและแบบจำลองควินเทสเซนซ์ที่ใช้ฟิสิกส์อนุภาค ได้แสดงการหาคำตอบสนาม (field solution) ของควินเทสเซนซ์ที่มีศักย์แบบเอกโพเนนเชียล (exponential) และ กำลังลบ (negative power law) โดยละเอียด จากคำตอบสนาม สามารถเห็นได้ว่าควินเทสเซนซ์ที่มีศักย์ดังกล่าวไม่เป็นไปตามเงื่อนไขที่ได้จากการสังเกตการณ์ ได้มีการรวบรวมและนำเสนอแนวทางในการปรับปรุงแบบจำลองเหล่านั้น ถึงแม้ว่าบางแบบจำลองควินเทสเซนซ์ที่เกิดจากทฤษฎี supersymmetry และ supergravity สามารถทำให้เงื่อนไขที่ได้จากการสังเกตการณ์เป็นจริง แต่แบบจำลองควินเทสเซนซ์เหล่านั้น มีปัญหาเมื่อสมมาตรแบบซูเปอร์ (supersymmetry) ของระบบเสียไป

ภาควิชา ฟิสิกส์	ลายมือชื่อนิสิต ....
สาขาวิชา ฟิสิกส์	ลายมือชื่ออาจารย์ที่ปรึกษา ....
ปีการศึกษา 2544	

##4172238123 : MAJOR PHYSICS
KEY WORD : QUINTESSENCE / COSMIC EVOLUTION

KHAMPHEE KARWAN : COMPARISON OF QUINTESSENCE MODELS. THESIS ADVISOR : AHPISIT UNGKITCHANUKIT, PhD., 74 pp. ISBN 974-03-0211-4

The purpose of this thesis is to compare the quintessence models. The ideas of quintessence and particle physics realization of quintessence models are reviewed. The field solutions of typical quintessence with exponential and negative power law potentials are derived in detail. From the field solutions it can be seen that the quintessence with these potentials cannot satisfy the observational constraints. The ways for modifying these quintessence models are reviewed. Although some quintessence models which arise from supersymmetry and supergravity theories can satisfy the observational conditions but these quintessence models have some problems when supersymmetry is broken.

Department ...Physics…	Student's signature……………………………………
Field of study…Physics...	Advisor's signature …………………………………
Academic year…2001…	

Acknowledgment

The author wishes to express his gratitude to his thesis advisor, Dr. Ahpisit Ungkitchanukit for his useful advice, encouragement and helps in various ways.

He would like to thank the thesis committee, Assistant Professor Pisistha Ratanavararak, Assistant Professor Pornchai Pacharintanakul and Dr. Rujikorn Dhanawittayapol, for their helps in reading and criticizing the manuscript.

He also would like to thank Miss Benjamas Tinanoi, Mr. Sarun Phibanchon, Mr. Kriangkrai Wantong, Mr. Itzadah Thongkul, Miss Warintorn Sreethawong, some students in the space energetic particle research group and many good friends for their helps.

CONTENTS

LIST OF FIGURES

Figure | Page

Chapter 1

Introduction

Recent observations suggest that the universe is currently expanding with acceleration. So the universe is dominated by an energy component with negative pressure at present [1]. Since this component has no effect on the dynamics of galaixes and does not cluster, its gravitational interactions with ordinary matter should be weak. This energy component may be called dark energy [2] due to its nonluminous nature and it cannot be observed directly. Dark energy is not like dark matter because dark matter affects the dynamics of galaxies. A simple model for the dark energy is vacuum energy, which in the theory of general relativity, is equivalent to the cosmological constant. The value of the dark energy density is very small compared with the theoretical value from cosmological constant model. The problem of how to obtain the observed value of the dark energy is known as the cosmological constant problem [3]. There are many ideas for solving the cosmological constant problem, for examples, the idea of time dependent cosmological "constant", and the idea in which the smallness of the cosmological constant arises from spontaneous symmetry breaking have been proposed as solutions to the problem. But these ideas have some problems which will be discussed in Section 1.3.

A good candidate for the solution of the cosmological constant problem is quintessence. Quintessence is the name given to models where the dark energy component of the universe is due to an inhomogeneous field which evolves in its self-interaction potential, it cannot be perfectly homogeneous because a time varying homogeneous component would be inconsistent with the equivalence principle [4]. However, the inhomogeneous part of quintessence is generally treated as a small perturbation. Since this perturbation component tends to decay inside the horizon [5] while the universe is in the backgound, (radiation, matter), dominated era, the energy density of the inhomogeneous part can be neglected when the evolution of the universe is considered. When the cosmic microwave background (CMB) anisotropy is studied the inhomogeneous part of quintessence has to be taken into account [4].

Quintessence models with suitable potentials have tracking behaviour. With this behaviour the final result of quintessence's evolution is the same for a wide range of initial conditions. Sometimes, the quintessence field for model with the tracking behaviour is called tracker field [6]. Note that some authors use the word "tracker" to refer to quintessence whose energy density decreases with the same rate as that of the background during the background dominated epoch, but in this thesis this word is not used in this sense. The evolution of tracker field depends only on the parameters of its potential. So the correct value of the dark energy density can be obtained by adjusting the values of the parameters. In the cases where the parameters arise naturally from particle physics models, the fine tuning problem is less severe than the fine tuning problem in cosmological constant model. Although dark energy is dominant at present, but its density is of the same order as matter energy density. Since the two energy components decrease with different rates as the universe evolves, the initial conditions of the dark energy must be set carefully to give an appropriate present value of the dark energy. This is the coincidence problem [6], and since tracker quintessence is not sensitive to the initial conditions this problem is avoided. In this thesis, quintessence models are compared and discussed.

The organization of this thesis is as follows. In Section 1.1 some important ideas and equations in cosmology are introduced. The observational results and the cosmological constant problem are discussed in Sections 1.2 and 1.3 respectively. The evolution of the quintessence field and the tracking behaviour of quintessence are discussed in Chapter 2. A selection of quintessence models are described and compared in Chapters 3 and 4 respectively. The conclusion is made in Chapter 5.

1.1 Introduction to some ideas of cosmology

The Friedmann equations which describe the evolution of the universe is derived in this section. The spacetime geometry is described by the metric tensor , which takes the form of the Robertson-Walker metric [7]:

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\}, \tag{1.1}$$

where $a(t)$ is the scale factor, k is the topological curvature which gives the spacetime geometry. The Robertson-Walker metric follows from the Cosmological Principle which assumes that the spacetime of the universe is homogeneous and isotropic.

The three possible geometries of spacetime, $k > 0$, $k = 0$ and $k < 0$ are shown in Fig. 1.1. Eq. (1.1) shows that the spatial part of the universe can expand or contract if the scale factor respectively increases or decreases with time. So the evolution of the spatial part of the universe is given by the time dependence of the scale factor as governed by the Friedmann equations. The Friedmann equations can be derived from Einstein equation by introducing appropriate model for the matter and energy in the universe and using the Robertson-Walker metric tensor to calculate the Ricci tensor which represents spacetime curvature. So before the Friedmann equations are derived the Ricci tensor is calculated and the model of matter and energy for the universe is studied. For the Ricci tensor, first the

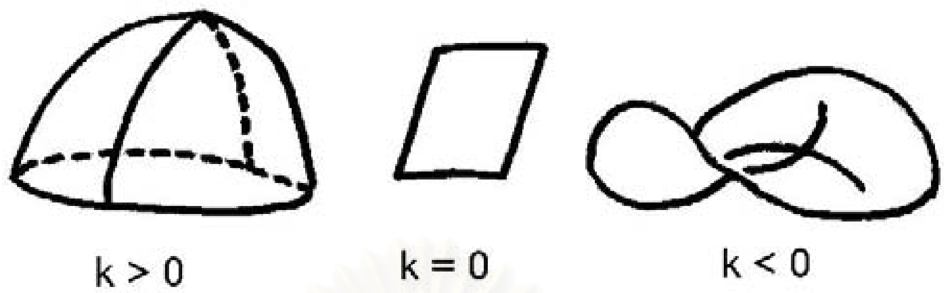

Figure 1.1: The three possible geometries of spacetime.

Riemann tensor must be calculated from the Christoffel symbols. The Christoffel symbols are given in terms of the metric tensor by [7]

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_{\mu} g_{\nu\rho} + \partial_{\nu} g_{\rho\mu} - \partial_{\rho} g_{\mu\nu}). \tag{1.2}$$

Using the Robertson-Walker metric for the metric tensor a sample calculation for the Christoffel symbol is shown below:

$$\Gamma^{0}_{11} = \frac{1}{2} g^{0\rho}(-\partial_{\rho} g_{11}) = \frac{\dot{a}a}{1-kr^2}, \tag{1.3}$$

and the non-zero components of the Christoffel symbols are obtained [7]

$$\begin{aligned}
\Gamma^{0}_{11} = \frac{\dot{a}a}{1-kr^2}, \qquad \Gamma^{0}_{22} = a\dot{a}r^2, \qquad \Gamma^{0}_{33} = a\dot{a}r^2 \sin^2\theta, \\
\Gamma^{1}_{01} = \Gamma^{1}_{10} = \Gamma^{2}_{20} = \Gamma^{2}_{02} = \Gamma^{3}_{03} = \Gamma^{3}_{30} = \frac{\dot{a}}{a}, \\
\Gamma^{1}_{22} = -r(1-kr^2), \quad \Gamma^{1}_{33} = -r(1-kr^2)\sin^2\theta, \\
\Gamma^{2}_{12} = \Gamma^{2}_{21} = \Gamma^{3}_{13} = \Gamma^{3}_{31} = \frac{1}{r}, \\
\Gamma^{2}_{33} = -\sin\theta\cos\theta, \quad \Gamma^{3}_{23} = \Gamma^{3}_{32} = \cot\theta.
\end{aligned} \tag{1.4}$$

The Riemann tensor can now be calculated using the relation [7]

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}. \tag{1.5}$$

The Ricci tensor is obtained by contracting two indices of the Riemann tensor

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \tag{1.6}$$

and by contracting the Ricci tensor with the metric tensor the Ricci scalar is obtained

$$R = g^{\mu\nu} R_{\mu\nu}. \tag{1.7}$$

The explicit calculation of R_{00} is shown below. First using (1.5) to calculate $R^{0}{}_{\sigma 0\sigma}$,

$$R^{0}{}_{\sigma 0\sigma} = \partial_0 \Gamma^0_{\sigma\sigma} - \partial_\sigma \Gamma^0_{0\sigma} + \Gamma^0_{0\lambda}\Gamma^\lambda_{\sigma\sigma} - \Gamma^0_{\sigma\lambda}\Gamma^\lambda_{0\sigma}. \tag{1.8}$$

For the Riemann tensor components we have

$$R^{0}{}_{000} = 0 \quad , \quad R^{0}{}_{101} = \frac{a\ddot{a} + \dot{a}^2}{1 - kr^2} - \frac{\dot{a}^2}{1 - kr^2},$$
$$R^{0}{}_{202} = r^2(a\ddot{a} + \dot{a}^2) - r^2\dot{a}^2, \quad , \quad R^{0}{}_{303} = r^2(a\ddot{a} + \dot{a}^2)\sin^2\theta - r^2\dot{a}^2\sin^2\theta, \tag{1.9}$$

and by lowering the index α

$$R_{0\sigma 0\sigma} = g_{0\alpha} R^{\alpha}{}_{\sigma 0\sigma}, \tag{1.10}$$

and contracting the index σ we obtain

$$R_{00} = g^{\sigma\sigma} R_{0\sigma 0\sigma} = -3\frac{\ddot{a}}{a}. \tag{1.11}$$

The other non-zero components of the Ricci tensor are

$$\begin{aligned} R_{11} &= \frac{a\ddot{a} + 2\dot{a}^2 + 2k}{1 - kr^2}, \\ R_{22} &= r^2(a\ddot{a} + 2\dot{a}^2 + 2k), \\ R_{33} &= r^2(a\ddot{a} + 2\dot{a}^2 + 2k)\sin^2\theta, \end{aligned} \tag{1.12}$$

and the Ricci scalar is given by

$$R = \frac{6}{a^2}(a\ddot{a} + \dot{a}^2 + k). \tag{1.13}$$

Matter and radiation in the universe can be described by perfect fluid which is isotropic in its rest frame. The energy-momentum tensor for a perfect fluid is [7]

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & -g^{ij}p \end{pmatrix}, \tag{1.14}$$

where ρ and p are the energy density and pressure of the perfect fluid respectively. The trace of $T^{\mu\nu}$ is

$$T = T_{\mu}{}^{\mu} = g_{\mu\nu}T^{\mu\nu}. \tag{1.15}$$

From energy conservation we get

$$\begin{aligned} 0 &= \nabla_{\mu}T^{\mu}{}_{0} = \partial_{\mu}T^{\mu}{}_{0} + \Gamma^{\mu}_{\mu\lambda}T^{\lambda}{}_{0} - \Gamma^{\lambda}_{\mu 0}T^{\mu}{}_{\lambda}, \\ &= -\partial_{0}\rho - 3\rho\frac{\dot{a}}{a} - 3p\frac{\dot{a}}{a}. \end{aligned} \tag{1.16}$$

It is useful to introduce the equation of state which is a relation between the energy density and the pressure. The most general equation of state in a spacetime with the Robertson-Walker metric can be written in a simple form as [7]:

$$p = \omega\rho, \tag{1.17}$$

where ω can be constant or time dependent.

Rewriting Eq. (1.16) by using Eq. (1.17)

$$\frac{\dot{\rho}}{\rho} = -3(\omega+1)\frac{\dot{a}}{a}, \tag{1.18}$$

it can be seen that for constant ω, the relation between ρ and a is

$$\rho \propto a^{-3(1+\omega)}. \tag{1.19}$$

The popular examples of perfect fluids are dust (sometimes called "matter") and radiation. Dust refers to nonrelativistic matter; examples of dust are ordinary stars and galaxies. Because the velocity of dust is low (nonrelativistic matter), the pressure due to dust in the universe is negligible in comparison with the energy density of the dust and in the equation of state ω=0. From Eq. (1.19) it can be seen that the energy density of matter decreases as

$$\rho \propto a^{-3}. \tag{1.20}$$

Examples of radiation are ordinary electromagnetic radiation and relativistic matter. Radiation has $\omega = \frac{1}{3}$ and its energy density decays as

$$\rho \propto a^{-4}. \tag{1.21}$$

The universe whose energy density is mostly due to matter or radiation are known respectively as matter or radiation dominated universe.

Let us consider Einstein equation with a positive cosmological constant Λ [7],

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \tag{1.22}$$

where $G_{\mu\nu}$ is the Einstein tensor and G is Newton's gravitational constant. The effective energy-momentum tensor due to the cosmological constant term is

$$T^{(\Lambda)}_{\mu\nu} = -\frac{\Lambda g_{\mu\nu}}{8\pi G}. \tag{1.23}$$

Eq. (1.22) becomes

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + T^{\Lambda}_{\mu\nu}). \tag{1.24}$$

It can be seen that the Einstein tensor does not vanish even in the vacuum where $T_{\mu\nu}$ vanishes. So this equation implies that the cosmological constant is equivalent to the vacuum energy. This vacuum energy is assumed to be a perfect fluid and the trace of the energy-momentum tensor is given by

$$T^{(\Lambda)\mu}_{\mu} = -\rho + 3p = -4\frac{\Lambda}{8\pi G}. \tag{1.25}$$

Then together with Eq. (1.23) the equation of state for the cosmological constant,

$$\rho = -p = \frac{\Lambda}{8\pi G}, \tag{1.26}$$

follows, showing that the cosmological constant has $\omega = -1$.

Now we are ready to derive Friedmann equations which are the dynamical equations of cosmology. From Einstein equation [7] (in the absence of the cosmological constant term),

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right), \tag{1.27}$$

where $R_{\mu\nu}$ and $T_{\mu\nu}$ are respectively the Ricci and the energy-momentum tensor. Using Eqs. (1.4) and (1.12), Eq. (1.27) becomes

$$-3\frac{\ddot{a}}{a} = 4\pi G(\rho + 3p), \tag{1.28}$$

for R_{00} and

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G(\rho - p) \tag{1.29}$$

for R_{ii}. Using Eq. (1.28) to eliminate $\frac{\ddot{a}}{a}$ from Eq. (1.29), one obtains Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p) \tag{1.30}$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2}. \tag{1.31}$$

If the cosmological constant is not zero, Friedmann equations become

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p) + \frac{\Lambda}{3}, \tag{1.32}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{1.33}$$

To make contact with observations, the following parameters are introduced:

$$\text{Hubble parameter:} \quad H = \frac{\dot{a}}{a}$$

$$\text{Redshift parameter:} \quad z + 1 = \frac{a_o}{a}$$

$$\text{Density parameter:} \quad \Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_c}$$

where the subscript "o" refers to the present value and $\rho_c = \frac{3H^2}{8\pi G}$ is the critical energy density. The Friedmann equation (1.33) can now be written as

$$1 = \Omega - \Omega_k + \Omega_\Lambda, \tag{1.34}$$

where

$$\Omega_k = \frac{k}{H^2 a^2} \quad \text{and} \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}. \tag{1.35}$$

From Eq. (1.33) it is seen that the universe will expand with acceleration if $\omega < -\frac{1}{3}$. The universe with a positive cosmological constant falls in this class as $\omega_\Lambda = -1$.

In the standard cosmological model [8], the universe began with a big bang followed by an inflationary epoch, a radiation dominated epoch and a matter dominated epoch respectively. The universe expanded rapidly in the inflationary epoch so the curvature (k) term, which depends on a^{-2}, quickly became negligible and universe became flat ($k = 0$).

1.2 Observational results

The observations which suggest that the universe is domimated by dark energy and is now undergoing accelerated expansion are discussed in this section. Results from these observations give constraints on the cosmological constant and quintessence.

The age of the universe

The relation between the age of the universe and Ω is calculated below. The Friedmann equation for a flat universe with a cosmological constant term can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda), \tag{1.36}$$

where ρ_m and ρ_Λ are, respectively, the matter and cosmological constant energy density. From Eq. (1.19) it follows that

$$\rho \propto a^{-3} \quad \longrightarrow \quad \frac{\rho_m}{\rho_{om}} = \left(\frac{a_o}{a}\right)^3, \tag{1.37}$$

$$\rho_\Lambda = constant = \rho_{o\Lambda}, \tag{1.38}$$

where the subscript "o" refers to the present value; this notation will be used throughout this thesis.

Substituting Eqs. (1.38) and (1.37) into Eq. (1.36) one gets

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\left(\rho_{om}\left(\frac{a_o}{a}\right)^3 + \rho_{o\Lambda}\right), \\
\left(\frac{\dot{a}}{a_o}\right)^2 &= \frac{8\pi G}{3}\left(\frac{a}{a_o}\right)^2\left(\rho_{om}\left(\frac{a_o}{a}\right)^3 + \rho_{o\Lambda}\right) \\
&= H_o^2\left(\Omega_{om}\frac{a_o}{a} + \Omega_{o\Lambda}\left(\frac{a}{a_o}\right)^2\right),
\end{aligned} \tag{1.39}$$

where $H_o = \dfrac{\dot{a}_o}{a_o}$. Introducing the dimensionless variable $x = \dfrac{a}{a_o}$, Eq. (1.39) can be put in the form

$$\begin{aligned}
\dot{x} &= H_o\sqrt{\Omega_{om}x^{-1} + \Omega_{o\Lambda}x^2}, \\
\frac{\sqrt{x}dx}{\sqrt{\Omega_{om} + \Omega_{o\Lambda}x^3}} &= H_o dt.
\end{aligned} \tag{1.40}$$

Letting $u^2 = x^3$, then $\frac{3}{2}\sqrt{x}dx = du$ and we obtain

$$\frac{2}{3}\frac{du}{\sqrt{\Omega_{om} + \Omega_{o\Lambda}u^2}} = H_o dt \tag{1.41}$$

so that

$$t_o - t = \frac{2}{3H_o\sqrt{\Omega_{o\Lambda}}}\left[\sinh^{-1}\left(\frac{\Omega_{o\Lambda}}{\Omega_{om}}\right)^{\frac{1}{2}} - \sinh^{-1}\left(\left(\frac{\Omega_{o\Lambda}}{\Omega_{om}}\right)^{\frac{1}{2}}\left(\frac{a}{a_o}\right)^{\frac{3}{2}}\right)\right]. \qquad (1.42)$$

Setting $t = 0$ when $a = 0$ and using the relation $\sqrt{x} + \sqrt{1+x} = \frac{1}{\sqrt{x}-\sqrt{1+x}}$ the age of the universe at persent time can be put in the form

$$t_o = \frac{2}{3H_o\sqrt{\Omega_{o\Lambda}}}\frac{1}{2}\ln\left(\frac{\sqrt{\Omega_{o\Lambda}}+1}{1-\sqrt{\Omega_{o\Lambda}}}\right), \qquad (1.43)$$

where no explicit dependence on Ω_m appears. For the Einstein-de Sitter universe $\Omega_m = 1$ and $\Omega_\Lambda = 0$, then the age of the universe is given by

$$t_o = \frac{2}{3H_o}. \qquad (1.44)$$

The experimental values of H_o are between 50 and 100 km/sec/Mpc or between 5.1×10^{-11} and 1.02×10^{-10} year^{-1}. So the age of the Einstein-de Sitter universe is bounded by

$$6.5 \text{ Gyr} < t_o < 12.1 \text{ Gyr}, \qquad (1.45)$$

these values are smaller than the age of the oldest stars (13-16 Gyr). However, by including the cosmological constant with a value of $\Omega_\Lambda = 0.7$ the range

$$9.4 \text{ Gyr} < t_o < 18 \text{ Gyr}, \qquad (1.46)$$

which is now consistent with the age of the oldest stars, is obtained for the age of the universe [9]. Note that if the cosmological constant is replaced by quintessence with constant ω, the age of universe will change very little for the same values of the parameters H_o, Ω_{om} [10] but the calculations become more complicated.

Luminosity distance

Consider a light source of absolute luminosity L located at r and an observer located at $r = 0$. Light emitted at time $t = t$ is received by the observer at time $t = t_o$, with t and t_o related by the equation $(1+z) = \frac{a(t_o)}{a(t)}$. The luminosity flux F at the position of the observer is

$$F = \frac{L}{4\pi d_L^2}, \tag{1.47}$$

where d_L is the luminosity distance given by [7]

$$d_L = ar(1+z), \tag{1.48}$$

where the product ar is the physical distance, the factor $(1+z)$ results from the photon redshift and the time dependence of a [7].

On a null radial geodesic, $ds^2 = 0$ and that

$$\int \frac{dt}{a(t)} = \int \frac{dr}{\sqrt{1-kr^2}}. \tag{1.49}$$

The integral on RHS of Eq. (1.49) gives

$$\int \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \frac{1}{\sqrt{k}} \sin^{-1}(\sqrt{k}r) & \text{for } k > 0 \\ r & \text{for } k = 0 \\ \frac{1}{\sqrt{|k|}} \sinh^{-1}(\sqrt{\mid k \mid} r) & \text{for } k < 0 \end{cases}, \tag{1.50}$$

and using the relation $\frac{dz}{dt} = (1+z)H$ for the left-hand side of Eq. (1.49) we have

$$\int \frac{dt}{a(t)} = \int \frac{dz}{(1+z)Ha}. \tag{1.51}$$

By using Friedmann equation

$$\frac{H^2}{H_o^2} = \Omega_{om} \left(\frac{a_o}{a}\right)^3 + \Omega_{odark} \left(\frac{a_o}{a}\right)^{3(\omega_{dark}+1)}, \tag{1.52}$$

where Ω_{odark} is the density parameter of dark energy with $\omega = \omega_{dark}$, Eq. (1.51) becomes

$$\int \frac{dt}{a(t)} = \frac{1}{H_o a_o} \int \frac{dz}{\sqrt{\Omega_{om}(1+z)^3 + \Omega_{odark}(1+z)^{3(1+\omega_{dark})}}}. \tag{1.53}$$

From Eqs. (1.49), (1.50) and (1.53), one gets

$$\sqrt{|k|}r = \sin_k \left[\frac{\sqrt{|k|}}{H_o a_o} \int \frac{dz}{\sqrt{\Omega_{om}(1+z)^3 + \Omega_{odark}(1+z)^{3(\omega_{dark}+1)}}} \right], \tag{1.54}$$

where $\sin_k = \sin$ for $k \geq 0$ and $\sin_k = \sinh$ for $k < 0$. The luminosity distance is then given by

$$d_L = \frac{1+z}{H_o\sqrt{\Omega_k}} \sin_k \left[\sqrt{\Omega_k} \int \frac{dz}{\sqrt{\Omega_{om}(1+z)^3 + \Omega_{odark}(1+z)^{3(\omega_{dark}+1)}}} \right]. \tag{1.55}$$

The apparent magnitude (m) is related to d_L through the relation [9]

$$m = M + 5\log_{10}\frac{d_L}{\mathrm{Mpc}} + 25, \tag{1.56}$$

where M is the absolute magnitude.

Using type Ia supernova as standard candles, the observed values of m_{ob} can be used to estimate the values of the parameters Ω_m and Ω_{dark} by maximizing the function

$$\chi = \prod_i \frac{1}{2\pi\sigma_i} \exp\left(\frac{(m_i - m_{iob})^2}{2\sigma_i^2} \right). \tag{1.57}$$

The parameter ω_{odark} can then be estimated with additional data from CMB anistropy observations. The current values are [11, 12]

$$\Omega_{om} \approx 0.3 \quad , \quad \Omega_{odark} \approx 0.7,$$

$$-1 < \omega_{odark} < -0.8 \quad \text{or} \quad -1 < \omega_{odark} < -0.6. \tag{1.58}$$

Baryon density

In a flat universe with $\Omega_m = 1$, the mass fraction in baryon in cluster is higher than the upper limit given by nucleosynthesis. The mass fraction can agree with nucleosynthesis only if $\Omega_{om} = 0.16$ [9] (Ω_{om} includes baryon and dark matter). Moreover, studies of the structure formations suggest that $\Omega_m = 0.2$ [9].

1.3 Cosmological constant problem

The cosmological constant problem and the basic idea of quintessence which is the way for solving the cosmological constant problem are reviewed in this section. The details of quintessence are discussed in the next two chapters.

From observations

$$\Omega_{odark} \approx 0.7 \quad \text{or} \quad \rho_{odark} \approx \rho_{oc}. \tag{1.59}$$

Consider Friedmann equation in the form

$$\begin{aligned} \rho_{oc} &= \rho_{om} + \langle \rho_{om} \rangle + \rho_{\Lambda}, \\ &= \rho_{om} + \frac{\Lambda_{eff}}{8\pi G}, \end{aligned} \tag{1.60}$$

where $\langle \rho_{om} \rangle$ is the vacuum expectation value of the matter energy density. From Eq. (1.59) we get

$$\rho_{odark} \approx \rho_{o\Lambda_{eff}} \approx \rho_{oc} \approx \frac{3H_o}{8\pi G} \approx 10^{-47}\,\text{GeV}^4. \tag{1.61}$$

In field theory $\langle \rho_{om} \rangle$ may arise from the zero-point energy of all the normal modes of some field with mass m,

$$\langle \rho_{om} \rangle = \frac{1}{(2\pi)^3} \int d^3k \frac{\omega}{2} = \frac{1}{2(2\pi)^3} \int_0^{k_c} dk k^2 4\pi \sqrt{k^2 + m^2} = \frac{\Lambda_c}{16\pi^2}, \tag{1.62}$$

where k_c is cut-off wave number. In general relativity k_c is of the order of Planck's mass. So

$$\langle \rho_{om} \rangle = \frac{(8\pi G)^{-2}}{16\pi^2} = 2^{-10}\pi^{-4}G^{-2} = 2 \times 10^{71}\,\text{GeV}^4. \tag{1.63}$$

The above simple calculation shows that $\rho_{\Lambda_{eff}}$ is very small compared with $\langle \rho_{om} \rangle$ and $\frac{\Lambda}{8\pi G}$ must cancel with $\langle \rho_{om} \rangle$ to better than 118 orders to give the very small value of $\rho_{\Lambda_{eff}}$. This is known as the cosmological constant problem. In other words, the cosmological constant problem is the problem of the unnaturally small value of $\rho_{\Lambda_{eff}}$. This problem may be solved by finding mechanisms or models that can give rise to the small value of $\rho_{\Lambda_{eff}}$.

Now we consider some ideas for the solution of the cosmological constant problem. The first example was suggested by Zeldovich [13]. He assumed that $\langle\rho_{om}\rangle$ arises from quantum fluctuations, and to the zeroth order $\langle\rho_{om}\rangle$ cancels with $\frac{\Lambda}{8\pi G}$ leaving only higher order terms. This leads to [3]

$$\langle\rho\rangle = 10^{-37}\,\mathrm{GeV}^4. \tag{1.64}$$

It can be seen that this value is still far larger than the value of $\rho_{\Lambda_{eff}}$.

The idea of spontaneous symmetry breaking has also been invoked. In the theory in which the symmetry is spontaneously broken, the scalar field potential takes the form

$$V = V_o - \mu^2\phi^\dagger\phi + g(\phi^\dagger\phi)^2, \tag{1.65}$$

where $\mu^2 > 0$ and $g > 0$. The value of this potential at its minimum is

$$V_{min} = V_o - \frac{\mu^4}{4g} = \frac{\Lambda}{8\pi G} - \langle\rho_{om}\rangle = \rho_{odark}. \tag{1.66}$$

The value of V_o must be close to $\dfrac{\mu^4}{4g}$ to have the right value of ρ_{odark}. So in this model the cosmological constant had large value at early epoch before the symmetry breaking. This is desirable because a large effective cosmological constant in the early epoch will give rise to inflation.

There are no symmetry in our 4-dimensional world which gives rise to the appropriate value of ρ_{odark}. But if the local supersymmetry is broken in higher dimensions the appropriate value of ρ_{odark} can be obtained in our 4-dimensional world [14].

The idea that the cosmological constant is time dependent has also been suggested as a way to solve the cosmological constant problem [9]. If the cosmological “constant” is time dependent, it can start with an appropriate initial value and decay to its correct present value. But a time dependent cosmological "constant" is a smooth time varying object and is inconsistent with the equivalence principle. Although the initial value of a time dependent cosmological "constant" is not too small, but it must be set carefully and it is hard to explain its origin. So the coincident problem persists.

Chapter 2

The evolution of quintessence field

In this chapter, the field equation for quintessence in a flat universe is derived. The solutions for the exponential and negetive power law potentials are found. Quintessence with these potentials are studied because many particle physics theories give rise to potentials of these forms. The solutions of the field equations can also be obtained analytically so the evolution of the universe and the quintessence field can be studied (in terms of the scale factor and energy density) explicitly from the field solutions. With these solutions the evolution of quintessence fields with similar potentials may be discussed qualitatively. The quintessence with exponential and negative power law potentials are good examples for the study of the tracking behaviour of quintessence. It can be shown explicitly that the field solutions converge to some common solution (a scaling solution in this case) for a wide range of initial conditions. In the flat Robertson-Walker universe which contains radiation, matter and quintessence, the evolution of $a(t)$ and $\rho_\phi(t)$ can be specified by $\omega_\phi(t)$ which is governed by the scalar field potential [4]. So it is sufficient to study the evolution of the universe using scalar field quintessence. From this point of view and the fact that the inhomogeneous part of quintessence can be neglected when CMB anisotropy is not considered, in this thesis and in many papers quintessence is assumed to be a homogeneous scalar field. Quintessence's equation of motion is derived in Section 2.1. The solution of this equation when the quintessence's kinetic energy is dominant is shown in Section 2.2. Scaling solutions of quintessence with negative power law and exponential potentals in background dominated universe are discussed in Section 2.3 and 2.4 respectively. The basic ideas of tracker solution are discussed in Section 2.5.

2.1 Homogeneous scalar field in a flat universe

A flat universe containing a perfect fluid with energy density ρ_b and pressure p_b, plus a scalar field ϕ with potential $V(\phi)$ satisfies the equation

$$H^2 = \frac{8\pi G}{3}(\rho_b + \rho_\phi), \tag{2.1}$$

where ρ_ϕ is the energy density of the field. Introducing the variable m through $m = 3(\omega + 1)$, the equation of state will become $p = (\frac{m}{3} - 1)\rho$ and from the conservation of energy ρ_b decreases as

$$\rho_b \propto a^{-m}. \tag{2.2}$$

The value of m will be 3 or 4 if ρ_b is respectively due to radiation or matter energy density.

The Lagrangian density of quintessence field is

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi), \tag{2.3}$$

where ∇_μ is the covariant derivative in general relativity [7]. From the Euler-Lagrange equation:

$$\begin{aligned} -\frac{\partial V}{\partial \phi} &= \nabla_\mu \frac{\partial L}{\partial(\nabla_\mu \phi)} \\ &= \nabla_\mu(g^{\mu\nu}\partial_\nu\phi), \\ &= g^{\mu\nu}[\partial_\mu\partial_\nu\phi - \Gamma^\lambda_{\mu\nu}\partial_\lambda\phi]. \end{aligned}$$

Using the condition that ϕ is homogeneous (space independent), we obtain

$$\begin{aligned} -\frac{\partial V}{\partial \phi} &= (\partial_0)^2\phi - g^{\mu\nu}\Gamma^0_{\mu\nu}\partial_0\phi \\ &= \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi}, \end{aligned} \tag{2.4}$$

which is the required field equation of motion.

Next, let us calculate the energy density and pressure of this field. From the energy-momentum tensor [15],

$$\begin{aligned} T^{\mu\nu} &= \left[\frac{\partial \mathcal{L}}{\partial \nabla_\mu \phi}\right] \nabla^\nu \phi - g^{\mu\nu}\mathcal{L} \\ &= \nabla^\mu \phi \nabla^\nu \phi - g^{\mu\nu}\mathcal{L}, \end{aligned} \tag{2.5}$$

the field energy density, the 00-component of $T^{\mu\nu}$, is given by:

$$\begin{aligned} T^{00} &= \dot{\phi}^2 - \frac{1}{2}(\nabla^0\phi\nabla^0\phi - \nabla^i\phi\nabla_i\phi) + V(\phi) \\ &= \frac{1}{2}\dot{\phi}^2 + V(\phi). \end{aligned} \tag{2.6}$$

If this field is assumed to be a perfect fluid, its energy momentum tensor can written as

$$T^{\mu\nu} = \begin{pmatrix} \rho_\phi & 0 \\ 0 & -g^{ij}p_\phi \end{pmatrix}. \tag{2.7}$$

From Eq. (2.5) we get

$$\begin{aligned} T^{ii} &= \partial^i\phi\partial^i\phi - g^{ii}\left[\frac{1}{2}(\partial^0\phi\partial_0\phi - \partial^i\phi\partial_i\phi) - V(\phi)\right] \\ &= -g^{ii}\left(\frac{\dot{\phi}^2}{2} - V(\phi)\right). \end{aligned} \tag{2.8}$$

Note that $\nabla^\mu = \partial^\mu$ when it acts on a scalar. So the pressure of the quintessence field is

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \tag{2.9}$$

Using the equation of state $p = \omega\rho$, the relations among ω_ϕ, $\dot{\phi}^2$ and V are given by

$$\frac{\dot{\phi}^2}{\rho_\phi} = 1 + \omega_\phi, \tag{2.10}$$

$$\frac{2V}{\rho_\phi} = 1 - \omega_\phi. \tag{2.11}$$

If ω_ϕ is constant, energy conservation suggests that

$$\rho_\phi \propto a^{-3(\omega_\phi+1)} \propto a^{-n}. \tag{2.12}$$

The field solution which gives this relation is called the scaling solution. From Eq. (2.6), the time derivative of ρ_ϕ is

$$\dot{\rho_\phi} = \dot{\phi}\ddot{\phi} + \dot{\phi}\frac{\partial V}{\partial \phi}. \tag{2.13}$$

Multiplying both sides of the field equation by $\dot{\phi}$ and using the above equation, we obtain

$$\dot{\rho_\phi} = -3H\dot{\phi}^2. \tag{2.14}$$

If ρ_ϕ decreases as Eq. (2.12), we get

$$-\rho_\phi n\frac{\dot{a}}{a} = -3H\dot{\phi}^2, \tag{2.15}$$

or

$$\frac{\dot{\phi}^2/2}{\rho_\phi} = \frac{n}{6}. \tag{2.16}$$

This equation shows that the power law behaviour of ρ_ϕ follows from the fix ratio between the kinetic and the total energy. The upper limit of n is 6 and corresponds to kinetic energy domination, and the lower limit of n is 0 and corresponds to potential enengy domination. At the lower limit of n, ρ_ϕ becomes constant, so the quintessence field behaves as the cosmological constant.

2.2 Kinetic energy dominated epoch

In some quintessence models (i.e., the models which connect quintessence to inflation [16]) or in some period of quintessence evolution, the quintessence kinetic energy can dominate the energy of the universe. The solution of the field equation for which the kinetic energy dominates is discussed briefly here for completeness.

If the kinetic energy dominates the field equation and Friedmann equation will become

$$\ddot{\phi} + 3H\dot{\phi} = 0, \tag{2.17}$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2\right). \tag{2.18}$$

Let $u = \dot{\phi}$, then Eq. (2.17) becomes

$$\frac{\dot{u}}{u} = -3\frac{\dot{a}}{a}. \tag{2.19}$$

So

$$u = \dot{\phi} \propto a^{-3}. \tag{2.20}$$

Substituting $\dot{\phi}$ from Eq. (2.20) into Eq. (2.18), we obtain

$$\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &\propto a^{-6}, \\ a^2 da &\propto dt, \\ a &\propto t^{\frac{1}{3}}. \end{aligned} \tag{2.21}$$

Then the field solution is

$$\begin{aligned} \dot{\phi} &\propto \frac{1}{t}, \\ \phi &\propto \ln(t). \end{aligned} \tag{2.22}$$

The field solution can be written in terms of a as

$$\phi \propto 3\ln(a). \tag{2.23}$$

The field energy density decreases as

$$\rho_\phi \propto \frac{\dot{\phi}^2}{2} \propto a^{-6}. \tag{2.24}$$

This result is the same as that in the previous section. Note that the variational constant of the above relations can be calculated by some conditions which is not discussed here.

2.3 Quintessence with negative power law potential

Standard cosmological model suggests that the universe is radiation dominated after inflation. After the radiation dominated epoch, the universe becomes matter dominated. So the field equation with background, (radiation,matter), dominance should be solved first. In the background dominated epoch, Friedmann equation (2.1) becomes

$$H^2 = \frac{8\pi G}{3}\rho_b. \tag{2.25}$$

From energy conservation, Eq. (2.2), the Hubble parameter can be written as

$$\frac{\dot{a}}{a} \propto a^{-\frac{m}{2}}. \tag{2.26}$$

Then the scale factor increases as

$$\begin{aligned} a^{\frac{m}{2}} &\propto t, \\ a &= At^{\frac{2}{m}}. \end{aligned} \tag{2.27}$$

It is easy to see that

$$\ddot{a} \propto \frac{2}{m}\left(\frac{2}{m} - 1\right) t^{\frac{2}{m}-2}. \tag{2.28}$$

Since the universe will expand with accelelation ($\ddot{a} > 0$) if $m < 2$, the universe decelerates in the background dominated epoch because $m = 4$ and $m = 3$ for radiation and matter respectively.

The background density decreases with time as

$$\rho_b \propto t^{-2}. \tag{2.29}$$

By using relation (2.27), the Hubble parameter can be written as

$$\frac{\dot{a}}{a} = \frac{2}{mt}. \tag{2.30}$$

Substituting Eq. (2.30) into the field equation, we obtain

$$\ddot{\phi} + \frac{6}{mt}\dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \tag{2.31}$$

If $V = \frac{\Lambda^{\alpha+4}}{\phi^{\alpha}}$ [5, 17], the above equation will become

$$\ddot{\phi} + \frac{6}{mt}\dot{\phi} - \alpha\frac{\Lambda^{\alpha+4}}{\phi^{\alpha+1}} = 0. \tag{2.32}$$

The solution of this equation can be written in the form $\phi = bt^p$. Substituting this trial solution into Eq. (2.32), we get

$$bp(p-1)t^{p-2} + \frac{6}{m}bpt^{p-2} = \alpha\frac{\Lambda^{\alpha+4}}{b^{\alpha+1}t^{p(\alpha+1)}}. \tag{2.33}$$

This equation can be satisfied if

$$p - 2 = -p(\alpha+1), \tag{2.34}$$

$$bp(p-1) + \frac{6}{m}bp = \alpha\frac{\Lambda^{\alpha+4}}{b^{\alpha+1}}. \tag{2.35}$$

Then

$$p = \frac{2}{\alpha+2}, \tag{2.36}$$

$$b = \left[\frac{2}{\alpha+2}\left(\frac{6}{m} - \frac{\alpha}{\alpha+2}\right)\right]^{-\frac{1}{\alpha+2}} \left[\alpha\Lambda^{4+\alpha}\right]^{\frac{1}{\alpha+2}}. \tag{2.37}$$

From the definition of the field energy density, we get

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V \propto t^{2(p-1)} \propto a^{m(p-1)} \propto a^{-n}. \tag{2.38}$$

So

$$n = -m(p-1) = \frac{m\alpha}{\alpha+2}. \tag{2.39}$$

The parameter ω_ϕ can be calculated by using the definitions of n and m as

$$\omega_\phi = \frac{\alpha\omega_b - 2}{\alpha+2}. \tag{2.40}$$

From Eq. (2.39), we see that

$$\frac{\rho_\phi}{\rho_b} \propto a^{m-n} \propto a^{\frac{2m}{\alpha+2}}. \tag{2.41}$$

This shows that ρ_ϕ increases in comparison with ρ_b because ρ_ϕ decreases slower than ρ_b. So there is some value of a, say, a_c, which gives $\rho_\phi = \rho_b$. Although Eq. (2.41) is derived from the assumsion $\rho_\phi < \rho_b$, but if this equation is assumed to be valid approximately for $\rho_\phi \leq \rho_b$, the scale factor a_c will satisfy

$$\frac{\rho_\phi}{\rho_b} = \left(\frac{a}{a_c}\right)^{\frac{2m}{\alpha+2}}. \tag{2.42}$$

If the universe evolves until $a > a_c$, it will enter the quintessence dominated epoch and the field will not have a scaling solution. From observations, we know that

$$\frac{\rho_{o\phi}}{\rho_{ob}} \approx 2.3. \tag{2.43}$$

This together with Eq. (2.42) imply that $a_o \approx a_c$. Note that generally $\log_{10}(\frac{a_o}{a_c}) \approx 2$ and $\log_{10}(\frac{a_o}{a_{rad}}) \approx 30$ [12]; a_{rad} is a scale factor at the beginning of the radiation epoch.

It is convenient to introduce the rescaled time variable t',

$$t' = t\left(\alpha\Lambda^{\alpha+4}\right)^{\frac{1}{2}}, \tag{2.44}$$

for the discussion of the tracking behaviour of the scaling solution of the field equation. With this variable, Eq. (2.32) becomes

$$\ddot{\phi} + \frac{6}{mt'}\dot{\phi} - \frac{1}{\phi^{\alpha+1}} = 0, \tag{2.45}$$

where now the " $\dot{}$ " and " $\ddot{}$ " refer to the derivatives with respect to t'. The solution of Eq. (2.45) is $\phi = b't'^p$ where

$$b' = \left[\frac{2}{\alpha+2}\left(\frac{6}{m} - \frac{\alpha}{\alpha+2}\right)\right]^{-\frac{1}{\alpha+2}}. \tag{2.46}$$

By another change of variables [17]

$$t' = e^{T} \quad ; \quad u(T) = \frac{\phi(T)}{\phi_c(T)}, \tag{2.47}$$

where ϕ_c is the scaling solution, Eq. (2.45) becomes

$$\begin{aligned}
\frac{1}{t'^2}\left(\frac{\partial^2}{\partial T^2} - \frac{\partial}{\partial T}\right)(u\phi_c) + \frac{6}{mt'^2}\frac{\partial}{\partial T}(u\phi_c) - \frac{1}{(u\phi_c)^{\alpha+1}} &= 0, \\
\frac{1}{t'}\left(u''\phi_c + 2u'\phi_c' - u'\phi_c + \frac{6}{m}u'\phi_c\right) + \frac{u}{\phi_c^{\alpha+1}} - (u\phi_c)^{-(\alpha+1)} &= 0, \\
\frac{\phi_c}{t'^2}u'' + \left(\frac{4}{\alpha+2} + \frac{6}{m} - 1\right)\frac{u'\phi_c}{t'^2} + \phi_c^{-\alpha-1}(u - u^{-\alpha-1}) &= 0, \\
u'' + \left(\frac{4}{\alpha+2} + \frac{6}{m} - 1\right)u' + \frac{2}{\alpha+2}\left(\frac{6}{m} - \frac{\alpha}{\alpha+2}\right)(u - u^{-\alpha-1}) &= 0.
\end{aligned} \tag{2.48}$$

Let $v = u'$ then the second order differential equation, Eq. (2.48), becomes the set of the first order equations

$$\begin{aligned}
u' &= v, \\
v' &= -\left(\frac{4}{\alpha+2} + \frac{6}{m} - 1\right)v - \frac{2}{\alpha+2}\left(\frac{6}{m} - \frac{\alpha}{\alpha+2}\right)(u - u^{-\alpha-1}).
\end{aligned} \tag{2.49}$$

The fixed points of above equations are

$$(v_c, u_c) = \begin{cases} (0,1) & \text{for even } \alpha \\ (0,\pm 1) & \text{for odd } \alpha \end{cases}. \tag{2.50}$$

Eq. (2.47) shows that at the fixed point (0,1), the field solution is

$$\phi(T) = \phi_c(T). \tag{2.51}$$

This means that quintessence ϕ will evolve into the scaling solution ϕ_c for a wide range of initial conditions; this is the tracking behaviour of quintessence. Linearizing Eq. (2.49) about the fixed point (0,1), we obtain

$$\begin{pmatrix} \delta u' \\ \delta v' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2\left(\frac{6}{m} - \frac{\alpha}{\alpha+2}\right) & \left(1 - \frac{4}{\alpha+2} - \frac{6}{m}\right) \end{pmatrix}\begin{pmatrix} \delta u \\ \delta v \end{pmatrix}, \tag{2.52}$$

where $\delta u = u - u_c$, $\delta v = v - v_c$, $\delta u' = u' - u'_c$, $\delta v' = v' - v'_c$; (u_c, v_c) being the fixed point. The eigenvalues $(\lambda_\pm)$ of Eq. (2.52) are

$$\lambda_\pm = \frac{1}{2} - \frac{3}{m} - \frac{2}{\alpha+2} \pm \left[\left(\frac{1}{2} - \frac{3}{m} - \frac{2}{\alpha+2}\right)^2 - 2\left(\frac{6}{m} - \frac{\alpha}{\alpha+2}\right)\right]^{\frac{1}{2}}. \tag{2.53}$$

The fixed point (0,1) will be stable if

$$-2\frac{6+m}{6-m} < \alpha. \tag{2.54}$$

This relation is always satisfied, since $\alpha > 0$ for negative power law potential. This means that the scaling solution discussed above has a tracking behaviour for all positive α.

Eq. (2.42) shows that if $a > a_c$, the universe will be in the quintessence field dominated epoch. In this epoch, the field is assumed to be slowly rolling down its potential ($\ddot{\phi} < \dot{\phi}$, $\dot{\phi}^2 < V$). So the Friedmann and field equations become

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{\Lambda^{\alpha+4}}{\phi^\alpha}\right), \tag{2.55}$$

$$3H\dot{\phi} = \alpha\frac{\Lambda^{\alpha+4}}{\phi^{\alpha+1}}. \tag{2.56}$$

Substituting H from Eq. (2.55) into Eq. (2.56), we obtain

$$3\left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} \frac{\Lambda^{\frac{4+\alpha}{2}}}{\phi^{\frac{\alpha}{2}}} d\phi = \alpha\frac{\Lambda^{\alpha+4}}{\phi^{\alpha+1}} dt, \tag{2.57}$$

so that

$$\frac{6}{\alpha+4}\left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} \frac{\phi_o^2}{\sqrt{V(\phi_o)}}\left[\left(\frac{\phi}{\phi_o}\right)^{\frac{\alpha+4}{2}} - 1\right] = \alpha(t - t_o). \tag{2.58}$$

So the field solution is

$$\phi = \phi_o\left[1 + \frac{\alpha(\alpha+4)}{6}\left(\frac{3}{8\pi G}\right)^{\frac{1}{2}} \frac{\sqrt{V_o}}{\phi_o^2}(t - t_o)\right]^{\frac{2}{\alpha+4}}. \tag{2.59}$$

From Eqs. (2.55) and (2.59), the scale factor can now be written as

$$a \propto \exp[\beta t^{\frac{4}{4+\alpha}}]. \tag{2.60}$$

So

$$\ddot{a} \propto \left[-\frac{4\alpha}{(4+\alpha)^2 t^{\frac{4}{\alpha+4}}} + \left(\frac{4}{4+\alpha}\right)^2 \right] t^{-\frac{2\alpha}{4+\alpha}} \exp[\beta t^{\frac{4}{4+\alpha}}], \tag{2.61}$$

showing that the universe will expand with acceleration if t is large. The parameter ω_ϕ can be calculated using Eq. (2.10)

$$\omega_\phi = \frac{\dot{\phi}^2}{\rho_\phi} - 1. \tag{2.62}$$

From the slowly rolling condition ($\ddot{\phi} < \dot{\phi},\ \dot{\phi}^2 < V$)

$$\omega_\phi \approx \frac{\dot{\phi_o}^2}{V_o} - 1, \tag{2.63}$$

and using Eq. (2.59), we get

$$\omega_\phi \approx \frac{\alpha^2}{9} \frac{3}{8\pi G \phi_o^2} - 1. \tag{2.64}$$

Since it is not easy to estimate ϕ_o, then it is better to find another form of ω_o. From Eq. (2.58), one may assume that

$$\frac{\phi_o^2}{\sqrt{V_0}} \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} \frac{6}{\alpha+4} = \alpha t_o \tag{2.65}$$

$$\phi = \left[\frac{\alpha(\alpha+4)}{6} \left(\frac{3}{8\pi G}\right)^{\frac{1}{2}} \Lambda^{\frac{\alpha+4}{2}} t \right]^{\frac{2}{\alpha+4}} \tag{2.66}$$

Since $\dot{\phi}_o = \dfrac{2}{\alpha+4} \dfrac{\phi_o}{t_o}$, then

$$\begin{aligned} \omega_\phi &\approx \frac{4}{(\alpha+4)^2 V_0} \left(\frac{\phi_o}{t_o}\right)^2 - 1 \\ &\approx \frac{2}{3} \frac{\alpha}{\alpha+4} \left(\frac{3}{8\pi G}\right)^{\frac{1}{2}} \frac{1}{\sqrt{V_o} t_o} - 1 \\ &\approx \frac{2}{3} \frac{\alpha}{\alpha+4} \frac{1}{H_o t_o} - 1. \end{aligned} \tag{2.67}$$

Since $t_o H_o$ can be approximated from the age of the universe, then the value of ω_ϕ may be estimated approximately.

2.4 Quintessence with exponential potential

As in the previous section, we start the dissussion with the background dominated epoch. In this epoch the field equation and Friedmann equation are

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \tag{2.68}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_b. \tag{2.69}$$

Eq. (2.69) can be solved using Eq. (2.2) and we obtain

$$a \propto t^{\frac{2}{m}}; \quad \rho_b \propto t^{-2}; \quad H = \frac{2}{mt}. \tag{2.70}$$

If the potential is exponential [5, 18–20] ($V = V_c e^{-k\phi}$; $k = \lambda\sqrt{8\pi G}$,), Eq. (2.68) will become

$$\ddot{\phi} + 3H\dot{\phi} - kV = 0. \tag{2.71}$$

A field solution ϕ of eq. (2.71) of the form $\phi = c\ln(bt)$ will give rise to a relation $\rho_\phi = a^{-n}$. Substituting $\phi = c\ln(bt)$ into Eq. (2.71), we obtain

$$-\frac{c}{t^2} + \frac{6c}{mt^2} = V_c k(bt)^{-ck}. \tag{2.72}$$

Eq. (2.72) is satisfied when

$$\begin{aligned} c &= \frac{2}{k}, \\ b &= \left[\frac{2}{V_c k^2}\left(\frac{6}{m} - 1\right)\right]^{-\frac{1}{2}}. \end{aligned} \tag{2.73}$$

For this field solution, the field energy density decreases as

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V \propto t^{-2}. \tag{2.74}$$

This has the same time dependence as ρ_b so that

$$\frac{\rho_b}{\rho_\phi} = constant, \tag{2.75}$$

and it follows that

$$\omega_b = \omega_\phi. \tag{2.76}$$

The constant ratio ρ_ϕ/ρ_b implies that the density parameter of field ϕ which is defined by

$$\Omega_\phi = \frac{\rho_\phi}{\rho_c} = \frac{\rho_\phi 8\pi G}{3H^2} \tag{2.77}$$

is also constant. From Eqs. (2.72) and (2.73), V is given by

$$V = \frac{2}{t^2k^2}\left(\frac{6}{m} - 1\right), \tag{2.78}$$

and ρ_ϕ is given by

$$\rho_\phi = \frac{1}{2}\left(\frac{2}{kt}\right)^2 + \frac{2}{k^2t^2}\left(\frac{6}{m} - 1\right). \tag{2.79}$$

Then density parameter of the field has the simple form

$$\Omega_\phi = \frac{8\pi G}{3H^2}\left[\frac{1}{2}\left(\frac{2}{kt}\right)^2 + \frac{2}{k^2t^2}\left(\frac{6}{m} - 1\right)\right], \tag{2.80}$$

which can be further simplified using H from Eq. (2.70) to give

$$\Omega_\phi = \frac{m}{k^2}(8\pi G) = \frac{m}{\lambda^2}. \tag{2.81}$$

In this case, we see that there can be no field dominated epoch for constant λ as the limit $\rho_b \gg \rho_\phi$ has been used for the Friedmann equation (Eq. (2.69)). This limit is consistent with the standard model for nucleosynthesis where in the earlier nucleosynthsis phase, the value $\Omega_\phi \approx 0.15$ is expected. So without time dependence, Ω_ϕ cannot satisfy the observational condition $\Omega_\phi \approx 0.7$ today. We will discuss this later.

Next, let us consider the limit $\rho_\phi \gg \rho_b$ with the field slowly rolling down its potential. The field equation and Friedmann equation now become

$$3H\dot{\phi} = kV_c e^{-k\phi}, \tag{2.82}$$

$$H^2 = \frac{8\pi G}{3} V_c e^{-k\phi}. \tag{2.83}$$

Solving for the field by substituting H from Eq. (2.83) into Eq. (2.82) and integrating, we obtain

$$3\left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} \sqrt{V_c} e^{-\frac{k\phi}{2}} d\phi = k V_c e^{-k\phi} dt \tag{2.84}$$

which gives

$$\phi = \frac{2}{k} \ln\left[\frac{k^2}{6}\left(\frac{3}{8\pi G}\right)^{\frac{1}{2}} t\sqrt{V_c}\right]. \tag{2.85}$$

Using this field solution ϕ in Eq. (2.83), the Hubble parameter is obtained

$$\begin{aligned} H^2 &= \frac{8\pi G}{3} V_c \frac{8\pi G}{3}\left(\frac{6}{k^2}\right)^2 \frac{1}{V_c t^2} \\ &= \frac{4}{\lambda^4 t^2} \end{aligned} \tag{2.86}$$

which implies that the scale factor increases as

$$a \propto t^{\frac{2}{\lambda^2}}. \tag{2.87}$$

From the above equation, we get

$$\ddot{a} \propto \frac{2}{\lambda^2}\left(\frac{2}{\lambda^2} - 1\right) t^{\frac{2}{\lambda^2} - 2}. \tag{2.88}$$

Like Eq. (2.28), the universe will expand with accelelation if $\lambda^2 < 2$ and the accelelation rate will increases if $\lambda^2 < 1$.

With the field solution, Eq. (2.85), it is seen that the field energy density decreases as

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V \propto t^{-2} \propto a^{-\lambda^2}. \tag{2.89}$$

Recalling the discussion about the limit of n after Eq. (2.16), we get

$$\lambda^2 \leq 6. \tag{2.90}$$

In this field dominated case, a small increase in the value of ρ_b will lead to an increase in the coefficient of the damping term of the field equation (2.71) because $H^2 \propto \rho_b$. It follows that as quintessence roles down its potential, the rate of increase of $\dot{\phi}$ will be lowered with increasing ρ_b. This leads to a lower value of $n = \frac{\dot{\phi}^2}{2\rho_\phi}$, and ρ_ϕ will decrease with a slower rate (i.e., $\rho_\phi = a^{-\lambda^2 - D}$ with $\lambda^2 > D > 0$). For $\lambda^2 < m$, ρ_ϕ will decrease slower than ρ_b, so the field energy always dominates. For $\lambda^2 > m$, ρ_ϕ will decrease faster than ρ_b and the background energy will eventually dominate. This rough consideration suggests that the solution (2.73) for the background dominated epoch can exist if $\lambda^2 > m$.

Next the stabillity of the field solution will be discussed. From

$$\frac{\partial}{\partial t}\left(\frac{\dot{a}}{a}\right) = \dot{H} = \frac{\ddot{a}}{a} - H^2, \tag{2.91}$$

the Friedmann equations, Eqs. (1.30) and (1.31), give

$$\begin{aligned} \dot{H} &= -4\pi G\left(\rho_b + p_b + \rho_\phi + p_\phi\right) \\ &= -4\pi G\left(\rho_b + p_b + \dot{\phi}^2\right). \end{aligned} \tag{2.92}$$

Introducing new variables

$$x = \frac{\sqrt{\kappa}\dot{\phi}}{\sqrt{6}H}; \quad y = \frac{\sqrt{\kappa}\sqrt{V}}{\sqrt{3}H}. \tag{2.93}$$

where $\kappa = 8\pi G$, it is easy to see that

$$\begin{aligned} x' = \frac{\partial x}{\partial \ln a} = H^{-1}\frac{\partial x}{\partial t} &= \frac{\sqrt{\kappa}}{H\sqrt{6}}\left(\frac{\ddot{\phi}}{H} - \frac{\dot{\phi}\dot{H}}{H^2}\right), \\ y' &= \frac{\sqrt{\kappa}}{H\sqrt{3}}\left(\frac{\dot{V}}{2\sqrt{V}H} - \frac{\sqrt{V}\dot{H}}{H^2}\right). \end{aligned} \tag{2.94}$$

Substituting $\ddot{\phi}$ from Eq. (2.71) and $\dot{H}$ from Eq. (2.92) into Eq. (2.94), we get

$$\begin{aligned} x' &= \frac{\sqrt{\kappa}}{\sqrt{6}H}\left(-3\dot{\phi} + \frac{\lambda\sqrt{\kappa}V}{H} + \frac{\kappa}{2H^2}\left[\dot{\phi}(\rho_b + p_b) + \dot{\phi}^3\right]\right) \\ &= -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\gamma - \frac{3}{2}x^3\gamma - \frac{3}{2}xy^2\gamma + 3x^3, \end{aligned} \tag{2.95}$$

where $\gamma = 1 + \omega$. Since $\dot{V} = \frac{\partial V}{\partial \phi}\dot{\phi}$, then

$$
\begin{aligned}
y' &= \frac{\sqrt{\kappa}}{\sqrt{3}H}\left(-\frac{\dot{\phi}\lambda\sqrt{\kappa}\sqrt{V}}{2H} + \frac{\kappa}{2H^2}\left[\sqrt{V}\gamma\rho_b + \sqrt{V}\dot{\phi}^2\right]\right) \\
&= -\sqrt{\frac{3}{2}}xy\lambda + \frac{3}{2}\gamma y - \frac{3}{2}\gamma x^2 y - \frac{3}{2}\gamma y^3 + 3x^2 y. \qquad (2.96)
\end{aligned}
$$

The fixed points of above set of equations are [18]

$$
(x_c, y_c) = (0,0), (\pm 1, 0), \left(\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}\right), \left(\sqrt{\frac{3}{2}}\frac{\gamma}{\lambda}, \left[\frac{3\gamma}{2\lambda^2}(2-\gamma)\right]^{\frac{1}{2}}\right). \qquad (2.97)
$$

These fixed points correspond to no-field, kinetic energy dominated, scalar field dominated and background energy dominated cases respectively.

The stability of these fixed points can be studied by linearizing Eqs. (2.95) and (2.96) as follows:

$$
\begin{aligned}
x' = (\delta x - x_c)' &= -3(\delta x - x_c) + \lambda\sqrt{\frac{3}{2}}(\delta y - y_c)^2 + \frac{3}{2}(\delta x - x_c)\gamma \\
&\quad -\frac{3}{2}(\delta x - x_c)^3\gamma - \frac{3}{2}(\delta x - x_c)(\delta y - y_c)^2\gamma + 3(\delta x - x_c)^3 \\
&= -3\delta x + 9x_c^2\delta x + \frac{3}{2}\gamma\delta x - \frac{9}{2}x_c^2\gamma\delta x - \frac{3}{2}y_c^2\gamma\delta x + \lambda\sqrt{6}y_c\delta y \\
&\quad -3x_c y_c\gamma\delta y \qquad (2.98) \\
y' = (\delta y - y_c)' &= -\sqrt{\frac{3}{2}}(\delta x - x_c)(\delta y - y_c)\lambda + \frac{3}{2}\gamma(\delta y - y_c) \\
&\quad -\frac{3}{2}\gamma(\delta x - x_c)^2(\delta y - y_c) - \frac{3}{2}\gamma(\delta y - y_c)^3 \\
&\quad +3(\delta x - x_c)^2(\delta y - y_c) \\
&= -\lambda\sqrt{\frac{3}{2}}y_c\delta x + 6y_c x_c\delta x - 3x_c y_c\gamma\delta x - \lambda\sqrt{\frac{3}{2}}x_c\delta y + 3x_c^2\delta y \\
&\quad +\frac{3}{2}\gamma\delta y - \frac{3}{2}\gamma x_c^2 - \frac{9}{2}\gamma y_c^2\delta y, \qquad (2.99)
\end{aligned}
$$

where $\delta x = x - x_c$, $\delta y = y - y_c$, $\delta x' = x' - x_c'$ and $\delta y' = y' - y_c'$. The above equations can be written in a matrix form as

$$
\begin{pmatrix} \delta x' \\ \delta y' \end{pmatrix} = \mathbf{M}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix}. \qquad (2.100)
$$

where

$$\mathbf{M} = \begin{pmatrix} -3+9x_c^2+\frac{3}{2}\gamma-\frac{9}{2}x_c^2\gamma-\frac{3}{2}y_c^2\gamma & \lambda\sqrt{6}y_c-3x_cy_c\gamma \\ -\lambda\sqrt{\frac{3}{2}}y_c+6y_cx_c-3x_cy_c\gamma & -\lambda\sqrt{\frac{3}{2}}x_c+3x_c^2+\frac{3}{2}\gamma-\frac{3}{2}\gamma x_c^2-\frac{9}{2}\gamma y_c^2 \end{pmatrix}.$$

The eigenvalues $e_{1,2}$ of $\mathbf{M}$ are [18]:

1. For the no-field case ($(x_c, y_c) = (0, 0)$):

$$e_1 = -\frac{3}{2}(2-\gamma); \quad e_2 = \frac{3}{2}\gamma. \tag{2.101}$$

   Since $0 \le \gamma \le 2$, then this fixed point is a saddle point.

2. For the kinetic energy dominated case ($(x_c, y_c) = (\pm 1, 0)$):

$$e_1 = \sqrt{\frac{3}{2}}(\sqrt{6} \mp \lambda); \quad e_2 = 3(2-\gamma). \tag{2.102}$$

   The fixed point (1,0) will be a saddle point if $\lambda > \sqrt{6}$ and it will be an unstable node if $\lambda < \sqrt{6}$. The point $(-1, 0)$ will be a saddle point if $\lambda < -\sqrt{6}$ and it will be an unstable node if $\lambda > -\sqrt{6}$.

3. For the scalar field dominated case ($(x_c, y_c) = (\frac{\lambda}{\sqrt{6}}, \sqrt{1-\frac{\lambda 2}{6}})$):

$$e_1 = \lambda^2 - 6; \quad e_2 = \lambda^2 - 3\gamma. \tag{2.103}$$

   We see from Eq. (2.90) that $\lambda^2 < 6$, so this fixed point will be a stable node if $\lambda^2 < 3\gamma$, and it will be a saddle point if $\lambda^2 > 3\gamma$. This agrees with the rough discussion above.

4. For the background dominated case ($(x_c, y_c) = (\sqrt{\frac{3}{2}}\frac{\gamma}{\lambda}, [\frac{3\gamma}{2\lambda^2}(2-\gamma)]^{\frac{1}{2}})$):

$$e_{1,2} = -\frac{3(2-\gamma)}{4}\left[1 \pm \left(1 - \frac{8\gamma(\lambda^2-3\gamma)}{\lambda^2(2-\gamma)}\right)^{\frac{1}{2}}\right]. \tag{2.104}$$

   This fixed point will be a stable node if $3\gamma < \lambda^2 < 24\gamma^2\sqrt{9\gamma-2}$ and it will be a stable spiral if $\lambda^2 > 24\gamma^2\sqrt{9\gamma-2}$.

2.5 Basic ideas of tracker solution

The conditions for the tracking behavior [6] will be derived after a brief introduction to some properties of the tracking behavior. First the field equation

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \tag{2.105}$$

where $V' = \dfrac{\partial V}{\partial \phi}$, is rewritten in an appropriate form. Defining T as $T = \dfrac{\dot{\phi}^2}{2}$, one gets

$$\frac{\dot{T}}{\sqrt{2T}} + 3H\sqrt{2T} + \frac{\dot{V}}{\sqrt{2T}} = 0, \tag{2.106}$$

and from the relation $\rho_\phi = T + V$ the equation becomes

$$\frac{\dot{T}}{\sqrt{2T}} + \frac{\dot{V}}{\sqrt{2T}V}\rho_\phi - \frac{\dot{V}\sqrt{2T}}{2V} + 3H\sqrt{2T} = 0. \tag{2.107}$$

So

$$\pm\frac{V'}{V} = \frac{3H\sqrt{2T}}{\rho_\phi}\left(1 + \frac{1}{6}\frac{\partial \ln x}{\partial \ln a}\right), \tag{2.108}$$

where $x = T/V$. Letting $\kappa = 8\pi G$, then $\rho_c = \dfrac{H^2}{\kappa}$ and

$$\pm\frac{V'}{V} = 3\frac{\sqrt{\kappa}}{\sqrt{\Omega_\phi}}\sqrt{1+\omega_\phi}\left(1 + \frac{1}{6}\frac{\partial \ln x}{\partial \ln a}\right). \tag{2.109}$$

It was shown in Sections 2.3 and 2.4 that there are solutions to the field equation that converge to the common solution which is a scaling solution. The solutions which spend a long time converging are called tracker solutions. Although the field solutions do not always converge to the tracker solutions, but if the converging period is long enough a wide range of initial conditions will give the same evolution of field. The field solution depends on the parameters of the potential. A good example of the tracker solution is the field solution in Section 2.3. When the background energy dominates, the field solution converges to a scaling solution. When the field energy dominates, although the field solution does not converge to a scaling solution but the tracking behaviour still exists. The converging can occur if ω_ϕ does not remain constant. Since $\omega_\phi = \frac{p_\phi}{\rho_\phi}$ slowly varies between -1 and 1 through a long period of the universe evolution, then one can assume that ω_ϕ is nearly constant [4,21]. This is a property of the tracker solution. From this property and $1 + \omega_\phi \sim O(1)$, one gets

$$\frac{V'}{V} \approx \frac{1}{\sqrt{\Omega_\phi}} \approx \frac{H}{\dot{\phi}}. \tag{2.110}$$

An important function for the tracker solution is the Γ function defined as

$$\Gamma = \frac{V''V}{V'^2} = 1 + \frac{\omega_b - \omega_\phi}{2(1+\omega_\phi)} - \frac{1+\omega_b - 2\omega_\phi}{2(1+\omega_\phi)}\frac{\dot{x}}{6+\dot{x}} - \frac{2}{1+\omega_\phi}\frac{\ddot{x}}{(6+\dot{x})^2}, \tag{2.111}$$

where $\dot{x} = \frac{\partial \ln x}{\partial \ln a}$ and $\ddot{x} = \frac{\partial^2 \ln x}{\partial (\ln a)^2}$. The above equation shows that for $\omega_b > \omega_\phi$, the tracker solution will exist if $\Gamma > 1$, and for $\omega_b < \omega_\phi < 1/2(1+\omega_b)$, the tracker solution will exist if $1 - (1-\omega_b)/(6+2\omega_\phi) < \Gamma < 1$. Note that the upper bound of ω_ϕ is required for the convergence of the field solutions to the tracker solution. The other condition for the tracker solution is that Γ is nearly constant, i.e., $\left|\frac{1}{H}\frac{d(\gamma-1)}{dt}\right| \ll |\Gamma - 1|$. From Eq. (2.110), one gets

$$\left|\frac{\Gamma^{-1}}{H}\frac{d(\Gamma-1)}{dt}\right| \approx \left|\frac{\Gamma'}{\Gamma(V'/V)}\right| \ll 1. \tag{2.112}$$

For examples if $V = \frac{K}{\phi^\alpha}$, Eq. (2.111) gives $\Gamma = 1 + \alpha^{-1}$ and the left hand side of Eq. (2.112) is zero. So this potential gives $\omega_\phi < \omega_b$. If $V = K\phi^\alpha$, then $\omega_\phi > \omega_b$. The general discussion on the convergence of field solutions to tracker solutions is given in Ref. [6].

Chapter 3

Quintessence models

Scalar field quintessence with exponential or negative power law potentials can arise from particle physics theory. The behaviour of these fields are discussed in this chapter. The details of particle physics theory are not discussed here because they are beyond the scope of this thesis.

The negative power law quintessence which arises from dynamical supersymmetry breaking is discussed in Section 3.1. In Section 3.2, supergravity quintessence which has the potential of the form $V = \frac{\Lambda^{\alpha+4}}{\phi^\alpha} e^{k\phi^2/2}$ is discussed. Quintessence with exponential potential and non-canonical kinetic term are discussed in Section 3.3.

3.1 Quintessence and Supersymmetric QCD

In this section we consider scalar field quintessence in which its negative power law potential arises from non-perturbative gauge dynamics that lead to supersymmetry breaking. The scalar field potential of supersymmetric QCD with N_c colors and N_f flavors is given by [22, 23]

$$\begin{aligned} V(\vec{\phi}_f, \vec{\overline{\phi}}^f) &= \sum_{i=1}^{N_f} \left(|F_{\phi_i}|^2 + |F_{\overline{\phi}^i}|^2 \right) + \frac{1}{2} \sum_\alpha (\lambda_c^{\alpha\, c'} D_{c'}^c)^2 \\ &= V_F + V_D, \end{aligned} \tag{3.1}$$

where λ^α are the generators of the group $SU(N_c)$, $F_{\phi_i} = \dfrac{\partial W}{\partial \phi_i}$, $F_{\overline{\phi}^i} = \dfrac{\partial W}{\partial \overline{\phi}^i}$ (W being the superpotential), $D_{c'}^c = \phi^{\dagger cf} \phi_{c'f} - \overline{\phi}^{cf} \overline{\phi}^\dagger_{c'f}$ (ϕ_{cf} and $\overline{\phi}^{cf}$ being respectively the scalar components of the chiral superfields Φ_{cf} and $\overline{\Phi}^{cf}$ with color index c

$(1 \leq c \leq N_c)$ and flavor index f $(1 \leq f \leq N_f)$),

$$\vec{\phi}_f = \begin{pmatrix} \phi_{1f} \\ \phi_{2f} \\ \vdots \\ \phi_{N_c f} \end{pmatrix} \quad , \quad \vec{\overline{\phi}}^f = \begin{pmatrix} \overline{\phi}^{1f} \\ \overline{\phi}^{2f} \\ \vdots \\ \overline{\phi}^{N_c f} \end{pmatrix},$$

and V_F and V_D are respectively the F term and D term potential.

Supersymmetry will be preserved if $V = 0$. Classically, one can set all the superpotential couplings to zero [24], so supersymmetry remains unbroken if there exists some field configurations which give a vanishing D term. These field configurations are not isolated points but form a subspace. This subspace is called the D-flat direction. From Ref. [25], the D-flat direction is given by

$$\phi_{cf} = \overline{\phi}^{\dagger}_{cf} = \begin{cases} q_f \delta_{cf} & \text{if } 1 < c < N_f \\ 0 & \text{if } N_f < c < N_c \end{cases} . \tag{3.2}$$

When supersymmetry is broken by the non-perturbative effect, the superpotential which arises has the form [24–26]

$$W(\vec{\overline{\Phi}}^f, \vec{\Phi}_f) = (N_c - N_f) \left[\frac{\Lambda^{3N_c - N_f}}{\det(\overline{\Phi}^{cf'} \Phi_{cf})} \right]^{\frac{1}{N_c - N_f}} . \tag{3.3}$$

Note that $\overline{\phi}^{cf'} \phi_{cf} = \vec{\overline{\phi}}^{f'} \vec{\phi}_f$.

From Eq. (3.2), the scalar potential is

$$V(\vec{\overline{\phi}}^f, \vec{\phi}_f) = \frac{\Lambda^{2a}}{[\det(\vec{\overline{\phi}}\vec{\phi})]^{d+1} [\det(\vec{\overline{\phi}}\vec{\phi})]^{\dagger\, d+1}} \sum_{i=1}^{N_f} \left[\left| \frac{\partial \det(\vec{\overline{\phi}}\vec{\phi})}{\partial \phi_i} \right|^2 + \left| \frac{\partial \det(\vec{\overline{\phi}}\vec{\phi})}{\partial \overline{\phi}^i} \right|^2 \right], \tag{3.4}$$

where $a = \dfrac{3N_c - N_f}{N_c - N_f}$ and $d = \dfrac{1}{N_c - N_f}$. Since $\vec{\overline{\phi}}^j \vec{\phi}_j$ can be diagonalized, on diagolization, we get

$$V(\vec{\overline{\phi}}^f, \vec{\phi}_f) \;\; = \;\; \frac{\Lambda^{2a}}{\left(\prod_j^{N_f} \vec{\overline{\phi}}^j \vec{\phi}_j\right)^{d+1} \left(\prod_j^{N_f} \vec{\overline{\phi}}^j \vec{\phi}_j\right)^{\dagger d+1}}$$

$$\times \sum_{i=1}^{N_f} \left[\frac{\left(\prod_j^{N_f} \vec{\bar{\phi}}^j \vec{\phi}_j\right)\left(\prod_j^{N_f} \vec{\bar{\phi}}^j \vec{\phi}_j\right)^\dagger}{\vec{\phi}_i \vec{\phi}^{i\dagger}} + \frac{\left(\prod_j^{N_f} \vec{\bar{\phi}}^j \vec{\phi}_j\right)\left(\prod_j^{N_f} \vec{\bar{\phi}}^j \vec{\phi}_j\right)^\dagger}{\vec{\bar{\phi}}^j \vec{\bar{\phi}}_i^\dagger} \right]$$

$$= \frac{\Lambda^{2a}}{\left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^d \left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^{\dagger d}} \sum_{i=1}^{N_f} \left[\frac{1}{\vec{\phi}_i \vec{\phi}^{i\dagger}} + \frac{1}{\vec{\bar{\phi}}^j \vec{\bar{\phi}}_i^\dagger} \right]. \tag{3.5}$$

If the q_fs are real fields, the potential in the D flat direction will be

$$V(q_f) = 2\frac{\Lambda^{2a}}{\left(\prod_j q_j\right)^{4d}} \sum_{i=1}^{N_f} \frac{1}{q_i^2}. \tag{3.6}$$

The evolution of fields q_fs in the universe can be described by

$$\ddot{q}_f + 3H\dot{q}_f + \left.\frac{\partial V}{\partial \vec{\phi}_f}\right|_D = 0, \tag{3.7}$$

where $\left.\frac{\partial V}{\partial \vec{\phi}_f}\right|_D$ is the derivative of V which is evaluated along the D-flat direction. From Eq. (3.5), the derivative of V with respect to ϕ_f is

$$\begin{aligned} \frac{\partial V}{\partial \vec{\phi}_f} &= -d\frac{\Lambda^{2a}}{\left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^{d+1} \left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^{\dagger d}} \frac{\prod \vec{\bar{\phi}}\vec{\phi}}{\vec{\phi}_f} \sum_{i=1}^{n_f} \left[\frac{1}{\vec{\phi}_i \vec{\phi}^{i\dagger}} + \frac{1}{\vec{\bar{\phi}}^j \vec{\bar{\phi}}_i^\dagger} \right] \\ &\quad - \frac{\Lambda^{2a}}{\left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^d \left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^{\dagger d}} \frac{1}{\vec{\phi}_f^2 \vec{\phi}_f^\dagger} \\ &= -\frac{\Lambda^{2a}}{\left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^d \left(\prod \vec{\bar{\phi}}\vec{\phi}\right)^{\dagger d}} \frac{1}{\vec{\phi}_f} \left[d\sum_{i=1}^{N_f} \left[\frac{1}{\vec{\phi}_i \vec{\phi}^{I\dagger}} + \frac{1}{\vec{\bar{\phi}}^j \vec{\bar{\phi}}_i^\dagger} \right] + \frac{1}{\vec{\phi}_f \vec{\phi}^{f\dagger}} \right]. \end{aligned} \tag{3.8}$$

So if the q_fs are real fields, we get

$$\left.\frac{\partial V}{\partial \vec{\phi}_f}\right|_D = -\frac{\Lambda^{2a}}{\left(\prod_j q_j\right)^{4d}} \frac{1}{q_f} \left[2d\sum_{i=1}^{N_f} \frac{1}{q_i^2} + \frac{1}{q_f^2} \right]. \tag{3.9}$$

If the initial conditions for all the q_fs are the same, one can see from Eqs. (3.7) and (3.9) that

$$\ddot{q}_f + 3H\dot{q}_f - g\frac{\Lambda^{2a}}{q_f^{2g+1}} = 0, \tag{3.10}$$

where $g = \dfrac{N_c + N_f}{N_c - N_f}$. The field q_f behaves as a single scalar field in the potential $V = \dfrac{\Lambda^{2a}}{2q^{2g}}$.

From Eqs. (2.36) and (2.37), the coefficients of the field solution $q = bt^p$ in the background dominated epoch are

$$p = \frac{1-r}{2}, \tag{3.11}$$

$$b = \left[\frac{4m(1+r)}{(1-r)^2[12 - m(1+r)]} \Lambda^{2a} \right]^{\frac{1-r}{4}}, \tag{3.12}$$

where $r = \dfrac{N_f}{N_c}$. If all the initial conditions for the q_fs are different, the evolution of the fields are described by N_f coupled differential equations. For example, if $N_f = 2$, the field equations will become

$$\begin{aligned} \ddot{q}_1 + 3H\dot{q}_1 - dq_1 \frac{\Lambda^{2a}}{(q_1 q_2)^2 dN_c} \left[2 + d\frac{q_2^2}{q_1^2} \right] &= 0, \\ \ddot{q}_2 + 3H\dot{q}_2 - dq_2 \frac{\Lambda^{2a}}{(q_1 q_2)^2 dN_c} \left[2 + d\frac{q_1^2}{q_2^2} \right] &= 0. \end{aligned} \tag{3.13}$$

As in the single field case, we look for the power law field solutions of the form

$$q_f = c_f t^{p_f}. \tag{3.14}$$

In the background dominated epoch, we know from (2.70) that

$$H = \frac{2}{mt}, \quad a \propto t^{\frac{2}{m}}, \tag{3.15}$$

so the field equations in this epoch are

$$\ddot{q}_f + \frac{6}{mt}\dot{q}_f - \frac{\Lambda^{2a}}{\left(\prod_j^{N_f} q_j\right)^{4d}} \frac{1}{q_f} \left[2d \sum_{i=1}^{N_f} \frac{1}{q_i} + \frac{1}{q_f} \right] = 0. \tag{3.16}$$

Substituting Eq. (3.14) into Eq. (3.16), we obtain

$$c_f p_f (p_f - 1) t^{p_f - 2} + \frac{6}{m} p_f c_f t^{p_f - 2} \tag{3.17}$$

$$- \frac{\Lambda^{2a}}{\left(\prod_j^{N_f} c_j\right)^{4d} t^{4d \sum_{j=1}^{N_f} p_j}} \frac{1}{c_f t^{p_f}} \left[2d \sum_{i=1}^{N_f} \frac{1}{c_i^2 t^{2p_f}} + \frac{1}{c_f^2 t^{2p_f}} \right] = 0. \tag{3.18}$$

This equation gives

$$p_f - 2 = -4d \sum_{i=1}^{N_f} p_i - 3p_f = -4d \sum_{i=1}^{N_f} p_i - p_f - 2p_{k(\neq f)}. \tag{3.19}$$

Since $k \neq f$, the above equation shows that $p_k = p_f$ for all possible values of k and

$$p_f - 2 = -4dN_f p_f - 3p_f \tag{3.20}$$

so that

$$p_f = \frac{1-r}{2} = p. \tag{3.21}$$

Because all the p_fs are equal, Eq. (3.18) becomes

$$p(p-1) + \frac{6p}{m} = \frac{\Lambda^{2a}}{(\prod_j c_j)^{4d}} \frac{1}{c_f^2} \left[2d \sum_{i=1}^{N_f} \frac{1}{c_i^2} + \frac{1}{c_f^2} \right]. \tag{3.22}$$

Taking the difference of the above equation with indices f and g, one obtains

$$\frac{1}{c_f^4} - \frac{1}{c_g^4} = \left(\frac{1}{c_g^2} - \frac{1}{c_f^2} \right) 2d \sum_{i=1}^{N_f} \frac{1}{c_i}. \tag{3.23}$$

This equation can be satisfied if all the c_fs are equal, so the subscripts for c_fs can be dropped, and Eqs. (3.21) and (3.22) give

$$c = \left[\frac{4m(1+r)}{(1-r)^2[12 - m(1+r)]} \Lambda^{2a} \right]^{\frac{1-r}{4}}. \tag{3.24}$$

From Eqs. (3.6), (3.14) and (3.15), one sees that

$$\rho_{q_f} \propto \frac{\dot{q_f}^2}{2} + V \propto t^{2p-2} \propto a^{m(p-1)}.$$

Since ρ_{q_f} is of the form of a perfect fluid

$$\omega_{q_f} = \omega_b \frac{1+r}{2} - \frac{1-r}{2}. \tag{3.25}$$

As in Section 2.3, one can show that Eq. (3.14) are the converging solutions of the field equations (3.16). The parameter Λ can be estimated by using the conditions that the fields have tracking behaviour and start to dominate the universe energy today. Both conditions can be satisfied if

$$V(q_{of}) \simeq \rho_{oc}, \quad V''(q_{of}) \simeq H_o. \tag{3.26}$$

From Eq. (3.6), one gets

$$2\frac{\Lambda^{2a}}{q_f^{2\frac{1-r}{1+r}}} = \rho_{oc}.$$

So

$$\frac{\Lambda}{m_p} = \left(\frac{q_{of}}{m_p}\right)^{\frac{1+r}{3-r}} \left(\frac{\rho_{oc}}{2rN_c m_p^4}\right)^{\frac{1-r}{2(3-r)}}, \tag{3.27}$$

where $m_p = G^{-\frac{1}{2}}$ is the Planck's mass. From [22]

$$V_o'' = \frac{2(3+r)(1+r)}{(1+r)^2}\frac{V_o}{q_{of}^2},$$

we obtain

$$\left(\frac{q_{of}}{m_p}\right)^2 = \frac{3(3+r)(1+r)}{4\pi(1-r)^2}\frac{1}{rN_c}. \tag{3.28}$$

Eqs. (3.27) and (3.28) show that the parameter Λ has a minimum value when $N_c \to \infty$ and fixed N_f. Since $\dfrac{\rho_{oc}}{m_p} \approx \dfrac{10^{-47}}{10^{71}} \approx 10^{-120}$, it follows that $\Lambda \approx \dfrac{10^{-2}}{N_c^{\frac{1}{6}}}$. For fixed N_c, Λ increases as N_f increases from 1 to its maxinum value $N_c - 1$. If $N_c = 20$ and N_f is close to N_c, the value of Λ will approximately be m_p. Eq. (3.27) shows that 10^{-1} GeV $< \Lambda < 10^3$ GeV for $3 < N_c < 20$ and $N_f = 1, 2$. This range of Λ is comparable with the particle energy scale (i.e., $m_w \approx 10^2$ GeV, $m_p \approx 10^{19}$ GeV). So the fine tuning problem in this case is less severe than the fine tuning problem of the cosmological constant.

The existence of the superpotential in Eq. (3.3) requires the weak coupling condition, this condition will be satisfied if $\Lambda < q_f$. From Eqs. (3.27) and (3.28), one sees that $\Lambda < q_f$ for any N_f as long as $N_c < 20$. Before supersymmetry is broken by non-perturbative effect, the potential for q_f vanishes. So these fields behave as free fields ($\dot{q_f}^2$ = constant) and q_f increases with t. Then this model can be used after the time t_e that gives $q_f(t_e) = \Lambda$. The values of q_f at the present time which can be estimated by Eq. (3.28) are approximately m_p. Eq. (3.10) shows that if the initial conditions for all the q_fs are the same, q_f will behave as a single scalar field in a negative power law potential. From Eqs. (2.38) and (3.11), we obtain

$$\omega_q = \frac{1+r}{2}\omega_b - \frac{1-r}{2} = \omega_{q_f}. \tag{3.29}$$

For different initial conditions of q_f, one sees from Eqs. (3.21), (3.24), (3.11) and (3.12) that all fields q_fs have the same scaling solutions. These fields all evolve in the same way and in the same manner as a single field at late time. But the time which the fields $q_f s$ are on track depends on the ratio of fields' initial values [22].

3.2 Supergravity and quintessence

Supergravity theories with suitable superpotential and Kähler potential can give rise to quintessence with appropriate potential. The properties of the superpotential are discussed but the required Kähler potential is written down without any discussion. In supergravity theory, the bosonic part of the Lagrangian can be derived from the potential [23, 27]

$$G = \kappa K + \ln\left(\kappa^3 |W|^2\right), \tag{3.30}$$

where $\kappa = 8\pi G$, K is the Kähler potential and W is the superpotential. The kinetic term is given by

$$K_i^j \partial^\mu \phi^{*i} \partial_\mu \phi_j, \tag{3.31}$$

where $K_i^j = \dfrac{\partial^2 K}{\partial \phi^{*i} \partial \phi_j}$. The scalar potential is

$$V = \frac{1}{\kappa^2} e^G (G^i (G^{-1})_i^j G_j - 3) + V_D, \tag{3.32}$$

where $V_D > 0$ is a D term potential as in Section 3.1.

If the form of the superpotential is the same as that in the previous section,

$$W \sim \frac{\Lambda^a}{\phi^{2dN_f}} = \frac{\Lambda^{s+3}}{\phi^s}; \qquad s = 2dN_f, \tag{3.33}$$

and the Kähler potential is flat ($K = \phi\phi^*$), we will obtain

$$\begin{aligned} G &= \kappa\phi\phi^* + \ln\left(\kappa^3 \frac{\Lambda^{2a}}{\phi^{4dN_f}}\right), \quad G_\phi = \kappa\phi^* - \frac{2s}{\phi}, \\ G_{\phi^*} &= \kappa\phi - \frac{2s}{\phi^*}, \qquad (G^{-1})_{\phi\phi^*} = \frac{1}{\kappa}. \end{aligned} \tag{3.34}$$

If V_D is set to be zero, the scalar potential becomes

$$V = e^{\kappa|\phi|^2} \frac{\Lambda^{b+4}}{\phi^b}\left[\frac{(b-2)^2}{4} - (b+1)\kappa|\phi|^2 + \kappa^2|\phi|^4\right], \tag{3.35}$$

where $b = 2s + 2$. The present value of ϕ, $\phi \approx m_p$, which can be estimated from Eq. (3.26) will lead to a negative scalar potential as can be seen from Eq. (3.35) above. This is a serious problem because it can lead to a negative energy density for the universe, since the field ϕ is slowly rolling ($\rho_\phi \approx V$) and the universe is dominated by the field energy density at present time ($\rho_{oc} \approx \rho_{o\phi} \approx V_o$). Generally, one can see from Eq. (3.32) that the negative contribution to V comes from the term $-\frac{3}{\kappa^2}e^G$.

The negative value of V can be avoided by imposing that the superpotential vanish and the Kähler potential is non-flat. Consider the supergravity model with two types of fields, the quintessence field q and the matter fields (X, Y_i). The superpotential W and V_D vanish when evaluated along the D-flat direction which is assumed to be along $X \neq 0, Y_i = 0$. The gauge symmetry is broken by $X \neq 0$. If one of $\frac{\partial W}{\partial Yi} = W_Y \neq 0$ when evaluated along the flat direction of the D term, one will get

$$\begin{aligned} G_{Y^\dagger} = \kappa K_{Y^\dagger} + \frac{W_{Y^\dagger}}{W}, \quad G_Y = \kappa K_Y + \frac{W_Y}{W}, \\ (G^{-1})_{Y^\dagger Y} = (\kappa K_{Y^\dagger Y})^{-1}. \end{aligned} \tag{3.36}$$

So the scalar potential can be written in the simple form

$$\begin{aligned} V &= e^{\kappa K}\kappa|W|^2\left[(\kappa K_Y + \frac{W_Y}{W})(\kappa K_{Y^\dagger Y})^{-1}\left(\kappa K_{Y^\dagger} + \frac{W_{Y^\dagger}}{W}\right) - 3\right] \\ &= e^{\kappa K}(K_{Y^\dagger Y})^{-1}|W_Y|^2. \end{aligned} \tag{3.37}$$

An example of high energy physics model which gives rise to this model is the string-inspired model with an anomalous $U(1)_X$ gauge symmetry [28, 29], when the gauge symmetry of this model factorises as $G \times U(1)_X$ where G contains the Standard Model gauge group and $U(1)_X$ is an anomalous abelian symmetry. The fields of this model are split into three groups; X, Y and Y_i. The D term potential of this model is [27]

$$V_D = \frac{g_X^2}{2}\left(K_X X - 2K_Y Y + \sum_i g_i K_{Y_i} Y_i - \zeta^2\right)^2, \tag{3.38}$$

where g_X is a $U(1)_X$ gauge coupling and ζ is the Fayet-Iliopoulos term. The Kähler potential of the effective supergravity is

$$K = XX^* + \frac{(qq^*)^p}{m_c^{2p-2}} + |Y|^2\frac{(qq^*)^n}{m_c^{2n}}, \tag{3.39}$$

where m_c is an UV cutoff. In this model, the potential V_D vanishes along the D-flat direction in which $X = \zeta$, $Y = Y_i = 0$. The $U(1)_X$ gauge symmetry is broken by a non zero value of X along the D-flat direction. The superpotential W can be written in the form of the Yukawa coupling as $W = \lambda X^2 Y + \ldots$. So $W = 0$ and $W_Y \neq 0$ when evaluated along the D-flat direction. From Eq. (3.39), we get

$$K_{YY^*} = \frac{(qq^*)^n}{m_c^{2n}}, \tag{3.40}$$

$$K_{qq^*}|_D = p^2\frac{(qq^*)^{p-1}}{m_c^{2p-2}}, \tag{3.41}$$

where $|_D$ means evaluated along the D flat direction. Eq. (3.41) shows that for $q = q^*$, the kinetic term of q will be canonical if we let

$$q \longrightarrow \frac{q}{\sqrt{2}} \quad \text{and} \quad p = 1. \tag{3.42}$$

From Eqs. (3.37) and (3.41), we get

$$\begin{aligned} V &= e^{\kappa q^2/2}\frac{m_c^{2n}2^{2n}}{q^{2n}}\lambda^2\zeta^4 e^{\kappa\zeta^2} \\ &= \frac{\Lambda^{4+\alpha}}{q^\alpha}e^{\kappa q^2/2}, \end{aligned} \tag{3.43}$$

where $\Lambda^{4+\alpha} = m_c^\alpha 2^\alpha \lambda^2\zeta^4$, $\alpha = 2n$ and $e^{\kappa\zeta^2} \sim 1$. The parameter λ is required to be approximately unity to avoid any fine-tuning of its value. The value of ζ should be larger than 10^2 GeV because $U(1)_X$ gauge symmetry should be broken above the weak interaction scale. Since the universe energy is dominated by the energy of the slowly rolling quintessence ($\rho_{oc} \simeq V(q_o \approx m_p)$), so for $\alpha > 11$ it follows that $\Lambda > 10^{15}$ GeV and $m_c > 10^{14}$ GeV, and the string scale (m_s), which is calculated from the relation $m_p^2 m_c^6 \approx m_s^8$ [27], will have value in the appropriate range (larger than 10^{15} GeV). The fine tuning problem can thus be avoided if $\alpha > 11$.

Let us consider the behaviour of quintessence field when its potential is given by Eq. (3.43). The field's value is small compared with m_p at a small red shift, so the exponential factor of the potential can be expanded in terms of polynomials with infinite degree, and the potential will become a positive power law potential with arbitrary degree. The tracking behaviour due to negative-power law potential is thus lost in this beginning stage. The tracking behaviour occurs at the very end of the field's evolution when $q \approx m_p$. The existence of tracking behaviour has been checked numerically by Brax and Martin [27]. Since the exponential factor of the potential increases when the value of the field increases, so the value of V/ρ_q increases. This pushes ω_q towards the value -1 as discussed in Section 2.1. When the field is on track ($q \approx m_p$), the value of ω_q mainly depends on the exponential factor of the potential. So ω_q does not depend on α as shown in Fig. 3.1. This is a very desirable property of this potential.

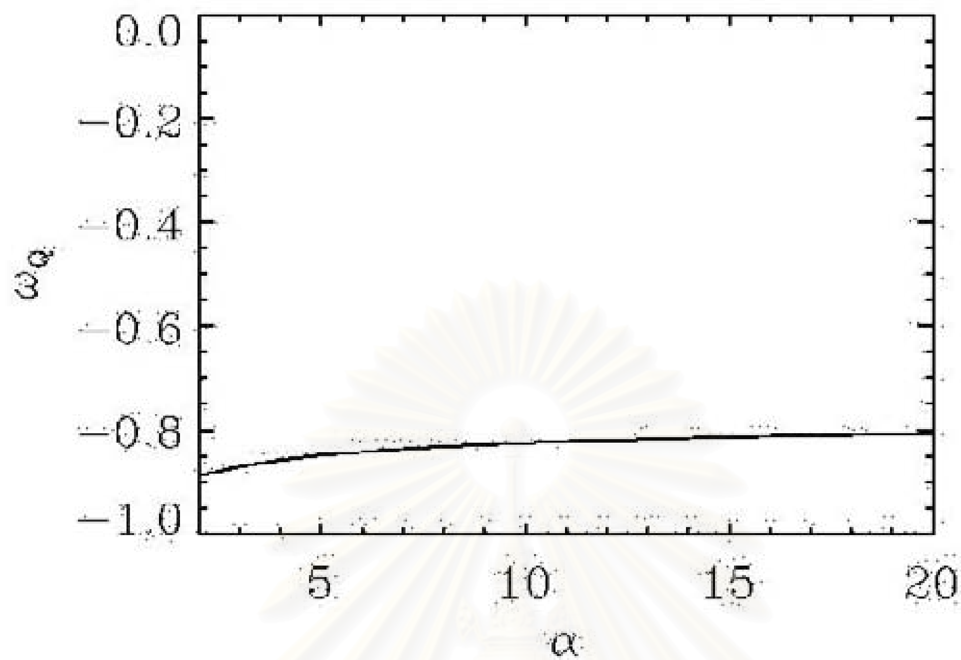

Figure 3.1: $\omega - \alpha$ relation for the potential in Eq. (3.43)

3.3 Quintessence with non-canonical kinetic term

In Section 2.4, one sees that quintessence with exponential potential cannot satisfy the constraints (1.58) from observation. But if its kinetic term is non-canonical, its behaviour will change and the conditions (1.58) can be satistied. Quintessence with non-canonical kinetic term can arise from supergravity theory [30].

Consider the Lagrangian of a scalar field with non-canonical kinetic term

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 k^2(\phi) - e^{-\phi}, \tag{3.44}$$

where $m_p = (8\pi G)^{-\frac{1}{2}}$ has been set to unity. By the change of variables

$$\chi = K(\phi), \quad k(\phi) = \frac{\partial K}{\partial \phi}, \tag{3.45}$$

the Lagrangian can be put in the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - e^{-K^{-1}(\chi)}, \tag{3.46}$$

where $K^{-1}(\chi) = \phi$. If $k(\phi)$ is constant $(= k)$, the above Lagrangian will become

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - e^{-\chi/k}. \tag{3.47}$$

The evolution of this field was discussed in Section 2.4.

It can be seen that the relation between the energy density of ϕ and χ is

$$\begin{aligned} \rho_\phi &= \frac{1}{2}(\partial_0 \phi)^2 k^2 + e^{-\phi} \\ &= \frac{1}{2}(\partial_0 \chi)^2 + e^{-\chi/k} = \rho_\chi. \end{aligned} \tag{3.48}$$

Then, from Section 2.4, for $k^2 < \dfrac{1}{m}$ (with $m = 3, 4$ for matter and radiation respectively), the field energy density decreases as

$$\rho_\phi \propto \rho_\chi \propto a^{-m}, \tag{3.49}$$

and

$$\Omega_\phi = \Omega_\chi = mk^2. \tag{3.50}$$

For $k^2 > \dfrac{1}{m}$, the universe is in the field dominated epoch and

$$\rho_\phi \propto \rho_\chi \propto a^{-\frac{1}{k^2}}; \quad \omega_\phi = \frac{1}{3k^2} - 1. \tag{3.51}$$

Since the universe is dominated by the background energy at early time, so $k^2 < \dfrac{1}{m}$ and $\omega_\phi = 0$ for constant k, because the ratio $\dfrac{\rho_\phi}{\rho_v}$ is constant. Eq. (3.51) shows that ω_ϕ can be close to -1 for $k^2 \gg \dfrac{1}{m}$ in the field dominated case. So k should be smaller than $\frac{1}{m}$ at early time and larger than $\frac{1}{m}$ at present time. An example for k with this property is given by [31]

$$k(\phi) = k_{min} + b(\tanh(\phi - \phi_1)\tanh(\phi - \phi_2) + 1), \tag{3.52}$$

where $k_{min} = 0.15$, $b = 0.25$, $\phi_1 = 40.0$ and $\phi_2 = 249.8$.

The field and Friedmann equations are solved numerically and the cosmological evolution is shown in Fig. 3.2. As ϕ increases, the coefficient $k(\phi)$, as given by Eq. (3.52), remains almost constant at first ($0 < \phi < 50$) and changes abruptly from the large initial value of 0.65 to the intermediate value of 0.15 and remains there ($50 < \phi < 250$) before abruptly changing back to the large value as shown in Fig. 3.3.

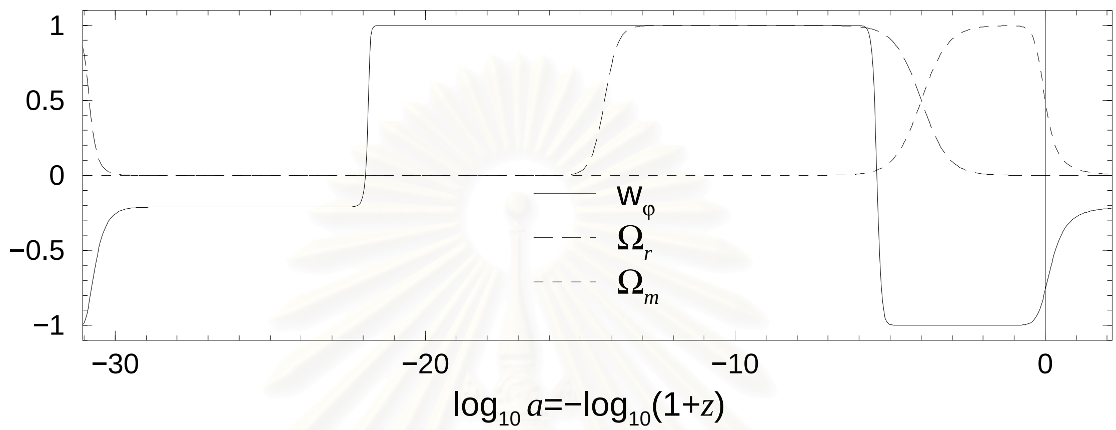

Figure 3.2: Cosmological evolution with quintessence field with k given by Eq. (3.52)

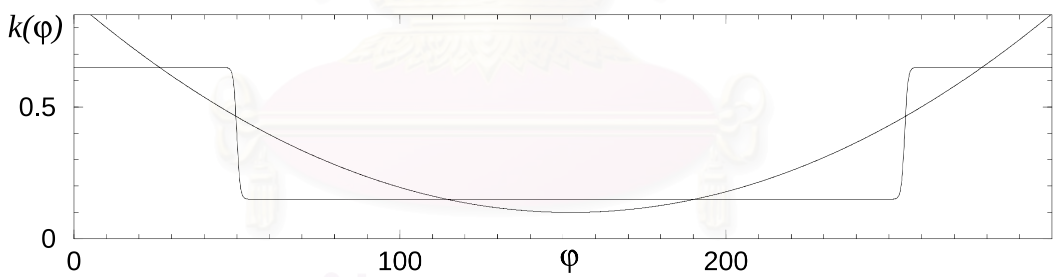

Figure 3.3: Two different $k\phi$s which give the same qualitative evolution in Fig. 3.2.

From Figs. 3.2 and 3.3, one sees that during the initial period with large k, the universe is field dominated and inflates because $k^2 > \dfrac{1}{m}$ and $\omega_\phi \to -1$ in this period. When k drops, the field accelerates because when k decreases the ratio of $\dfrac{\dot{\phi}^2}{2\phi} = n = k^{-2}$ increases ($\rho_\phi \propto a^{-k^{-2}} \propto a^{-n}$), so the universe enters the kinetic energy dominated epoch. As the universe evolves further, since the kinetic energy density decays faster than the radiation density, the universe becomes radiation dominated. Since ρ_ϕ is small in this epoch, the evolution of the universe follows the standard cosmological model and the radiation dominated epoch is followed by the matter dominated epoch. When k changes to a large value approximately at the present time, the ratio $\frac{V}{\rho_\phi}$ increases and ω_ϕ gets close to -1, so the universe becomes field dominated because ρ_ϕ decreases slower than ρ_b at this time. The initial conditions for cosmological evolution in Fig. 3.2 are $\rho_{int} = 1$, $\phi = 2$ and $\dot{\phi} = 0$. The evolution gives $\Omega_{o\phi} \approx 0.5$ and $\omega_{o\phi} \approx -0.96$. It is interesting to observe that the same cosmological evolution as in Fig. 3.2 can be obtained without any abrupt change of k. For example,

$$k = k_{min} + \left(\frac{\phi - \phi_1}{\phi_2}\right)^2, \tag{3.53}$$

where $k_{min} = 0.1$, $\phi_1 = 42.5$ and $\phi_2 = 160.0$.

Let us consider the cosmic evolution for smooth function k and smooth evolution of ϕ. If the validity of Eq. (3.50) with $k = k(\phi)$ is assumed, one gets

$$\begin{aligned} d\sqrt{\Omega_\phi} &= \sqrt{m}\, dk, \\ \frac{d\sqrt{\Omega_\phi}}{d\ln a} &= \sqrt{m}\,\frac{dk}{d\phi}\frac{d\phi}{d\ln a}. \end{aligned} \tag{3.54}$$

From

$$3H^2 = \rho_c, \tag{3.55}$$

one gets

$$\frac{\dot{\phi}^2}{H^2} = \frac{3\dot{\phi}^2}{\rho_c}. \tag{3.56}$$

Since $\frac{1}{H}\frac{\partial}{\partial t} = \frac{\partial}{\partial \ln a}$, then

$$\frac{d\phi}{d\ln a} = \sqrt{\frac{6}{k^2}\left[\frac{k^2\dot{\phi}^2}{2\rho_c}\right]}. \tag{3.57}$$

By using the relation $\dot{\phi}^2 = (1+\omega_\phi)\rho_\phi$, one gets

$$\frac{d\phi}{d\ln a} = \sqrt{\frac{3}{k^2}(1+\omega_\phi)\Omega_\phi}. \tag{3.58}$$

Substituting the above equation into Eq. (3.54), one gets

$$\frac{d\sqrt{\Omega_\phi}}{d\ln a} = \sqrt{m}\frac{dk}{d\phi}\sqrt{3(1+\omega_\phi)\frac{\Omega_\phi}{k^2}}. \tag{3.59}$$

In the case of matter domination ($\omega_\phi = 0$), one gets

$$\frac{dk}{d\phi} = \frac{1}{3\sqrt{3}}\frac{d\sqrt{\Omega_\phi}}{d\ln a}. \tag{3.60}$$

To preserve the standard nucleosynthesis and structure formation processes, Ω_ϕ should be less than 0.2 during these periods. Since the univerese is field dominated today, then $\Omega_{o\phi} > 0.5$. The value of $\log_{10}(a)$ changes approximately 3 when the universe evolves from the structure formation epoch to the present. So one gets

$$\frac{dk}{d\phi} = \frac{1}{3\sqrt{3}}\frac{d\sqrt{\Omega_\phi}}{d\ln a} \geq 0.007. \tag{3.61}$$

If the field in this case can be approximated by Eq. (2.73) (Because Eq. (3.50) is assumed to be valid), one gets

$$\Delta\phi \sim 6\Delta\ln a \sim 6 \times 3 \times \ln 10 \approx 41.4\ . \tag{3.62}$$

By using the allowed values of ϕ and k in the structure formation period which are 250 and 0.26 respectively, the present value of k as given by the lower limit of the relation (3.61) is too small ($k \approx 41.4 \times 0.007 + 0.26 \approx 0.5 < 1/\sqrt{2}$) to accelerate the universe, since k does not grow fast enough when ϕ increases.

Next consider the exponential form of k,

$$k = e^{\frac{\phi-\phi_1}{a}}, \tag{3.63}$$

which allows strong growth of k during cosmic evolution. From Eq. (3.45), one gets

$$K = \alpha e^{\frac{\phi-\phi_1}{\alpha}}. \tag{3.64}$$

The canonical Lagrangian has the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - \alpha^\alpha e^{\phi_1} \chi^{-\alpha}. \tag{3.65}$$

This is a well-known Lagrangian for negative power law quintessence. When α is large, the growth rate of k with ϕ is slow and the approximation $\Delta\phi \sim \Delta \ln a$ can be used. One gets

$$k_o - k_i = \Delta k \approx e^{\frac{\phi_i-\phi_1}{\alpha}} \left[e^{\frac{\Delta\phi}{\alpha}} - 1 \right] \approx k(\phi_i)[(\frac{a_o}{a_i})^\alpha - 1] \approx k(\phi)(\frac{a_o}{a_i})^\alpha, \tag{3.66}$$

where k_o and k_i are the values of k evaluated, respectively, in the present and in the past. If $\Delta k \approx 2$ for $\Delta \log_{10}(a) = 3$, then through the whole evolution of the universe (where $\Delta \log_{10}(a) \approx 30$), $\Delta k \approx 2^{10} \approx 1000$. The initial value of k is not too small and there is no fine tuning problem.

In the next chapter, one will see that a large value of α cannot accelerate the universe today. The universe can expand with acceleration if α is small ($\alpha < 6$). By using the allowed value of $k = 0.26$ and $\phi = 250$ during the structure formation period and setting $\alpha = 6$, Eq. (3.63) gives $\phi_1 = 258.1$ and from Eq. (3.66), one gets

$$k(0) = e^{-258.1/6} \approx 10^{-19}. \tag{3.67}$$

So k which is given by Eq. (3.63) has the fine tuning problem because its value at the early time (when $\phi \approx 0$) is too small compared with m_p (≈ 1) which is the particle physics mass scale at that time.

Chapter 4

Comparison of quintessence models

The quintessence models discussed in Chapter 3 will be compared in Section 4.1 by studying the existence of their scaling solutions and tracking behaviour. In Section 4.2, the values of $\omega_{o\phi}$ for each models will be compared. The modifications and some particle physics problems of these quintessence models are studied in Section 4.3.

4.1 The existence of scaling solution and tracking behaviour of quintessence

The existence of the scaling solution can be studied by using the relation between V, Ω and (constant) ω_ϕ [32], so this relation will first be derived.

From the energy conservation of quintessence field, if ω_ϕ is constant and the field behaves as a perfect fluid, the field energy density will decrease as

$$\rho_\phi \propto a^{-n}. \tag{4.1}$$

This relation implies that the field solution is a scaling solution as discussed in Chapters 2 and 3. If the universe contains quintessence and perfect fluid the Friedmann equation Eq. (1.31) becomes

$$\dot{a}^2 = a_o^2 H_o^2 \left(\Omega_{o\phi} \left(\frac{a_o}{a} \right)^{3\omega_\phi + 1} + \Omega_{om} \frac{a_o}{a}. \right). \tag{4.2}$$

From Eqs. (2.11) and (4.1), one obtains

$$\frac{V_o}{V} = \frac{\rho_{o\phi}}{\rho_\phi}; \quad \frac{V_o}{V} = \left(\frac{a}{a_o} \right)^{3(\omega_\phi + 1)}. \tag{4.3}$$

Another expression of $\dot{a}$ is required to eliminate $\dot{a}$ from Eq. (4.2), it is obtained by finding the derivative with respect to t of the above equation and is given by

$$\frac{\dot{a}}{a_o} = \dot{\phi} \frac{-1}{3(\omega_\phi + 1)} \left(\frac{V}{V_o} \right)^{-\frac{1}{3(\omega_\phi+1)}} \frac{V'}{V}, \tag{4.4}$$

where "$\dot{}$" $= \frac{\partial}{\partial t}$ and "$\,'\,$" $= \frac{\partial}{\partial \phi}$. At present $t = t_o$ and $\phi = \phi_o$, and one gets

$$\dot{a}_o = a_o \dot{\phi}_o \left(-\frac{1}{3(\omega_\phi + 1)} \right) \frac{V_o'}{V_o}. \tag{4.5}$$

By dividing Eq. (4.4) by Eq. (4.5), one obtains

$$\frac{\dot{a}}{\dot{a}_o} = \frac{\dot{\phi}}{\dot{\phi}_o} \left(\frac{V}{V_o} \right)^{-\frac{1}{3(\omega_\phi + 1)} - 1} \frac{V'}{V_o'}. \tag{4.6}$$

The variable $\dot{\phi}$ in Eq. (4.6) can be eliminated by using Eq. (2.10) which can be written as

$$\frac{\dot{\phi}}{\dot{\phi}_o} = \left(\frac{a_o}{a} \right)^{\frac{1}{6(\omega_\phi + 1)}} = \left(\frac{V}{V_o} \right)^{\frac{1}{2}}. \tag{4.7}$$

Substituting Eq. (4.6) into Eq. (4.7), one gets

$$\frac{\dot{a}}{\dot{a}_o} = \left(\frac{V}{V_o} \right)^{-\frac{3\omega_\phi + 5}{6(\omega_\phi + 1)}} \frac{V'}{V_o'}, \tag{4.8}$$

and at present $a = a_o$ and $\phi = \phi_o$, and Eq. (4.2) becomes

$$\dot{a}_o{}^2 = a_o^2 H_o^2 \left(\Omega_{o\phi} \left(\frac{a_o}{a} \right)^{3\omega_\phi + 1} + \Omega_{om} \frac{a_o}{a} \right) = a_o^2 H_o^2. \tag{4.9}$$

Using Eqs. (4.3) and (4.8), the Friedmann equation (4.2) can be written as

$$\begin{aligned} \frac{V'}{V_o'} &= \left(\frac{V}{V_o} \right)^{\frac{3\omega_\phi + 5}{6(\omega_\phi + 1)}} \left(\Omega_{om} \left(\frac{V}{V_o} \right)^{\frac{1}{3(\omega_\phi + 1)}} + \Omega_{o\phi} \left(\frac{V}{V_o} \right)^{\frac{3\omega_\phi + 1}{3(\omega_\phi + 1)}} \right)^{\frac{1}{2}} \\ &= \left(\Omega_{om} \left(\frac{V}{V_o} \right)^{\frac{\omega_\phi + 2}{\omega_\phi + 1}} + \Omega_{o\phi} \left(\frac{V}{V_o} \right)^2 \right)^{\frac{1}{2}}. \end{aligned} \tag{4.10}$$

Next, we study the existence of scaling solutions and tracking behaviour for the negative power law, exponential, and the negative power times exponential potentials. Consider first the negative power law potential ($V = \dfrac{\Lambda^{\alpha+4}}{\phi^\alpha}$). Using Eq. (4.10), one gets

$$\left(\frac{\phi_o}{\phi} \right)^{2\alpha + 1} = \Omega_{om} \left(\frac{\phi_o}{\phi} \right)^{\alpha \frac{\omega_\phi + 2}{\omega + 1}} + \Omega_{o\phi} \left(\frac{\phi_o}{\phi} \right)^{2\alpha}. \tag{4.11}$$

It can be seen that the above equation cannot be satisfied by any value of α and ω_ϕ. But if the matter energy dominates, the above equation gives $\omega_\phi = -\frac{2}{\alpha+2}$, which agrees with the result in Section 2.3. When the field energy dominates, no value of ω_ϕ and α can satisfy Eq. (4.11). This implies that ω_ϕ is not constant because the scaling solution does not exist in this case. But from Section 2.5, one sees that tracking behaviour still exists although the field energy dominates.

In the case of the exponential potential ($V = e^{-k\phi}$), by using Eq. (4.10) one gets

$$e^{-2k(\phi-\phi_o)} = \Omega_{om} e^{-k\frac{\omega_\phi+2}{\omega_\phi+1}(\phi-\phi_o)} + \Omega_{o\phi} e^{-2k(\phi-\phi_o)}. \tag{4.12}$$

For matter energy dominated case, Eq. (4.12) gives $\omega_\phi = 0$. For field energy dominated case, Eq. (4.12) can still be satisfied, but the value of ω_ϕ cannot be determined. Moreover, one can see that Eq. (4.12) can be satisfied for all values of Ω_{om} and $\Omega_{o\phi}$ for which $\Omega_{om} + \Omega_{o\phi} = 1$, but only the case of $\Omega_{om} \gg \Omega_{o\phi}$ and $\Omega_{o\phi} = 1$ gives stable field solutions as shown in Section 2.4. If the kinetic term of the quintessence field is non-canonical, its potential can be written as $V \sim e^{-K(\phi)}$, and the left-hand side of Eq. (4.12) becomes $\frac{K'(\phi)}{K'(\phi_o)} e^{-K(\phi)+K(\phi_o)}$. Because K is not a linear function of ϕ, the scaling solution does not exist in this case. The tracker solution are studied by using Eqs. (2.111) and (2.112), from which one gets

$$\Gamma = 1 - \frac{K''}{K'^2}, \tag{4.13}$$

$$\frac{\Gamma'}{\Gamma(V'/V)} = d\Gamma = \frac{K'''K' - 2K''^2}{K'^2(K'^2 - K'')}. \tag{4.14}$$

If $K = \kappa\phi^n$, $\kappa^2 = G^n$, Eq. (4.13) gives $\Gamma < 1$ and $\omega_\phi > \omega_m$. This value of ω_ϕ cannot satisfy (or does not agree with) the condition from observation. If $K = -\kappa\phi^n$, Eqs. (4.13) and (4.14) give

$$\Gamma = 1 + \frac{(n-1)}{n|K|}; \quad d\Gamma = \frac{(n-1)(n-2) - 2(n-1)^2}{(n^2 + n(n-1))K} = -\frac{n+7}{(2n-1)K}. \tag{4.15}$$

The above equation gives $\Gamma > 1$ and $|d\Gamma| \ll 1$ for large value of n or ϕ. So this potential can give $\omega_\phi < \omega_m$ as desired and quintessence with this potential has tracking behaviour for large n or ϕ. If n is negative, Eq. (4.15) shows that the condition for tracking behaviour can be satisfied if the value of ϕ is small. It can be seen that for $n = 2$ the form of this potential is similar to the exponential factor of the potential in Section 3.3. Eq. (4.15) shows that for $n = 2$ the condition for tracking behaviour can be satisfied if the value of ϕ is large. Note that the condition for tracker behaviour can be satisfied for all values of ϕ if n large. If $K = m_p\phi_1^{n-1}(\phi - \phi_2)^{\frac{1}{n}}$, Eqs. (4.13) and (4.14) will become

$$\Gamma = 1 - \frac{(1-n)}{K}; \quad d\Gamma = \frac{(1-n)(1-2n) - 2(1-n)^2}{(1-(1-n))K} = -\frac{1+7n}{nK}. \tag{4.16}$$

It is easy to see that the condition $|d\Gamma| \ll 1$ for tracking behaviour can be satisfied if ϕ is large. One can see that the kinetic coefficient $k(\phi)$ which is given by Eq. (3.52) can lead to this type of potential if $n = 3$ and $k_{min} = 0$. This means that quintessence with this kinetic coefficient has tracking behaviour at large value of ϕ. For $K = \pm\alpha\ln\phi$, it can be seen that $\Gamma = 1 \pm \alpha^{-1}$ similar to the power law potential case.

Now we come to the third potential of the form $V = \dfrac{\Lambda^{\alpha+4}}{\phi^\alpha}e^{k\phi^2/2}$. By using Eq. (4.10), one gets

$$\begin{aligned}\left(\frac{k\phi^2-\alpha}{k\phi_o^2}\right)^2\left(\frac{\phi_o}{\phi}\right)^{2\alpha+2} e^{k\phi^2-k\phi_o^2} &= \Omega_{om}\left(\frac{\phi_o}{\phi}\right)^{\alpha\frac{\omega_\phi+2}{\omega_\phi+1}} e^{k\frac{\omega_\phi+2}{2(\omega_\phi+1)}(\phi^2-\phi_o^2)} \\ &\quad +\Omega_{o\phi}\left(\frac{\phi_o}{\phi}\right)^{2\alpha} e^{k(\phi^2-\phi_o^2)}.\end{aligned} \tag{4.17}$$

It is easy to see that scaling solution dose not exist in this type of potential. But from Eqs. (2.111) and (2.112), one gets

$$\Gamma = 1 + \frac{k\phi^2+\alpha}{(k\phi^2-\alpha)^2} > 1, \tag{4.18}$$

$$d\Gamma = -\frac{2k\phi(k\phi^2+3\alpha)}{(k\phi^2-\alpha)^2[k\phi^2+\alpha+(k\phi^2-\alpha)^2]} < 1 \tag{4.19}$$

if ϕ is large. So the field solution has tracking behaviour and $\omega_\phi < \omega_m$ for large value of ϕ. The above examples show that tracking behaviour may exist although there are no scaling solution for the field equation.

4.2 The value of $\omega_{o\phi}$

The values of $\omega_{o\phi}$ from the potentials discussed in the previous section are considered in this section. We begin with the potential $V = \dfrac{\Lambda^{\alpha+4}}{\phi^\alpha}$. In this case, the field energy starts to dominate the universe energy near the present time. Then the scaling solution may be used approximately and we may assume that

$$\omega_\phi = -\frac{2}{\alpha+2}. \tag{4.20}$$

The lower limit of $\omega_{o\phi}$ is -0.6 and occurs when $\alpha = 1$. The value of $\omega_{o\phi}$ increases as α increases. The lower limit of $\omega_{o\phi}$ is larger than observational value ($\omega_{o\phi} < -0.7$). So this approximation is not valid. When the field energy dominates and the slow rolling condition is applied, one obtains from Eq. (2.67)

$$\omega_{o\phi} \approx \frac{2}{3}\frac{\alpha}{\alpha+4}\frac{1}{H_o t_o} - 1. \tag{4.21}$$

One can see from Eq. (1.39) that it is not easy to estimate $H_o t_o$, because the $\Omega_{o\Lambda}\left(\dfrac{a}{a_o}\right)^2$ term in this equation becomes $\Omega_{o\phi}\left(\dfrac{a_o}{a}\right)^{3\omega_\phi+1}$ in this case. If the approximate values of t_o ($\approx$ 15 Gyr) and $50 < H_o < 100$ (km/sec/Mpc) are used, the value of $H_o t_o$ becomes

$$0.77 = \frac{15\times 10^9}{9.8\times 10^9}\times 0.50 < H_o t_o < \frac{15\times 10^9}{9.8\times 10^9}\times 1.00 = 1.53. \tag{4.22}$$

For $\alpha = 5$, Eq. (4.21) becomes

$$-0.52 > \omega_{o\phi} > -0.76\,. \tag{4.23}$$

This value is near the value given in Ref. [6] ($\omega_{o\phi} \approx -0.56$) if H_o is near 0.5. For other values of α, the value of $\omega_{o\phi}$ as given by Eq. (4.21) are not close to the value given in Ref. [6] and referred to by Ref. [27]. Note that the values of $\omega_{o\phi}$ from [6] and [27] are quite different. The relation between $\omega_{u\phi}$ and Ω_{om} as given by Ref. [6] are shown in Fig. 4.1.

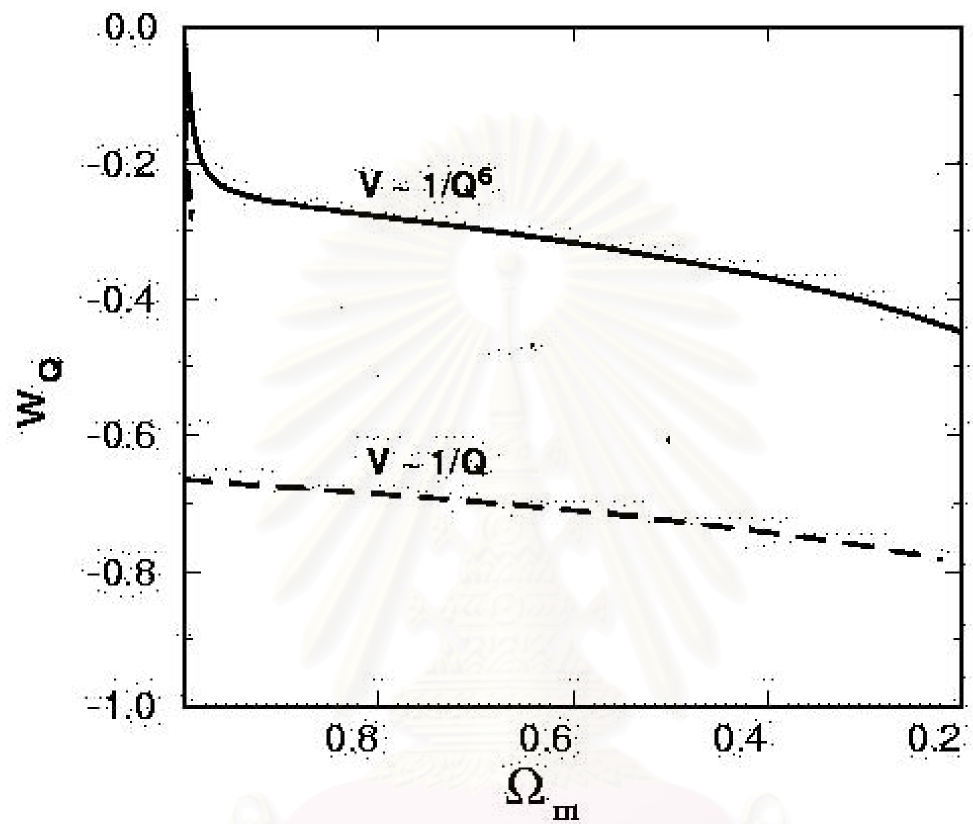

Figure 4.1: $\Omega_{om} - \omega_{o\phi}$ relation for various α.

This figure shows that the universe will expand with acceleration if $\alpha < 6$. From Eqs. (2.27) and (2.29), the density parameter of quintessence with negative power law potential increases as

$$\Omega_\phi \propto \frac{\rho_\phi}{\rho_b} \propto t^{-\frac{2n}{m}+2} \propto t^{2\frac{m-n}{m}}. \tag{4.24}$$

By using Eq. (2.39), we get

$$\Omega_\phi \propto t^{\frac{4}{\alpha+2}}. \tag{4.25}$$

If this equation is assumed to be valid through the universe evolution, we get

$$\begin{aligned}\Omega_{o\phi} &= \Omega_{i\phi}\left(\frac{t_o}{t_i}\right)^{\frac{4}{\alpha+2}} \\ &= 0.2\times\left(\frac{15\times 10^9}{10^8}\right)^{\frac{4}{\alpha+2}} \\ &= 0.2\times(1.5\times 10^2)^{\frac{4}{\alpha+2}}, \qquad (4.26)\end{aligned}$$

where $\Omega_{i\phi}$ (≈ 0.2) is the density parameter of quintessence during the time of structure formation ($t \approx t_i \approx 10^8$ year). This equation shows that quintessence energy can dominate the universe energy ($\Omega_\phi > 0.5$) at present (not before or after) if $\alpha > 19$. So although quintessence with this potential can dominate the universe energy today but it cannot accelerate the expansion of the universe.

Next we consider the exponential potential ($V = V_c e^{-k\phi}$). Since field energy subdominates in the radiation dominated epoch and the ratio $\frac{\rho_\phi}{\rho_b}$ is constant, then the field energy never dominates and $\omega_{o\phi} = \omega_m = 0$. But if the kinetic term of this quintessence field has non-canonical form, the value of $\omega_{o\phi}$ can be lower than -0.7 which agrees with the observational data as shown in Section 3.3. Although the value of $\Omega_{o\phi}$ which is given in Section 3.3 is lower than the observational value (≈ 0.7), but by an appropriate change in the form of $k(\phi)$ the value of $\Omega_{o\phi}$ can be made higher. The model of quintessence field with exponential potential can be improved in many ways to give better values of $\omega_{o\phi}$ and $\Omega_{o\phi}$. This will be shown in the next section.

In the case of the potential $V = \frac{K}{\phi^\alpha}e^{k\phi^2/2}$, a good value of $\omega_{o\phi}$ (≈ -0.8) is obtained and unlike the negative power potential, the $\omega_{o\phi}$ from this potential does not depend on α. The relation between Ω_{om} and $\omega_{o\phi}$ is shown in Fig. 4.2.

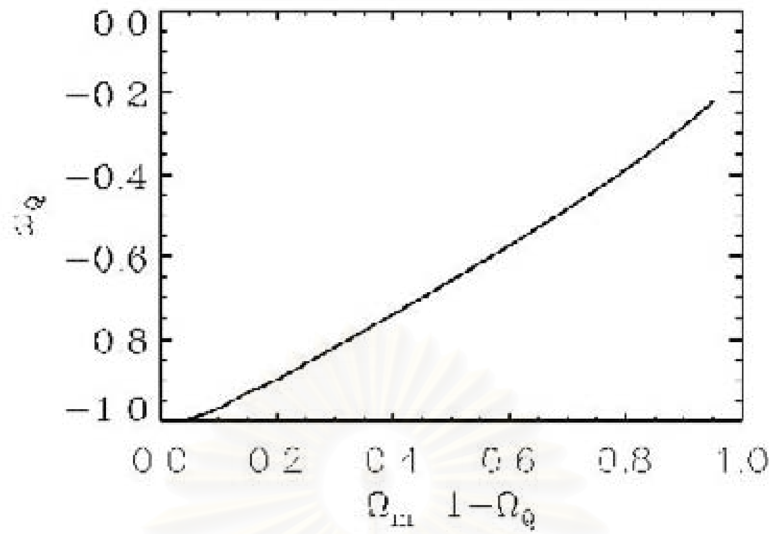

Figure 4.2: $\Omega_{om} - \omega_{o\phi}$ relation for potential $V = \frac{K}{\phi^{\alpha}} e^{k\phi^2/2}$.

4.3 The modifications and problems of some quintessence models

The modifications of quintessence models from Chapters 2 and 3 are discussed in this section.

We start with the exponential potential, $V = V_c e^{-k\phi}$. Although quintessence with this potential cannot accommodate the present values of $\omega_{o\phi}$ and $\Omega_{o\phi}$, there are many ways to improve this model. For example, if quintessence couples to the invisible matter as

$$\mathcal{L} = \frac{\dot{\phi}^2}{2} - V_c e^{-k\phi} + \mathcal{L}_v + f(\phi)\mathcal{L}_i, \tag{4.27}$$

where $\mathcal{L}_v$ is the Lagrangian of the visible matter (photon, baryons) and $\mathcal{L}_i$ is the Lagrangian of the invisible matter (non-baryonic dark matter). In the radiation dominated epoch the invisible matter subdominates, so quintessence has a scaling solution and the ratio $\frac{\rho_{o\phi}}{\rho_{or}}$ is constant. In the matter dominated epoch the invibible matter dominates, and due to the presence of $f(\phi)$ the quintessence solution can deviate from the scaling solution. By a suitable choice of the form of $f(\phi)$, the value of $\omega_{o\phi}$ can be lower than -0.7 [33].

As the next example, consider quintessence with power times exponential potential $V = V_c(\phi)e^{-k\phi}$, where $V_c(\phi)$ has the general form $(\phi + B)^\alpha + A$ [34]. From this potential, one gets

$$\Gamma = \frac{(\alpha(\alpha-1)B\phi^{\alpha-2} - 2k\alpha B\phi^{\alpha-1} + k^2V_c)V_c}{(\alpha B\phi^{\alpha-1} - kV_c)^2}. \tag{4.28}$$

Since $\Gamma > 1$ and $\Gamma \sim$ constant for $\phi > 1$, then quintessence with this potential has tracking behaviour and its energy can dominate the universe energy at present.

In the case of the potential $V = \dfrac{\Lambda^{\alpha+4}}{\phi^\alpha}$, one can see from Section 3.2 that the negative power law potential of quintessence can arise from supersymmetric QCD. In this model, the field value at the present time is close to the value of Planck mass ($q_o \sim m_p$) as can be seen from Eq. (3.28). For this reason, some authors [27, 35] suggest that the quintessence models should be based on supergravity. So the supergravity correction of the model in Section 3.2 should be considered. Eq. (3.35) shows that the scalar potential of supergravity theory which arises from the flat Kähler potential and the superpotential $W = \dfrac{\Lambda^{\alpha+3}}{\phi^\alpha}$ can have negative values when quintessence field takes its present value. This problem can be solved by using an appropriate form of the Kähler potential. For example, if the Kähler potential is [35]

$$K = \frac{1}{\kappa}[\ln(\sqrt{\kappa}\bar{q} + \sqrt{\kappa}\bar{q}^*)]^2, \tag{4.29}$$

where $\kappa = 8\pi G$. The canonical kinetic term of the field q can be obtained by using the relation

$$\begin{aligned} K_{\bar{q}^*\bar{q}}(\partial_\mu\bar{q}^*\partial^\mu\bar{q}) &= \frac{1}{2}(\partial_\mu q^*\partial^\mu q), \\ \sqrt{2K_{\bar{q}^*\bar{q}}}\partial^\mu\bar{q} &= \partial^\mu q. \end{aligned} \tag{4.30}$$

Since

$$K_{\bar{q}^*\bar{q}} = \frac{2[1 - \ln(\sqrt{\kappa}\bar{q} + \sqrt{\kappa}\bar{q}^*)]}{\kappa(\bar{q} + \bar{q}^*)^2}, \tag{4.31}$$

by setting $\bar{q} = \bar{q}^*$ one gets

$$q = -\frac{2}{3\sqrt{\kappa}}[1 - \ln(2\sqrt{\kappa}\bar{q})]^{\frac{3}{2}}. \tag{4.32}$$

The scalar potential for this model can be obtained from the above Kähler potential and the superpotential $W = \dfrac{\Lambda^{\alpha+3}}{\bar{q}^{\alpha}}$ through

$$\begin{aligned} G_{\bar{q}} &= \frac{\ln(2\sqrt{\kappa}\bar{q})}{\bar{q}} - \frac{\alpha}{\bar{q}}, \quad G_{\bar{q}^*} = \frac{\ln(2\sqrt{\kappa}\bar{q})}{\bar{q}} - \frac{\alpha}{\bar{q}}, \\ (G^{-1})_{\bar{q}\bar{q}^*} &= \frac{2\bar{q}^2}{1 - \ln(2\sqrt{\kappa}q)}. \end{aligned} \tag{4.33}$$

Using the dimensionless field χ as $\chi = \left(\frac{3}{2}\sqrt{\kappa}q\right)^{2/3} = 1 - \ln(2\sqrt{\kappa}\bar{q})$, the scalar potential for this model assumes the form

$$\begin{aligned} V &= e^{(1-\chi)^2}\kappa^2\frac{\Lambda^{2\alpha+6}}{\bar{q}^{2\alpha}}[(1-\chi-\alpha)(2\chi^{-1})(1-\chi-\alpha) - 3], \\ &= M^4[2\chi^2 + (4\alpha - 7) + 2(\alpha-1)^2]\exp[(1-\chi)^2 - 2\alpha(1-\chi)], \end{aligned} \tag{4.34}$$

where $M^4 = \Lambda^{6+2\alpha}\kappa^{2+2\alpha}2^{2\alpha}$. The evolution of the field q is not discussed here. The evolution of this field is discussed in Ref. [35].

Let us consider the breaking of supersymmetry at the mass scale $m_{su} \geq 10^{10}$ GeV [35]. Rewriting the scalar potential from Eq. (3.2) as

$$V = |F| - e^{\kappa K}3\kappa^2|W|^2, \tag{4.35}$$

it can be seen that the superpotential $W = \dfrac{\Lambda^{\alpha+3}}{\phi^{\alpha}}$ cannot be a source of supersymmetry breaking, because its value is very small compared with m_{su} at the present time. A suitably modified superpotential is given by

$$W = \frac{\Lambda^{3+\alpha}}{q^{\alpha}} + m_{3/2}\kappa^{-2}, \tag{4.36}$$

where $m_{3/2}$ is the gravitino mass. This superpotential gives a correction

$$\delta V \sim \frac{\Lambda^{3+\alpha} m_{3/2}}{\kappa^{\alpha}} + m_{3/2}^2 \kappa^{-2}, \tag{4.37}$$

for $q \sim m_p$, to the potential in Eq. (4.34). This correction gives a large value of V or ρ_{oq} when the field takes its present value ($q \sim m_p$). This does not agree with the observational data which requires a very small value of ρ_{oq}. This is a serious problem in all supergravity models of quintessence [35]. This problem can be avoided in many ways but a good way is building quintessence from string theory [35, 36].

Next consider the potential $V = \frac{K}{\phi^{\alpha}} e^{k\phi^2/2}$. Since this potential arises in supergravity theory, when supersymmetry is broken, a large correction

$$\delta V \sim \mathcal{O}(m_{3/2}^2 \kappa^{-2}). \tag{4.38}$$

will be given to the scalar potential and the same problem as in the previous discussion exists in this case.

Chapter 5

Conclusion

In this thesis, the basic ideas of quintessence are reviewed. Only the homogeneous part of scalar field quintessence is studied, because the energy density of the inhomogeneous part is small and has no effect on the expansion rate of the universe. Generally, quintessence is assumed to be a perfect fluid, so if $p_\phi/\rho_\phi = \omega_\phi$ is constant the energy density of quintessence will decrease as $\rho_\phi \propto a^{-3(\omega_\phi+1)}$. The behaviour of quintessence is characterized by its potential. The field solutions of the typical quintessence with negative power law and exponential potentials are derived. Both types of quintessence have scaling solutions and tracking behaviour. The scaling solution of quintessence implies that $\rho_\phi \propto a^{-3(\omega_\phi+1)}$. For the exponential potential the quintessence has two interesting stable solutions. The first solution occurs when the universe is in the background dominated epoch. The energy density of quintessence decreases with the same rate as the background energy density in this case, so the ratio between ρ_b and ρ_ϕ is constant. The second stable solution occurs when the quintessence energy dominates the universe energy. The energy density of quintessence decreases with slower rate than the background energy density, so $\omega_\phi < \omega_b$ in this case. From the standard cosmological model, $\rho_\phi < \rho_b$ in the radiation dominated epoch, so the energy density of quintessence decreases with the same rate as the background energy density and never dominate the universe energy. The universe cannot expand with acceleration in this case. Quintessence with simple exponential potential cannot satisfy the conditions, Eq. (1.58), from observations. For the negative power law potential, the stable solution of quintessence occurs in the radiation dominated epoch. The energy density of quintessence decreases slower than the background energy density for this solution, and the quintessence energy can dominate the universe energy at the present time. The time of quintessence domination can be estimated from the parameters of the potential. Although the final result of quintessence evolution does not change for a wide range of the initial conditions of ρ_ϕ, but if the initial value of Ω_ϕ ($\Omega_{i\phi} \leq 0.2$) is considered, the quintessence energy will dominate the universe energy too early unless $\alpha > 18$. For $\alpha > 6$ one has $\omega_\phi > -1/3$, so although this quintessence can dominate the universe energy but it cannot lead to expansion with acceleration of the universe.

The negative power law potential quintessence can arise from supersymmetric QCD. If the initial value of Ω_ϕ is not considered and quintessence is assumed to dominate at present time, the appropriate value of the parameter Λ of the potential can be estimated by choosing the values of N_c and N_f. Although the negative power law potential cannot give $\omega_\phi < -0.7$, but if the interactions between the quintessence and the Standard Model particles are taken into account, the value of ω_ϕ can be less than -0.7. Generally the coupling constants of the quintessence to the matter fields is very small, so the fine tuning of their values is required. This problem can be avoided by introducing some symmetry (for example, see [37]). Since $\phi_o \approx m_p$, then the supergravity correction of this model should be considered. There are some problems in the supergravity version of this model. If supersymmetry is assumed to be explicitly broken at a mass scale m_s, the superpotential must be modified. The modification of the superpotential leads to large correction to the quintessence potential and the observational condition cannot be satisfied. The potential $V = \dfrac{\Lambda^{\alpha+4}}{\phi^\alpha} e^{\kappa\phi^2/2}$ can be derived from supergravity theory. The value of $\omega_{o\phi}$ can be less than -0.7 (≈ -0.8) for this potential. Moreover, $\omega_{o\phi}$ in this case does not depend on α. This is a good property of this potential. But if supersymmetry is assumed to be broken at the mass scale m_s, this gives large correction to the quintessence potential. Although, the quintessence with simple exponential potential cannot satisfy observational conditions, but if its kinetic term is non-canonical the observational conditions can be satisfied as shown in Section 3.3. Note that the kinetic coefficient there is written down without any physical motivations, so this model is only a mathematical model.

If the parameter ω_ϕ of quintessence is constant, the relation between V, Ω_ϕ and ω_ϕ can be derived. The existence of scaling solution can be studied by using this relation. If the conditions for tracking behavior are used, one can see that the potentials $V = \dfrac{\Lambda^{\alpha+4}}{\phi^\alpha}$ and $V \sim \exp[-k\phi]$ have scaling solutions and tracking behavior, and the potentials $V = \frac{\Lambda^{\alpha+4}}{\phi^\alpha} e^{\kappa\phi^2/2}$ and $V \sim \exp[-K(\phi)]$ have tracking behavior, but do not have scaling solutions.

Although quintessence models which are reviewed in this thesis are not very satisfactory from the observational viewpoint, but they are reasonably adequate to describe the accelerated expansion of the universe qualitatively, and the coincident problem can be avoided in these models. Quintessence scenario can be developed in many ways. In string/brane theory, gravity has a special property and scalar field with various properties can exist, so quintessence scenario can be modified in many ways by using string/brane theory. Three examples are shown briefly below.

In the first example, our 3-dimensional universe including the usual matter fields and forces except for gravity are considered to be confined on the brane in the higher dimensions of superstring theory, the gravity in this brane will be different from the 4-dimensional Einstein theory. The Friedmann equation on the brane is different from Eq. (1.31) and its form is [38]

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_4^2}{3}\rho + \frac{\kappa_5^4}{36}\rho^2 + \frac{C}{a^4}, \tag{5.1}$$

where $\kappa_4^2 = 8\pi G$ and κ_5^4 are the 4- and 5-dimensional gravitational constants respectively and C is a constant which denotes "dark" radiation. The ρ^2 term plays an important role in the quintessence scenario [38]. When this term dominates at the early time of the universe evolution, the energy parameter of quintessence (Ω_ϕ) with negative power law potential decreases in time. When time increases the ρ term will dominate, quintessence will behave as ordinary quintessence with negative power law and Ω_ϕ will increase up to the present. Since quintessence with negative power law potential will start to dominate the universe energy too early if $\Omega_{i\phi}$ is not sufficiently small, then this model has a good property because Ω_ϕ decreases to a small value before increasing again.

Consider the next example, where quintessence is assumed to be a free field with non-canonical kinetic term. The Lagrangian of this quintessence is [39, 40]

$$\mathcal{L} = \bar{P}(\chi); \qquad \chi = \frac{1}{2}(\partial\phi)^2. \tag{5.2}$$

The form of $\bar{P}(\chi)$ can be estimated by using the condition that the model should be stable [39, 40]. In this model, $p_\phi = \bar{P}$ and $\rho_\phi = \frac{\partial \bar{P}}{\partial \chi}\chi - \bar{P}$. The energy density of quintessence (ρ_ϕ) decreases at the same rate as radiation in the radiation dominated epoch. At the beginning of the matter dominated epoch, the quintessence energy density decreases rapidly to a small value. After that it will decrease slower than matter energy density and dominates the universe energy at the present time. Note that some authors call this quintessence as k-essence [40].

Now we come to the example where the equation of state of quintessence takes the form

$$\rho_\phi = -\frac{A}{p_\phi}, \tag{5.3}$$

where A is a constant. Fluid with this equation of state was first introduced by Chaplygin as a model to study lifting force on a plane wing [41] and is known as Chaplygin's gas. Fluid with this equation of state also exists in high energy physics (see Ref. [42]).

From Eq. (5.3) and energy conservation Eq. (1.16), the energy density of this quintessence decreases as

$$\frac{\rho_\phi d\rho_\phi}{\rho_\phi^2 - A} = -3da, \tag{5.4}$$
$$\rho_\phi = \sqrt{A + \frac{B}{\rho^{-6}}},$$

where B is an integration constant. If $B > 0$, one sees from the above equation that for small a (i.e., $a^6 \ll B/A$) the energy density of this quintessence decreases with the same rate as matter as

$$\rho_\phi \sim \frac{\sqrt{B}}{a^3}. \tag{5.5}$$

For very large a (i.e., $a^6 \gg B/A$), this quintessence behaves as a cosmological constant as

$$\rho_\phi \sim \sqrt{A}, \quad p_\phi \sim -\sqrt{A}. \tag{5.6}$$

For large a eq. (5.5) can be expanded as

$$\rho_\phi \approx \sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6}, \tag{5.7}$$
$$p_\phi \approx -\sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6}.$$

The above equation describes the mixture of cosmological constant $\sqrt{A}$ with a scalar field with $\frac{1}{2}\dot{\phi}^2 \gg V(\phi)$. So by choosing A and B, the energy density of this quintessence decreases as matter at early time, has the form of a mixture at the end of matter dominated epoch and behaves as a cosmological constant at present.

The potential of a scalar field which corresponds to the equation of state (5.3) can be calculated as shown below. Since

$$\begin{aligned} \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho_\phi &= \sqrt{A + \frac{B}{a^6}}, \\ \frac{1}{2}\dot{\phi}^2 = p_\phi &= -\frac{A}{\sqrt{A + Ba^{-6}}}, \end{aligned} \tag{5.8}$$

then

$$\dot{\phi}^2 = \frac{B}{\sqrt{A + Ba^{-6}}}, \tag{5.9}$$

and

$$V(\phi) = \frac{2a^6(A + Ba^{-6}) - B}{2a^6\sqrt{A + Ba^{-6}}}. \tag{5.10}$$

If the universe is flat ($\dot{a} = a\sqrt{\rho_\phi}$), one gets

$$\frac{\partial \phi}{\partial a} = \dot{a}\dot{\phi} = \frac{\sqrt{B}}{a(Aa^6 + B)^{1/2}}. \tag{5.11}$$

By integrating the above equation, one gets

$$a^6 = \frac{4B\exp(6\phi)}{A(1 - \exp(6\phi))^2}. \tag{5.12}$$

Substituting this equation into Eq. (5.10), one obtains

$$V(\phi) = \frac{\sqrt{A}}{2}\left(\cosh 3\phi + \frac{1}{\cosh 3\phi}\right). \tag{5.13}$$

Most quintessence models assume that the cosmological constant is zero without any explanation. The cosmological constant can be zero by imposing some unknown symmetry. Some authors [43, 44] suggest that the cosmological constant can be zero by some adjustment mechanisms, where the cosmological constant is charaterized by a scalar field which is slowly rolling down its potential with special form of non-canonical kinetic term [43, 44]. In brane theory the cosmological constant can be small on the brane (our four dimensional world) but still naturally large in the bulk (hidden dimensions) [14].

Quintessence is an active area of research in cosmo-particle physics. The progress in brane theory provides new ideas and impetus for the construction of quintessence models. Models where dissipative effects [45] and models where the addition of tensor field [46] to the scalar field have been investigated. The inhomogeneous part of quintessence is also being actively investigated to understand the CMB spectrum, in particular the first Doppler peak [47].

REFERENCES

[1] S. Perlmutter et al., Astrophys. J. **517**, (1999): 565;
J. G. Bartlett, A. Blanchard, M. Le Dour, M. Douspis and D. Barbosa, astro-ph/9812226.

[2] N. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science, **284**, (1999): 1481.

[3] S. Weinberg, Rev. Mod. Phys., **61**, (1989); 1.

[4] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett., **80**, (1998): 1582.

[5] B. Ratra and P. J. E. Peebles, Phys. Rev. **D37**, (1988): 3406.

[6] P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. **D59**, (1999): 123504.

[7] S. M. Carroll, Lecture Notes on General Relativity, gr-qc/9712019.

[8] P. Coles and F. Lucchin, Cosmology : The Origin and Evolution of Cosmic Structure, John Wiley & Sons, (1995).

[9] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. **D9**, (2000): 373-444.

[10] S. Bludman and M. Roos, astro-ph/0004328.

[11] G. Efstathiou, astro-ph/9904356.

[12] L. Wang, R. R. Caldwell, J. P. Ostriker and P. J. Steinhardt, Astrophys. J. **530**, (2000): 17.

[13] Ya. B. Zeldovich, JETP Lett., **6**, (1967): 316.

[14] C. Schmidhuber, hep-th/0104131.

[15] L. H. Ryder, Quantum Field Theory, 2*nd* edition, Cambridge University Press, (1996).

[16] P. J. E. Peebles and A. Vilenkin, Phys. Rev. **D59**, (1999): 063505.

[17] A. R. Liddle and R. J. Scherrer, Phys. Rev. **D59**, (1999): 023509.

[18] E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. **D57**, (1998): 4686.

[19] C. Wetterich, Astron. Astrophys., **301**, (1995): 321.

[20] P. G. Ferreira and M. Joyce, Phys. Rev. **D58**, (1998): 023503.

[21] V. B. Johri, Phys. Rev. **D63**, (2001): 103504.

[22] A. Masiero, A. Pietroni and F. Rosati, Phys. Rev. **D61**, (2000): 023504.

[23] D. Bailin and A. Love, Supersymmetric Gauge Field Theory and String Theory, Institute of Physics Publishing, (1994).

[24] Y. Shadmi, Rev. Mod. Phys. **72**, (2000): 25.

[25] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B256**, (1985): 557.

[26] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B241**, (1984): 493.

[27] P. Brax and J. Martin, Phys. Lett. **B468**, (1999): 40.

[28] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. **B289**, (1987): 589.

[29] P. Brax and J. Martin, Phys. Rev. **D61**, (2000): 103502.

[30] P. Binétruy, Phys. Rev. **D60**, (1999): 063502.

[31] A. Hebecker and C. Wetterich, Phys. Lett. **B497**, (2001): 281.

[32] E. Di Pietro and J. Demaret, Int. J. Mod. Phys. **D10**, (2001): 231.

[33] R. Bean and J. Megueijo, astro-ph/0007199.

[34] A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, (2000): 2076.

[35] E. J. Copeland, N. J. Nunes and F. Rosati, Phys. Rev. **D62**, (2000): 123503; F. Rosati, Nucl. Phys. Proc. Suppl. **95**, (2001): 74.

[36] K. Choi, Phys. Rev. **D62**, (2000): 043509.

[37] C. Kolda and D. Lyth, Phys. Lett. **B458**, (1999): 197.

[38] K. Maeda, astro-ph/0012313.

[39] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. **D62**, (2000): 023511.

[40] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, (2000): 4438.

[41] S. Chaplygin, Sci. Mem. Moscow Univ. Math. Phys., **21**, (1904): 1.

[42] A. Kamenshchik, U. Moschella, V. Pasquier, Phys. Rev. Lett. **B511**, (2001): 265.

[43] V. A. Rubakov, Phys. Rev. **D61**, (2000): 061501.

[44] A. Hebecker and C. Wetterich, Phys. Rev. Lett. **85**, (2000): 3339.

[45] L. P. Chimento, A. S. Jakubi and D. Pavón, Proc. COSMOLOGY 2000, Lisbon, Portugal, 12-15 July 2000, astro-ph/0010079.

[46] W. Hu, D. J. Eisenstein, M. Tegmark and M. White, Phys. Rev. **D59**, (1999): 023512.

[47] P. F. González-Díaz, astro-ph/0103194.

Curriculum Vitae

Name : Khamphee KARWAN Sex : Male

Date and Place of Birth : 14 July 1975, Bangkok, Thailand.

Education :

High School : May 1991 - March 1994, Bodindecha School,
Bangkok, Thailand.

M6 Certificate (1994).

University : June 1994 - September 2001, Physics Department,
Faculty of Science, Chulalongkorn University,
Bangkok, Thailand.

B.Sc.(Physics) (May 1998)

Scholarship : May 1991 - May 2001, The Development and Promotion
of Science and Technology Talent Project.