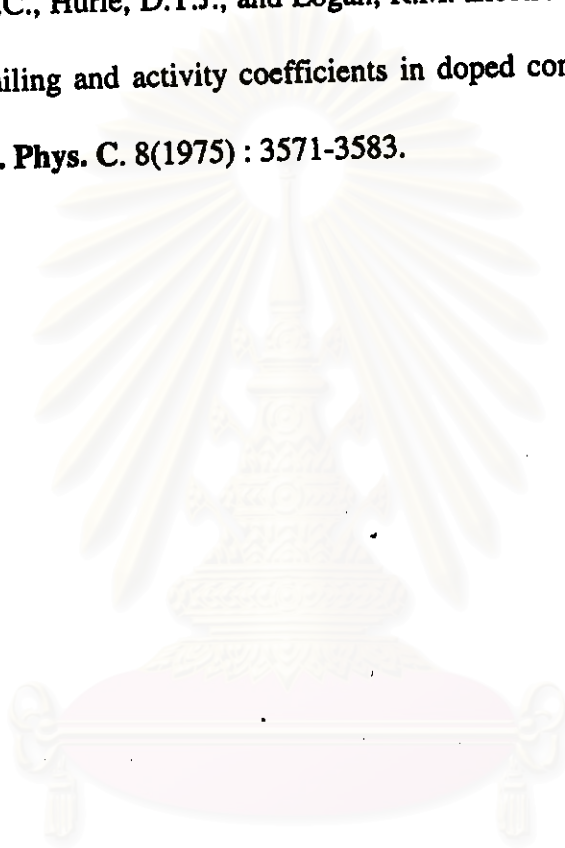


## REFERENCES

1. Kane, E.O. Thomas-Fermi approach to impure semiconductor band structure. **Phys. Rev.** 131(1963) : 79-88.
2. Sa-yakanit, V., and Glyde, H.R. Impurity-band density of states in heavily doped semiconductors : A variational calculation. **Phys. Rev. B** 22(1980) : 6222-6232.
3. Van Cong, H., and Debiais, G. An accurate empirical generalized Einstein relation and its applications to the n(p)-type silicon at high temperatures. **Solid-State Electronics** 38(1995) : 83-87.
4. Van Cong, H., and Debiais, G. A simple accurate expression of the reduced Fermi energy for any reduced carrier density. **J. Appl. Phys.** 75(1993) : 1545-1546.
5. Landsberg, P.T. **Recombination in semiconductors**. Cambridge : Cambridge university press, 1991.
6. Landsberg, P. T. On the diffusion theory of rectification. **Proc. R. Soc. A** 213(1952) : 226-237.

7. Aschroft, N.W., and Mermin, N.D. **Solid State Physics**. Philadelphia : W.B. Saunders company, 1991.
8. Joyce, W.B., and Dixon, R.W. Analytic approximations for the Fermi energy of ideal Fermi gas. **Appl. Phys. Lett.** 31(1977) : 354-356.
9. Chang, T.Y., and Izabelle, A. Full range analytic approximations for Fermi-Dirac integral  $F_{1/2}$  in term of  $F_{1/2}$  . **J. Appl. Phys.** 65(1989) : 2162-2164.
10. Fistul, V.I. **Heavily doped semiconductors**. New York : Plenum,1969.
11. Halperin, B.I.,and Lax, M. Impurity-band tails in the high-density limit. I. Minimum counting methods. **Phys. Rev.** 148(1966) : 722-740.
12. Wolff, P.A. Theory of the band structure of very degenerate semiconductors. **Phys. Rev.** 126(1962) : 405-412.
13. Ehrenreich, H. Band structure and electron transport of GaAs. **Phys. Rev.** 120(1960) : 1951-1963.
14. Hwang, C.J. Calculation of Fermi energy and bandtail parameters in heavily doped and degenerate n-type GaAs. **J. Appl. Phys.** 41(1970) : 2668.

15. Sritrakool, W., Glyde, H.R., and Sa-yakanit, V. The Fermi-energy and screening length in n-type GaAs. *Can. J. Phys.* 60(1982) : 373-378.
16. Herbert, D.C., Hurle, D.T.J., and Logan, R.M. Electron-electron interaction, band tailing and activity coefficients in doped compensated semiconductors. *J. Phys. C.* 8(1975) : 3571-3583.



สถาบันวิทยบริการ  
จุฬาลงกรณ์มหาวิทยาลัย



**APPENDIX**

สถาบันวิทยบริการ  
จุฬาลงกรณ์มหาวิทยาลัย

## APPENDIX

### THE SOMMERFELD EXPANSION

The Sommerfeld expansion is applied to integrals of the form'

$$\int_{-\infty}^{\infty} dE H(E) f(E) \quad , \quad f(E) = \frac{1}{\exp\left(\frac{E - E_f}{k_B T}\right) + 1} \quad , \quad (1)$$

where  $H(E)$  vanishes as  $E \rightarrow -\infty$  and diverges no longer rapidly than some power of  $E$  as  $E \rightarrow \infty$ . If one defines

$$K(E) = \int_{-\infty}^E H(E') dE' \quad , \quad (2)$$

so that

$$H(E) = \frac{dK(E)}{dE} \quad , \quad (3)$$

then one can integrate by parts in (1) to get

$$\int_{-\infty}^{\infty} H(E) f(E) dE = \int_{-\infty}^{\infty} K(E) \left( -\frac{\partial f}{\partial E} \right) dE . \quad (4)$$

Since  $f$  is indistinguishable from zero when  $E$  is more than a few  $k_B T$  greater than  $E_f$ , and indistinguishable from unity when  $E$  is more than a few  $k_B T$  less than  $E_f$ , its  $E$ -derivative will be appreciable only within a few  $k_B T$  of  $E_f$ . Provided that  $H$  is nonsingular and not too rapidly varying in the neighborhood of  $E = E_f$ , it is very reasonable to evaluate (4) by expanding  $K(E)$  in the Taylor series about  $E = E_f$ , with the expectation that only the first few terms will be of importance

:

$$K(E) = K(E_f) + \sum_{n=1}^{\infty} \left[ \frac{(E - E_f)^n}{n!} \left[ \frac{d^n K(E)}{dE^n} \right]_{E=E_f} \right] . \quad (5)$$

When we substitute (5) in (4), the leading term gives just  $K(E_f)$ , since

$$\int_{-\infty}^{\infty} \left( -\frac{\partial f}{\partial E} \right) dE = 1 .$$

Furthermore, since  $\partial f / \partial E$  is an even function of  $E - E_f$ , only terms with even  $n$  in (5) contribute to (4), and if we reexpress  $K$  in terms of the original function  $H$  through (2), we find that :

$$\int_{-\infty}^{\infty} dE H(E) f(E) = \int_{-\infty}^{E_f} H(E) dE + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{(E - E_f)^{2n}}{(2n)!} \left( -\frac{\partial f}{\partial E} \right) \frac{d^{2n-1}}{dE^{2n-1}} H(E) \Big|_{E=E_f} .(6)$$

Finally, making the substitution  $(E - E_f)/k_B T = x$  , we find that

$$\int_{-\infty}^{\infty} dE H(E) f(E) = \int_{-\infty}^{E_f} H(E) dE + \sum_{n=1}^{\infty} a_n (k_B T)^{2n} \frac{d^{2n-1}}{dE^{2n-1}} H(E) \Big|_{E=E_f} , (7)$$

where the  $a_n$  are dimensionless numbers given by

$$a_n = \int_{-\infty}^{\infty} \frac{x^{2n}}{(2n)!} \left( -\frac{d}{dx} \frac{1}{e^x + 1} \right) dx . (8)$$

This is usually written in terms of the Riemann zeta function,  $\zeta(n)$ , as

$$a_n = \left( 2 - \frac{1}{2^{2(n-1)}} \right) \zeta(2n) , (9)$$

where  $\zeta(n) = 1 + 1/2^n + 1/3^n + 1/4^n + \dots$  (10)

## CURRICULUM VITAE

Mr. Jessada Sukpitak was born on September 19, 1972 in Bangkok. He received his B.Sc. degree in physics from Chulalongkorn University in 1994.



สถาบันวิทยบริการ  
จุฬาลงกรณ์มหาวิทยาลัย