พฤติกรรมสัญญาณขาดหายของแผนการสัญญาณซ้ำแบบร่วมมือพร้อมด้วยการเลือกรีเลย์

นายกำพล วรดิษฐ์

ศูนย์วิทยทรัพยากร
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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรดุษฎีบัณฑิต

สาขาวิชาวิศวกรรมไฟฟ้า ภาควิชาวิศวกรรมไฟฟ้า

คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

ปีการศึกษา 2552

ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

OUTAGE BEHAVIOR OF COOPERATIVE DIVERSITY SCHEMES WITH RELAY SELECTION

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A Dissertation Submitted in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy Program in Electrical Engineering

Department of Electrical Engineering
Faculty of Engineering
Chulalongkorn University

Academic year 2009

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Thesis Title	Outage Behavior of Cooperative Diversity Schemes with Relay Selection
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Field of Study	Electrical Engineering
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กำพล วรดิษฐ์ : พฤติกรรมสัญญาณขาดหายของแผนการสัญญาณซ้ำแบบร่วมมือพร้อมด้วยการเลือกรีเลย์. (Outage Behavior of Cooperative Diversity Schemes with Relay Selection) อ.ที่ปรึกษาวิทยานิพนธ์หลัก : รศ.ดร.ลัญฉกร วุฒิสิทธิกุลกิจ, 189 หน้า.

ในการพัฒนาทางด้านโทรคมนาคม ได้มีการผลักดันมาตรฐานการสื่อสารดิจิตอลใหม่มาอย่างต่อเนื่อง เพื่อรองรับปริมาณความต้องการใช้งานที่มากขึ้นอย่างรวดเร็วและเพื่อแก้ปัญหาการมีบริเวณที่เป็นจุดบอดสัญญาณ จากแรงขับนี้ ได้มีความพยายามที่จะแก้ปัญหาทางวิศวกรรมที่จุดเชื่อมต่อไร้สาย ซึ่งเป็นจุดติดขัดของระบบสื่อสารด้วยขีดจำกัดของช่องสัญญาณ แนวคิดสำคัญที่จะปรับปรุงจุดติดขัดนี้คือการใช้สัญญาณซ้ำแบบร่วมมือ โดยเฉพาะอย่างยิ่งในสถานการณ์ที่เวลาร่วมนัยยาวนานเมื่อเทียบกับระยะเวลาของบล็อกของการส่งสัญญาณ ซึ่งเทคนิคการแทรกสลับหรือการเข้ารหัสช่องสัญญาณไม่ยาวนานพอที่จะบรรเทาปัญหาเฟดดิงได้อย่างมีประสิทธิภาพ

วิทยานิพนธ์นี้เลือกพิจารณาแผนการสัญญาณซ้ำแบบร่วมมือซึ่งใช้รีเลย์หลายตัวพร้อมด้วยเกณฑ์วิธีการเลือกรีเลย์ ความน่าจะเป็นที่สัญญาณขาดหายและความจุที่สัญญาณขาดหายเป็นปริมาณบ่งชี้สมรรถนะของระบบที่ใช้เทคนิคสัญญาณซ้ำ วิทยานิพนธ์นี้จึงเสนอการวิเคราะห์ปริมาณทั้งสองของแผนการสัญญาณซ้ำแบบร่วมมือที่พิจารณา ผลการวิเคราะห์ให้แง่มุมหลายประการ เช่น มีขีดเริ่มเปลี่ยนทางอัตราส่วนสัญญาณต่อสัญญาณรบกวนที่บางแผนการสัญญาณซ้ำแบบร่วมมือจะมีสรรมถนะที่ดีกว่าหรือแย่กว่าการส่งสัญญาณตามปกติ โดยนิพจน์ที่วิเคราะห์ได้มีความแม่นตรง, คำนวณได้โดยง่าย, และใช้งานได้กับทอพอโลยีและอัตราส่วนสัญญาณต่อสัญญาณรบกวนใดๆในช่องสัญญาณแบบเรย์ลีเฟดดิง ผลที่ได้สามารถชี้นำการออกแบบระบบสัญญาณซ้ำแบบร่วมมือที่มีประสิทธิภาพและการวางเครือข่ายที่ใช้เทคนิคสัญญาณซ้ำแบบร่วมมือ

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ภาควิชา วิศวกรรมไฟฟ้า ลายมือชื่อนิสิต ....
สาขาวิชา วิศวกรรมไฟฟ้า ลายมือชื่ออ.ที่ปรึกษาวิทยานิพนธ์หลัก ....
ปีการศึกษา 2552

4571802221 : MAJOR ELECTRICAL ENGINEERING

KEYWORDS : COOPERATIVE DIVERSITY / SELECTIVE RELAYING / OUTAGE PROBABILITY / OUTAGE CAPACITY

KAMPOL WORADIT : OUTAGE BEHAVIOR OF COOPERATIVE DIVERSITY SCHEMES WITH RELAY SELECTION. THESIS ADVISOR : Assoc. Prof. LUNCHAKORN WUTTISITTIKULKIJ, Ph.D., 189 pp.

New digital communication standards have been raised to support the dramatic rise of the capacity demand and to provide the comprehensive coverage. Thereby, many researchers have taken on the technical challenges in the part of wireless interface, which is the bottleneck of a communication system, due to its limited channel. The important idea that can improve the system in a major way is to use cooperative diversity, especially when the coherence time is too long compared to the transmission block, and the interleaver technique or channel coding is not long enough to combat the fading efficiently.

In this thesis, we are particularly interested in the cooperative diversity schemes that employ multiple relay nodes with relay selection protocol. For systems employing diversity techniques, the outage probability and the outage capacity are the important figures of merit. We analyze both the outage probability and the outage capacity for such cooperative diversity schemes. The analyses show many insights, for example, there exists a signal-to-noise ratio threshold, below which some cooperative diversity schemes outperform direct communication. The obtained expressions are exact, simple, and applicable to arbitrary network topologies and signal-to-noise ratios in Rayleigh fading channels. Our results can serve as guidelines for efficient system design of cooperative diversity schemes and deployment of cooperative networks.

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Field of Study : Electrical Engineering ...... Advisor's Signature ....

Academic Year : 2009 ....

Acknowledgements

First, I would like to thank my advisor, Prof. Lunchakorn Wuttisittikulkij, and also my co-advisors, Prof. Andreas F. Molisch, Prof. Prasit Prapinmongkolkarn, Dr. Tony Q.S. Quek and Prof. Moe Z. Win. My gratitude is extended to Ulric Ferner, Wesley Gifford, Pedro C. Pinto, Yuan Shen, Dr. Siwaruk Siwamogsatham and Watcharaphan Suwansantisuk for valuable discussions and comments, especially, Watcharaphan Suwansantisuk, who helps me on Lemma 4 and Proposition 7. Also, I thank the dissertation committee, Prof. Somchai Jitapunkul, Prof. Tuptim Angkaew, Prof. Watit Benjapolakul, Prof. Phaophak Sirisuk and Dr. Siwaruk Siwamogsatham.

I would like to acknowledge the financial supports from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program (Grant No.PHD/0010/2546).

I would like to thank my colleagues, Dr. Henk A. Wymeersch, Dr. Cedric Herzet, Prof. Poompat Saengudomlert, Matthieu Guyot and Pannawit Ekkapat.

Finally, I would like to express my gratitude to my parents, teachers, schools and universities.

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CHAPTER I

INTRODUCTION

1.1 Cooperative Diversity and Its Commercial Applications

Signal attenuation or fading limits the data transmission rate and introduces errors to the transmitted signal. The undesired effect of fading can be mitigated by channel coding [1], time diversity [2], frequency diversity [3], spatial diversity [4,5,6], or cooperative diversity [7,8,9,10]. Cooperative diversity can improve transmission reliability and increase capacities of wireless networks, especially in large-scale fading, in which spatial diversity from the cooperation schemes can improve the performance of the systems significantly [11]. The common concept is to employ the relay node(s) in the network so that the relay node relays the signals transmitted from the source node to the destination node when the direct link between the source node and the destination node is poor.

Cooperative diversity has been applied commercially in the broadband wireless access system, for example, the Broadband Wireless Metropolitan Area Network, which has been standardized in IEEE 802.16j, where one or more relay stations can be employed to provide additional coverage or performance advantage [12]. When the mobile station has a poor link to the base station, the mobile station can communicate with the base station via the relay station instead. Service operators have to upgrade the base stations so that the base stations can recognize the relay stations. The relay stations are classified into two different types: transparent and non-transparent. A non-transparent relay station communicates with the base station and mobile stations using the same carrier frequency, and can operate in both centralized and distributed scheduling mode. A transparent relay station communicates with the base station and mobile stations using the same or different carrier frequencies, and can only operate in centralized scheduling mode. In addition, cooperative diversity will be applied commercially in the cellular system, for example, the Long Term Evolution Advanced, which is being standardized by the 3rd Generation Partnership Project to serve as a 4G mobile communication standard [13,14]. The Long Term Evolution Advanced is compatible with the first release Long Term Evolution equipment and can share the same frequency bands. Basically, the relay nodes help the base station to extend the range to the mobile station. This reduces the propagation path loss, and lowers the interference.

1.2 Research Motivation

1.2.1 Scheme Comparison

Various cooperative diversity schemes have been proposed in the literature. The primary work considers the single-relay cooperative diversity schemes [10,15,16], where a system is composed of one source node, one destination node, and single relay node. The source node can transmit signals to the destination using the two-hop link via the relay node instead of the direct link. The performance is improved because the chance that the two-hop link and the direct link become poor at the same time is smaller than the chance that the direct link alone becomes poor.

It is natural to further improve the performance by increasing the number of relay nodes. The later work addresses the multiple-relay cooperative diversity schemes without relay selection [17,18,19], where a system uses all available relay nodes to relay the signals. The available relay nodes mean the relay nodes that can decode the signals from the source node correctly. The performance of such systems increases monotonically with the number of relays.

Actually, the multiple-relay cooperative diversity schemes without relay selection suffer the loss in spectrum resource, that is,

$$\text{Transmission Rate} \propto \frac{1}{\text{No. of Selected Relays}}.$$

Accordingly, the recent work proposes the multiple-relay cooperative diversity schemes with relay selection [20,21,22,23,24], where a system selects only the best relay node to relay the signals. Even though not all relays are used, the loss in the asymptotic performance does not incur because having several relay nodes to select already provided diversity orders [20].

From the aforementioned comparison, the multiple-relay cooperative diversity schemes with relay selection provide diversity gain without harming the transmission rate. Therefore, this thesis focuses on considering the multiple-relay cooperative diversity schemes with relay selection, which have been proposed in three different schemes in the literature: the fixed selective decode-and-forward without direct link combining scheme [21], the fixed selective decode-and-forward with direct link combining scheme [22], and the smart selective decode-and-forward scheme [24].

In fact, there are schemes that allow the relaying for more than two hops, but these schemes are proposed for using in ad hoc networks, which are the different kind of application, not the scope of this thesis.

1.2.2 Analysis versus Simulation

Since we have three different cooperative diversity schemes to consider and to benchmark with the non-cooperative system, we have to quantify their performance for doing comparison and gaining insights. The improvement in reliability and capacity of the cooperative diversity schemes can be well measured by their outage capacities and outage probabilities [25]. Both performance measures are also applicable for the non-cooperative system [26,27], which will be served as a benchmark (see section 1.3).

However, the results in the literature are incomplete as shown in Table 1. By observing some special cases, we can be misled or make some conclusions too generally. As a result, we do not truly understand the gain offered from the cooperative diversity schemes and how to choose the best scheme. In order to really understand all schemes and apply them properly, the outage probability and outage capacity should be computed for all schemes with any topology, any number of relays, and any signal-to-noise ratio regime. Unfortunately, it takes too long time to do Monte Carlo simulations.

For example, in 9-relay case, the Intel Pentium M processor 1.6 GHz with 768 MB of RAM takes 2 days to yield the results for only one topology setting.

Therefore, we decided to find the analytical results, which can be obtained without waiting time, as proposed in [28,29].

Table 1 The available results in the literature

Scheme	Outage Probability	Outage Capacity
Fixed selective decode-and-forward without direct link combining	Exact analysis [21]	Not available
Fixed selective decode-and-forward with direct link combining	Approximated analysis [22,23]	Simulation [24]
Smart selective decode-and-forward	Not available	Simulation [24]

Another advantage of the analytical results is that the obtained mathematical expressions can be used later to prove theory to gain insights. By observing the simulation results alone, we can only conclude some trends without proof. For example, we have limitation in discussing the influence of the signal-to-noise ratio regime, the relay network topology, and the number of relays. On the other hand, the analytical result in [30] leads to the theory with proof that the fixed selective decode-and-forward without direct link combining scheme achieves the same diversity-multiplexing tradeoff as achieved by more complex protocols.

1.3 Outage Probability and Outage Capacity

The definitions of the outage probability and the outage capacity are given below.

- **Outage probability** The probability that the maximum instantaneous end-to-end mutual information falls below a certain specified threshold, which is a transmission rate.

- **Outage capacity** The maximum transmission rate that is guaranteed to be supported if outages are allowed to occur with a certain specified probability.

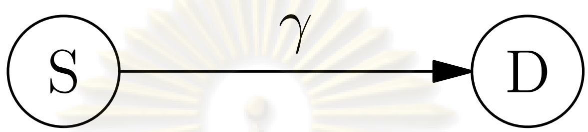

Figure 1 The point-to-point transmission: The source node transmits signals to the destination node, where the link has an instantaneous signal-to-noise ratio of γ, which is a random variable with probability density function $p_\gamma(\gamma)$

For point-to-point transmission as shown in Figure 1, the outage probability and outage capacity can be calculated with the standard formulas. When the instantaneous signal-to-noise ratio is denoted by γ and its probability density function is known to be $p_\gamma(\gamma)$, the outage probability at a transmission rate of R is given by [26]

$$\mathbb{P}_{\text{out}}^{\text{DC}}(R) = \Pr\{\log_2(1+\gamma) < R\} = \int_0^{\gamma_{\text{th}}} p_\gamma(\gamma)\,d\gamma, \tag{1.1}$$

where $\gamma_{\text{th}} = 2^R - 1$. When the outages are allowed to occur with probability ϵ, the outage capacity is given by [27]

$$C_{\text{out}}^{\text{DC}}(\epsilon) = \log_2(1 + F^{-1}(1-\epsilon)\mathbb{E}\{\gamma\}), \tag{1.2}$$

where F is the complementary cumulative distribution function of the instantaneous signal-to-noise ratio, i.e.,

$$F(x) := \Pr\{\gamma < x\},$$

and $\mathbb{E}\{\gamma\}$ is the expectation of the instantaneous signal-to-noise ratio, i.e.,

$$\mathbb{E}\{\gamma\} = \int_0^{\infty} p_\gamma(\gamma)\,d\gamma.$$

1.4 Outline

Chapter II describes the system model and channel model, and describes the considered cooperative diversity schemes, namely, the fixed selective decode-and-forward without direct link combining scheme, the fixed selective decode-and-forward with direct link combining scheme, and the smart selective decode-and-forward scheme. Chapter III presents the analysis of the outage probabilities for all considered cooperative diversity schemes as well as discussions. Chapter IV presents the analysis of the outage capacities for all considered cooperative diversity schemes as well as discussions. Chapter V concludes the dissertation.

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CHAPTER II

MODELS AND SCHEMES

For convenience, all variables are listed in Table 2.

Table 2 Variable list

Notation	Description
K	number of relay nodes
k	$\in \{1, 2, 3, \ldots K\}$, index of relay node
$k_{\max}$	selected relay node
$\mathcal{K}$	$= \{1, 2, 3, \ldots K\}$, set of indexes of relay nodes
$\mathcal{S}$	$\subset \{1, 2, 3, \ldots K\}$ subset of indexes of relay nodes
l	$= \vert \mathcal{S} \vert$, cardinality of $\mathcal{S}$
S	source node
D	destination node
y_j	received signal at node j
h_{ij}	channel gain between node i and node j
x_i	signal transmitted by node i
n_j	noise at node j
N_0	variance of noise
Ω_{ij}	path loss in linear scale
d_{ij}	distance between node i and node j
α	path loss exponent
P	average transmit power
W	transmission bandwidth
ϵ	acceptable outage probability
SNR_{ij}	instantaneous signal-to-noise ratio between node i and node j

Table 2 Variable list (continued)

Notation	Description
SNR	average signal-to-noise ratio between the source node and the destination node
SNR_{ij}	average signal-to-noise ratio between node i and node j
$\text{SNR}_{\text{threshold}}$	threshold of average signal-to-noise ratio that two performance curves cross
I_{ij}	instantaneous mutual information between node i and node j
I_{SD}	instantaneous mutual information between source node and destination node
$I_{\text{S}k}$	instantaneous mutual information between source node and k th relay node
$I_{k\text{D}}$	instantaneous mutual information between k th relay node and destination node
R	maximum supported rate
R_{ij}	maximum supported rate between node i and node j
R_{DC}	maximum supported rate of direct communication
$R_{\text{FSDF-nodirect}}$	maximum supported rate of fixed selective decode-and-forward without direct link combining scheme
$R_{\text{FSDF-direct}}$	maximum supported rate of fixed selective decode-and-forward with direct link combining scheme
R_{SSDF}	maximum supported rate of smart selective decode-and-forward

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Table 2 Variable list (continued)

Notation	Description
R_{MRC}	maximum supported rate of combined links using maximum ratio combining
$\mathbb{P}_{\mathrm{out}}^{\mathrm{DC}}$	outage probability of direct communication
$\mathbb{P}_{\mathrm{out}}^{\mathrm{FSDF\text{-}nodirect}}$	outage probability of fixed selective decode-and-forward without direct link combining scheme
$\mathbb{P}_{\mathrm{out}}^{\mathrm{FSDF\text{-}direct}}$	outage probability of fixed selective decode-and-forward with direct link combining scheme
$\mathbb{P}_{\mathrm{out}}^{\mathrm{SSDF}}$	outage probability of smart selective decode-and-forward
$C_{\mathrm{out}}^{\mathrm{DC}}$	outage capacity of direct communication
$C_{\mathrm{out}}^{\mathrm{FSDF\text{-}nodirect}}$	outage capacity of fixed selective decode-and-forward without direct link combining scheme
$C_{\mathrm{out}}^{\mathrm{FSDF\text{-}direct}}$	outage capacity of fixed selective decode-and-forward with direct link combining scheme
$C_{\mathrm{out}}^{\mathrm{SSDF}}$	outage capacity of smart selective decode-and-forward
$w_{\epsilon}^{\mathrm{FSDF\text{-}nodirect}}$	$\in (0,1]$, transformation variable in fixed selective decode-and-forward without direct link combining scheme
$w_{\epsilon}^{\mathrm{FSDF\text{-}direct}}$	$\in (0,1]$, transformation variable in fixed selective decode-and-forward with direct link combining scheme
$w_{\epsilon}^{\mathrm{SSDF}}$	$\in (0,1]$, transformation variable in smart selective decode-and-forward

2.1 System Model

We consider a wireless relay network consisting of $K+2$ single-antenna nodes: a designated source-destination node pair and K relay nodes. The locations of all nodes are arbitrary but deterministically determined and all nodes do not move during the consideration period. Without loss of generality, the distance between the source and the destination nodes is normalized to one unit. The source and the destination nodes are denoted by S and D, respectively. The source node can transmit signals directly to the destination node or transmit signals to the destination node via a relay. All relays are operating in decode-and-forward mode, and only a single relay will be selected to regenerate the signals. All nodes operate in a common frequency band. The system uses the time-division-multiplexing (TDM) protocol, and does not allow signal collision, that is, only one node can transmit signals at a time. Hence, if the system selects a relay to regenerate the signals, the transmission will be divided into two time slots so that the source and the relay nodes can transmit signals in separate time. The system is subject to the decrease of capacity to a half from dividing the transmission into two time slots.

2.2 Channel Model

We consider Rayleigh frequency-flat fading with the coherence time that is long enough for the system to complete transmitting a block of data. The model for the received signal and the channel for a link between any pair of nodes i and j is given by

$$y_j = h_{ij} x_i + n_j, \tag{2.1}$$

where x_i is the signal transmitted by node i, $h_{ij} \sim \mathcal{CN}(0,\Omega_{ij})$ is the complex channel gain over the link $i \to j$, $n_j \sim \mathcal{CN}(0,N_0)$ is additive white Gaussian noise at node j. We will denote by $\mathcal{CN}(\mu,\sigma^2)$ a complex circularly symmetric Gaussian distribution with mean μ and variance $\sigma^2/2$ for the real and imaginary components. The channel gains, noise, and transmitted signals are independent. The channel gain h_{ij} captures the effects of fading as well as path loss by setting $\Omega_{ij} = d_{ij}^{-\alpha}$, where d_{ij} denotes the distance between node i and node j, and α is the path loss exponent. We denote the channel from S to the k th relay by $h_{\mathrm{S}k}$, and the channel from the k th relay to D by $h_{k\mathrm{D}}$. Every node i transmits with the same average transmit signal power $P \triangleq \mathbb{E}\left\{|x_i|^2\right\}$. Finally, we define SNR_{ij} as the instantaneous signal-to-noise ratio between node i and node j, and define SNR as the average signal-to-noise ratio from the source node to the destination node, given by

$$\begin{aligned}\mathsf{SNR} &\triangleq \mathbb{E}\left\{SNR_{\mathrm{SD}}\right\} \\ &= \frac{\mathbb{E}\left\{\left|h_{\mathrm{SD}}\right|^2\right\}P}{N_0 W} \\ &= \frac{\Omega_{\mathrm{SD}}P}{N_0 W} \\ &= d_{\mathrm{SD}}^{-\alpha}\frac{P}{N_0 W}.\end{aligned} \tag{2.2}$$

where W is the transmission bandwidth. With respect to the average signal-to-noise ratio from the source node to the destination node, the average signal-to-noise ratio from node i to node j can be written as

$$\begin{aligned}\mathbb{E}\left\{SNR_{ij}\right\} &\triangleq \mathbb{E}\left\{SNR_{ij}\right\} \\ &= \frac{\mathbb{E}\left\{\left|h_{ij}\right|^2\right\}P}{N_0 W} \\ &= \frac{\Omega_{ij}P}{N_0 W} \\ &= d_{ij}^{-\alpha}\frac{P}{N_0 W} \\ &= \frac{d_{ij}^{-\alpha}}{d_{\mathrm{SD}}^{-\alpha}}\mathsf{SNR}.\end{aligned} \tag{2.3}$$

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2.3 Selective Relaying Schemes

In this section, we will describe the cooperative diversity schemes, which select a best relay from a set of relays to forward the information, including (i) fixed selective decode-and-forward without direct link combining, (ii) fixed selective decode-and-forward with direct link combining, and (iii) smart selective decode-and-forward. The selection is carried out to maximize the instantaneous end-to-end mutual information, as explained in the following subsections. As a benchmark, we describe the direct communication without cooperative diversity here.

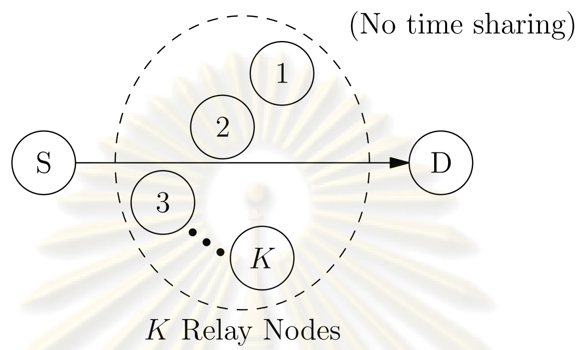

Figure 2 The direct communication scheme: The source node transmits signals directly to the destination node without any help from the relay nodes for the entire transmission period.

Without any cooperative diversity, the source node transmits the signals directly to the destination node as shown in Figure 2. In this case, it is not necessary to divide the channel into two time slots. So, the source node can transmit signals to the destination node for the entire frame without loss from time sharing. The instantaneous end-to-end mutual information (between y_{D} and x_{S}), conditioned on the instantaneous channel, is given by

$$
\begin{aligned}
R_{\mathrm{DC}} &= \log_2\left(1+SNR_{\mathrm{SD}}\right) \\
&= \log_2\left(1+\frac{|h_{\mathrm{SD}}|^2 P}{N_0 W}\right) \\
&= \log_2\left(1+\frac{|h_{\mathrm{SD}}|^2 \,\mathrm{SNR}}{d_{\mathrm{SD}}^{-\alpha}}\right).
\end{aligned}
\tag{2.4}
$$

We refer to SNR_{SD} as the instantaneous SNR between the source and destination.

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2.3.1 Fixed Selective Decode-and-forward without Direct Link Combining

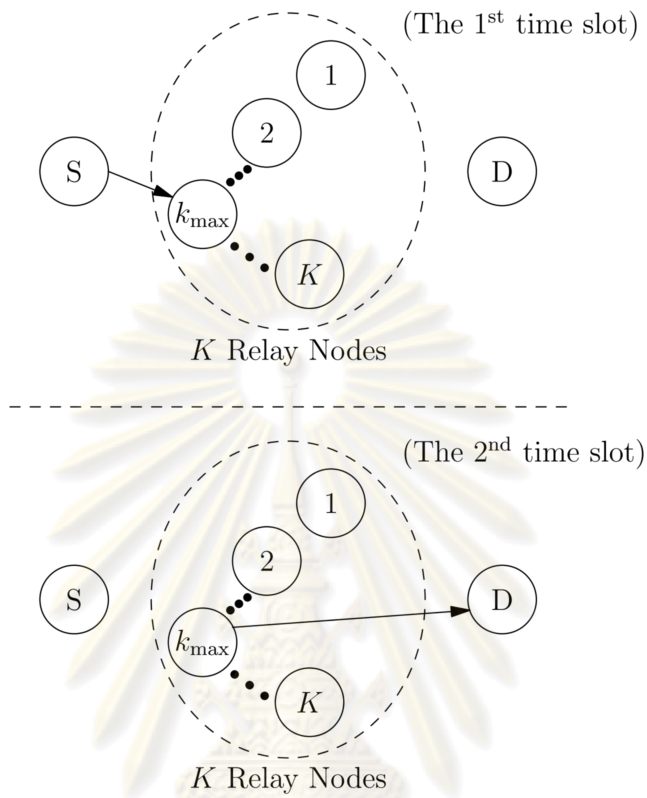

Figure 3 The fixed selective decode-and-forward without direct-link combining scheme: The transmission is divided into two time slots. During the first time slot, the source node transmits signals, and only the selected relay node listens. During the second time slot, the selected relay node decodes and regenerates the signals to the destination node, which decodes the received signals.

The first cooperative scheme considered here is the fixed selective decode-and-forward without direct-link combining, proposed in [21], as illustrated in Figure 3. This scheme always selects one relay to establish cooperation. If the selected relay fails to decode, the transmission is declared unsuccessful. If the decoding succeeds, the relay forwards the information to the destination.

The selection is based on maximizing the instantaneous end-to-end mutual information. First, each relay is considered based on the following transmission scheme: The transmission is divided into two time slots. In the first time slot, the source node transmits symbols, and the selected relay node listens. Given that the kth relay node is being considered, the maximum rate supported by the channel between the source and the relay nodes is

$$\begin{aligned} R_{\mathrm{S}k} &= \frac{1}{2} I_{\mathrm{S}k} \\ &= \frac{1}{2}\log_2\left(1+SNR_{\mathrm{S}k}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 \,\mathrm{SNR}}{d_{\mathrm{SD}}^{-\alpha}}\right), \end{aligned} \tag{2.5}$$

where we designate the channel gain from the source node to the kth relay by $h_{\mathrm{S}k}$. Likewise, the channel gain from the kth relay node to the destination node is designated by $h_{k\mathrm{D}}$. Also, the factor $1/2$ takes into account the loss from the time sharing due to dividing the channel into two time slots. If the transmission rate is not higher than the maximum rate, the relay will be able to decode the received signals and will transmit the regenerated version of the signals to the destination node during the second time slot while the source node will remain silent. The destination node decodes the signals transmitted from the selected relay node with an associated maximum rate of

$$\begin{aligned} R_{k\mathrm{D}} &= \frac{1}{2} I_{k\mathrm{D}} \\ &= \frac{1}{2}\log_2\left(1+SNR_{k\mathrm{D}}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{k\mathrm{D}}|^2 P}{N_0 W}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{k\mathrm{D}}|^2 \,\mathrm{SNR}}{d_{\mathrm{SD}}^{-\alpha}}\right). \end{aligned} \tag{2.6}$$

Thus, the end-to-end maximum rate supported by using the kth relay node is given by

$$R_{\text{flow-nodirect}}\left(k\right) = \min\left\{R_{\mathrm{S}k}, R_{k\mathrm{D}}\right\}. \tag{2.7}$$

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After considering all relays, the relay that offers the highest maximum rate is selected, say the $k_{\max}$ th relay, that is,

$$k_{\max} = \arg\max_{k\in\mathcal{K}} R_{\text{flow-nodirect}}(k), \tag{2.8}$$

where $\mathcal{K}$ is the set of all relay indexes: $\mathcal{K} \in \{1, 2, 3, \ldots, K\}$ and the $k_{\max}$ th relay node is the selected relay node. The maximum rate supported by the fixed selective decode-and-forward without direct link combining scheme is given by

$$R_{\text{FSDF-nodirect}} = \max_{k\in\mathcal{K}} \min\{R_{\text{S}k}, R_{k\text{D}}\}. \tag{2.9}$$

To implement the fixed selective decode-and-forward without direct-link combining scheme, the system must have the ability to select the best relay to regenerate the signals and to keep other relays silent. One possible way is to use the relay selection protocol proposed in [30]. In the overhead period, all relay nodes listen to the ready-to-send (RTS) signal from the source node and listen to the clear-to-send (CTS) signal from the destination. By receiving RTS signal from the source node and receiving CTS signal from the destination node, all relay nodes know the channel strengths from the source node and to the destination node. Then, all relay nodes wait for a period of time, which is reciprocal to the channel strength either from the source node or to the destination node, by using the weaker one. Then, the best relay node, which has the shortest waiting time, regenerates the signals. Other relay nodes can sense the transmitted signals from the best relay node and keep silent. The length of the overhead can be set small as long as the probability that several relay nodes have equal waiting time is negligible.

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2.3.2 Fixed Selective Decode-and-forward with Direct Link Combining

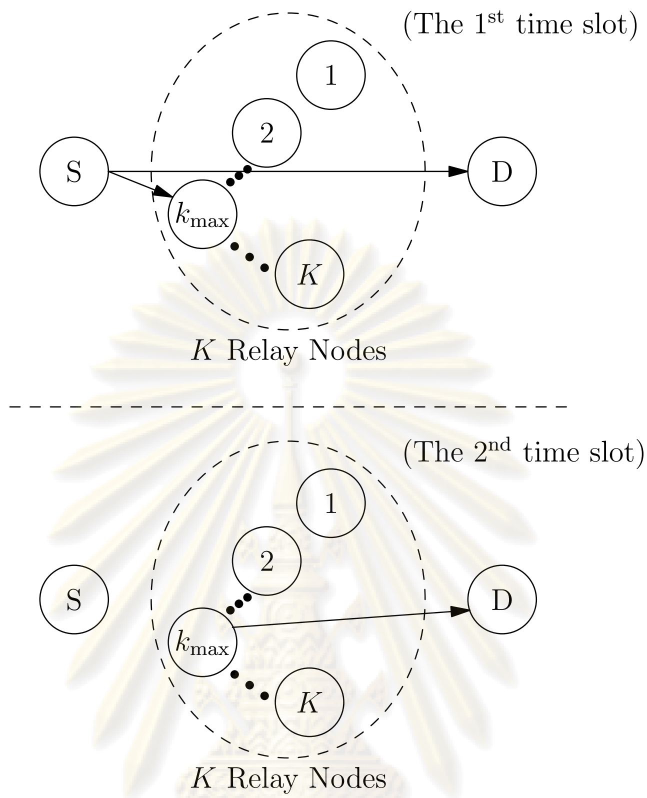

Figure 4 The fixed selective decode-and-forward with direct-link combining scheme: The transmission is divided into two time slots. During the first time slot, the source node transmits signals, and the selected relay and the destination nodes listen. During the second time slot, the selected relay node decodes and regenerates signals to the destination node, which combines and decodes the received signals from both time slots.

The second cooperative scheme considered here is the fixed selective decode-and-forward with direct-link combining, proposed in [22], as illustrated in Figure 4. This scheme always selects one relay to establish cooperation. If the selected relay fails to decode, the transmission is declared unsuccessful. If the decoding succeeds, the relay forwards the information to the destination. The destination node combines the signal received from both the source node and the selected relay node at the different time slots using maximum ratio combining.

The relay node selection is based on maximizing the instantaneous end-to-end mutual information. First, each relay is considered based on the following transmission scheme: The transmission is divided into two time slots. In the first time slot, the source node transmits symbols, and both the selected relay node and the destination node listen. Given that the kth relay node is being considered, the maximum rate supported by the channel between the source and the relay nodes is

$$\begin{aligned} R_{\mathrm{S}k} &= \frac{1}{2} I_{\mathrm{S}k} \\ &= \frac{1}{2}\log_2\left(1+SNR_{\mathrm{S}k}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 \,\mathrm{SNR}}{d_{\mathrm{SD}}^{-\alpha}}\right), \end{aligned} \tag{2.10}$$

where we designate the channel gain from the source node to the kth relay by $h_{\mathrm{S}k}$. Likewise, the channel gain from the kth relay node to the destination node is designated by $h_{k\mathrm{D}}$. Also, the factor $1/2$ takes into account the loss from the time sharing due to dividing the channel into two time slots. If the transmission rate is not higher than the maximum rate, the relay will be able to decode the received signals and will transmit the regenerated version of the signals to the destination node during the second time slot while the source node will remain silent. The destination node performs maximum ratio combining on two signals received from the source node and the selected relay node at the different time slots, and then decodes the combined signal. The maximum rate supported by combining both channels is given by

$$\begin{aligned} R_{\mathrm{MRC}}(k) &= \frac{1}{2}\log_2\left(1+SNR_{\mathrm{S}k}+SNR_{k\mathrm{D}}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}+\frac{|h_{k\mathrm{D}}|^2 P}{N_0 W}\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{\left(|h_{\mathrm{SD}}|^2+|h_{k\mathrm{D}}|^2\right)\mathrm{SNR}}{d_{\mathrm{SD}}^{-\alpha}}\right). \end{aligned} \tag{2.11}$$

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Therefore, the maximum instantaneous end-to-end mutual information supported by using the k th relay node is given by

$$R_{\text{flow-direct}}(k) = \min\{R_{Sk}, R_{MRC}(k)\}. \tag{2.12}$$

After considering all relays, the relay that offers the highest maximum end-to-end rate, say the $k_{\max}$ th relay, is selected, that is,

$$k_{\max} = \arg\max_{k\in\mathcal{K}} R_{\text{flow-direct}}(k), \tag{2.13}$$

The maximum rate supported by the fixed selective decode-and-forward with direct link combining scheme is given by

$$R_{\text{FSDF-direct}} = \max_{k\in\mathcal{K}} \min\{R_{Sk}, R_{MRC}(k)\} \tag{2.14}$$

To implement the fixed selective decode-and-forward with direct-link combining scheme, the system must have the ability to select the best relay to regenerate the signals and to keep other relays silent. One possible way is to use the relay selection protocol proposed in [29]. In the overhead period, all relay nodes listen to the RTS signal from the source node and listen to the CTS signal from the destination. Also, the destination node, which knows the channel strength from the source node by receiving the RTS signal from the source node, broadcasts the channel strength between the source node and the destination. By receiving RTS signal from the source node and receiving CTS signal from the destination node as well as the broadcasted channel strength between the source node to the destination node, all relay nodes know the channel strengths from the source node and to the destination node as well as the link between the source node and the destination node. Then, all relay nodes wait for a period of time, which is reciprocal to the channel strength either from the source node or of the combined channel between the link from the source node to the destination node and the link from itself to the destination node, by using the weaker one. Then, the best relay node, which has the shortest waiting time, regenerates the signals. Other relay nodes can sense the transmitted signals from the best relay node and keep silent. The length of the overhead can be set small as long as the probability that several relay nodes have equal waiting time is negligible.

2.3.3 Smart Selective Decode-and-forward

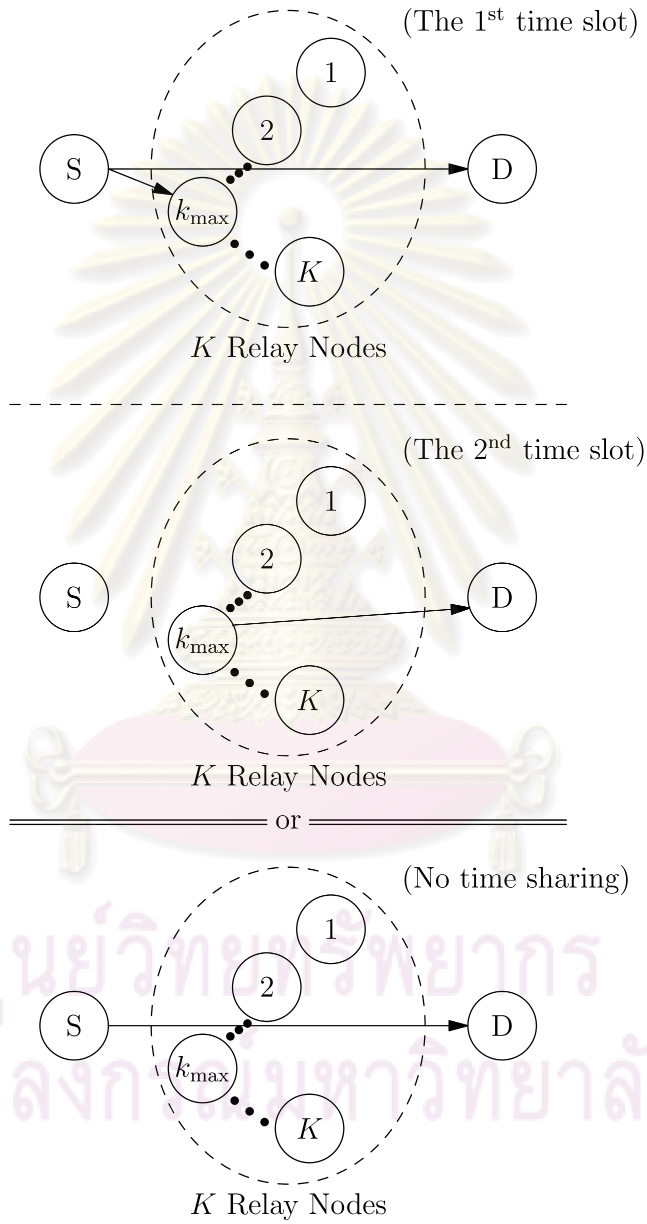

Figure 5 The smart selective decode-and-forward scheme: The scheme performs as either the fixed selective decode-and-forward with direct-link combining scheme or the direct communication scheme by choosing the scheme that offers the higher maximum rate.

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The third cooperative scheme considered here is the smart selective decode-and-forward, proposed in [24], as illustrated in Figure 5. This scheme is inspired from the loss from time sharing due to dividing the channel into two time slots in both fixed selective decode-and-forward schemes. This loss can affect the performance of the system more severely than the obtained benefit from performing cooperation. The third considered scheme is an adaptive cooperative scheme which decides when to use the fixed selective decode-and-forward with direct-link combining scheme or direct communication without cooperation based on the channel conditions. In this case, the maximum achievable rate is given by

$$R_{\text{SSDF}} = \max\{R_{\text{FSDF-direct}}, R_{\text{DC}}\}, \tag{2.15}$$

where R_{DC} and $R_{\text{FSDF-direct}}$ are given in (2.4) and (2.14), respectively.

To implement the fixed selective decode-and-forward with direct-link combining scheme, the system must have the ability to select the best relay to regenerate the signals and to keep other relays silent. One possible way is to use the relay selection protocol proposed in [29]. In the overhead period, all relay nodes listen to the RTS signal from the source node and listen to the CTS signal from the destination. Also, the destination node, which knows the channel strength from the source node by receiving the RTS signal from the source node, broadcasts the channel strength between the source node and the destination. By receiving RTS signal from the source node and receiving CTS signal from the destination node as well as the broadcasted channel strength between the source node to the destination node, all relay nodes know the channel strengths from the source node and to the destination node as well as the link between the source node and the destination node. Then, all relay nodes wait for a period of time, which is reciprocal to the channel strength either from the source node or of the combined channel between the link from the source node to the destination node and the link from itself to the destination node, by using the weaker one. Other relay nodes can sense the transmitted signals from the best relay node, which has the shortest waiting time, and keep silent. Then, the best relay node compare itself with the channel strength between the source node and the destination node with compensation for the time sharing free. After the comparison, the best relay node notifies the source node whether the cooperation should be used. If not, the best relay goes to sleep mode and the source node transmit the signals without cooperation and time sharing. Otherwise, the transmission is divided into two time slots, where the source node transmits the signals in the first time slot and the best relay node regenerates the signals in the second time slot. The length of the overhead can be set small as long as the probability that several relay nodes have equal waiting time is negligible.

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CHAPTER III

OUTAGE PROBABILITY

The cooperative diversity has been expected to leverage the reliability of the wireless communications, which are exposed to the path loss and fading. In order to examine this expectation, we have to quantify the reliability for explicitly measuring how much the improvement is obtained and for comparing the considered schemes with different sets of factors. One of the important measures of reliability is the outage probability. This chapter first analyzes the outage probabilities of the considered cooperative diversity schemes. The obtained formulas are verified by computer simulations with several sets of parameters. Then, the formulas are used to provide numerical results with comparisons and discussions to gain insights.

3.1 Direct Communication

In order to examine whether the used cooperative diversity scheme really provides advantage in the employed environment and how much the improvement is, we have to compare the performance when the cooperative diversity scheme is being used to the performance when the cooperative diversity scheme is not being used. Accordingly, before we consider the outage probabilities of the cooperative diversity schemes, the outage probability of the direct communication is analyzed here to serve as a benchmarking.

In the direct communication case, the formula for calculating the outage probability is straightforward and is available in standard materials. The outage probability is a function of a desired transmission rate R and an average signal-to-noise ratio SNR. The formula is provided in Proposition 1.

Proposition 1 : The outage probability, which is obtained from direct communication between the source node and the destination node, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [27]

$$\mathbb{P}_{\text{out}}^{\text{DC}}\left(R,\text{SNR}\right)=1-\exp\left(-\left(\frac{2^{R}-1}{\text{SNR}}\right)\right),$$
$$\text{SNR}\triangleq d_{\text{SD}}^{-\alpha}\frac{P}{N_0W} \tag{3.1}$$

where $\mathbb{P}_{\text{out}}^{\text{DC}}$ denote the outage probability of the direct communication, d_{SD} is the distance between the source node and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage probability is the probability that the instantaneous mutual information between the source node and the destination node falls below the desired transmission rate.

$$\mathbb{P}_{\text{out}}^{\text{DC}}\left(R,\text{SNR}\right)=\Pr\left\{I_{\text{SD}}\left(SNR\right)<R\right\}, \tag{3.2}$$

where I_{SD} denotes the instantaneous mutual information between the source node and the destination node, and is a function of the instantaneous signal-to-noise ratio, denoted by SNR , between the source node and the destination node.

Given the instantaneous signal-to-noise ratio, the instantaneous mutual information can be calculated by [31]

$$I_{\text{SD}}\left(SNR\right)=\log_2\left(1+SNR\right), \tag{3.3}$$

So,

$$\Pr\left\{I_{\text{SD}}\left(SNR\right)<R\right\}=\Pr\left\{\log_2\left(1+SNR\right)<R\right\}. \tag{3.4}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the destination node, multiplied by the average transmit signal power at the source node $P\triangleq\mathbb{E}\left\{\left|x_{\text{S}}\right|^2\right\}$, and divided by the noise power spectral density at the destination node N_0 , and divided by the transmission bandwidth W .

$$SNR=\frac{\left|h_{\text{SD}}\right|^2 P}{N_0W}, \tag{3.5}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{SD}}\right|^2$ is a random variable. Then,

$$\begin{aligned}\Pr\left\{\log_2\left(1+SNR\right)<R\right\}&=\Pr\left\{\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0W}\right)<R\right\}\\&=\Pr\left\{\left|h_{\mathrm{SD}}\right|^2<\left(2^R-1\right)\frac{N_0W}{P}\right\}.\end{aligned} \tag{3.6}$$

Since h_{SD} is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{SD}}\right)$, $\left|h_{\mathrm{SD}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{SD}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)=\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right). \tag{3.7}$$

Then,

$$\begin{aligned}\Pr\left\{\left|h_{\mathrm{SD}}\right|^2<\left(2^R-1\right)\frac{N_0W}{P}\right\}&=\int_0^{\left(2^R-1\right)\frac{N_0W}{P}} p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx\\&=\int_0^{\left(2^R-1\right)\frac{N_0W}{P}}\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx\\&=-\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)\Bigg|_0^{\left(2^R-1\right)\frac{N_0W}{P}}\\&=1-\exp\left(-\frac{\left(2^R-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)\end{aligned} \tag{3.8}$$

Since $\Omega_{\mathrm{SD}}=d_{\mathrm{SD}}^{-\alpha}$ and is the mean of $\left|h_{\mathrm{SD}}\right|^2$, the average signal-to-noise ratio of the link between the source node and the destination node can be written as

$$\begin{aligned}\mathrm{SNR}&=\mathbb{E}\left\{\left|h_{\mathrm{SD}}\right|^2\right\}\frac{P}{N_0W}\\&=\Omega_{\mathrm{SD}}\frac{P}{N_0W}\\&=d_{\mathrm{SD}}^{-\alpha}\frac{P}{N_0W}.\end{aligned} \tag{3.9}$$

Substituting (3.9) into (3.8) yields (3.1). Q.E.D.

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Lemma 1 : Let x be a random variable with a complex circularly symmetric Gaussian distribution with mean 0 and variance Ω, i.e., $x \sim \mathcal{CN}(0,\Omega)$. Let $y = |x|^2$. Then, y is an exponentially distributed random variable with a rate parameter of Ω^{-1}, i.e., with mean Ω.

Proof : There are two approaches to prove the lemma. We will show both of them as follows.

Approach 1: Let x_1 be the real part of x.

$$x_1 = \mathrm{Re}\{x\}, \tag{3.10}$$

and let x_2 be the imaginary part of x.

$$x_2 = \mathrm{Im}\{x\}. \tag{3.11}$$

Then, the probability density function of x_1 is normal distribution with mean 0 and variance $\frac{\Omega}{2}$.

$$p_{x_1}(x_1) = \frac{1}{\sqrt{2\pi\frac{\Omega}{2}}}\exp\left(-\frac{x_1^2}{2\frac{\Omega}{2}}\right), \tag{3.12}$$

and the probability density function of x_2 is also normal distribution with mean 0 and variance $\frac{\Omega}{2}$.

$$p_{x_2}(x_2) = \frac{1}{\sqrt{\pi\Omega}}\exp\left(-\frac{x_2^2}{\Omega}\right), \tag{3.13}$$

Considering changing the Cartesian coordinate system (x_1, x_2) to the polar coordinate system (ρ,θ), the relations include

$$\rho = \sqrt{x_1^2 + x_2^2}, \tag{3.14}$$

$$\theta = \arctan\left(\frac{x_2}{x_1}\right), \tag{3.15}$$

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$$x_1 = \rho \cos(\theta), \tag{3.16}$$

$$x_2 = \rho \sin(\theta), \tag{3.17}$$

and the Jacobian of changing the differential in the Cartesian coordinate system (x_1, x_2) to the differential in the polar coordinate system (ρ, θ) can be calculated by

$$\begin{aligned} J\left((x_1, x_2),(\rho,\theta)\right) &= \det\left(\begin{bmatrix} \frac{\partial x_1}{\partial \rho} & \frac{\partial x_2}{\partial \rho} \\ \frac{\partial x_1}{\partial \theta} & \frac{\partial x_2}{\partial \theta} \end{bmatrix}\right) \\ &= \det\left(\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\rho\sin(\theta) & \rho\cos(\theta) \end{bmatrix}\right) \\ &= \rho\left(\cos^2(\theta)+\sin^2(\theta)\right) \\ &= \rho. \end{aligned} \tag{3.18}$$

Hence, we obtain

$$dx_1 dx_2 = \rho d\rho d\theta. \tag{3.19}$$

The joint probability of an infinitesimal area at the point (x_1, x_2) can be written as $p_{x_1,x_2}(x_1, x_2)dx_1dx_2$, and is related to the polar coordinate system (ρ,θ) by

$$p_{x_1,x_2}(x_1, x_2)dx_1dx_2 = p_{\rho,\theta}(\rho,\theta)\rho d\rho d\theta. \tag{3.20}$$

Since x_1 and x_2 are statistically independent, the joint probability can be written as the product of marginal probabilities

$$p_{x_1,x_2}(x_1, x_2)dx_1dx_2 = p_{x_1}(x_1)p_{x_2}(x_2)dx_1dx_2. \tag{3.21}$$

Equating the right hand side expressions of (3.20) and (3.21) gives

$$\begin{aligned} p_{\rho,\theta}(\rho,\theta)\rho d\rho d\theta &= \frac{1}{\sqrt{\pi\Omega}}\exp\left(-\frac{x_1^2}{\Omega}\right)\frac{1}{\sqrt{\pi\Omega}}\exp\left(-\frac{x_2^2}{\Omega}\right)dx_1dx_2 \\ &= \frac{1}{\pi\Omega}\exp\left(-\frac{x_1^2+x_2^2}{\Omega}\right)dx_1dx_2 \end{aligned}$$

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$$= \frac{1}{\pi\Omega} \exp\left(-\frac{\rho^2}{\Omega} \right) \rho d\rho d\theta. \tag{3.22}$$

Then, we take the integration over θ in order to obtain the marginal probability of ρ.

$$\begin{aligned} p_\rho(\rho) d\rho &= \int_0^{2\pi} p_{\rho,\theta}(\rho,\theta) \rho d\rho d\theta \\ &= \frac{1}{\pi\Omega} \exp\left(-\frac{\rho^2}{\Omega} \right) \rho d\rho \left[\theta\right]_0^{2\pi} \\ &= \frac{2}{\Omega} \exp\left(-\frac{\rho^2}{\Omega} \right) \rho d\rho, \end{aligned} \tag{3.23}$$

and the cumulative distribution function of ρ can be written as

$$\begin{aligned} F_{\mathrm{P}}(\rho) &= \Pr\{\mathrm{P} \le \rho\} \\ &= \int_0^\rho \frac{2}{\Omega} \exp\left(-\frac{\rho^2}{\Omega} \right) \rho d\rho \\ &= \int_0^\rho \exp\left(-\frac{\rho^2}{\Omega} \right) d\frac{\rho^2}{\Omega} \\ &= -\exp\left(-\frac{\rho^2}{\Omega} \right)\Big|_0^\rho \\ &= 1 - \exp\left(-\frac{\rho^2}{\Omega} \right). \end{aligned} \tag{3.24}$$

Then, we take differentiation with chain rule in order to obtain the probability density function of ρ.

$$\begin{aligned} \frac{dF_{\mathrm{P}}(\rho)}{d\rho} &= 0 - \exp\left(-\frac{\rho^2}{\Omega} \right) \frac{d}{d\rho}\left(-\frac{\rho^2}{\Omega} \right) \\ &= \frac{2\rho}{\Omega} \exp\left(-\frac{\rho^2}{\Omega} \right). \end{aligned} \tag{3.25}$$

Let

$$s = \rho^2. \tag{3.26}$$

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Hence, $\rho = \sqrt{s}$. The Jacobian of changing the differential in ρ-domain to the differential in s-domain can be written as

$$\begin{aligned} J(\rho \to s) &= \left| \frac{d\rho}{ds} \right| \\ &= \left| \frac{d}{ds}\sqrt{s} \right| \qquad (3.27) \\ &= \frac{1}{2\sqrt{s}}. \end{aligned}$$

Therefore, the probability density function of s is given by

$$\begin{aligned} p_s(s) &= J(\rho \to s)\, p_\rho\left(\rho = \sqrt{s}\right) \\ &= \frac{1}{2\sqrt{s}} \frac{2\sqrt{s}}{\Omega} \exp\left(-\frac{s}{\Omega}\right) \qquad (3.28) \\ &= \frac{1}{\Omega} \exp\left(-\frac{s}{\Omega}\right), \end{aligned}$$

which is the exponential distribution with rate parameter Ω^{-1}, i.e., with mean Ω. Since

$$s = \rho^2 = x_1^2 + x_2^2 = |x|^2 = y, \qquad (3.29)$$

it follows that the lemma is true. Q.E.D.

Approach 2 : Let x_1 be the real part of x, and x_2 be the imaginary part of x, as shown in (3.10) and (3.11), respectively. Then, the probability density functions of both x_1 and x_2 are normal distribution with mean 0 and variance $\frac{\Omega}{2}$, as shown in (3.12) and (3.13), respectively. Let

$$s_1 = x_1^2, \qquad (3.30)$$

and let

$$s_2 = x_2^2. \qquad (3.31)$$

It follows that $x_1 = \sqrt{s_1}$ and $x_2 = \sqrt{s_2}$. The Jacobian of changing the differential in x_1-domain to the differential in s_1-domain can be written as

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$$
\begin{aligned}
J(x_1 \to s_1) &= \left|\frac{dx_1}{ds_1}\right| \\
&= \left|\frac{d}{ds_1}\sqrt{s_1}\right| \\
&= \frac{1}{2\sqrt{s_1}}.
\end{aligned}
\tag{3.32}
$$

Similarly, the Jacobian of changing the differential in x_2-domain to the differential in s_2-domain can be written as

$$
\begin{aligned}
J(x_2 \to s_2) &= \left|\frac{dx_2}{ds_2}\right| \\
&= \left|\frac{d}{ds_2}\sqrt{s_2}\right| \\
&= \frac{1}{2\sqrt{s_2}}.
\end{aligned}
\tag{3.33}
$$

Therefore, the probability density function of s_1 is given by

$$
\begin{aligned}
p_{s_1}(s_1) &= 2J(x_1 \to s_1)\, p_{x_1}\left(x_1 = \sqrt{s_1}\right) \\
&= \frac{2}{2\sqrt{\pi\Omega s_1}}\exp\left(-\frac{s_1}{\Omega}\right),
\end{aligned}
\tag{3.34}
$$

and the probability density function of s_2 is given by

$$
\begin{aligned}
p_{s_2}(s_2) &= 2J(x_2 \to s_2)\, p_{x_2}\left(x_2 = \sqrt{s_2}\right) \\
&= \frac{1}{\sqrt{\pi\Omega s_2}}\exp\left(-\frac{s_2}{\Omega}\right),
\end{aligned}
\tag{3.35}
$$

where the factor of 2 is introduced due to the addition between two equal probabilities on the negative side and the positive side in x_1-domain and in x_2-domain, respectively. Let

$$
s = x_1^2 + x_2^2 = s_1 + s_2. \tag{3.36}
$$

It follows that s is a random variable, which is an addition between s_1 and s_2, which are statistically independent because both of them are a function of x_1 and x_2, respectively, which are in turn statistically independent. Therefore, the probability density function of s can be written as the convolution between the probability density function of s_1 and the probability density function of s_2.

$$\begin{aligned} p_s(s) &= p_{s_1}(s) * p_{s_2}(s) \\ &= \int_{-\infty}^{\infty} p_{s_1}(\tau) * p_{s_2}(s-\tau)\, d\tau \\ &= \int_0^s \frac{1}{\sqrt{\pi\Omega\tau}} \exp\left(-\frac{\tau}{\Omega}\right) \frac{1}{\sqrt{\pi\Omega(s-\tau)}} \exp\left(-\frac{s-\tau}{\Omega}\right) d\tau \\ &= \int_0^s \frac{1}{\pi\Omega\sqrt{(s-\tau)\tau}} \exp\left(-\frac{s}{\Omega}\right) d\tau \\ &= \frac{1}{\pi\Omega} \exp\left(-\frac{s}{\Omega}\right) \int_0^s \frac{1}{\sqrt{(s-\tau)\tau}}\, d\tau \\ &= \frac{1}{\pi\Omega} \exp\left(-\frac{s}{\Omega}\right) [\pi] \\ &= \frac{1}{\Omega} \exp\left(-\frac{s}{\Omega}\right), \end{aligned} \tag{3.37}$$

which is the exponential distribution with rate parameter Ω^{-1}, i.e., with mean Ω. Since

$$s = x_1^2 + x_2^2 = |x|^2 = y, \tag{3.38}$$

it follows that the lemma is true. Q.E.D.

3.2 Fixed Selective Decode-and-forward without Direct Link Combining Scheme

The formula for calculating the fixed selective decode-and-forward without direct link combining scheme does not exist in standard materials. In the literature, there is a formula that can calculate the exact outage probability of the fixed selective decode-and-forward without direct link combining scheme. We propose a different formula that can also calculate the exact value, but is simpler for computation. We will present both of them in this dissertation. The first formula contains the double summations, in which the inner summation grows combinatorial with the number of relays. On the other hand, the second formula contains the product that grows linearly with the number of relays. Therefore, it is more convenient to use the second formula even though both formulas provide the same result.

3.2.1 Exact Formula : Approach 1

For doing comparisons among different schemes later, the outage probability is still a function of a desired transmission rate R and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The first formula is provided in Theorem 1.

Theorem 1 : The outage probability, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward without direct link combining scheme in the wireless relay network having K relay nodes, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [21]

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\mathsf{SNR}) = 1 + \sum_{l=0}^{K} \sum_{\substack{\mathcal{S}\subseteq\mathcal{K} \\ |\mathcal{S}|=l}} (-1)^l \prod_{k\in\mathcal{S}} \left[\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}} \frac{\left(2^{2R}-1\right)}{\mathsf{SNR}} \right) \exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}} \frac{\left(2^{2R}-1\right)}{\mathsf{SNR}} \right) \right] \tag{3.39}$$

where $\mathcal{K} \in \{1,2,3,\ldots,K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}$ denotes the outage probability of the fixed selective decode-and-forward without direct link combining scheme, d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay node and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage probability is the probability that the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the link from the relay node to the destination node falls below the desired transmission rate.

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R, \text{SNR}) = \Pr\left\{ \max_k \min\left\{ \frac{1}{2} I_{\text{S}k}\left(SNR_{\text{S}k}\right), \frac{1}{2} I_{k\text{D}}\left(SNR_{k\text{D}}\right) \right\} < R \right\} \quad (3.40)$$

where $I_{\text{S}k}$ denotes the instantaneous mutual information between the source node and the k th relay node, $I_{k\text{D}}$ denotes the instantaneous mutual information between the k th relay node and the destination node, where both of them are the functions of the instantaneous signal-to-noise ratio, denoted by $SNR_{\text{S}k}$ and $SNR_{k\text{D}}$, respectively, between the source node and the k th relay node and between the k th relay node and the destination node. The factor of 1/2 takes into account the loss due to dividing the transmission into two time slots. The maximum of the minimum can be written as the joint maximums.

$$\begin{aligned} &\Pr\left\{ \max_k \min\left\{ \frac{1}{2} I_{\text{S}k}\left(SNR_{\text{S}k}\right), \frac{1}{2} I_{k\text{D}}\left(SNR_{k\text{D}}\right) \right\} < R \right\} = \\ &\qquad \Pr\left\{ \max_k \left\{ \frac{1}{2} I_{\text{S}k}\left(SNR_{\text{S}k}\right) \right\} < R \wedge \max_k \left\{ \frac{1}{2} I_{k\text{D}}\left(SNR_{k\text{D}}\right) \right\} < R \right\}. \end{aligned} \quad (3.41)$$

Furthermore, the joint probability can be written as the conditional probability with the normalization. Accordingly, we guarantee that the links from the source node to the designated subset of relay nodes can support the desired transmission rate first, and then calculate the probability that the strongest link from the relay node to the destination node can support the desired transmission rate. Thus, we have to consider all possible disjoint subsets of the relay nodes, i.e., all combinations of the relay nodes that can receive information from the source node successfully. For the sake of presentation, we refer to this disjoint subset as decoding subset.

$$\begin{aligned}
&\Pr\left\{\max_k\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\right\}<R\wedge\max_k\left\{\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}\\
&=\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\,\middle|\,\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\\
&\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\\
&=\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\,\middle|\,\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\\
&\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\right\}\Pr\left\{\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\},
\end{aligned} \tag{3.42}$$

where $\mathcal{S}$ is the decoding subset, which has a varying cardinality ranging from 0 until K, and is defined by

$$\mathcal{S}=\left\{k\in\mathcal{K}\,\middle|\,\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\geq R\right\}. \tag{3.43}$$

Let first consider the normalization probability. Given the instantaneous signal-to-noise ratio between the source node and the k th relay node, the instantaneous mutual information between the source node and the k th relay node can be calculated by

$$I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)=\log_2\left(1+SNR_{\mathrm{S}k}\right). \tag{3.44}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the k th relay node, multiplied by the average transmit signal power at the source node $P\triangleq\mathbb{E}\left\{\left|x_{\mathrm{S}}\right|^2\right\}$, and divided by the noise power spectral density at the k th relay node, which is assumed to be N_0 for all relay nodes, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{S}k}=\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}, \tag{3.45}$$

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where P, N_0, and W are constant, and $|h_{Sk}|^2$ is a random variable. Thus,

$$\begin{aligned}\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{Sk}\left(SNR_{Sk}\right)\not< R\right\} &= \Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+SNR_{Sk}\right)\geq R\right\}\\ &= \Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{|h_{Sk}|^2 P}{N_0 W}\right)\geq R\right\}.\end{aligned} \tag{3.46}$$

Since h_{Sk} are statistically independent among $k\in\mathcal{K}$, the expression is distributive and can be further manipulated.

$$\begin{aligned}\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{|h_{Sk}|^2 P}{N_0 W}\right)\geq R\right\} &= \bigcap_{k\in\mathcal{S}}\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{Sk}|^2 P}{N_0 W}\right)\geq R\right\}\\ &= \prod_{k\in\mathcal{S}}\Pr\left\{|h_{Sk}|^2\geq\left(2^{2R}-1\right)\frac{N_0 W}{P}\right\}\\ &= \prod_{k\in\mathcal{S}}\left[1-\Pr\left\{|h_{Sk}|^2<\left(2^{2R}-1\right)\frac{N_0 W}{P}\right\}\right].\end{aligned} \tag{3.47}$$

Since h_{Sk} is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{Sk}\right)$, $|h_{Sk}|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{Sk}^{-1}\right)$ (see Lemma 1).

$$p_{|h_{Sk}|^2}\left(x\right)=\frac{1}{\Omega_{Sk}}\exp\left(-\frac{x}{\Omega_{Sk}}\right). \tag{3.48}$$

Then,

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$$\begin{aligned}\Pr\left\{|h_{Sk}|^2<\left(2^{2R}-1\right)\frac{N_0 W}{P}\right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} p_{|h_{Sk}|^2}\left(x\right)dx\\ &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \frac{1}{\Omega_{Sk}}\exp\left(-\frac{x}{\Omega_{Sk}}\right)dx\\ &= -\exp\left(-\frac{x}{\Omega_{Sk}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}}\\ &= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0 W}{\Omega_{Sk}P}\right).\end{aligned} \tag{3.49}$$

Therefore,

$$\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\not< R\right\}=\prod_{k\in\mathcal{S}}\left[1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\right]$$
$$=\prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]. \quad (3.50)$$

Likewise,

$$\Pr\left\{\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)< R\right\}=\prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]. \quad (3.51)$$

Now, let consider the conditional probability. Given the instantaneous signal-to-noise ratio between the kth relay node and the destination node, the instantaneous mutual information between the kth relay node and the destination node can be calculated by

$$I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)=\log_2\left(1+SNR_{k\mathrm{D}}\right). \quad (3.52)$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the kth relay node and the destination node, multiplied by the average transmit signal power at the kth relay node, which is assume to equal that of the source node $\mathbb{E}\left\{\left|x_k\right|^2\right\}=P$, and divided by the noise power spectral density at the destination N_0, and divided by the transmission bandwidth W.

$$SNR_{k\mathrm{D}}=\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0W}, \quad (3.53)$$

where P, N_0, and W are constant, and $\left|h_{k\mathrm{D}}\right|^2$ is a random variable. Thus,

$$\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}< R\left|\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\not< R\right.\right\}$$

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$$= \Pr\left\{ \max_{k\in\mathcal{S}} \left\{ \frac{1}{2}\log_2\left(1+SNR_{k\mathrm{D}}\right) \right\} < R \middle| \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+SNR_{\mathrm{S}k}\right) \geq R \wedge \bigcap_{k\notin\mathcal{S}} \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\}$$

$$= \Pr\left\{ \max_{k\in\mathcal{S}} \left\{ \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) \right\} < R \middle| \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) \geq R \wedge \bigcap_{k\notin\mathcal{S}} \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\}. \tag{3.54}$$

Since the channel coefficients of the links from the source node to every relay node and the channel coefficients of the links from every relay node to the destination node are statistically independent, i.e., $h_{\mathrm{S}k}, k \in \mathcal{K}$ and $h_{k\mathrm{D}}, k \in \mathcal{K}$ are statistically independent, the conditional probability becomes probability.

$$\begin{aligned}
&\Pr\left\{ \max_{k\in\mathcal{S}} \left\{ \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) \right\} < R \middle| \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) \geq R \wedge \bigcap_{k\notin\mathcal{S}} \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \\
&\quad = \Pr\left\{ \max_{k\in\mathcal{S}} \left\{ \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) \right\} < R \right\} \\
&\quad = \Pr\left\{ \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) < R \right\}.
\end{aligned} \tag{3.55}$$

Since $h_{k\mathrm{D}}$ are statistically independent among $k \in \mathcal{K}$, the expression is distributive and can be further manipulated.

$$\begin{aligned}
\Pr\left\{ \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) < R \right\} &= \prod_{k\in\mathcal{S}} \Pr\left\{ \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) < R \right\} \\
&= \prod_{k\in\mathcal{S}} \Pr\left\{ \left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\}.
\end{aligned} \tag{3.56}$$

Since $h_{k\mathrm{D}}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{k\mathrm{D}}\right)$, $\left|h_{k\mathrm{D}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{k\mathrm{D}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right) = \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right). \tag{3.57}$$

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Then,

$$\begin{aligned}
\Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P}\right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right)dx \\
&= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right)dx \\
&= -\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} \\
&= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0 W}{\Omega_{k\mathrm{D}}P}\right).
\end{aligned} \tag{3.58}$$

Therefore,

$$\begin{aligned}
&\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\} < R \,\Bigg|\, \bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\right\} \\
&= \prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right)\right].
\end{aligned} \tag{3.59}$$

Substituting (3.50), (3.51) and (3.59) into (3.42), we obtain

$$\begin{aligned}
&\Pr\left\{\max_k\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\right\} < R \wedge \max_k\left\{\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\} < R\right\} \\
&= \sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right)\right] \\
&\quad \prod_{k\in\mathcal{S}}\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right] \\
&= \sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\prod_{k\in\mathcal{S}}\left[\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right)\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right] \\
&\quad \prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right].
\end{aligned} \tag{3.60}$$

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Since $\Omega_{Sk} = d_{Sk}^{-\alpha}$ and is the mean of $|h_{Sk}|^2$, the average signal-to-noise ratio of the link between the source node and the k th relay node can be written as

$$\begin{aligned}\mathbb{E}\{SNR_{Sk}\} &= \mathbb{E}\left\{|h_{Sk}|^2\right\}\frac{P}{N_0W} \\ &= \Omega_{Sk}\frac{P}{N_0W} \\ &= d_{Sk}^{-\alpha}\frac{P}{N_0W}.\end{aligned} \tag{3.61}$$

Comparing (3.61) with (3.9), we obtain

$$\frac{\Omega_{Sk}P}{N_0W} = \frac{d_{Sk}^{-\alpha}}{d_{SD}^{-\alpha}}\text{SNR}. \tag{3.62}$$

Since $\Omega_{kD} = d_{kD}^{-\alpha}$ and is the mean of $|h_{kD}|^2$, the average signal-to-noise ratio of the link between the k th relay node and the destination node can be written as

$$\begin{aligned}\mathbb{E}\{SNR_{kD}\} &= \mathbb{E}\left\{|h_{kD}|^2\right\}\frac{P}{N_0W} \\ &= \Omega_{kD}\frac{P}{N_0W} \\ &= d_{kD}^{-\alpha}\frac{P}{N_0W}.\end{aligned} \tag{3.63}$$

Comparing (3.63) with (3.9), we obtain

$$\frac{\Omega_{kD}P}{N_0W} = \frac{d_{kD}^{-\alpha}}{d_{SD}^{-\alpha}}\text{SNR}. \tag{3.64}$$

Substituting (3.62) and (3.64) into (3.60) yields (3.39). Q.E.D.

3.2.2 Exact Formula : Approach 2

Now, we provide the second formula in Theorem 2. Instead of relying on the double summations among decoding subsets, we consider the product among the flows from the source node to the destination node via the relay nodes. Hence, the

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product grows linearly with the number of relays, and the obtained result takes less computation time when the number of relays increases.

Theorem 2: The outage probability, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward without direct link combining scheme in the wireless relay network having K relay nodes, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [29]

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\text{SNR}) = \prod_{k=1}^{K}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\right]. \tag{3.65}$$

where d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage probability is the probability that the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the link from the relay node to the destination node falls below the desired transmission rate.

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\text{SNR}) = \Pr\left\{\max_{k}\min\left\{\frac{1}{2}I_{\text{S}k}\left(SNR_{\text{S}k}\right),\frac{1}{2}I_{k\text{D}}\left(SNR_{k\text{D}}\right)\right\}<R\right\} \tag{3.66}$$

where $I_{\text{S}k}$ denotes the instantaneous mutual information between the source node and the k th relay node, $I_{k\text{D}}$ denotes the instantaneous mutual information between the k th relay node and the destination node, where both of them are the functions of the instantaneous signal-to-noise ratio, denoted by $SNR_{\text{S}k}$ and $SNR_{k\text{D}}$, respectively, between the source node and the k th relay node and between the k th relay node and the destination node. The factor of 1/2 takes into account the loss due to dividing the transmission into two time slots. Then,

$$\Pr\left\{\max_{k}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}= \\ \Pr\left\{\bigcap_{k=1}^{K}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}. \tag{3.67}$$

Consider the flows from the source node to the destination node via the relay nodes. Each of them experiences the instantaneous signal-to-noise ratios that are statistical independent. Therefore, the expression is distributive, and the minimum can be manipulated further.

$$\begin{aligned}&\Pr\left\{\bigcap_{k=1}^{K}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}\\ &=\prod_{k=1}^{K}\Pr\left\{\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}\\ &=\prod_{k=1}^{K}\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\vee\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)<R\right\}.\end{aligned} \tag{3.68}$$

Without altering the expression, we take double negations, which change the union probability to joint probability.

$$\begin{aligned}&\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\vee\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)<R\right\}\\ &=\Pr\left\{\sim\left(\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\nless R\right)\right\}\\ &=1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\nless R\right\}.\end{aligned} \tag{3.69}$$

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Given the instantaneous signal-to-noise ratio between the source node and the k th relay node, the instantaneous mutual information between the source node and the k th relay node can be calculated by

$$I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)=\log_2\left(1+SNR_{\mathrm{S}k}\right). \tag{3.70}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the k th relay node, multiplied by the average transmit signal power at the source node $P\triangleq\mathbb{E}\left\{\left|x_{\mathrm{S}}\right|^2\right\}$, and divided by the noise power spectral density at the k th relay node, which is assumed to be N_0 for all relay nodes, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{S}k} = \frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}, \tag{3.71}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{S}k}\right|^2$ is a random variable. Likewise, given the instantaneous signal-to-noise ratio between the k th relay node and the destination node, the instantaneous mutual information between the k th relay node and the destination node can be calculated by

$$I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right) = \log_2\left(1 + SNR_{k\mathrm{D}}\right). \tag{3.72}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the k th relay node and the destination node, multiplied by the average transmit signal power at the k th relay node, which is assume to equal that of the source node $\mathbb{E}\left\{\left|x_k\right|^2\right\} = P$, and divided by the noise power spectral density at the destination N_0, and divided by the transmission bandwidth W.

$$SNR_{k\mathrm{D}} = \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}, \tag{3.73}$$

where P, N_0, and W are constant, and $\left|h_{k\mathrm{D}}\right|^2$ is a random variable. Thus,

$$\begin{aligned} &\Pr\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) \nless R \wedge \frac{1}{2} I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right) \nless R\right\} = \\ &\qquad \Pr\left\{\frac{1}{2}\log_2\left(1 + \frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) \nless R \wedge \frac{1}{2}\log_2\left(1 + \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}\right) \nless R\right\}. \end{aligned} \tag{3.74}$$

Since the channel coefficient of the link from the source node to the k th relay node and the channel coefficient of the link from the k th relay node to the destination node are statistically independent, i.e., $h_{\mathrm{S}k}$ and $h_{k\mathrm{D}}$ are statistically independent for any $k \in \{1, 2, \ldots, K\}$, the joint probability becomes the product between probabilities.

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$$\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0W}\right) \nless R \wedge \frac{1}{2}\log_2\left(1+\frac{|h_{k\mathrm{D}}|^2 P}{N_0W}\right) \nless R\right\}$$
$$= \Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0W}\right) \nless R\right\} \Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{k\mathrm{D}}|^2 P}{N_0W}\right) \nless R\right\}, \tag{3.75}$$

where both probabilities can be manipulated further.

$$\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0W}\right) \nless R\right\} = 1-\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0W}\right) < R\right\}$$
$$= 1-\Pr\left\{|h_{\mathrm{S}k}|^2 < \left(2^{2R}-1\right)\frac{N_0W}{P}\right\}, \tag{3.76}$$

and likewise,

$$\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{k\mathrm{D}}|^2 P}{N_0W}\right) \nless R\right\} = 1-\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{|h_{k\mathrm{D}}|^2 P}{N_0W}\right) < R\right\}$$
$$= 1-\Pr\left\{|h_{k\mathrm{D}}|^2 < \left(2^{2R}-1\right)\frac{N_0W}{P}\right\}. \tag{3.77}$$

Since $h_{\mathrm{S}k}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{S}k}\right)$, $|h_{\mathrm{S}k}|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{S}k}^{-1}\right)$ (see Lemma 1).

$$p_{|h_{\mathrm{S}k}|^2}(x) = \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right). \tag{3.78}$$

Then,

$$\Pr\left\{|h_{\mathrm{S}k}|^2 < \left(2^{2R}-1\right)\frac{N_0W}{P}\right\} = \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} p_{|h_{\mathrm{S}k}|^2}(x)\,dx$$
$$= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)dx$$
$$= -\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}}$$

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$$= 1 - \exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right). \tag{3.79}$$

Since $h_{k\mathrm{D}}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{k\mathrm{D}}\right)$, $\left|h_{k\mathrm{D}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{k\mathrm{D}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right) = \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right). \tag{3.80}$$

Then,

$$\begin{aligned}
\Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R}-1\right)\frac{N_0W}{P}\right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right)dx \\
&= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right)dx \\
&= -\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}} \\
&= 1 - \exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right).
\end{aligned} \tag{3.81}$$

Substituting (3.79) into (3.76) and substituting (3.81) into (3.77), the expression in (3.75) can be further manipulated.

$$\begin{aligned}
&\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2P}{N_0W}\right) \nless R \wedge \frac{1}{2}\log_2\left(1+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right) \nless R\right\} \\
&= \left[1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\right]\left[1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right)\right)\right] \\
&= \exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right).
\end{aligned} \tag{3.82}$$

Substituting (3.82) into (3.69), (3.68) is given by

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$$\Pr\left\{\bigcap_{k=1}^{K}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}$$
$$=\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}\right)\right]. \tag{3.83}$$

Since $\Omega_{\mathrm{S}k}=d_{\mathrm{S}k}^{-\alpha}$ and is the mean of $\left|h_{\mathrm{S}k}\right|^2$, the average signal-to-noise ratio of the link between the source node and the k th relay node can be written as

$$\begin{aligned}\mathbb{E}\left\{SNR_{\mathrm{S}k}\right\}&=\mathbb{E}\left\{\left|h_{\mathrm{S}k}\right|^2\right\}\frac{P}{N_0W}\\&=\Omega_{\mathrm{S}k}\frac{P}{N_0W}\\&=d_{\mathrm{S}k}^{-\alpha}\frac{P}{N_0W}.\end{aligned} \tag{3.84}$$

Comparing (3.84) with (3.9), we obtain

$$\frac{\Omega_{\mathrm{S}k}P}{N_0W}=\frac{d_{\mathrm{S}k}^{-\alpha}}{d_{\mathrm{SD}}^{-\alpha}}\mathrm{SNR}. \tag{3.85}$$

Since $\Omega_{k\mathrm{D}}=d_{k\mathrm{D}}^{-\alpha}$ and is the mean of $\left|h_{k\mathrm{D}}\right|^2$, the average signal-to-noise ratio of the link between the k th relay node and the destination node can be written as

$$\begin{aligned}\mathbb{E}\left\{SNR_{k\mathrm{D}}\right\}&=\mathbb{E}\left\{\left|h_{k\mathrm{D}}\right|^2\right\}\frac{P}{N_0W}\\&=\Omega_{k\mathrm{D}}\frac{P}{N_0W}\\&=d_{k\mathrm{D}}^{-\alpha}\frac{P}{N_0W}.\end{aligned} \tag{3.86}$$

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Comparing (3.86) with (3.9), we obtain

$$\frac{\Omega_{k\mathrm{D}}P}{N_0W}=\frac{d_{k\mathrm{D}}^{-\alpha}}{d_{\mathrm{SD}}^{-\alpha}}\mathrm{SNR}. \tag{3.87}$$

Substituting (3.85) and (3.87) into (3.83), the expression can be further manipulated.

$$\Pr\left\{\bigcap_{k=1}^{K}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{k\mathrm{D}}\left(SNR_{k\mathrm{D}}\right)\right\}<R\right\}$$

$$=\prod_{k=1}^{K}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right] \quad (3.88)$$

$$=\prod_{k=1}^{K}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right].$$

Substituting (3.88) into (3.67) solves (3.66), which yields (3.65). Q.E.D.

3.2.3 Checking Approach 2 with Approach 1

The formula (3.39) contains the combinatorial calculation, which makes the approach 1 more complicated than the approach 2, which results in the formula (3.65) containing only the product of K terms. We will check whether the simpler formula (3.65) obtained from the approach 2 gives the same result as the conventional formula (3.39) obtained from the approach 1. We did not find a way to directly manipulate the simpler formula (3.65) to be the conventional formula (3.39). What we can do is to first specify K, and then manipulate both formulas to the point that both expressions are the same. Here, we show the cases that $K=1$, $K=2$, and $K=3$.

Let start from $K=1$. The conventional formula gives

$$\mathbb{P}_{\mathrm{out}}^{\text{FSDF-nodirect}}(R,\mathsf{SNR})$$

$$=\sum_{l=0}^{1}\sum_{\substack{\mathcal{S}\subseteq\{1\}\\|\mathcal{S}|=l}}\prod_{k\in\mathcal{S}}\left[\left(1-\exp\left(-\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]$$

$$=(1)\prod_{k\in\{1\}}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]+\prod_{k\in\{1\}}\left[\left(1-\exp\left(-\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right](1)$$

$$=\left[1-\exp\left(-\frac{d_{\mathrm{S}1}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]+\left(1-\exp\left(-\frac{d_{1\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S}1}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)$$

$$=1-\exp\left(-\frac{d_{\mathrm{S}1}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{1\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right),$$

(3.89)

where $\prod_{k\in\varnothing}[\bullet]=1$. The simpler formula gives

$$\begin{aligned}\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\text{SNR}) &= \prod_{k=1}^{1}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\right]\\ &= 1-\exp\left(-\frac{d_{\text{S1}}^{\alpha}+d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\\ &= 1-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right),\end{aligned} \tag{3.90}$$

which is the same as (3.89).

Then, at $K=2$, the conventional formula gives

$$\begin{aligned}&\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\text{SNR})\\ &=\sum_{l=0}^{2}\sum_{\substack{\mathcal{S}\subseteq\{1,2\}\\|\mathcal{S}|=l}}\prod_{k\in\mathcal{S}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]\\ &=\prod_{k\in\{1,2\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{1\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]\\ &\prod_{k\in\{2\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{2\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]\\ &\prod_{k\in\{1\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{1,2\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]\\ &=Term1\left(\mathcal{S}=\varnothing\right)+Term2\left(\mathcal{S}=\{1\}\right)+Term3\left(\mathcal{S}=\{2\}\right)+Term4\left(\mathcal{S}=\{1,2\}\right)\\ &=1-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\\ &-\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\\ &+\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right),\end{aligned} \tag{3.91}$$

where

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$$Term1\left(\mathcal{S}=\varnothing\right)=$$

$$1-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right), \tag{3.92}$$

$$\begin{aligned}Term2\left(\mathcal{S}=\{1\}\right)=&\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right),\end{aligned} \tag{3.93}$$

$$\begin{aligned}Term3\left(\mathcal{S}=\{2\}\right)=&\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right),\end{aligned} \tag{3.94}$$

$$\begin{aligned}Term4\left(\mathcal{S}=\{1,2\}\right)=&\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\\
&+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right),\end{aligned} \tag{3.95}$$

and the simpler formula gives

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$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\text{SNR}) = \prod_{k=1}^{2}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\right]$$

$$=\left[1-\exp\left(-\frac{d_{\text{S1}}^{\alpha}+d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\right]\left[1-\exp\left(-\frac{d_{\text{S2}}^{\alpha}+d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\right]$$

$$=1-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)$$

$$-\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)$$

$$+\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right),$$

(3.96)

which is the same as (3.91).

Last, at $K=3$, the conventional formula gives

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R,\text{SNR})$$

$$=\sum_{l=0}^{3}\sum_{\substack{\mathcal{S}\subseteq\{1,2,3\}\\|\mathcal{S}|=l}}\prod_{k\in\mathcal{S}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]$$

$$=\prod_{k\in\{1,2,3\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{1\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]$$

$$\prod_{k\in\{2,3\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{2\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]$$

$$\prod_{k\in\{1,3\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{3\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]$$

$$\prod_{k\in\{1,2\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{1,2\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]$$

$$\prod_{k\in\{3\}}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]+\prod_{k\in\{1,3\}}\left[\left(1-\exp\left(-\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right)\exp\left(-\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\right]$$

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$$\prod_{k\in\{2\}}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]+\prod_{k\in\{2,3\}}\left[\left(1-\exp\left(-\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]$$

$$\prod_{k\in\{1\}}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]+\prod_{k\in\{1,2,3\}}\left[\left(1-\exp\left(-\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\right]$$

$$=Term1\left(\mathcal{S}=\varnothing\right)+Term2\left(\mathcal{S}=\{1\}\right)+Term3\left(\mathcal{S}=\{2\}\right)+Term4\left(\mathcal{S}=\{3\}\right)$$

$$+Term5\left(\mathcal{S}=\{1,2\}\right)+Term6\left(\mathcal{S}=\{1,3\}\right)+Term7\left(\mathcal{S}=\{2,3\}\right)+Term8\left(\mathcal{S}=\{1,2,3\}\right)$$

$$=1-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)$$

$$-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)$$

$$-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)$$

$$\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right), \tag{3.97}$$

where

$$Term1\left(\mathcal{S}=\varnothing\right)=1-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)$$

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$$-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right), \tag{3.98}$$

$$\begin{aligned}
Term2\left(\mathcal{S}=\{1\}\right) &= \exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&+\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&+\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&+\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&-\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{1D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right),
\end{aligned} \tag{3.99}$$

$$\begin{aligned}
Term3\left(\mathcal{S}=\{2\}\right) &= \exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)-\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&-\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)-\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&+\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&+\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{2D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\\
&+\exp\left(-\frac{d_{\text{S2}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S1}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)\exp\left(-\frac{d_{\text{S3}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\text{SNR}}\right)
\end{aligned}$$

$$-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right),$$

(3.100)

$$\begin{aligned}
Term4\left(\mathcal{S}=\{3\}\right) &= \exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&+\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&+\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&+\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right),
\end{aligned}$$

(3.101)

$$\begin{aligned}
Term5\left(\mathcal{S}=\{1,2\}\right) &= \exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right) \\
&+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathsf{SNR}}\right)
\end{aligned}$$

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$$+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right),$$

(3.102)

$$Term6\left(\mathcal{S}=\{1,3\}\right)=\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

$$\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right),$$

(3.103)

$$Term7\left(\mathcal{S}=\{2,3\}\right)=\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)$$

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$$
\begin{aligned}
&\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right),
\end{aligned}
\tag{3.104}
$$

$$
\begin{aligned}
&Term8\left(\mathcal{S}=\{1,2,3\}\right)=\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\\
&\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)
\end{aligned}
$$

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$$
\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)
$$

$$
\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)
$$

$$
\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right)
$$

$$
\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{\left(2^{2R}-1\right)}{\mathrm{SNR}}\right),
\tag{3.105}
$$

and the simpler formula gives

$$
\mathbb{P}_{\mathrm{out}}^{\mathrm{FSDF\text{-}nodirect}}(R,\mathrm{SNR})=\prod_{k=1}^{3}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\right]
$$

$$
=\left[1-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}+d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\right]\left[1-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}+d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\right]
$$

$$
\left[1-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}+d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\right]
$$

$$
=1-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)
$$

$$
-\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)-\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)
$$

$$
+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)
$$

$$
+\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)
$$

$$
+\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)
$$

$$
-\exp\left(-\frac{d_{\mathrm{S1}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{1D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{S2}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{2D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)
$$

$$\exp\left(-\frac{d_{\mathrm{S3}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right)\exp\left(-\frac{d_{\mathrm{3D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right), \tag{3.106}$$

which is the same as (3.97).

3.2.4 Computation time comparison

After checking that the proposed formula gives the answer equal to the counterpart formula in the literature, we then compare the computation time used by the proposed formula and the counterpart formula. The testing computer is the Intel Pentium M processor 1.6 GHz with 768 MB of RAM. The testing software is MATLAB version 7.2.9.232 (R2006a) using the command cputime, which returns the CPU time that has been used by the MATLAB process in seconds. The comparison is shown in Table 3. When we increase the number of relays, the increased computation time required by the proposed formula is not observable because the number of terms in the formula increases linearly. On the other hand, the computation time required by the counterpart formula in the literature gets long abruptly because the number of terms in the formula increases combinatorially.

Table 3 Computation time comparison between two exact formulas

Number of relays	In this thesis	In the literature
1	0.01	0.02
2	0.01	0.02
3	0.01	0.02
4	0.01	0.02
5	0.01	0.02
6	0.01	0.02
7	0.01	0.03
8	0.01	0.03
9	0.01	0.06
10	0.01	0.09

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11	0.01	0.18
12	0.01	0.34
13	0.01	0.66
14	0.01	1.31
15	0.01	2.66
16	0.01	5.37
17	0.01	10.69
18	0.01	21.82
19	0.01	43.82
20	0.01	88.61

3.3 Fixed Selective Decode-and-forward with Direct Link Combining Scheme

The formula for calculating the outage probability of the fixed selective decode-and-forward with direct link combining scheme does not exist in standard materials. Even though introducing the direct link combining does not completely change the fixed selective decode-and-forward without direct link combining scheme, it becomes much more involved to analyze the outage probability. In the literature, there are two different formulas that can calculate the approximated outage probability of the fixed selective decode-and-forward without direct link combining scheme in the low SNR regime and in the high SNR regime, respectively. We propose a formula that can calculate the exact value. We will present three of them in this dissertation.

3.3.1 Approximation in the low SNR regime

For doing comparisons among different schemes later, the outage probability is still a function of a desired transmission rate R and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula for approximation in the low SNR regime is provided in Proposition 2.

Proposition 2 : The outage probability, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward with direct link combining scheme in the wireless relay network having K relay nodes, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is approximated in the low SNR regime by [23]

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\mathsf{SNR}) \approx \sum_{l=0}^{K} \sum_{\substack{\mathcal{S}\subseteq\mathcal{K} \\ |\mathcal{S}|=l}} \Bigg\{ d_{\text{SD}}^{\alpha} \left[\prod_{k\in\mathcal{S}} f_r \exp\left(-\frac{d_{\text{kD}}^{\alpha}}{d_{\text{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathsf{SNR}} \right) \right] \sum_{i=0}^{l} \binom{l}{i} \left(\frac{1}{d_{\text{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathsf{SNR}} \right)^{3(l-i)} (-1)^{i}$$
$$\left(\frac{L^{3i+1}}{3i+1} - \frac{d_{\text{SD}}^{\alpha} L^{3i+2}}{(3i+2)} + \frac{1}{2} \frac{d_{\text{SD}}^{2\alpha} L^{3i+3}}{(3i+3)} - \frac{1}{6} \frac{d_{\text{SD}}^{3\alpha} L^{3i+4}}{(3i+4)} \right)$$
$$\prod_{k\in\mathcal{S}} \left[\exp\left(-\frac{d_{\text{Sk}}^{\alpha}}{d_{\text{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathsf{SNR}} \right) \right] \prod_{k\notin\mathcal{S}} \left[1-\exp\left(-\frac{d_{\text{Sk}}^{\alpha}}{d_{\text{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathsf{SNR}} \right) \right] \Bigg\} \tag{3.107}$$

where $\mathcal{K} \in \{1,2,3,\ldots,K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}$ denotes the outage probability of the fixed selective decode-and-forward with direct link combining scheme, d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay node and the destination node, and α is the path loss exponent, and

$$L = \frac{1}{d_{\text{SD}}^{\alpha}} \min\left(\frac{\left(1+\sqrt{2}\right)^{\frac{2}{3}} - 1 + \left(1+\sqrt{2}\right)^{\frac{1}{3}}}{\left(1+\sqrt{2}\right)^{\frac{1}{3}}}, \frac{2^{2R}-1}{\mathsf{SNR}} \right). \tag{3.108}$$

Proof : From the definition, the outage probability is the probability that the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links from the source node to the destination node and from the relay node to the destination node falls below the desired transmission rate.

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) = \Pr\left\{\max_k \min\left\{\frac{1}{2}I_{Sk}\left(SNR_{Sk}\right), \frac{1}{2}I_{\text{SD}+k\text{D}}\left(SNR_{k\text{D}}+SNR_{\text{SD}}\right)\right\} < R\right\} \tag{3.109}$$

where I_{Sk} denotes the instantaneous mutual information between the source node and the k th relay node, $I_{\text{SD}+k\text{D}}$ denotes the instantaneous mutual information between the source node combined with the k th relay node and the destination node, where both of them are the functions of the instantaneous signal-to-noise ratio, denoted by SNR_{Sk} and SNR_{SD}, $SNR_{k\text{D}}$, respectively, between the source node and the k th relay node, and between the source node and the destination node and between the k th relay node and the destination node. The factor of 1/2 takes into account the loss due to dividing the transmission into two time slots. The maximum of the minimum can be written as the joint maximums.

$$\begin{aligned}\Pr\left\{\max_k \min\left\{\frac{1}{2}I_{Sk}\left(SNR_{Sk}\right), \frac{1}{2}I_{\text{SD}+k\text{D}}\left(SNR_{k\text{D}}+SNR_{\text{SD}}\right)\right\} < R\right\} = \\ \Pr\left\{\max_k\left\{\frac{1}{2}I_{Sk}\left(SNR_{Sk}\right)\right\} < R \wedge \max_k\left\{\frac{1}{2}I_{\text{SD}+k\text{D}}\left(SNR_{k\text{D}}+SNR_{\text{SD}}\right)\right\} < R\right\}.\end{aligned} \tag{3.110}$$

Furthermore, the joint probability can be written as the conditional probability with the normalization. Accordingly, we guarantee that the links from the source node to the designated subset of relay nodes can support the desired transmission rate first, and then calculate the probability that the direct link combined with the strongest link from the relay node to the destination node can support the desired transmission rate. Thus, we have to consider all possible disjoint subsets of the relay nodes, i.e., all combinations of the relay nodes that can receive information from the source node successfully. For the sake of presentation, we refer to this disjoint subset as decoding subset.

$$\Pr\left\{\max_k\left\{\frac{1}{2}I_{Sk}\left(SNR_{Sk}\right)\right\} < R \wedge \max_k\left\{\frac{1}{2}I_{\text{SD}+k\text{D}}\left(SNR_{\text{SD}}+SNR_{k\text{D}}\right)\right\} < R\right\}$$

$$= \sum_{l=0}^{K} \sum_{\substack{\mathcal{S} \subseteq \mathcal{K} \\ |\mathcal{S}|=l}} \Pr\left\{ \max_{k \in \mathcal{S}} \left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}} \left(SNR_{\mathrm{SD}} + SNR_{k\mathrm{D}} \right) \right\} < R \middle| \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \wedge \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}$$

$$\Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \wedge \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}$$

$$= \sum_{l=0}^{K} \sum_{\substack{\mathcal{S} \subseteq \mathcal{K} \\ |\mathcal{S}|=l}} \Pr\left\{ \max_{k \in \mathcal{S}} \left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}} \left(SNR_{\mathrm{SD}} + SNR_{k\mathrm{D}} \right) \right\} < R \middle| \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \wedge \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}$$

$$\Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \right\} \Pr\left\{ \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}, \tag{3.111}$$

where $\mathcal{S}$ is the decoding subset, which has a varying cardinality ranging from 0 until K, and is defined by

$$\mathcal{S} = \left\{ k \in \mathcal{K} \middle| \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \geq R \right\}. \tag{3.112}$$

Given the instantaneous signal-to-noise ratio between the source node and the k th relay node, the instantaneous mutual information between the source node and the k th relay node can be calculated by

$$I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) = \log_2 \left(1 + SNR_{\mathrm{S}k} \right). \tag{3.113}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the k th relay node, multiplied by the average transmit signal power at the source node $P \triangleq \mathbb{E}\left\{ \left| x_{\mathrm{S}} \right|^2 \right\}$, and divided by the noise power spectral density at the k th relay node, which is assumed to be N_0 for all relay nodes, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{S}k} = \frac{\left| h_{\mathrm{S}k} \right|^2 P}{N_0 W}, \tag{3.114}$$

where P, N_0, and W are constant, and $\left| h_{\mathrm{S}k} \right|^2$ is a random variable. Thus,

$$\Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \right\} = \Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} \log_2 \left(1 + SNR_{\mathrm{S}k} \right) \geq R \right\}$$

$$= \Pr\left\{ \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) \geq R \right\}. \tag{3.115}$$

Since $h_{\mathrm{S}k}$ are statistically independent among $k \in \mathcal{K}$, the expression is distributive and can be further manipulated.

$$\begin{aligned}
\Pr\left\{ \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) \geq R \right\} &= \bigcap_{k\in\mathcal{S}} \Pr\left\{ \frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) \geq R \right\} \\
&= \prod_{k\in\mathcal{S}} \Pr\left\{ \left|h_{\mathrm{S}k}\right|^2 \geq \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\} \\
&= \prod_{k\in\mathcal{S}} \left[1-\Pr\left\{ \left|h_{\mathrm{S}k}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\} \right].
\end{aligned} \tag{3.116}$$

Since $h_{\mathrm{S}k}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{S}k}\right)$, $\left|h_{\mathrm{S}k}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{S}k}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{\mathrm{S}k}\right|^2}\left(x\right) = \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right). \tag{3.117}$$

Then,

$$\begin{aligned}
\Pr\left\{ \left|h_{\mathrm{S}k}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} p_{\left|h_{\mathrm{S}k}\right|^2}\left(x\right) dx \\
&= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right) dx \\
&= -\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \\
&= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0 W}{\Omega_{\mathrm{S}k} P}\right).
\end{aligned} \tag{3.118}$$

Therefore,

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$$\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\right\}=\prod_{k\in\mathcal{S}}\left[1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\right]$$
$$=\prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]. \quad (3.119)$$

Likewise,

$$\Pr\left\{\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)< R\right\}=\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]. \quad (3.120)$$

Now, let consider the conditional probability. Given the instantaneous signal-to-noise ratio between the k th relay node and the destination node and the instantaneous signal-to-noise ratio between the source node and the destination node, the instantaneous mutual information between source node combined with the k th relay node and the destination node can be calculated by

$$I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)=\log_2\left(1+SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right). \quad (3.121)$$

The instantaneous signal-to-noise ratios are the squared modulus of the complex channel coefficient between the source node and the destination node, and the k th relay node and the destination node, respectively, multiplied by the average transmit signal power at the k th relay node, which is assume to equal that of the source node $\mathbb{E}\left\{\left|x_k\right|^2\right\}=P$, and divided by the noise power spectral density at the destination N_0, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{SD}}=\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}, \quad (3.122)$$

$$SNR_{k\mathrm{D}}=\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}, \quad (3.123)$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{SD}}\right|^2$ and $\left|h_{k\mathrm{D}}\right|^2$ are random variables. Thus,

$$\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)\right\}<R\left|\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right.\right\}$$

$$=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)\right\}<R\left|\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right.\right\}$$

$$=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)\right\}<R\left|\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right.\right\}$$

(3.124)

Since the channel coefficients of the links from the source node to every relay node are statistically independent from the channel coefficients of the links from the source node to the destination node and from every relay node to the destination node, the conditional probability becomes probability.

$$\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)\right\}<R\left|\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right.\right\}$$

$$=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)\right\}<R\right\}$$

$$=\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)<R\right\}.$$

(3.125)

By introducing the condition on the channel coefficients of the links from the source node to the destination node with marginalization, the new expression equals the previous expression but becomes distributive and can be further manipulated.

$$\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)<R\right\}$$

$$=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{xP}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)<R\left|\left|h_{\mathrm{SD}}\right|^2=x\right.\right\}p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx$$

$$=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\prod_{k\in\mathcal{S}}\Pr\left\{\frac{1}{2}\log_2\left(1+\frac{xP}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)<R\right\}p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx$$

$$= \int_{0}^{\frac{\left(2^{2R}-1\right)N_0 W}{P}} \prod_{k\in\mathcal{S}} \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \frac{\left(2^{2R}-1\right)N_0 W}{P} - x\right\} p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx \tag{3.126}$$

Notice that the upper limit of integration is not infinity because the conditioned probability becomes zero after the instantaneous signal-to-noise ratio from the source node to the destination node is greater than the upper limit, i.e., the strength of the channel from the source node to the destination node alone guarantees the success of transmission regardless the help from any relay node. Since $h_{k\mathrm{D}}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{k\mathrm{D}}\right)$, $\left|h_{k\mathrm{D}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{k\mathrm{D}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right) = \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right). \tag{3.127}$$

Likewise,

$$p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right) = \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right). \tag{3.128}$$

Then,

$$\begin{aligned}
\Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P} - x\right\} &= \int_{0}^{\left(2^{2R}-1\right)\frac{N_0 W}{P}-x} p_{\left|h_{k\mathrm{D}}\right|^2}\left(x'\right)dx' \\
&= \int_{0}^{\left(2^{2R}-1\right)\frac{N_0 W}{P}-x} \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)dx' \\
&= -\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)\Bigg|_{0}^{\left(2^{2R}-1\right)\frac{N_0 W}{P}-x} \\
&= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0 W}{\Omega_{k\mathrm{D}}P} + \frac{x}{\Omega_{k\mathrm{D}}}\right).
\end{aligned} \tag{3.129}$$

Therefore,

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$$\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{SD}}|^2 P}{N_0W}+\frac{|h_{k\mathrm{D}}|^2 P}{N_0W}\right)<R\right\}$$
$$=\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{(2^{2R}-1)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx$$
$$=\frac{1}{\Omega_{\mathrm{SD}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{(2^{2R}-1)N_0W}{\Omega_{k\mathrm{D}}P}\right)\left(\exp\left(\frac{(2^{2R}-1)N_0W}{\Omega_{k\mathrm{D}}P}\right)-\exp\left(\frac{x}{\Omega_{k\mathrm{D}}}\right)\right)\right]\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx.$$

(3.130)

Since $\Omega_{k\mathrm{D}}=d_{k\mathrm{D}}^{-\alpha}$ and is the mean of $|h_{k\mathrm{D}}|^2$, the average signal-to-noise ratio of the link between the k th relay node and the destination node can be written as

$$\begin{aligned}\mathbb{E}\{SNR_{k\mathrm{D}}\}&=\mathbb{E}\{|h_{k\mathrm{D}}|^2\}\frac{P}{N_0W}\\&=\Omega_{k\mathrm{D}}\frac{P}{N_0W}\\&=d_{k\mathrm{D}}^{-\alpha}\frac{P}{N_0W}.\end{aligned} \tag{3.131}$$

Comparing (3.131) with (3.9), we obtain

$$\frac{\Omega_{k\mathrm{D}}P}{N_0W}=\frac{d_{k\mathrm{D}}^{-\alpha}}{d_{\mathrm{SD}}^{-\alpha}}\mathrm{SNR}. \tag{3.132}$$

Substituting (3.132) into (3.130), we obtain

$$\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{SD}}|^2 P}{N_0W}+\frac{|h_{k\mathrm{D}}|^2 P}{N_0W}\right)<R\right\}$$
$$=\frac{1}{\Omega_{\mathrm{SD}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\left(\exp\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)-\exp\left(\frac{x}{\Omega_{k\mathrm{D}}}\right)\right)\right]\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx.$$

(3.133)

The evaluation of this integral is difficult due to the product of the exponential. We thus make two approximations:

1) Tayloy Series expansion of

$$\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right) \approx \begin{cases} \left(1-\frac{x}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^2}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^3}{\Omega_{\mathrm{SD}}^3}\right) & ,0 \le x \le x_0 \\ 0 & ,x > x_0 \end{cases} \quad (3.134)$$

Because Ω_{SD} is always positive, the expression on the right side of (3.134) is a decreasing function of x. This is readily shown by examining its derivative with respect to x:

$$\frac{d}{dx}\left(1-\frac{x}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^2}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^3}{\Omega_{\mathrm{SD}}^3}\right)=-\frac{1}{2}\frac{1}{\Omega_{\mathrm{SD}}^3}\left(x-\frac{1}{\Omega_{\mathrm{SD}}^2}\right)^2-\frac{1}{2}\frac{1}{\Omega_{\mathrm{SD}}}<0, \forall x, \Omega_{\mathrm{SD}} \ge 0. \quad (3.135)$$

x_0 is the point where this expression becomes negative, i.e.,

$$\left(1-\frac{x}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^2}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^3}{\Omega_{\mathrm{SD}}^3}\right)=0, \quad (3.136)$$

which occurs at

$$x_0 = \Omega_{\mathrm{SD}} \frac{\left(1+\sqrt{2}\right)^{\frac{2}{3}}-1+\left(1+\sqrt{2}\right)^{\frac{1}{3}}}{\left(1+\sqrt{2}\right)^{\frac{1}{3}}} \quad (3.137)$$

2) Third-order approximation of

$$\exp\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)-\exp\left(\frac{x}{\Omega_{k\mathrm{D}}}\right) \approx f_r\left(\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3-x^3\right), \quad (3.138)$$

where f_r will be determined later.

The second term could also be expanded using Taylor series; the product of the terms, however, would significantly increase the number of terms in the approximation. We choose order-three approximations which yield good results. Clearly, the accuracy of the approximations could be further increased by increasing the approximation order.

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Using the approximation in (3.134), the upper limit of the integral in (3.133) is L, where

$$L = \min\left(x_0, \frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right). \tag{3.139}$$

The parameter f_r is then obtained by minimizing $E\left(f_r\right)$, the total squared error

$$es\left(x, f_r\right) = \left| \left(\exp\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right) - \exp\left(\frac{x}{\Omega_{k\mathrm{D}}} \right) \right) - f_r \left(\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^3 - x^3 \right) \right|^2$$

over the range $\left(0, L\right)$.

$$\begin{aligned} E\left(f_r\right) &= \int_0^L es\left(x, f_r\right) dx \\ &= \int_0^L \left| \left(\exp\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right) - \exp\left(\frac{x}{\Omega_{k\mathrm{D}}} \right) \right) - f_r \left(\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^3 - x^3 \right) \right|^2 dx \qquad (3.140) \\ &= \frac{\Omega_{k\mathrm{D}}^4}{14} \left[f_r\left(\Theta_1\right) + f_r^2\left(\Theta_2\right) + \Theta_3 \right], \end{aligned}$$

where

$$\begin{aligned} \Theta_1 = &-28 \exp\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right) \left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^3 \frac{L}{\Omega_{k\mathrm{D}}^4} + 7 \exp\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right) \frac{L^4}{\Omega_{k\mathrm{D}}^4} \\ &-28 \left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^3 \frac{1}{\Omega_{k\mathrm{D}}^3} - 168 + 28 \exp\left(\frac{L}{\Omega_{k\mathrm{D}}} \right) \left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^3 \frac{1}{\Omega_{k\mathrm{D}}^3} - 28 \exp\left(\frac{L}{\Omega_{k\mathrm{D}}} \right) \frac{L^3}{\Omega_{k\mathrm{D}}^3} \\ &+84 \exp\left(\frac{L}{\Omega_{k\mathrm{D}}} \right) \frac{L^2}{\Omega_{k\mathrm{D}}^2} - 168 \exp\left(\frac{L}{\Omega_{k\mathrm{D}}} \right) \frac{L}{\Omega_{k\mathrm{D}}} + 168 \exp\left(\frac{L}{\Omega_{k\mathrm{D}}} \right), \end{aligned}$$

$$\Theta_2 = 14 \left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^6 \frac{L}{\Omega_{k\mathrm{D}}^4} - 7 \left(\frac{1}{d_{\mathrm{SD}}^{\alpha}} \frac{2^{2R}-1}{\mathrm{SNR}} \right)^3 \frac{L^4}{\Omega_{k\mathrm{D}}^4} + 2 \frac{L^7}{\Omega_{k\mathrm{D}}^4},$$

and

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$$\Theta_3 = 28\exp\left(\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\frac{1}{\Omega_{k\mathrm{D}}^3} - 7\frac{1}{\Omega_{k\mathrm{D}}^3} + 14\exp\left(2\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\frac{L}{\Omega_{k\mathrm{D}}^4}$$
$$-28\exp\left(\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\exp\left(\frac{L}{\Omega_{k\mathrm{D}}}\right)\frac{1}{\Omega_{k\mathrm{D}}^3} + 7\exp\left(2\frac{L}{\Omega_{k\mathrm{D}}}\right)\frac{1}{\Omega_{k\mathrm{D}}^3}.$$

To minimize $E(f_r)$, we set the derivative of this expression with respect to f_r to zero and obtain

$$f_r = \frac{-7\Omega_{k\mathrm{D}}^4}{2L\left(7\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3 L^3 + 2L^6 + 14\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^6\right)}$$
$$\left[\begin{array}{l}\frac{L}{\Omega_{k\mathrm{D}}^4}\exp\left(\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\left(-4\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3 + L^3\right) + 4\frac{1}{\Omega_{k\mathrm{D}}^3}\left(\begin{array}{l}\exp\left(\frac{L}{\Omega_{k\mathrm{D}}}\right)\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3\\ -\exp\left(\frac{L}{\Omega_{k\mathrm{D}}}\right)L^3 - \left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3\end{array}\right)\\ +12\frac{L^2}{\Omega_{k\mathrm{D}}^2}\exp\left(\frac{L}{\Omega_{k\mathrm{D}}}\right) - 24\frac{L}{\Omega_{k\mathrm{D}}}\exp\left(\frac{L}{\Omega_{k\mathrm{D}}}\right) + 24\exp\left(\frac{L}{\Omega_{k\mathrm{D}}}\right) - 24\end{array}\right]. \tag{3.141}$$

Using Binomial Expansion, the resulting outage probability approximation can thus be written as

$$\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{SD}}|^2 P}{N_0 W}+\frac{|h_{k\mathrm{D}}|^2 P}{N_0 W}\right) < R\right\}$$
$$\approx \frac{1}{\Omega_{\mathrm{SD}}}\int_0^L\left[\prod_{k\in\mathcal{S}}\exp\left(-\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)f_r\left(\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3 - x^3\right)\right]\left(1-\frac{x}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^2}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^3}{\Omega_{\mathrm{SD}}^3}\right)dx$$
$$= \frac{1}{\Omega_{\mathrm{SD}}}\left[\prod_{k\in\mathcal{S}}f_r\exp\left(-\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right]\int_0^L\left(\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^3 - x^3\right)^l\left(1-\frac{x}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^2}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^3}{\Omega_{\mathrm{SD}}^3}\right)dx$$
$$= \frac{1}{\Omega_{\mathrm{SD}}}\left[\prod_{k\in\mathcal{S}}f_r\exp\left(-\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right]\int_0^L\sum_{i=0}^{l}\binom{l}{i}\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^{3(l-i)}\left(-x^3\right)^i\left(1-\frac{x}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^2}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^3}{\Omega_{\mathrm{SD}}^3}\right)dx$$
$$= \frac{1}{\Omega_{\mathrm{SD}}}\left[\prod_{k\in\mathcal{S}}f_r\exp\left(-\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right]\sum_{i=0}^{l}\binom{l}{i}\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^{3(l-i)}(-1)^i\int_0^L\left(x^{3i}-\frac{x^{3i+1}}{\Omega_{\mathrm{SD}}}+\frac{1}{2}\frac{x^{3i+2}}{\Omega_{\mathrm{SD}}^2}-\frac{1}{6}\frac{x^{3i+3}}{\Omega_{\mathrm{SD}}^3}\right)dx$$

$$
\begin{aligned}
&= \frac{1}{\Omega_{\mathrm{SD}}}\left[\prod_{k\in\mathcal{S}} f_r \exp\left(-\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right]\sum_{i=0}^{l}\binom{l}{i}\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^{3(l-i)}(-1)^{i} \\
&\left(\frac{L^{3i+1}}{3i+1}-\frac{L^{3i+2}}{\Omega_{\mathrm{SD}}(3i+2)}+\frac{1}{2}\frac{L^{3i+3}}{\Omega_{\mathrm{SD}}^{2}(3i+3)}-\frac{1}{6}\frac{L^{3i+4}}{\Omega_{\mathrm{SD}}^{3}(3i+4)}\right)
\end{aligned}
\qquad (3.142)
$$

Substituting (3.119), (3.120) and (3.142) into (3.111), we obtain

$$
\begin{aligned}
&\Pr\left\{\max_{k}\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\right\}<R\wedge\max_{k}\left\{\frac{1}{2}I_{\mathrm{SD+kD}}\left(SNR_{\mathrm{SD}}+SNR_{\mathrm{kD}}\right)\right\}<R\right\} \\
&=\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\left\{\frac{1}{\Omega_{\mathrm{SD}}}\left[\prod_{k\in\mathcal{S}} f_r \exp\left(-\frac{d_{\mathrm{kD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right]\sum_{i=0}^{l}\binom{l}{i}\left(\frac{1}{d_{\mathrm{SD}}^{\alpha}}\frac{2^{2R}-1}{\mathrm{SNR}}\right)^{3(l-i)}(-1)^{i}\right. \\
&\left(\frac{L^{3i+1}}{3i+1}-\frac{L^{3i+2}}{\Omega_{\mathrm{SD}}(3i+2)}+\frac{1}{2}\frac{L^{3i+3}}{\Omega_{\mathrm{SD}}^{2}(3i+3)}-\frac{1}{6}\frac{L^{3i+4}}{\Omega_{\mathrm{SD}}^{3}(3i+4)}\right) \\
&\left.\prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]\right\}
\end{aligned}
\qquad (3.143)
$$

Since $\Omega_{\mathrm{S}k}=d_{\mathrm{S}k}^{-\alpha}$ and is the mean of $|h_{\mathrm{S}k}|^2$, the average signal-to-noise ratio of the link between the source node and the k th relay node can be written as

$$
\begin{aligned}
\mathbb{E}\left\{SNR_{\mathrm{S}k}\right\} &= \mathbb{E}\left\{|h_{\mathrm{S}k}|^{2}\right\}\frac{P}{N_0W} \\
&= \Omega_{\mathrm{S}k}\frac{P}{N_0W} \\
&= d_{\mathrm{S}k}^{-\alpha}\frac{P}{N_0W}.
\end{aligned}
\qquad (3.144)
$$

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Comparing (3.144) with (3.9), we obtain

$$
\frac{\Omega_{\mathrm{S}k}P}{N_0W}=\frac{d_{\mathrm{S}k}^{-\alpha}}{d_{\mathrm{SD}}^{-\alpha}}\mathrm{SNR}. \qquad (3.145)
$$

Substituting $\Omega_{\mathrm{SD}}=d_{\mathrm{SD}}^{-\alpha}$ and (3.145) into (3.143) yields (3.107), in which L in (3.108) is obtained by substituting $\Omega_{\mathrm{SD}}=d_{\mathrm{SD}}^{-\alpha}$ into (3.137) and in turn substituting x_0 from (3.137) into (3.139). Q.E.D.

3.3.2 Approximation in the high SNR regime

For doing comparisons among different schemes later, the outage probability is still a function of a desired transmission rate R and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula for approximation in the high SNR regime is provided in Proposition 3.

Proposition 3 : The outage probability, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward with direct link combining scheme in the wireless relay network having K relay nodes, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is approximated in the high SNR regime by [22]

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\mathsf{SNR}) \approx \left(d_{\text{SD}}^{\alpha} \frac{2^{2R}-1}{\mathsf{SNR}} \right)^{K+1} d_{\text{SD}}^{\alpha} \sum_{l=0}^{K} \sum_{\substack{\mathcal{S}\subseteq\mathcal{K} \\ |\mathcal{S}|=l}} \frac{1}{l+1} \prod_{k\in\mathcal{S}} d_{k\text{D}}^{\alpha} \prod_{k\notin\mathcal{S}} d_{\text{S}k}^{\alpha}. \quad (3.146)$$

where $\mathcal{K} \in \{1,2,3,\ldots,K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}$ denotes the outage probability of the fixed selective decode-and-forward with direct link combining scheme, d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay node and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage probability is the probability that the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links from the source node to the destination node and from the relay node to the destination node falls below the desired transmission rate.

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) = \Pr\left\{\max_k \min\left\{\frac{1}{2}I_{\text{S}k}\left(SNR_{\text{S}k}\right), \frac{1}{2}I_{\text{SD+}k\text{D}}\left(SNR_{k\text{D}} + SNR_{\text{SD}}\right)\right\} < R\right\} \tag{3.147}$$

where $I_{\text{S}k}$ denotes the instantaneous mutual information between the source node and the k th relay node, $I_{\text{SD+}k\text{D}}$ denotes the instantaneous mutual information between the source node combined with the k th relay node and the destination node, where both of them are the functions of the instantaneous signal-to-noise ratio, denoted by $SNR_{\text{S}k}$ and SNR_{SD}, $SNR_{k\text{D}}$, respectively, between the source node and the k th relay node, and between the source node and the destination node and between the k th relay node and the destination node. The factor of 1/2 takes into account the loss due to dividing the transmission into two time slots. The maximum of the minimum can be written as the joint maximums.

$$\begin{aligned} &\Pr\left\{\max_k \min\left\{\frac{1}{2}I_{\text{S}k}\left(SNR_{\text{S}k}\right), \frac{1}{2}I_{\text{SD+}k\text{D}}\left(SNR_{k\text{D}} + SNR_{\text{SD}}\right)\right\} < R\right\} = \\ &\qquad \Pr\left\{\max_k\left\{\frac{1}{2}I_{\text{S}k}\left(SNR_{\text{S}k}\right)\right\} < R \wedge \max_k\left\{\frac{1}{2}I_{\text{SD+}k\text{D}}\left(SNR_{k\text{D}} + SNR_{\text{SD}}\right)\right\} < R\right\}. \end{aligned} \tag{3.148}$$

Furthermore, the joint probability can be written as the conditional probability with the normalization. Accordingly, we guarantee that the links from the source node to the designated subset of relay nodes can support the desired transmission rate first, and then calculate the probability that the direct link combined with the strongest link from the relay node to the destination node can support the desired transmission rate. Thus, we have to consider all possible disjoint subsets of the relay nodes, i.e., all combinations of the relay nodes that can receive information from the source node successfully. For the sake of presentation, we refer to this disjoint subset as decoding subset.

$$\Pr\left\{\max_k\left\{\frac{1}{2}I_{\text{S}k}\left(SNR_{\text{S}k}\right)\right\} < R \wedge \max_k\left\{\frac{1}{2}I_{\text{SD+}k\text{D}}\left(SNR_{\text{SD}} + SNR_{k\text{D}}\right)\right\} < R\right\}$$

$$= \sum_{l=0}^{K} \sum_{\substack{\mathcal{S} \subseteq \mathcal{K} \\ |\mathcal{S}|=l}} \Pr\left\{ \max_{k \in \mathcal{S}} \left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}} \left(SNR_{\mathrm{SD}} + SNR_{k\mathrm{D}} \right) \right\} < R \middle| \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \wedge \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}$$

$$\Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \wedge \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}$$

$$= \sum_{l=0}^{K} \sum_{\substack{\mathcal{S} \subseteq \mathcal{K} \\ |\mathcal{S}|=l}} \Pr\left\{ \max_{k \in \mathcal{S}} \left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}} \left(SNR_{\mathrm{SD}} + SNR_{k\mathrm{D}} \right) \right\} < R \middle| \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \wedge \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\}$$

$$\Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \right\} \Pr\left\{ \bigcap_{k \notin \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) < R \right\},$$

(3.149)

where $\mathcal{S}$ is the decoding subset, which has a varying cardinality ranging from 0 until K, and is defined by

$$\mathcal{S} = \left\{ k \in \mathcal{K} \middle| \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \geq R \right\}. \tag{3.150}$$

Given the instantaneous signal-to-noise ratio between the source node and the k th relay node, the instantaneous mutual information between the source node and the k th relay node can be calculated by

$$I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) = \log_2 \left(1 + SNR_{\mathrm{S}k} \right). \tag{3.151}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the k th relay node, multiplied by the average transmit signal power at the source node $P \triangleq \mathbb{E}\left\{ |x_{\mathrm{S}}|^2 \right\}$, and divided by the noise power spectral density at the k th relay node, which is assumed to be N_0 for all relay nodes, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{S}k} = \frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}, \tag{3.152}$$

where P, N_0, and W are constant, and $|h_{\mathrm{S}k}|^2$ is a random variable. Thus,

$$\Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} I_{\mathrm{S}k} \left(SNR_{\mathrm{S}k} \right) \nless R \right\} = \Pr\left\{ \bigcap_{k \in \mathcal{S}} \frac{1}{2} \log_2 \left(1 + SNR_{\mathrm{S}k} \right) \geq R \right\}$$

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$$= \Pr\left\{ \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}\right) \geq R \right\}. \qquad (3.153)$$

Since $h_{\mathrm{S}k}$ are statistically independent among $k \in \mathcal{K}$, the expression is distributive and can be further manipulated.

$$\begin{aligned}
\Pr\left\{ \bigcap_{k\in\mathcal{S}} \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}\right) \geq R \right\} &= \bigcap_{k\in\mathcal{S}} \Pr\left\{ \frac{1}{2}\log_2\left(1+\frac{|h_{\mathrm{S}k}|^2 P}{N_0 W}\right) \geq R \right\} \\
&= \prod_{k\in\mathcal{S}} \Pr\left\{ |h_{\mathrm{S}k}|^2 \geq \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\} \qquad (3.154) \\
&= \prod_{k\in\mathcal{S}} \left[1-\Pr\left\{ |h_{\mathrm{S}k}|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\} \right].
\end{aligned}$$

Since $h_{\mathrm{S}k}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{S}k}\right)$, $|h_{\mathrm{S}k}|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{S}k}^{-1}\right)$ (see Lemma 1).

$$p_{|h_{\mathrm{S}k}|^2}(x) = \frac{1}{\Omega_{\mathrm{S}k}} \exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right). \qquad (3.155)$$

Then,

$$\begin{aligned}
\Pr\left\{ |h_{\mathrm{S}k}|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P} \right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} p_{|h_{\mathrm{S}k}|^2}(x)\,dx \\
&= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \frac{1}{\Omega_{\mathrm{S}k}} \exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right) dx \qquad (3.156) \\
&= -\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \\
&= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0 W}{\Omega_{\mathrm{S}k} P}\right).
\end{aligned}$$

Therefore,

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$$\begin{aligned}\Pr\left\{\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\nless R\right\} &= \prod_{k\in\mathcal{S}}\left[1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\right] \\ &= \prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right].\end{aligned} \tag{3.157}$$

Likewise,

$$\Pr\left\{\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)< R\right\} = \prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]. \tag{3.158}$$

In the high SNR regime, the multiplication between (3.157) and (3.158) can be approximated by

$$\prod_{k\in\mathcal{S}}\left[\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right]\prod_{k\notin\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right] \approx \left[\frac{\left(2^{2R}-1\right)N_0W}{P}\right]^{K-l}\prod_{k\notin\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}}. \tag{3.159}$$

Now, let consider the conditional probability. Given the instantaneous signal-to-noise ratio between the k th relay node and the destination node and the instantaneous signal-to-noise ratio between the source node and the destination node, the instantaneous mutual information between source node combined with the k th relay node and the destination node can be calculated by

$$I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)=\log_2\left(1+SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right). \tag{3.160}$$

The instantaneous signal-to-noise ratios are the squared modulus of the complex channel coefficient between the source node and the destination node, and the k th relay node and the destination node, respectively, multiplied by the average transmit signal power at the k th relay node, which is assume to equal that of the source node $\mathbb{E}\left\{\left|x_k\right|^2\right\}=P$, and divided by the noise power spectral density at the destination N_0, and divided by the transmission bandwidth W.

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$$SNR_{\mathrm{SD}} = \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}, \tag{3.161}$$

$$SNR_{k\mathrm{D}} = \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}, \tag{3.162}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{SD}}\right|^2$ and $\left|h_{k\mathrm{D}}\right|^2$ are random variables. Thus,

$$\begin{aligned}
&\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)\right\}<R\,\middle|\,\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\not<R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\\
&=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)\right\}<R\,\middle|\,\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\not<R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\\
&=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0W}\right)\right\}<R\,\middle|\,\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\not<R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}
\end{aligned} \tag{3.163}$$

Since the channel coefficients of the links from the source node to every relay node are statistically independent from the channel coefficients of the links from the source node to the destination node and from every relay node to the destination node, the conditional probability becomes probability.

$$\begin{aligned}
&\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0W}\right)\right\}<R\,\middle|\,\bigcap_{k\in\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\not<R\wedge\bigcap_{k\notin\mathcal{S}}\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\\
&=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0W}\right)\right\}<R\right\}.
\end{aligned} \tag{3.164}$$

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Define the random variables X and Y as

$$X = \max_{k\in\mathcal{S}}\left\{\left|h_{k\mathrm{D}}\right|^2\right\}, \tag{3.165}$$

and

$$Y = \left|h_{\mathrm{SD}}\right|^2. \tag{3.166}$$

Since $h_{k\mathrm{D}}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{k\mathrm{D}}\right)$, $\left|h_{k\mathrm{D}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{k\mathrm{D}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right)=\frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right). \tag{3.167}$$

Likewise,

$$p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)=\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right). \tag{3.168}$$

Then, the cumulative distribution function of X can be derived as follows.

$$\begin{aligned}
F_X\left(x\right)&=\Pr\left\{\max_{k\in\mathcal{S}}\left\{\left|h_{k\mathrm{D}}\right|^2\right\}<x\right\}\\
&=\Pr\left\{\bigcap_{k\in\mathcal{S}}\left|h_{k\mathrm{D}}\right|^2<x\right\}\\
&=\prod_{k\in\mathcal{S}}\Pr\left\{\left|h_{k\mathrm{D}}\right|^2<x\right\}\\
&=\prod_{k\in\mathcal{S}}\int_0^x p_{\left|h_{k\mathrm{D}}\right|^2}\left(x'\right)dx'\\
&=\prod_{k\in\mathcal{S}}\int_0^x \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)dx'\\
&=\prod_{k\in\mathcal{S}}-\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)\Bigg|_0^x\\
&=\prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right)\right],
\end{aligned} \tag{3.169}$$

where the equality between the second line and the third line is due to the independence of $\left|h_{k\mathrm{D}}\right|^2,\forall k$. Consider the expression in (3.164),

$$\begin{aligned}
\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)\right\}&=\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{SD}}\right|^2P}{N_0W}+\max_{k\in\mathcal{S}}\frac{\left|h_{k\mathrm{D}}\right|^2P}{N_0W}\right)\\
&=\frac{1}{2}\log_2\left(1+\left(\left|h_{\mathrm{SD}}\right|^2+\max_{k\in\mathcal{S}}\left|h_{k\mathrm{D}}\right|^2\right)\frac{P}{N_0W}\right)
\end{aligned}$$

$$=\frac{1}{2}\log_2\left(1+\left(Y+X\right)\frac{P}{N_0W}\right). \tag{3.170}$$

Then,

$$\Pr\left\{\max_{k\in\mathcal{S}}\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\text{SD}}\right|^2P}{N_0W}+\frac{\left|h_{k\text{D}}\right|^2P}{N_0W}\right)\right\}<R\right\}=\Pr\left\{\frac{1}{2}\log_2\left(1+\left(Y+X\right)\frac{P}{N_0W}\right)<R\right\}$$
$$=\Pr\left\{Y+X<\frac{\left(2^{2R}-1\right)N_0W}{P}\right\}. \tag{3.171}$$

By introducing the condition on the channel coefficients of the links from the source node to the destination node with marginalization, the new expression equals the previous expression but becomes distributive and can be further manipulated.

$$\Pr\left\{Y+X<\frac{\left(2^{2R}-1\right)N_0W}{P}\right\}=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\Pr\left\{Y+X<\frac{\left(2^{2R}-1\right)N_0W}{P}\middle|y\right\}p_Y\left(y\right)dy$$
$$=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\Pr\left\{X<\frac{\left(2^{2R}-1\right)N_0W}{P}-y\right\}p_Y\left(y\right)dy. \tag{3.172}$$

Notice that the upper limit of integration is not infinity because the conditioned probability becomes zero after the instantaneous signal-to-noise ratio from the source node to the destination node is greater than the upper limit, i.e., the strength of the channel from the source node to the destination node alone guarantees the success of transmission regardless the help from any relay node. Using (3.168) and (3.169), the integration becomes

$$\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\Pr\left\{X<\frac{\left(2^{2R}-1\right)N_0W}{P}-y\right\}p_Y\left(y\right)dy$$
$$=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\text{D}}P}+\frac{y}{\Omega_{k\text{D}}}\right)\right]\frac{1}{\Omega_{\text{SD}}}\exp\left(-\frac{y}{\Omega_{\text{SD}}}\right)dy \tag{3.173}$$

When $\frac{P}{N_0W}\rightarrow\infty$, the argument of the exponential function becomes small. Therefore, the approximation $\exp(x)\approx 1+x$ can be applied.

$$\begin{aligned}&\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[1-\exp\left(-\frac{(2^{2R}-1)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{y}{\Omega_{k\mathrm{D}}}\right)\right]\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy\\ \approx&\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[\frac{(2^{2R}-1)N_0W}{\Omega_{k\mathrm{D}}P}-\frac{y}{\Omega_{k\mathrm{D}}}\right]\frac{1}{\Omega_{\mathrm{SD}}}\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy.\end{aligned}\tag{3.174}$$

With some manipulations,

$$\begin{aligned}&\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[\frac{(2^{2R}-1)N_0W}{\Omega_{k\mathrm{D}}P}-\frac{y}{\Omega_{k\mathrm{D}}}\right]\frac{1}{\Omega_{\mathrm{SD}}}\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy\\ &=\frac{1}{\Omega_{\mathrm{SD}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\prod_{k\in\mathcal{S}}\left[\frac{1}{\Omega_{k\mathrm{D}}}\left(\frac{(2^{2R}-1)N_0W}{P}-y\right)\right]\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy\\ &=\frac{1}{\Omega_{\mathrm{SD}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\left(\frac{(2^{2R}-1)N_0W}{P}-y\right)^{l}\prod_{k\in\mathcal{S}}\left[\frac{1}{\Omega_{k\mathrm{D}}}\right]\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy\\ &=\frac{1}{\Omega_{\mathrm{SD}}}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\left[\frac{(2^{2R}-1)N_0W}{P}-y\right]^{l}\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy.\end{aligned}\tag{3.175}$$

Using the binomial expansion,

$$\begin{aligned}&\frac{1}{\Omega_{\mathrm{SD}}}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\left[\frac{(2^{2R}-1)N_0W}{P}-y\right]^{l}\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy\\ &=\frac{1}{\Omega_{\mathrm{SD}}}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\int_0^{\frac{(2^{2R}-1)N_0W}{P}}\sum_{i=0}^{l}\binom{l}{i}\left(\frac{(2^{2R}-1)N_0W}{P}\right)^{l-i}(-y)^{l}\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy.\end{aligned}\tag{3.176}$$

After solving the integral, some manipulations and the use of identity 0.155 from [32], this expression reduces to

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$$\frac{1}{\Omega_{\mathrm{SD}}}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\int_{0}^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\sum_{i=0}^{l}\binom{l}{i}\left(\frac{\left(2^{2R}-1\right)N_0W}{P}\right)^{l-i}(-y)^{i}\left(1-\frac{y}{\Omega_{\mathrm{SD}}}\right)dy$$

$$=\left(\frac{\left(2^{2R}-1\right)N_0W}{P}\right)^{l+1}\frac{1}{\Omega_{\mathrm{SD}}}\left[\sum_{i=0}^{l}\binom{l}{i}\frac{(-1)^{i}}{i+1}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\right] \tag{3.177}$$

$$=\left(\frac{\left(2^{2R}-1\right)N_0W}{P}\right)^{l+1}\frac{1}{\Omega_{\mathrm{SD}}}\frac{1}{l+1}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}.$$

Substituting (3.159) and (3.177) into (3.149), we obtain

$$\Pr\left\{\max_{k}\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\right\}<R\wedge\max_{k}\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{\mathrm{SD}}+SNR_{k\mathrm{D}}\right)\right\}<R\right\}$$

$$=\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\left(\frac{\left(2^{2R}-1\right)N_0W}{P}\right)^{l+1}\frac{1}{\Omega_{\mathrm{SD}}}\frac{1}{l+1}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\left[\frac{\left(2^{2R}-1\right)N_0W}{P}\right]^{K-l}\prod_{k\notin\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}} \tag{3.178}$$

$$=\left(\frac{\left(2^{2R}-1\right)N_0W}{P}\right)^{K+1}\frac{1}{\Omega_{\mathrm{SD}}}\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\frac{1}{l+1}\prod_{k\in\mathcal{S}}\frac{1}{\Omega_{k\mathrm{D}}}\prod_{k\notin\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}}.$$

Substituting (3.9), $\Omega_{\mathrm{SD}}=d_{\mathrm{SD}}^{-\alpha}$, $\Omega_{\mathrm{S}k}=d_{\mathrm{S}k}^{-\alpha}$, and $\Omega_{k\mathrm{D}}=d_{k\mathrm{D}}^{-\alpha}$ into (3.178) yields (3.146). Q.E.D.

3.3.3 Exact Formula

For doing comparisons among different schemes later, the outage probability is still a function of a desired transmission rate R and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula for exact value at any SNR is provided in Theorem 3.

Theorem 3 : The outage probability, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward with direct link combining scheme in the wireless relay network having K relay nodes, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [28]

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) = 1 + \sum_{l=1}^{K} \sum_{\substack{\mathcal{S}\subseteq\mathcal{K} \\ |\mathcal{S}|=l}} (-1)^l \frac{\sum_{k\in\mathcal{S}} \left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right) \exp\left(-\left(1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\right)\frac{2^{2R}-1}{\text{SNR}}\right) - \exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\text{SNR}}\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1} \tag{3.179}$$

where $\mathcal{K} \in \{1,2,3,\ldots,K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}$ denotes the outage probability of the fixed selective decode-and-forward with direct link combining scheme, d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay node and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage probability is the probability that the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links from the source node to the destination node and from the relay node to the destination node falls below the desired transmission rate.

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) = \Pr\left\{\max_{k} \min\left\{\frac{1}{2} I_{\text{S}k}\left(SNR_{\text{S}k}\right), \frac{1}{2} I_{\text{SD}+k\text{D}}\left(SNR_{k\text{D}} + SNR_{\text{SD}}\right)\right\} < R\right\} \tag{3.180}$$

where $I_{\text{S}k}$ denotes the instantaneous mutual information between the source node and the k th relay node, $I_{\text{SD}+k\text{D}}$ denotes the instantaneous mutual information between the source node combined with the k th relay node and the destination node, where both of them are the functions of the instantaneous signal-to-noise ratio, denoted by $SNR_{\text{S}k}$ and SNR_{SD}, $SNR_{k\text{D}}$, respectively, between the source node and the k th relay node, and between the source node and the destination node and between the k th relay node and the destination node. The factor of 1/2 takes into account the loss due to dividing the transmission into two time slots.

By introducing the condition on the channel coefficients of the links from the source node to the destination node with marginalization, the new expression equals the previous expression but becomes distributive and can be further manipulated.

$$\begin{aligned}
&\Pr\left\{\max_k \min\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right), \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right)\right\} < R\right\} \\
&= \Pr\left\{\bigcap_k \min\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right), \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right)\right\} < R\right\} \\
&= \int_0^\infty \Pr\left\{\bigcap_k \min\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right), \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right)\right\} < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma\right\} p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma \\
&= \int_0^\infty \prod_{k=1}^{K} \Pr\left\{\min\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right), \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right)\right\} < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma\right\} p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma \\
&= \int_0^\infty \prod_{k=1}^{K} \left[1 - \Pr\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) \geq R \wedge \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) \geq R \,\middle|\, SNR_{\mathrm{SD}} = \gamma\right\}\right] p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma
\end{aligned} \tag{3.181}$$

The joint probability becomes a product of probabilities because the condition on the instantaneous signal-to-noise ratio between the source node and the destination node introduces the independence between the two events, which depend on the instantaneous signal-to-noise ration between the source node and the k th relay node, and the instantaneous signal-to-noise ratio between the k th relay node and the destination node and the instantaneous signal-to-noise ratio between the source and the destination node.

$$\int_0^\infty \prod_{k=1}^{K} \left[1 - \Pr\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) \geq R \wedge \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) \geq R \,\middle|\, SNR_{\mathrm{SD}} = \gamma\right\}\right] p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma$$

$$= \int_0^\infty \prod_{k=1}^{K} \begin{bmatrix} 1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) \geq R \middle| SNR_{\mathrm{SD}} = \gamma \right\} \\ \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) \geq R \middle| SNR_{\mathrm{SD}} = \gamma \right\} \end{bmatrix} p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma$$

$$= \int_0^\infty \prod_{k=1}^{K} \begin{bmatrix} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\}\right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\}\right) \end{bmatrix} p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma \quad (3.182)$$

$$= \int_0^\infty \prod_{k=1}^{K} \begin{bmatrix} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\}\right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\}\right) \end{bmatrix} p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma$$

where the last line comes from the independence between the channel coefficient from the source node to the k th relay node and the channel coefficient from the source node to the destination node. Considering the remaining conditional probability, it becomes zero when

$$\frac{1}{2} I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) \geq R \quad (3.183)$$

because the strength of the channel from the source node to the destination node alone guarantees that the mutual information of the combined links exceeds the targeted transmission rate. Given the instantaneous signal-to-noise ratio between the k th relay node and the destination node and the instantaneous signal-to-noise ratio between the source node and the destination node, the instantaneous mutual information between source node combined with the k th relay node and the destination node can be calculated by

$$I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) = \log_2\left(1 + SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right). \quad (3.184)$$

Therefore,

$$\Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\} = \begin{cases} 0, \gamma \geq 2^{2R} - 1 \\ 1, \gamma < 2^{2R} - 1 \end{cases}. \quad (3.185)$$

Hence, the integration can be split into two intervals with different integrands.

$$\int_0^{\infty}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\right)\\ \left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)<R\middle|SNR_{\mathrm{SD}}=\gamma\right\}\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma$$

$$=\int_0^{2^{2R}-1}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\right)\\ \left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)<R\middle|SNR_{\mathrm{SD}}=\gamma\right\}\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma$$

$$+\int_{2^{2R}-1}^{\infty}\prod_{k=1}^{K}\left[1-\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\right)\left(1-0\right)\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma$$

$$=\int_0^{2^{2R}-1}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\right)\\ \left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)<R\middle|SNR_{\mathrm{SD}}=\gamma\right\}\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma$$

$$+\int_{2^{2R}-1}^{\infty}\prod_{k=1}^{K}\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma \tag{3.186}$$

Given the instantaneous signal-to-noise ratio between the source node and the k th relay node, the instantaneous mutual information between the source node and the k th relay node can be calculated by

$$I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)=\log_2\left(1+SNR_{\mathrm{S}k}\right). \tag{3.187}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the k th relay node, multiplied by the average transmit signal power at the source node $P\triangleq\mathbb{E}\left\{\left|x_{\mathrm{S}}\right|^2\right\}$, and divided by the noise power spectral density at the k th relay node, which is assumed to be N_0 for all relay nodes, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{S}k}=\frac{\left|h_{\mathrm{S}k}\right|^2P}{N_0W}, \tag{3.188}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{S}k}\right|^2$ is a random variable. Thus,

$$
\begin{aligned}
\Pr\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R\right\} &= \Pr\left\{\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) < R\right\} \\
&= \Pr\left\{\left|h_{\mathrm{S}k}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P}\right\}.
\end{aligned}
\tag{3.189}
$$

Since $h_{\mathrm{S}k}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{S}k}\right)$, $\left|h_{\mathrm{S}k}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{S}k}^{-1}\right)$ (see Lemma 1).

$$
p_{\left|h_{\mathrm{S}k}\right|^2}\left(x\right) = \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right).
\tag{3.190}
$$

Therefore,

$$
\begin{aligned}
\Pr\left\{\left|h_{\mathrm{S}k}\right|^2 < \left(2^{2R}-1\right)\frac{N_0 W}{P}\right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} p_{\left|h_{\mathrm{S}k}\right|^2}\left(x\right)dx \\
&= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)dx \\
&= -\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \\
&= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0 W}{\Omega_{\mathrm{S}k} P}\right).
\end{aligned}
\tag{3.191}
$$

Now, let consider the conditional probability. The instantaneous signal-to-noise ratios are the squared modulus of the complex channel coefficient between the source node and the destination node, and the k th relay node and the destination node, respectively, multiplied by the average transmit signal power at the k th relay node, which is assume to equal that of the source node $\mathbb{E}\left\{\left|x_k\right|^2\right\} = P$, and divided by the noise power spectral density at the destination N_0, and divided by the transmission bandwidth W.

$$
SNR_{\mathrm{SD}} = \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W},
\tag{3.192}
$$

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$$SNR_{k\mathrm{D}} = \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}, \tag{3.193}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{SD}}\right|^2$ and $\left|h_{k\mathrm{D}}\right|^2$ are random variables. Thus,

$$\begin{aligned}
&\Pr\left\{\frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma\right\} \\
&\quad = \Pr\left\{\frac{1}{2}\log_2\left(1 + \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W} + \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right) < R \,\middle|\, \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W} = \gamma\right\} \\
&\quad = \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R} - 1 - \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right)\frac{N_0 W}{P} \,\middle|\, \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W} = \gamma\right\}
\end{aligned} \tag{3.194}$$

where the last line comes from the fact that the condition changes $\left|h_{\mathrm{SD}}\right|^2$ from being the random variable to the constant. For convenience, we define x such that

$$\gamma = \frac{xP}{N_0 W}. \tag{3.195}$$

Hence,

$$\begin{aligned}
&\Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \frac{N_0 W}{P}\left(2^{2R} - 1 - \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right) \,\middle|\, \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W} = \gamma\right\} \\
&\quad = \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R} - 1 - \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right)\frac{N_0 W}{P} \,\middle|\, \left|h_{\mathrm{SD}}\right|^2 = x\right\} \\
&\quad = \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R} - 1\right)\frac{N_0 W}{P} - x\right\}.
\end{aligned} \tag{3.196}$$

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Since $h_{k\mathrm{D}}$ is a random variable with the probability density function $\mathcal{CN}\left(0, \Omega_{k\mathrm{D}}\right)$, $\left|h_{k\mathrm{D}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{k\mathrm{D}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{k\mathrm{D}}\right|^2}(x) = \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right). \tag{3.197}$$

Then,

$$
\begin{aligned}
\Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R}-1\right)\frac{N_0W}{P}-x\right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}-x} p_{\left|h_{k\mathrm{D}}\right|^2}\left(x'\right)dx' \\
&= \int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}-x} \frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)dx' \\
&= -\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}-x} \\
&= 1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right).
\end{aligned}
\tag{3.198}
$$

Then, using (3.191) and (3.198) in (3.186), we obtain

$$
\begin{aligned}
&\int_0^{2^{2R}-1}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\right) \\ \left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)<R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma \\
&+\int_{2^{2R}-1}^{\infty}\prod_{k=1}^{K}\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma \\
&=\int_0^{2^{2R}-1}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\right) \\ \left(1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right)\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma \\
&+\int_{2^{2R}-1}^{\infty}\prod_{k=1}^{K}\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma \\
&=\int_0^{2^{2R}-1}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma \\
&+\int_{2^{2R}-1}^{\infty}\prod_{k=1}^{K}\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma
\end{aligned}
\tag{3.199}
$$

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Since h_{SD} is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{SD}}\right)$, $\left|h_{\mathrm{SD}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{SD}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)=\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right). \tag{3.200}$$

From (3.192) and (3.200), we obtain

$$p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma=p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx. \tag{3.201}$$

Applying (3.201) in (3.199), the limits of the integration change accordingly.

$$\begin{aligned}
&\int_{0}^{2^{2R}-1}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\
&+\int_{2^{2R}-1}^{\infty}\prod_{k=1}^{K}\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\
&=\int_{0}^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx\\
&+\int_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}\prod_{k=1}^{K}\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx\\
&=\int_{0}^{\frac{\left(2^{2R}-1\right)N_0W}{P}}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx\\
&+\int_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}\prod_{k=1}^{K}\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx.
\end{aligned} \tag{3.202}$$

By expanding the products and distributing the terms, we obtain

$$
\begin{aligned}
&\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}} \prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx \\
&+\int_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty} \prod_{k=1}^{K}\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx \\
&=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}} \left[1+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\exp\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)x\right)\right] \\
&\qquad \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx \\
&+\int_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}\left(1+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}}\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\right)\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx \\
&=\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}} \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right) \\
&\qquad \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)x\right)dx \\
&+\int_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}}\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx.
\end{aligned}
\tag{3.203}
$$

Integrating both integrands and manipulating them further, we obtain

$$
\begin{aligned}
&\int_0^{\frac{\left(2^{2R}-1\right)N_0W}{P}} \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right) \\
&\qquad \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)x\right)dx \\
&+\int_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}}\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx
\end{aligned}
$$

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$$= -\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)\Bigg|_{0}^{\frac{\left(2^{2R}-1\right)N_0W}{P}}$$

$$+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\frac{(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)x\right)}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}\Bigg|_{0}^{\frac{\left(2^{2R}-1\right)N_0W}{P}}$$

$$-\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)\Bigg|_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\sum_{k\in\mathcal{S}}\frac{1}{\Omega_{\mathrm{S}k}}\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\frac{\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)}{-1}\Bigg|_{\frac{\left(2^{2R}-1\right)N_0W}{P}}^{\infty}$$

$$= -\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)+1$$

$$+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\,\frac{\exp\left(-\left(\frac{1}{\Omega_{\mathrm{SD}}}+\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}\right)\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)-\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}$$

$$+\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\exp\left(-\left(\frac{1}{\Omega_{\mathrm{SD}}}+\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}\right)\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)$$

$$=1+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\,\frac{\left[\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)\exp\left(-\left(\frac{1}{\Omega_{\mathrm{SD}}}+\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}\right)\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)-\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\right]}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}$$

$$= 1 + \sum_{l=1}^{K} \sum_{\substack{\mathcal{S} \subseteq \mathcal{K} \\ |\mathcal{S}| = l}} (-1)^l \frac{\left[\begin{array}{l} \sum_{k \in \mathcal{S}} \left(\frac{\Omega_{\text{SD}}}{\Omega_{k\text{D}}} \right) \exp\left(-\left(1 + \sum_{k \in \mathcal{S}} \left(\frac{\Omega_{\text{SD}}}{\Omega_{\text{S}k}} \right) \right) \frac{\left(2^{2R} - 1\right) N_0 W}{\Omega_{\text{SD}} P} \right) \\ - \exp\left(-\sum_{k \in \mathcal{S}} \left(\frac{\Omega_{\text{SD}}}{\Omega_{\text{S}k}} + \frac{\Omega_{\text{SD}}}{\Omega_{k\text{D}}} \right) \frac{\left(2^{2R} - 1\right) N_0 W}{\Omega_{\text{SD}} P} \right) \end{array} \right]}{\sum_{k \in \mathcal{S}} \left(\frac{\Omega_{\text{SD}}}{\Omega_{k\text{D}}} \right) - 1} \quad (3.204)$$

Substituting $\Omega_{\text{S}k} = d_{\text{S}k}^{-\alpha}$, $\Omega_{k\text{D}} = d_{k\text{D}}^{-\alpha}$, $\Omega_{\text{SD}} = d_{\text{SD}}^{-\alpha}$ into (3.204), and comparing it with (3.9) yields (3.179). Q.E.D.

3.3.4 Computation time comparison

It should be noted that the approximations in the literature were not done for reducing computational complexity. The approximations were done only because it was difficult to analyze the exact formula, and can even use longer computation time. We compare the computation time used by the proposed exact formula and the approximated formulas in the literature as shown in Table 4. The testing computer is the Intel Pentium M processor 1.6 GHz with 768 MB of RAM. The testing software is MATLAB version 7.2.9.232 (R2006a) using the command cputime, which returns the CPU time that has been used by the MATLAB process in seconds. It can be observed that the approximation formula for low SNR regime uses much longer computation time than the exact formula. Therefore, there is no need to apply the approximation formula for low SNR because it is not only less accurate but also less efficient in computation, especially when the number of relays gets large. Compared to the approximation formula for high SNR, the exact formula uses around the same computation time. Therefore, the exact formula provides accuracy without introducing the computational burden.

Table 4 Computation time comparison between exact and approximated formulas

Number of relays	Exact	Approx. low SNR	Approx. high SNR
2	0.02	0.13	0.02
3	0.02	0.16	0.02

4	0.02	0.18	0.02
5	0.03	0.23	0.02
6	0.03	0.36	0.02
7	0.03	0.59	0.03
8	0.03	0.98	0.03
9	0.04	1.82	0.03
10	0.07	3.72	0.06
11	0.07	6.82	0.06
12	0.13	13.75	0.10
13	0.23	27.38	0.21
14	0.46	55.91	0.41

3.4 Smart Selective Decode-and-forward Scheme

The formula for calculating the outage probability of the smart selective decode-and-forward scheme does not exist in standard materials due to the complication, which is more than all fixed selective decode-and-forward schemes. Without an analytical result, the smart selective decode-and-forward scheme can only be studied by computer simulations. In this thesis, we propose a formula that can calculate the exact value at any SNR.

For doing comparisons among different schemes later, the outage probability is still a function of a desired transmission rate R and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula for exact value at any SNR is provided in Theorem 4.

Theorem 4 : The outage probability, which is obtained from employing the cooperative diversity with the smart selective decode-and-forward scheme in the wireless relay network having K relay nodes, at a transmission rate of R and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [28]

$$\mathbb{P}_{\mathrm{out}}^{\mathrm{SSDF}}(R,\mathrm{SNR}) = 1-\exp\left(-\frac{2^{R}-1}{\mathrm{SNR}}\right) + \sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\mathrm{SNR}}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1\right)\frac{2^{R}-1}{\mathrm{SNR}}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1} \quad (3.205)$$

where $\mathcal{K}\in\{1,2,3,\ldots,K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $\mathbb{P}_{\mathrm{out}}^{\mathrm{SSDF}}$ denotes the outage probability of the smart selective decode-and-forward scheme, d_{SD} is the distance between the source node and the destination node, $d_{\mathrm{S}k}$ is the distance between the source node and the k th relay node, $d_{k\mathrm{D}}$ is the distance between the k th relay node and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage probability is the probability that the higher between the instantaneous mutual information of the link from the source node to the destination node without transmission time dividing and the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links from the source node to the destination node and from the relay node to the destination node with two time slots dividing falls below the desired transmission rate.

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$$\mathbb{P}_{\mathrm{out}}^{\mathrm{SSDF}}(R,\mathrm{SNR}) = \Pr\left\{\max\left\{I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right),\max_{k}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)\right\}\right\}<R\right\} \quad (3.206)$$

where I_{SD} denotes the instantaneous mutual information between the source node and the destination node, $I_{\mathrm{S}k}$ denotes the instantaneous mutual information between the source node and the k th relay node, $I_{\mathrm{SD}+k\mathrm{D}}$ denotes the instantaneous mutual information between the source node combined with the k th relay node and the destination node, where both of them are the functions of the instantaneous signal-to-noise ratio, denoted by $SNR_{\mathrm{S}k}$ and SNR_{SD}, $SNR_{k\mathrm{D}}$, respectively, between the source node and the k th relay node, and between the source node and the destination node and between the k th relay node and the destination node. The factor of 1/2 takes into account the loss due to dividing the transmission into two time slots.

By introducing the condition on the channel coefficients of the links from the source node to the destination node with marginalization, the new expression equals the previous expression but becomes distributive and can be further manipulated.

$$
\begin{aligned}
&\Pr\left\{\max\left\{I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right),\max_{k}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)\right\}\right\}<R\right\}\\
&=\Pr\left\{\bigcap_{k}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)\right\}<R\cap I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right)<R\right\}\\
&=\int_{0}^{\infty}\Pr\left\{\bigcap_{k}\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)\right\}<R\cap I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right)<R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}\\
&\quad p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\
&=\int_{0}^{\infty}\Pr\left\{I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right)<R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}\\
&\quad\prod_{k=1}^{K}\Pr\left\{\min\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right),\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)\right\}<R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\
&=\int_{0}^{\infty}\Pr\left\{I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right)<R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}\\
&\quad\prod_{k=1}^{K}\left[1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)\geq R\wedge\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)\geq R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma
\end{aligned}
\tag{3.207}
$$

The joint probability becomes a product of probabilities because the condition on the instantaneous signal-to-noise ratio between the source node and the destination node introduces the independence between the two events, which depend on the instantaneous signal-to-noise ration between the source node and the k th relay node, and the instantaneous signal-to-noise ratio between the k th relay node and the destination node and the instantaneous signal-to-noise ratio between the source and the destination node.

$$
\begin{aligned}
&\int_0^\infty \Pr\left\{ I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \\
&\prod_{k=1}^{K} \left[1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) \geq R \wedge \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) \geq R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \right] p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma \\
&= \int_0^\infty \Pr\left\{ I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \\
&\quad \prod_{k=1}^{K} \left[\begin{array}{l} 1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) \geq R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \\ \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) \geq R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \end{array} \right] p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma \\
&= \int_0^\infty \Pr\left\{ I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \\
&\quad \prod_{k=1}^{K} \left[\begin{array}{l} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \right) \end{array} \right] p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma \\
&= \int_0^\infty \Pr\left\{ I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \\
&\quad \prod_{k=1}^{K} \left[\begin{array}{l} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma \right\} \right) \end{array} \right] p_{SNR_{\mathrm{SD}}}\left(\gamma\right) d\gamma
\end{aligned}
\tag{3.208}
$$

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where the last line comes from the independence between the channel coefficient from the source node to the k th relay node and the channel coefficient from the source node to the destination node. Considering the remaining conditional probabilities, they becomes zero when

$$I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) \geq R, \tag{3.209}$$

and

$$\frac{1}{2} I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) \geq R, \tag{3.210}$$

respectively, because the strength of the channel from the source node to the destination node alone guarantees that the mutual information of the direct link and the mutual information of the combined links exceed the targeted transmission rate. Given the instantaneous signal-to-noise ratio between the source node and the destination node, the instantaneous mutual information between the source node and the destination node can be calculated by

$$I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) = \log_2\left(1 + SNR_{\mathrm{SD}}\right). \tag{3.211}$$

Also, given the instantaneous signal-to-noise ratio between the k th relay node and the destination node and the instantaneous signal-to-noise ratio between the source node and the destination node, the instantaneous mutual information between source node combined with the k th relay node and the destination node can be calculated by

$$I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{\mathrm{SD}} + SNR_{k\mathrm{D}}\right) = \log_2\left(1 + SNR_{\mathrm{SD}} + SNR_{k\mathrm{D}}\right). \tag{3.212}$$

Therefore,

$$\Pr\left\{I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma\right\} = \begin{cases} 0, \gamma \geq 2^R - 1 \\ 1, \gamma < 2^R - 1 \end{cases}, \tag{3.213}$$

and

$$\Pr\left\{\frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma\right\} = \begin{cases} 0, \gamma \geq 2^{2R} - 1 \\ 1, \gamma < 2^{2R} - 1 \end{cases}. \tag{3.214}$$

Hence, the integration can be split into three intervals with different integrands.

$$\int_0^\infty \Pr\left\{ I_{\mathrm{SD}}\left(SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\}$$

$$\prod_{k=1}^{K} \left[\begin{array}{l} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\} \right) \end{array} \right] p_{SNR_{\mathrm{SD}}}(\gamma)\, d\gamma$$

$$= \int_0^{2^R-1} (1) \prod_{k=1}^{K} \left[\begin{array}{l} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\} \right) \end{array} \right] p_{SNR_{\mathrm{SD}}}(\gamma)\, d\gamma$$

$$+ \int_{2^R-1}^{2^{2R}-1} (0) \prod_{k=1}^{K} \left[\begin{array}{l} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\} \right) \end{array} \right] p_{SNR_{\mathrm{SD}}}(\gamma)\, d\gamma$$

$$+ \int_{2^{2R}-1}^{\infty} (0) \prod_{k=1}^{K} \left[1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \right) (1-0) \right] p_{SNR_{\mathrm{SD}}}(\gamma)\, d\gamma$$

$$= \int_0^{2^R-1} \prod_{k=1}^{K} \left[\begin{array}{l} 1 - \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R \right\} \right) \\ \left(1 - \Pr\left\{ \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \middle| SNR_{\mathrm{SD}} = \gamma \right\} \right) \end{array} \right] p_{SNR_{\mathrm{SD}}}(\gamma)\, d\gamma. \tag{3.215}$$

Given the instantaneous signal-to-noise ratio between the source node and the k th relay node, the instantaneous mutual information between the source node and the k th relay node can be calculated by

$$I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) = \log_2\left(1 + SNR_{\mathrm{S}k}\right). \tag{3.216}$$

The instantaneous signal-to-noise ratio is the squared modulus of the complex channel coefficient between the source node and the k th relay node, multiplied by the average transmit signal power at the source node $P \triangleq \mathbb{E}\left\{ \left| x_{\mathrm{S}} \right|^2 \right\}$, and divided by the noise power spectral density at the k th relay node, which is assumed to be N_0 for all relay nodes, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{S}k} = \frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}, \tag{3.217}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{S}k}\right|^2$ is a random variable. Thus,

$$\begin{aligned} \Pr\left\{\frac{1}{2} I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right) < R\right\} &= \Pr\left\{\frac{1}{2}\log_2\left(1 + \frac{\left|h_{\mathrm{S}k}\right|^2 P}{N_0 W}\right) < R\right\} \\ &= \Pr\left\{\left|h_{\mathrm{S}k}\right|^2 < \left(2^{2R} - 1\right)\frac{N_0 W}{P}\right\}. \end{aligned} \tag{3.218}$$

Since $h_{\mathrm{S}k}$ is a random variable with the probability density function $\mathcal{CN}\left(0, \Omega_{\mathrm{S}k}\right)$, $\left|h_{\mathrm{S}k}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{S}k}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{\mathrm{S}k}\right|^2}\left(x\right) = \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right). \tag{3.219}$$

Therefore,

$$\begin{aligned} \Pr\left\{\left|h_{\mathrm{S}k}\right|^2 < \left(2^{2R} - 1\right)\frac{N_0 W}{P}\right\} &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} p_{\left|h_{\mathrm{S}k}\right|^2}\left(x\right) dx \\ &= \int_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \frac{1}{\Omega_{\mathrm{S}k}}\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right) dx \\ &= -\exp\left(-\frac{x}{\Omega_{\mathrm{S}k}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0 W}{P}} \\ &= 1 - \exp\left(-\frac{\left(2^{2R} - 1\right) N_0 W}{\Omega_{\mathrm{S}k} P}\right). \end{aligned} \tag{3.220}$$

Now, let consider the conditional probability. The instantaneous signal-to-noise ratios are the squared modulus of the complex channel coefficient between the source node and the destination node, and the kth relay node and the destination node, respectively, multiplied by the average transmit signal power at the kth relay node, which is assume to equal that of the source node $\mathbb{E}\left\{\left|x_k\right|^2\right\} = P$, and divided by

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the noise power spectral density at the destination N_0, and divided by the transmission bandwidth W.

$$SNR_{\mathrm{SD}} = \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}, \tag{3.221}$$

$$SNR_{k\mathrm{D}} = \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W}, \tag{3.222}$$

where P, N_0, and W are constant, and $\left|h_{\mathrm{SD}}\right|^2$ and $\left|h_{k\mathrm{D}}\right|^2$ are random variables. Thus,

$$\begin{aligned}
&\Pr\left\{\frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}} + SNR_{\mathrm{SD}}\right) < R \,\middle|\, SNR_{\mathrm{SD}} = \gamma\right\} \\
&= \Pr\left\{\frac{1}{2}\log_2\left(1 + \frac{\left|h_{k\mathrm{D}}\right|^2 P}{N_0 W} + \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right) < R \,\middle|\, \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W} = \gamma\right\} \\
&= \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R} - 1 - \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right)\frac{N_0 W}{P} \,\middle|\, \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W} = \gamma\right\}
\end{aligned} \tag{3.223}$$

where the last line comes from the fact that the condition changes $\left|h_{\mathrm{SD}}\right|^2$ from being the random variable to the constant. For convenience, we define x such that

$$\gamma = \frac{xP}{N_0 W}. \tag{3.224}$$

Hence,

$$\begin{aligned}
&\Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \frac{N_0 W}{P}\left(2^{2R} - 1 - \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right) \,\middle|\, \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W} = \gamma\right\} \\
&= \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R} - 1 - \frac{\left|h_{\mathrm{SD}}\right|^2 P}{N_0 W}\right)\frac{N_0 W}{P} \,\middle|\, \left|h_{\mathrm{SD}}\right|^2 = x\right\} \\
&= \Pr\left\{\left|h_{k\mathrm{D}}\right|^2 < \left(2^{2R} - 1\right)\frac{N_0 W}{P} - x\right\}.
\end{aligned} \tag{3.225}$$

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Since $h_{k\mathrm{D}}$ is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{k\mathrm{D}}\right)$, $\left|h_{k\mathrm{D}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{k\mathrm{D}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{k\mathrm{D}}\right|^2}\left(x\right)=\frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x}{\Omega_{k\mathrm{D}}}\right). \tag{3.226}$$

Then,

$$\begin{aligned}\Pr\left\{\left|h_{k\mathrm{D}}\right|^2<\left(2^{2R}-1\right)\frac{N_0W}{P}-x\right\}&=\int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}-x}p_{\left|h_{k\mathrm{D}}\right|^2}\left(x'\right)dx'\\&=\int_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}-x}\frac{1}{\Omega_{k\mathrm{D}}}\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)dx'\\&=-\exp\left(-\frac{x'}{\Omega_{k\mathrm{D}}}\right)\Bigg|_0^{\left(2^{2R}-1\right)\frac{N_0W}{P}-x}\\&=1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right).\end{aligned} \tag{3.227}$$

Then, using (3.220) and (3.227) in (3.215), we obtain

$$\begin{aligned}&\int_0^{2^R-1}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{S}k}\left(SNR_{\mathrm{S}k}\right)<R\right\}\right)\\\left(1-\Pr\left\{\frac{1}{2}I_{\mathrm{SD}+k\mathrm{D}}\left(SNR_{k\mathrm{D}}+SNR_{\mathrm{SD}}\right)<R\,\middle|\,SNR_{\mathrm{SD}}=\gamma\right\}\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\&=\int_0^{2^R-1}\prod_{k=1}^{K}\left[\begin{array}{l}1-\left(1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\right)\right)\\\left(1-\left(1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right)\right)\end{array}\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\&=\int_0^{2^R-1}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma.\end{aligned} \tag{3.228}$$

Since h_{SD} is a random variable with the probability density function $\mathcal{CN}\left(0,\Omega_{\mathrm{SD}}\right)$, $\left|h_{\mathrm{SD}}\right|^2$ is an exponentially distributed random variable with the probability density function $\exp\left(\Omega_{\mathrm{SD}}^{-1}\right)$ (see Lemma 1).

$$p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)=\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right). \tag{3.229}$$

From (3.221) and (3.229), we obtain

$$p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma=p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx. \tag{3.230}$$

Applying (3.230) in (3.228), the limits of the integration change accordingly.

$$\begin{aligned}
&\int_0^{2^R-1}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]p_{SNR_{\mathrm{SD}}}\left(\gamma\right)d\gamma\\
&=\int_0^{\frac{\left(2^R-1\right)N_0W}{P}}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]p_{\left|h_{\mathrm{SD}}\right|^2}\left(x\right)dx\\
&=\int_0^{\frac{\left(2^R-1\right)N_0W}{P}}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx
\end{aligned} \tag{3.231}$$

By expanding the products and distributing the terms, we obtain

$$\begin{aligned}
&\int_0^{\frac{\left(2^R-1\right)N_0W}{P}}\prod_{k=1}^{K}\left[1-\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{S}k}P}\right)\exp\left(-\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{k\mathrm{D}}P}+\frac{x}{\Omega_{k\mathrm{D}}}\right)\right]\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx\\
&=\int_0^{\frac{\left(2^R-1\right)N_0W}{P}}\left[1+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}\left(-1\right)^l\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\exp\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)x\right)\right]\\
&\quad\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)dx
\end{aligned}$$

$$= \int_{0}^{\frac{\left(2^{R}-1\right)N_0W}{P}} \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right) + \sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}} (-1)^{l}\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)$$

$$\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)x\right)dx. \tag{3.232}$$

Integrating the integrand and manipulating them further, we obtain

$$\int_{0}^{\frac{\left(2^{R}-1\right)N_0W}{P}} \frac{1}{\Omega_{\mathrm{SD}}}\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right) + \sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}} (-1)^{l}\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)$$

$$\frac{1}{\Omega_{\mathrm{SD}}}\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)x\right)dx$$

$$= -\exp\left(-\frac{x}{\Omega_{\mathrm{SD}}}\right)\Bigg|_{0}^{\frac{\left(2^{R}-1\right)N_0W}{P}}$$

$$+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}} \frac{(-1)^{l}\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)x\right)}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}\Bigg|_{0}^{\frac{\left(2^{R}-1\right)N_0W}{P}}$$

$$= -\exp\left(-\frac{\left(2^{R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)+1$$

$$+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}} (-1)^{l}\frac{\left[\begin{array}{l}\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)\frac{\left(2^{R}-1\right)N_0W}{P}\right)\\ -\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\end{array}\right]}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}$$

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$$=1-\exp\left(-\frac{\left(2^{R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)$$

$$+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{\mathrm{S}k}}+\frac{1}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{P}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{1}{\Omega_{k\mathrm{D}}}\right)-\frac{1}{\Omega_{\mathrm{SD}}}\right)\frac{\left(2^{R}-1\right)N_0W}{P}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}$$

$$=1-\exp\left(-\frac{\left(2^{R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)$$

$$+\sum_{l=1}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{\mathrm{S}k}}+\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)\frac{\left(2^{2R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1\right)\frac{\left(2^{R}-1\right)N_0W}{\Omega_{\mathrm{SD}}P}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{\Omega_{\mathrm{SD}}}{\Omega_{k\mathrm{D}}}\right)-1}.$$

(3.233)

Substituting $\Omega_{\mathrm{S}k}=d_{\mathrm{S}k}^{-\alpha}$, $\Omega_{k\mathrm{D}}=d_{k\mathrm{D}}^{-\alpha}$, $\Omega_{\mathrm{SD}}=d_{\mathrm{SD}}^{-\alpha}$ into (3.233), and comparing it with (3.9) yields (3.205). Q.E.D.

3.5 Verification with Computer Simulations

In this section, we verify the analytical outage probabilities with Monte Carlo simulations. To cover several cases, we vary the number of relay nodes at 1, 4, or 9, and the target transmission rate at 1 b/s/Hz, 2 b/s/Hz, or 4 b/s/Hz. The relay nodes are arranged in grid topology between the source node and the destination node, between which the distance is 1000 m. All curves are plotted as a function of SNR, which is the average signal-to-noise ratio between the source node and the destination node.

In Figure 6 – Figure 14, we plot the analytical and simulated outage probabilities of the fixed selective decode-and-forward without direct link combining scheme, the fixed selective decode-and-forward with direct link combining scheme, and the smart selective decode-and-forward scheme with the number of relay nodes $K=\{1,4,9\}$ and at a rate $R=\{1,2,4\}$ b/s/Hz, as a function of the SNR. It can be observed that the analytical results are in good agreement with the simulation results, showing the validity of our analytical expressions in Theorem 1 – Theorem 4. Note that the outage probabilities of the fixed selective decode-and-forward without direct link combining obtained from Theorem 1 and Theorem 2 are exactly the same and give the overlap curves, which can be seen as one curve. This verifies that Theorem 2 simplifies the calculation in Theorem 1 without injuring exactness.

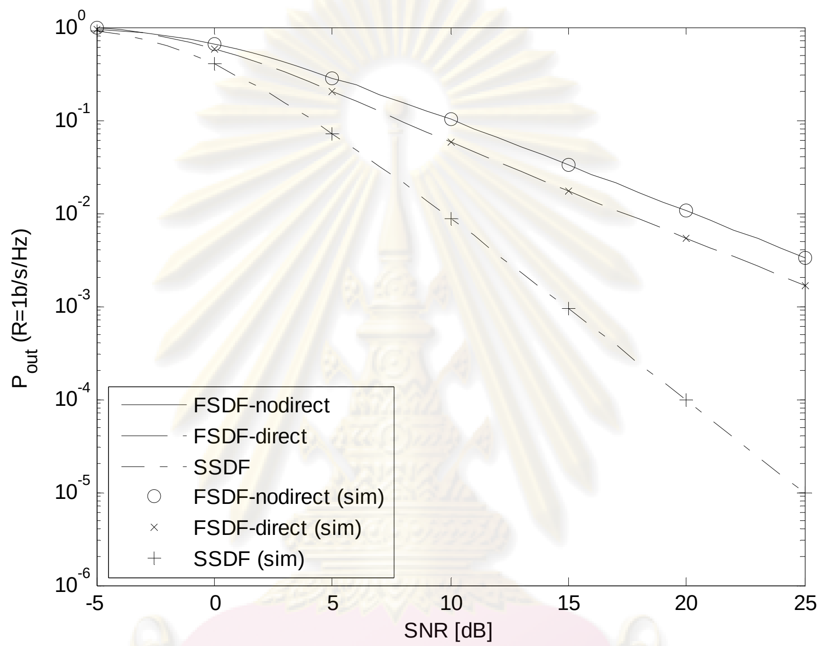

Figure 6 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 1 b/s/Hz for 1-relay network

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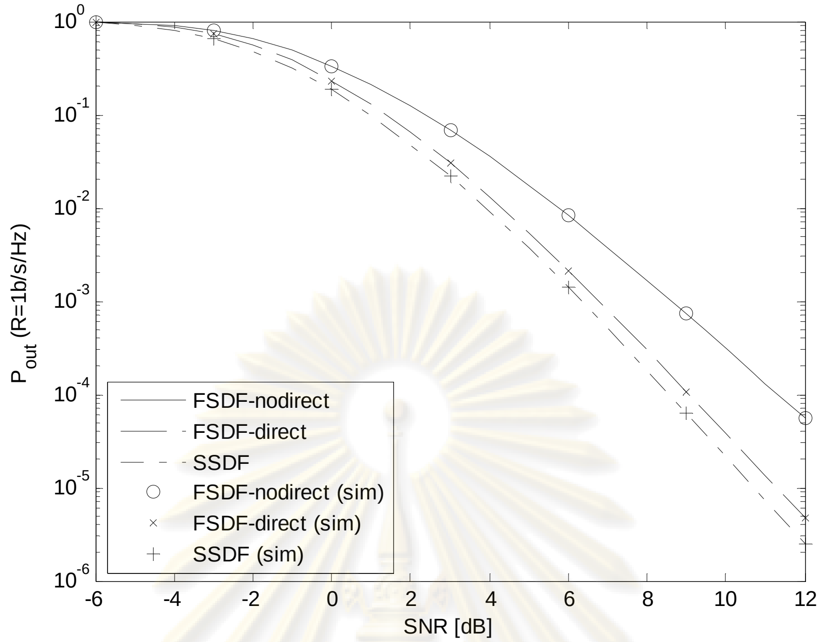

Figure 7 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 1 b/s/Hz for 4-relay network

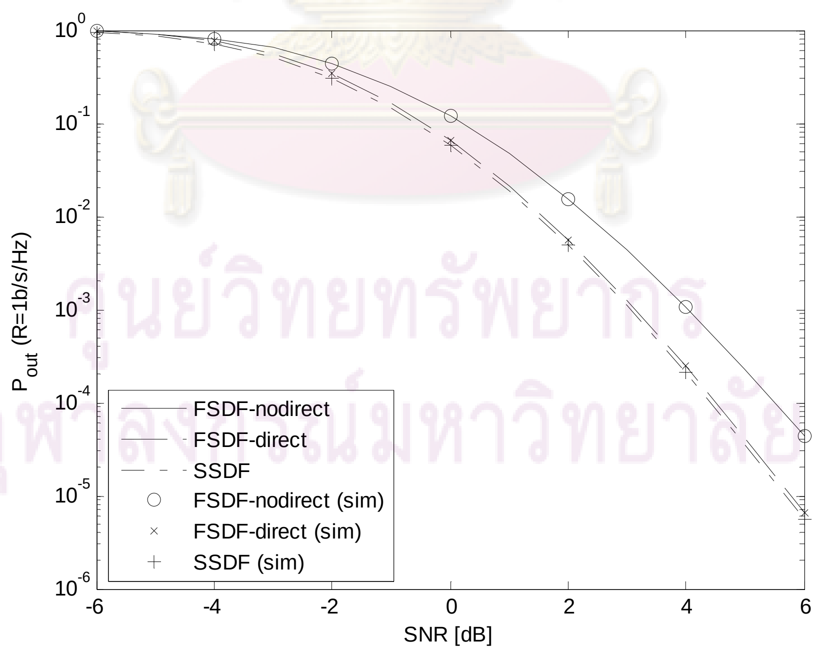

Figure 8 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 1 b/s/Hz for 9-relay network

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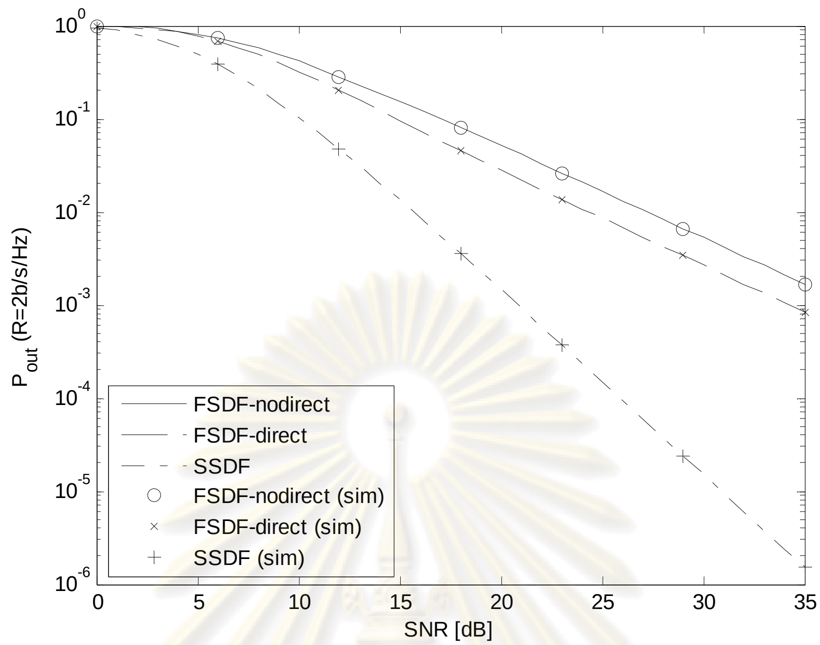

Figure 9 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 2 b/s/Hz for 1-relay network

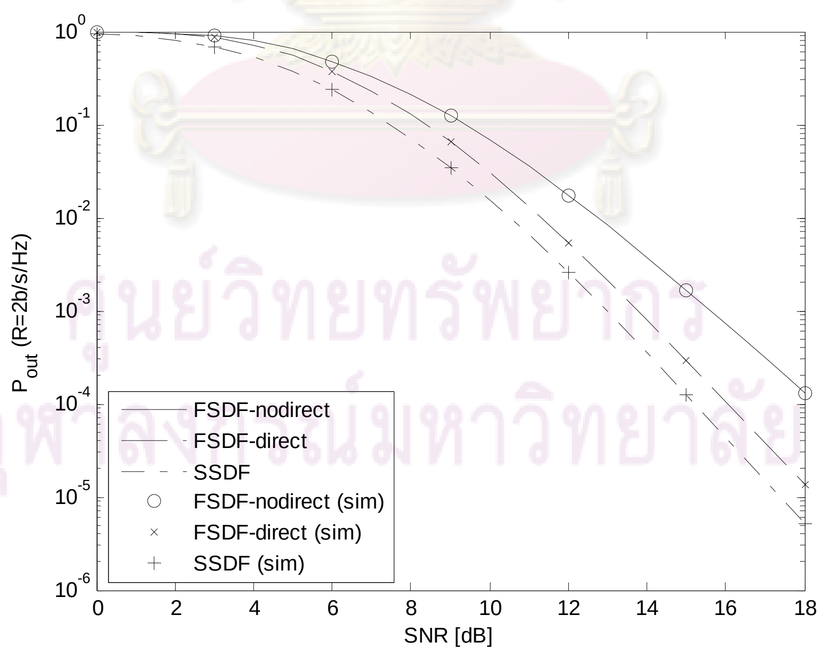

Figure 10 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 2 b/s/Hz for 4-relay network

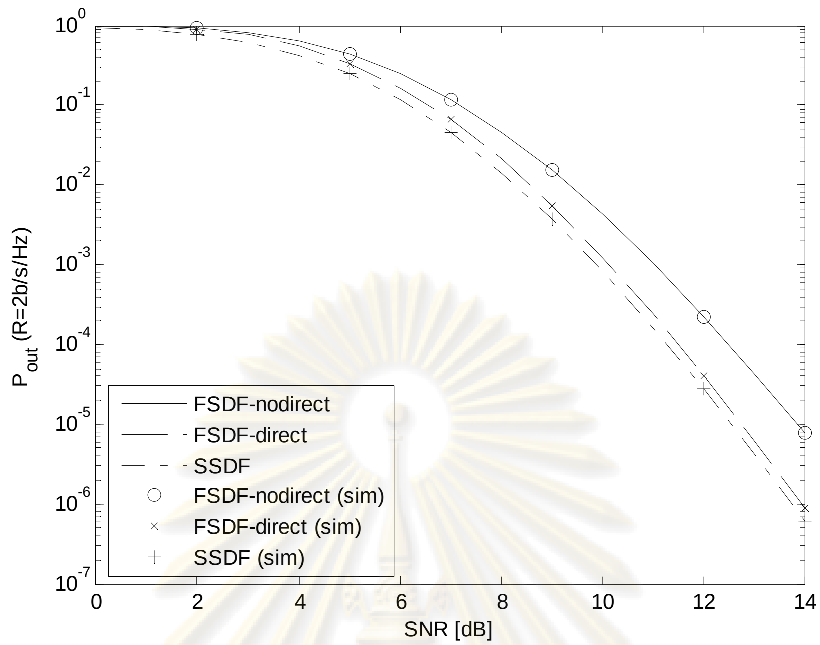

Figure 11 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 2 b/s/Hz for 9-relay network

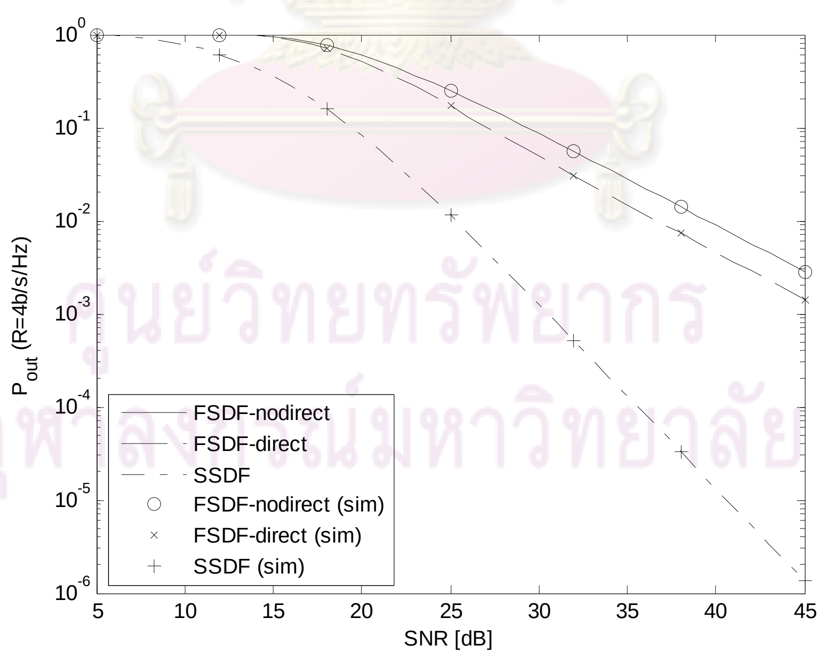

Figure 12 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 4 b/s/Hz for 1-relay network

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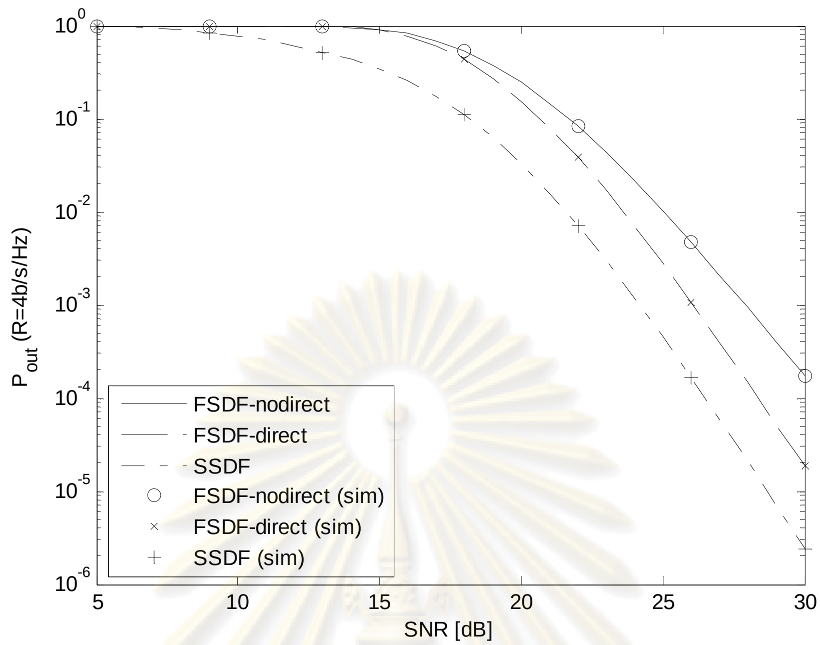

Figure 13 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 4 b/s/Hz for 4-relay network

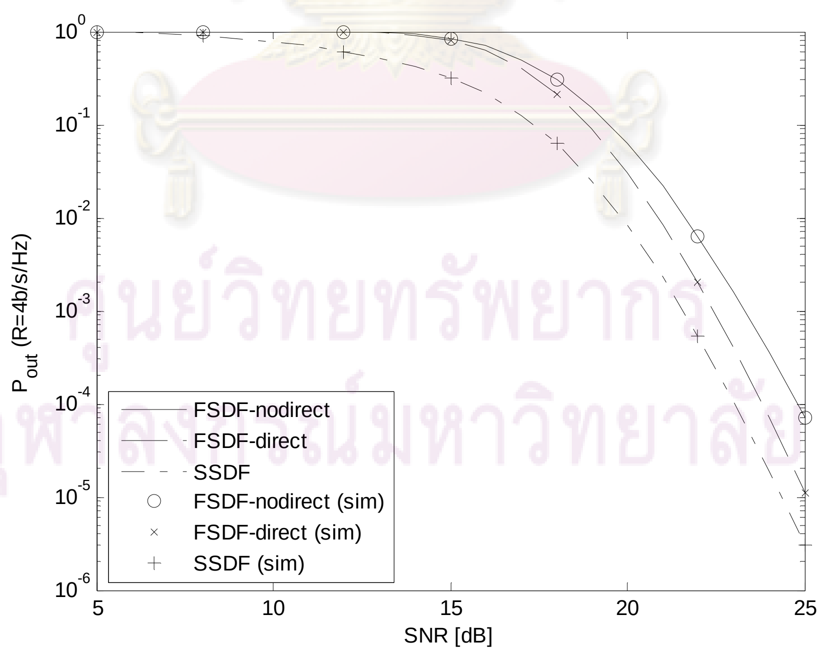

Figure 14 Verification of the outage probabilities of all analyzed cooperative diversity schemes at a rate of 4 b/s/Hz for 9-relay network

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3.6 Results and Discussions

First, we show that the approximated outage probabilities of the fixed selective decode-and-forward with direct link combining scheme provided in Proposition 2 and Proposition 3 only work in specific cases, and become not handy in general. In Figure 15, we reproduce the parameters used to demonstrate the approximations in the literature, namely, at a rate of 1 b/s/Hz and 4 relay nodes with an artificial topology where all links have equal average signal-to-noise ratio. The approximation in low SNR regime is from Proposition 2, and the approximation in high SNR regime is from Proposition 3, and the exact value is from our analysis in Theorem 3. The approximation from Proposition 2 gets close to the exact value in low SNR regime, while the approximation from Proposition 3 gets close to the exact value in high SNR regime. In Figure 16, we change the topology to grid topology, and the approximation in low SNR regime does not work, while the approximation in high SNR is still reasonable. In Figure 17, we change the number of relay nodes to 1 relay node, and the approximation in high SNR does not work either. Therefore, the approximations in the literature are not reliable, and our exact analyses are necessary.

Then, we examine the performance gain of using cooperative diversity schemes over direct communication in terms of outage probability with various parameters. The outage probability of direct communication is provided in Proposition 1. In Figure 18, the target rate is 2 b/s/Hz, and the number of relay nodes is 4. Compared to direct communication, all cooperative diversity schemes offer diversity gain with the same diversity gain order, which can be observed from the slopes of the curves. Among the cooperative diversity schemes, the fixed selective decode-and-forward with direct link combining scheme is superior to the fixed selective decode-and-forward without direct link combining scheme, and the smart selective decode-and-forward scheme is in turn superior to the fixed selective decode-and-forward with direct link combining scheme. As we increase the number of relay nodes, both the diversity gain order and the performance gain are larger for all cooperative diversity schemes as shown in Figure 19 and Figure 20, where we increase the number of relay nodes to 9 relay nodes and 15 relay nodes, respectively. Also, the performance gap between the fixed selective decode-and-forward with direct link combining scheme and the smart selective decode-and-forward scheme becomes smaller. Therefore, the ability, to decide to cooperate or not cooperate, introduced in the smart selective decode-and-forward scheme can be overlooked in favor of the simpler fixed selective decode-and-forward with direct link combining scheme in such environment. Another cause that can give the same effect is the decreased target rate as shown in Figure 21, where we decrease the target rate to 1 b/s/Hz. It can be observed that the gap between the fixed selective decode-and-forward with direct link combining scheme and the smart selective decode-and-forward scheme becomes smaller.

In Figure 22, we increase the target rate to 4 b/s/Hz. It can be observed that the diversity gain obtained from cooperative diversity schemes is still preserved, but the cooperative diversity schemes, without the ability to decide to cooperate or not cooperate, namely, the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme, offer the performance gain over direct communication after the thresholds of average signal-to-noise ratio. This is because both fixed cooperative diversity schemes require higher received energy per bit to compensate the transmission rate due to dividing the transmission into two time slots. The similar problem can occur when we decrease the number of relay nodes as shown in Figure 23, where we keep the target rate at 2 b/s/Hz but decrease the number of relay nodes to 2 relay nodes. It can be observed that the diversity gain obtained from cooperative diversity schemes is still preserved, but the cooperative diversity schemes, without the ability to decide to cooperate or not cooperate, namely, the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme, offer the performance gain over direct communication after the thresholds of average signal-to-noise ratio. In any cases, the smart selective decode-and-forward scheme always offer good performance gain over direct communication as shown in Figure 24, where we further increase the target rate to 4 b/s/Hz.

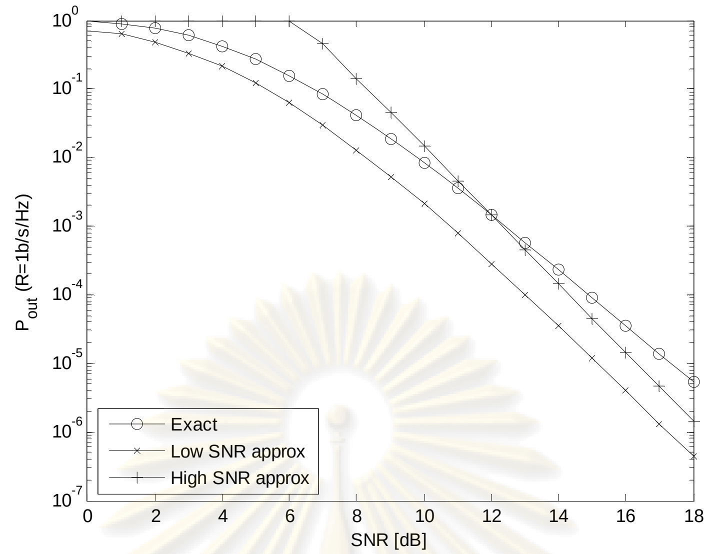

Figure 15 The case that the approximated outage probabilities of the fixed selective decode-and-forward with direct link combining scheme in the literature are satisfying: a rate of 1 b/s/Hz for 4-relay network where all link have equal average signal-to-noise ratio

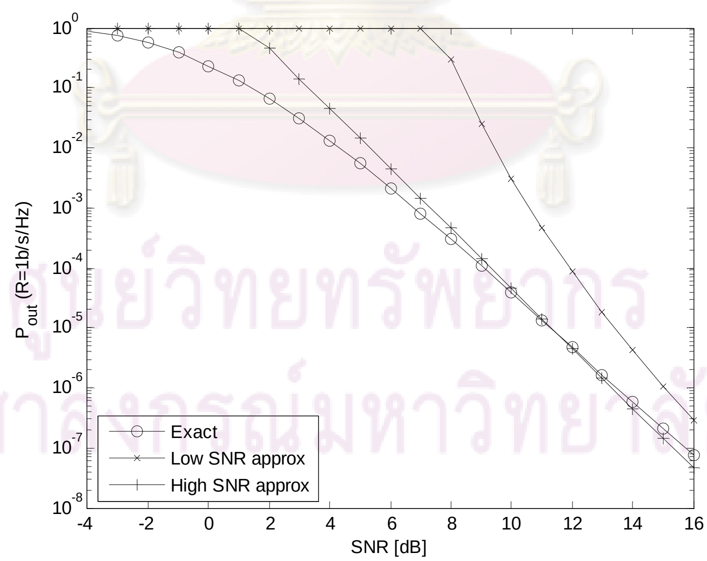

Figure 16 The case that the approximated outage probability of the fixed selective decode-and-forward with direct link combining scheme in the literature for low SNR regime is unsatisfying: a rate of 1 b/s/Hz for 4-relay network with grid topology

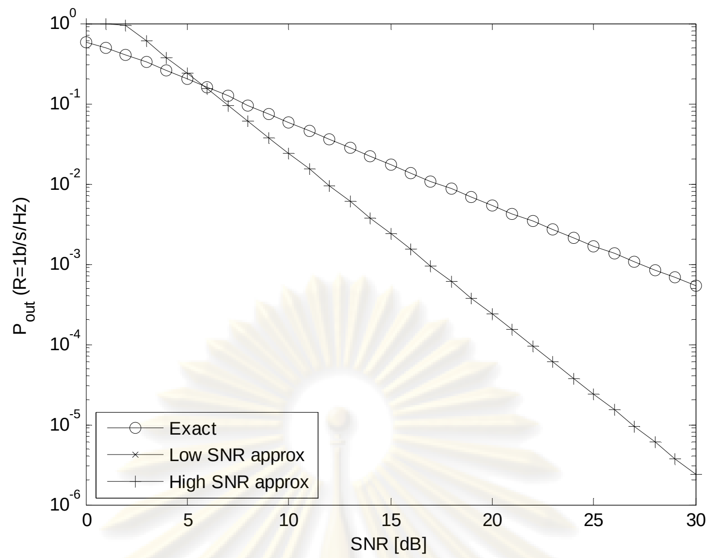

Figure 17 The case that the approximated outage probability of the fixed selective decode-and-forward with direct link combining scheme in the literature for high SNR regime is unsatisfying: a rate of 1 b/s/Hz for 1-relay network with grid topology

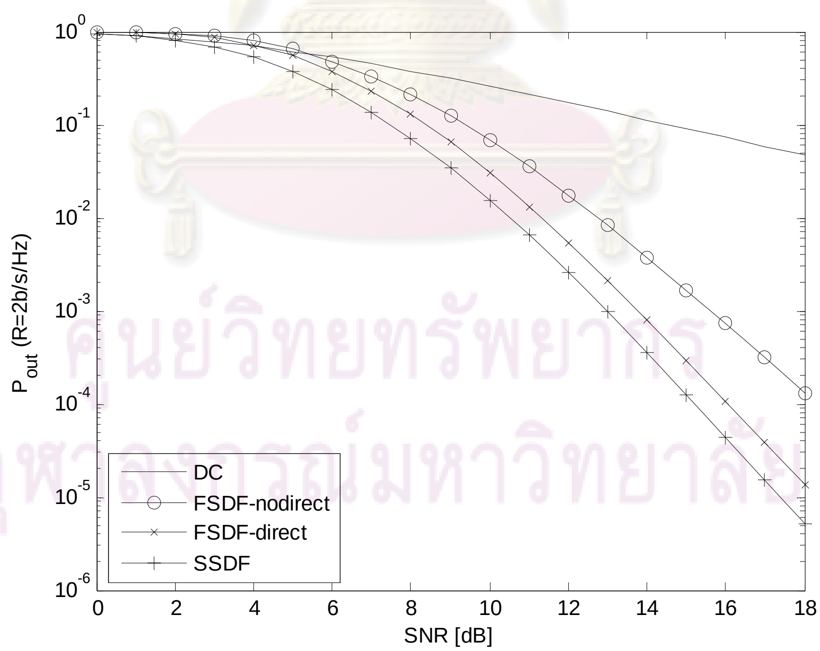

Figure 18 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 2 b/s/Hz for 4-relay network with direct communication

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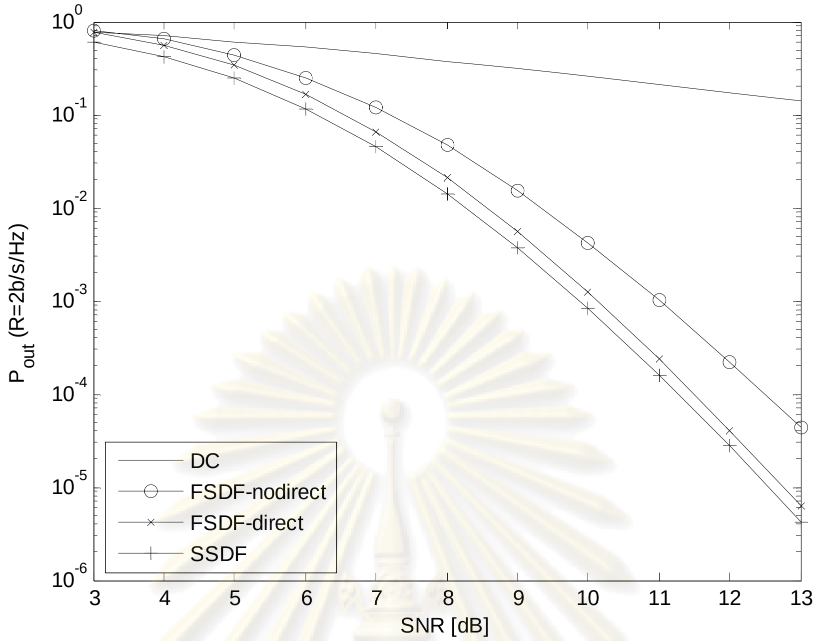

Figure 19 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 2 b/s/Hz for 9-relay network with direct communication

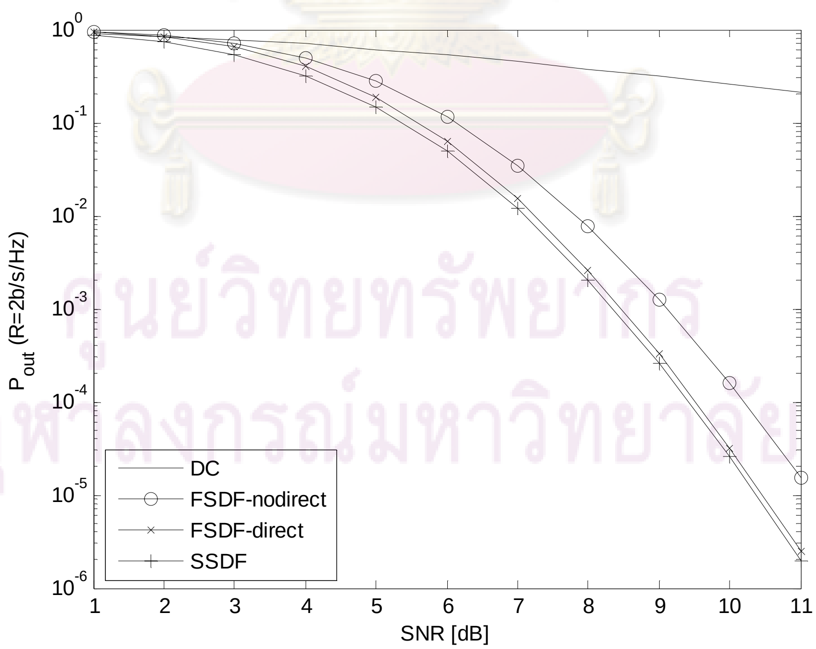

Figure 20 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 2 b/s/Hz for 15-relay network with direct communication

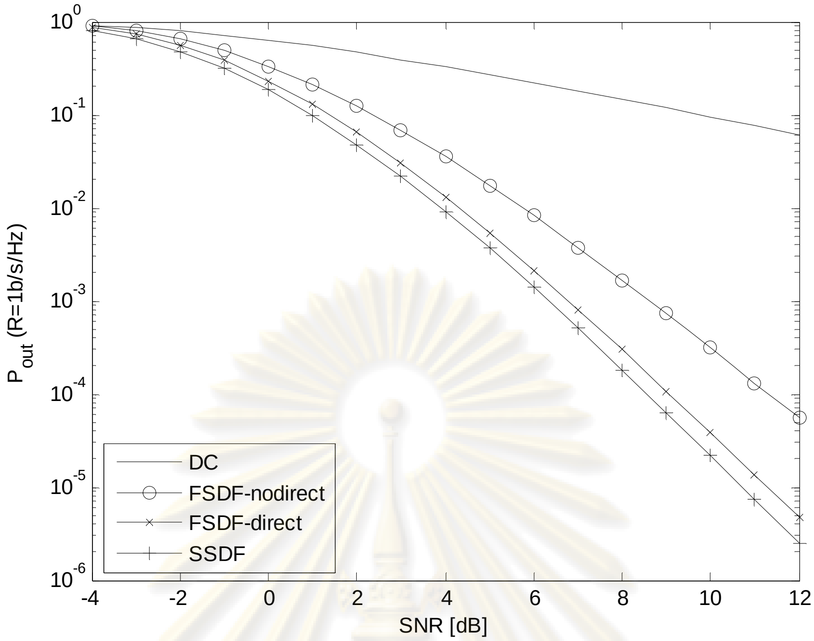

Figure 21 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 1 b/s/Hz for 4-relay network with direct communication

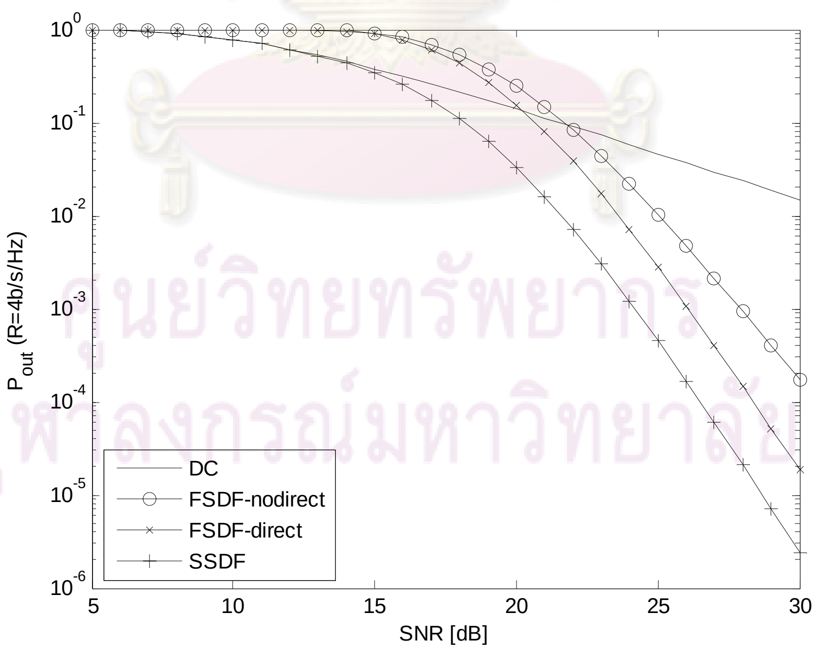

Figure 22 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 4 b/s/Hz for 4-relay network with direct communication

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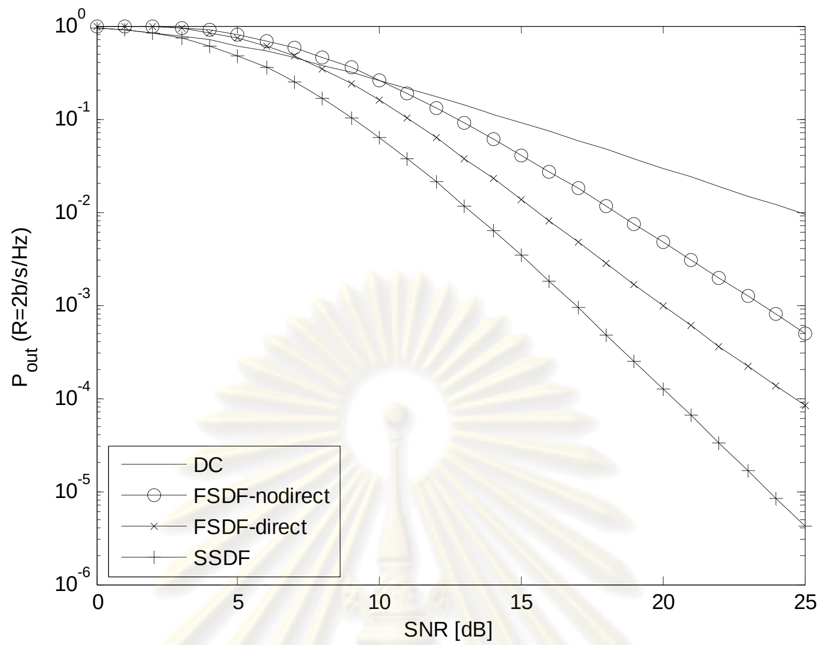

Figure 23 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 2 b/s/Hz for 2-relay network with direct communication

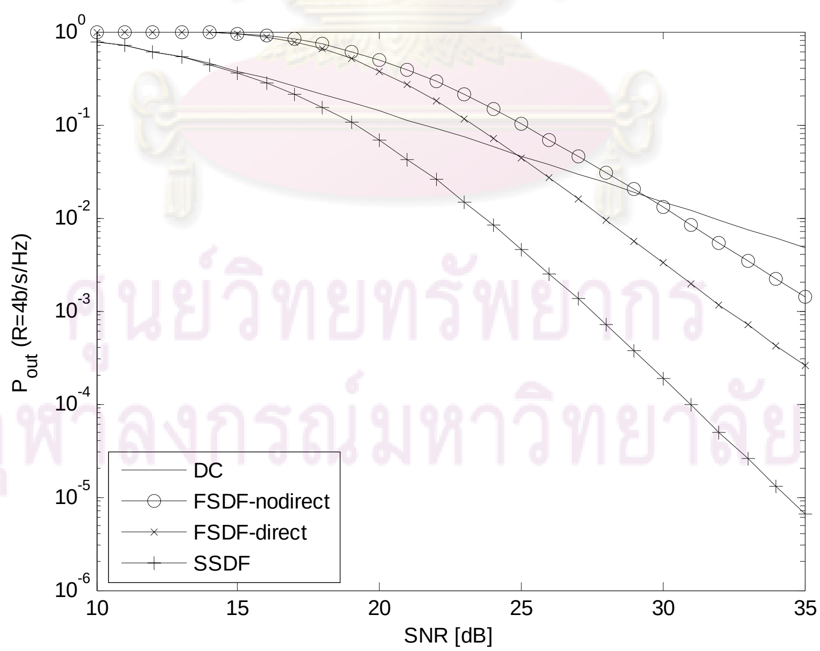

Figure 24 Outage probability comparisons of all considered cooperative diversity schemes at a rate of 4 b/s/Hz for 2-relay network with direct communication

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CHAPTER IV
OUTAGE CAPACITY

We have seen that the cooperative diversity can really improve the reliability. Another desirable aspect of the cooperative diversity is the improvement in capacity. In order to examine this expectation, we have to quantify the capacity for explicitly measuring how much the improvement is obtained and for comparing the considered schemes with different sets of factors. One of the important measures of capacity is the outage capacity. This chapter first analyzes the outage capacities of the considered cooperative diversity schemes. Then, the obtained formulas lead to the follow-up theories that draw insights with analytical proofs. The obtained formulas as well as the follow-up theories are verified by computer simulations with several sets of parameters. Then, the formulas are used to provide numerical results with comparisons and discussions.

4.1 Direct Communication

In order to examine whether the used cooperative diversity scheme really provides advantage in the employed environment and how much the improvement is, we have to compare the performance when the cooperative diversity scheme is being used to the performance when the cooperative diversity scheme is not being used. Accordingly, before we consider the outage capacities of the cooperative diversity schemes, the outage capacity of the direct communication is analyzed here to serve as a benchmarking.

In the direct communication case, the formula for calculating the outage capacity is straightforward and is available in standard materials. The outage capacity is a function of an acceptable outage probability ε and an average signal-to-noise ratio SNR. The formula is provided in Proposition 4.

Proposition 4 : The outage capacity, which is obtained from direct communication between the source node and the destination node, at an outage probability of ε and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [27]

$$C_{\text{out}}^{\text{DC}}(\varepsilon, \mathsf{SNR}) = \log_2\left(1 + \mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right),$$
$$\mathsf{SNR} \triangleq d_{\text{SD}}^{-\alpha}\frac{P}{N_0 W} \quad (4.1)$$

where $C_{\text{out}}^{\text{DC}}$ denote the outage capacity of the direct communication, d_{SD} is the distance between the source node and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage capacity is the maximum transmission rate, between the source node and the destination node, that is guaranteed to be supported if outages are allowed to occur within a determined probability.

$$C_{\text{out}}^{\text{DC}}(\varepsilon, \mathsf{SNR}) = \max R$$
$$\text{subject to } \mathbb{P}_{\text{out}}^{\text{DC}}(R, \mathsf{SNR}) \le \epsilon \quad (4.2)$$

which can be solved by first finding the cumulative distribution function of the mutual information between the source node and the destination node, that is,

$$F(x) = \Pr\{I_{\text{SD}} < x\}.$$

Then, solving x^* such that $F(x^*) = \varepsilon$ yields the outage capacity. Since we already knew the outage probability, it is not necessary to calculate the cumulative distribution function of the mutual information between the source node and the destination node, and the optimizer is the transmission rate R such that

$$\mathbb{P}_{\text{out}}^{\text{DC}}(R, \mathsf{SNR}) = \epsilon. \quad (4.3)$$

From Proposition 1,

$$\mathbb{P}_{\text{out}}^{\text{DC}}(R, \mathsf{SNR}) = 1 - \exp\left(-\left(\frac{2^R - 1}{\mathsf{SNR}}\right)\right). \quad (4.4)$$

By equating (4.4) to ϵ, that is,

$$1-\exp\left(-\left(\frac{2^{R}-1}{\mathsf{SNR}}\right)\right)=\epsilon. \tag{4.5}$$

Then, R becomes $C_{\text{out}}^{\text{DC}}\left(\varepsilon,\mathsf{SNR}\right)$.

$$1-\exp\left(-\left(\frac{2^{C_{\text{out}}^{\text{DC}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)\right)=\epsilon. \tag{4.6}$$

By arranging the terms, we obtain

$$\exp\left(-\left(\frac{2^{C_{\text{out}}^{\text{DC}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)\right)=1-\epsilon. \tag{4.7}$$

By taking logarithms on both sides, we obtain

$$-\left(\frac{2^{C_{\text{out}}^{\text{DC}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)=\ln\left(1-\epsilon\right), \tag{4.8}$$

which can be manipulated further to

$$2^{C_{\text{out}}^{\text{DC}}(\varepsilon,\mathsf{SNR})}=1+\mathsf{SNR}\ln\left(\frac{1}{1-\epsilon}\right). \tag{4.9}$$

Taking logarithms on both sides again yields (4.1). Q.E.D.

4.2 Fixed Selective Decode-and-forward without Direct Link Combining Scheme

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The formula for calculating the fixed selective decode-and-forward without direct link combining scheme does not exist in standard materials, due to the complication, which is more than finding the average capacity. Without an analytical result, the outage capacity can only be studied by computer simulations. In this thesis, we propose a formula that can calculate the value analytically.

For doing comparisons among different schemes later, the outage capacity is still a function of an acceptable outage probability ϵ and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula is provided in Theorem 5.

Theorem 5 : The outage capacity, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward without direct link combining scheme in the wireless relay network having K relay nodes, at an acceptable outage probability of ϵ and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [29]

$$C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon, \mathsf{SNR}) = \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right), \tag{4.10}$$

in which $w_{\varepsilon}^{\text{FSDF-nodirect}}$ is solved from $w^{\text{FSDF-nodirect}}$ in the equation

$$\epsilon = \prod_{k=1}^{K}\left[1-\exp\left(-\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\right], \tag{4.11}$$

after applying the transformation

$$w^{\text{FSDF-nodirect}} = \exp\left(-\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right), \tag{4.12}$$

where $C_{\text{out}}^{\text{FSDF-nodirect}}$ denotes the outage capacity of the fixed selective decode-and-forward without direct link combining scheme, d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage capacity is the maximum end-to-end transmission rate, trough the flow from the source node via the selected relay node to the destination node, that is guaranteed to be supported if outages are allowed to occur within a determined probability.

$$C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon, \text{SNR}) = \max R \quad \text{subject to } \mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R, \text{SNR}) \le \epsilon \tag{4.13}$$

which can be solved by first finding the cumulative distribution function of the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the link from the relay node to the destination node, that is,

$$F(x) = \Pr\left\{\max_k \min\left\{\frac{1}{2} I_{\text{S}k}, \frac{1}{2} I_{k\text{D}}\right\} < x\right\}.$$

Then, solving x^* such that $F(x^*) = \varepsilon$ yields the outage capacity. Since we already knew the outage probability, it is not necessary to calculate the cumulative distribution function of the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the link from the relay node to the destination node, and the optimizer is the transmission rate R such that

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R, \text{SNR}) = \epsilon. \tag{4.14}$$

From Theorem 2,

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R, \text{SNR}) = \prod_{k=1}^{K}\left[1 - \exp\left(-\frac{d_{\text{S}k}^{\alpha} + d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R} - 1}{\text{SNR}}\right)\right)\right]. \tag{4.15}$$

By equating (4.15) to ϵ, that is,

$$\prod_{k=1}^{K}\left[1 - \exp\left(-\frac{d_{\text{S}k}^{\alpha} + d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2R} - 1}{\text{SNR}}\right)\right)\right] = \epsilon. \tag{4.16}$$

Then, R becomes $C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon, \text{SNR})$.

$$\prod_{k=1}^{K}\left[1 - \exp\left(-\frac{d_{\text{S}k}^{\alpha} + d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\left(\frac{2^{2C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon, \text{SNR})} - 1}{\text{SNR}}\right)\right)\right] = \epsilon. \tag{4.17}$$

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Unfortunately, the expression cannot be manipulated further to solve for $C_{\mathrm{out}}^{\mathrm{FSDF\text{-}nodirect}}(\epsilon,\mathsf{SNR})$. With careful inspection, we propose a transformation

$$w^{\mathrm{FSDF\text{-}nodirect}} = \exp\left(-\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right), \tag{4.18}$$

to apply to the left-hand-side expression in (4.16), denoted as the function $f(R,\mathsf{SNR})$.

The intuitive reasons behind the proposed transformation are as follows. First, the domain of the transformed function is bounded between 0 and 1. Therefore, efficient solving is promising. Second, the transformation maps both R and SNR into a single domain. Hence, writing R as a function of SNR by knowing a single parameter can be expected. Third, as will be shown later, the transformed function is continuous and strictly decreasing. So, the equation solving can be done.

By applying the transformation to the function $f(R,\mathsf{SNR})$, we obtain

$$\prod_{k=1}^{K}\left[1-\exp\left(-\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right)\right] \leftrightarrow \prod_{k=1}^{K}\left[1-w^{\mathrm{FSDF\text{-}nodirect}\,\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}}\right]. \tag{4.19}$$

The transformed function, denoted by $F\left(w^{\mathrm{FSDF\text{-}nodirect}}\right)$, is now in $w^{\mathrm{FSDF\text{-}nodirect}}$-domain. Then, we show that $F\left(w^{\mathrm{FSDF\text{-}nodirect}}\right)$ is continuous. Since $\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}$ is a constant, $w^{\mathrm{FSDF\text{-}nodirect}\,\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}}$ is a parabolic function, which is continuous. Also, due to the additive rule, the function $1-x$ is continuous. Hence,

$$1-w^{\mathrm{FSDF\text{-}nodirect}\,\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}},$$

which is the composite function of those two continuous functions, is continuous. Last, due to the multiplicative rule,

$$\prod_{k=1}^{K}\left[1-w^{\mathrm{FSDF\text{-}nodirect}\,\frac{d_{\mathrm{S}k}^{\alpha}+d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}}\right]$$

is continuous, so $F\left(w^{\text{FSDF-nodirect}}\right)$ is continuous.

From Lemma 2, $F\left(w^{\text{FSDF-nodirect}}\right)$ is strictly decreasing and is bounded between 0 and 1. Hence, we can efficiently solve for the unique $w_{\varepsilon}^{\text{FSDF-nodirect}} \in (0,1]$ in

$$F\left(w_{\varepsilon}^{\text{FSDF-nodirect}}\right) = \epsilon, \tag{4.20}$$

that is,

$$\prod_{k=1}^{K}\left[1 - w_{\epsilon}^{\text{FSDF-nodirect}\,\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\right] = \epsilon. \tag{4.21}$$

After solving for $w_{\varepsilon}^{\text{FSDF-nodirect}}$, by taking inverse transformation to

$$w^{\text{FSDF-nodirect}} = w_{\varepsilon}^{\text{FSDF-nodirect}}, \tag{4.22}$$

we obtain

$$\exp\left(-\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right) = w_{\varepsilon}^{\text{FSDF-nodirect}}, \tag{4.23}$$

which means

$$\exp\left(-\left(\frac{2^{2C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon,\text{SNR})}-1}{\text{SNR}}\right)\right) = w_{\varepsilon}^{\text{FSDF-nodirect}}. \tag{4.24}$$

Therefore, at a given ε, $C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon,\text{SNR})$ can be written as a function of SNR via a single parameter, which is $w_{\varepsilon}^{\text{FSDF-nodirect}}$. This indicates that the characteristic of the wireless relay network, parameterized by $\left\{d_{\text{SD}}^{\alpha}, d_{\text{S}k}^{\alpha}, d_{k\text{D}}^{\alpha}; \forall k \in \mathcal{K}\right\}$, is lumped into the single parameter. By taking logarithms on both sides,

$$-\left(\frac{2^{2C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon,\text{SNR})}-1}{\text{SNR}}\right) = \ln\left(w_{\varepsilon}^{\text{FSDF-nodirect}}\right), \tag{4.25}$$

which can be manipulated further to

$$2^{2C_{\text{out}}^{\text{FSDF-nodirect}}(\varepsilon,\text{SNR})} = 1 + \text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right). \tag{4.26}$$

Taking logarithms on both sides again and dividing both sides by 2 yield (4.10). Q.E.D.

Lemma 2 : The function $F(x):(0,1]\rightarrow[0,1)$ defined by

$$F(x) \triangleq \prod_{k=1}^{K}\left[1 - x^{\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\right] \tag{4.27}$$

is strictly decreasing, and is bounded between 0 and 1.

Proof : Let $x, y \in \mathbb{R}$, where

$$0 < x < y < 1. \tag{4.28}$$

By powering thoroughly with a constant c_1,

$$0 < x^{c_1} < y^{c_1} < 1. \tag{4.29}$$

By multiplying -1 thoroughly,

$$0 > -x^{c_1} > -y^{c_1} > -1. \tag{4.30}$$

By adding 1 thoroughly,

$$1 > 1 - x^{c_1} > 1 - y^{c_1} > 0. \tag{4.31}$$

Changing the constant from c_1 to c_2, we obtain

$$1 > 1 - x^{c_2} > 1 - y^{c_2} > 0. \tag{4.32}$$

By multiplying (4.31) and (4.32), we obtain

$$1 > \left(1 - x^{c_1}\right)\left(1 - x^{c_2}\right) > \left(1 - y^{c_1}\right)\left(1 - y^{c_2}\right) > 0. \tag{4.33}$$

Introducing c_3 until c_K and using mathematical induction, we obtain

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$$1 > \prod_{k=1}^{K}\left(1 - x^{c_k}\right) > \prod_{k=1}^{K}\left(1 - y^{c_k}\right) > 0. \quad (4.34)$$

By choosing

$$c_k = \frac{d_{\mathrm{S}k}^{\alpha} + d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}, \quad (4.35)$$

we obtain

$$1 > \prod_{k=1}^{K}\left(1 - x^{\frac{d_{\mathrm{S}k}^{\alpha} + d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}}\right) > \prod_{k=1}^{K}\left(1 - y^{\frac{d_{\mathrm{S}k}^{\alpha} + d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}}\right) > 0, \quad (4.36)$$

which means

$$1 > F\left(x\right) > F\left(y\right) > 0. \quad (4.37)$$

Since we defined $x < y$, $F\left(w^{\text{FSDF-nodirect}}\right)$ is strictly decreasing and is bounded between 0 and 1. Q.E.D.

4.3 Fixed Selective Decode-and-forward with Direct Link Combining Scheme

The formula for calculating the fixed selective decode-and-forward with direct link combining scheme does not exist in standard materials, due to the complication, which is more than finding the average capacity. Without an analytical result, the outage capacity can only be studied by computer simulations. In this thesis, we propose a formula that can calculate the value analytically.

For doing comparisons among different schemes later, the outage capacity is still a function of an acceptable outage probability ϵ and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula is provided in Theorem 6.

Theorem 6 : The outage capacity, which is obtained from employing the cooperative diversity with the fixed selective decode-and-forward with direct link combining scheme in the wireless relay network having K relay nodes, at an acceptable outage probability of ϵ and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [28]

$$C_{\text{out}}^{\text{FSDF-direct}}\left(\varepsilon, \mathsf{SNR}\right) = \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right), \tag{4.38}$$

in which $w_{\varepsilon}^{\text{FSDF-direct}}$ is solved from $w^{\text{FSDF-direct}}$ in the equation

$$\epsilon = 1 + \sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}}\left(-1\right)^{l}\frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\exp\left(-\left(1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\right)\frac{2^{2R}-1}{\mathsf{SNR}}\right)-\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\mathsf{SNR}}\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1}, \tag{4.39}$$

after applying the transformation

$$w^{\text{FSDF-direct}} = \exp\left(-\left(\frac{2^{2R}-1}{\mathsf{SNR}}\right)\right), \tag{4.40}$$

where $\mathcal{K}\in\{1,2,3,\ldots,K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $C_{\text{out}}^{\text{FSDF-direct}}$ denotes the outage capacity of the fixed selective decode-and-forward with direct link combining scheme, d_{SD} is the distance between the source node and the destination node, $d_{\text{S}k}$ is the distance between the source node and the k th relay node, $d_{k\text{D}}$ is the distance between the k th relay and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage capacity is the maximum end-to-end transmission rate, trough the flow from the link between the source node and the selected relay node to the combined links between the selected relay node and the destination node and between the source node and the destination node, that is guaranteed to be supported if outages are allowed to occur within a determined probability.

$$C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\text{SNR}) = \max R \quad \text{subject to } \mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) \le \epsilon \tag{4.41}$$

which can be solved by first finding the cumulative distribution function of the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links between the relay node and the destination node and between the source node and the destination node, that is,

$$F(x) = \Pr\left\{\max_k \min\left\{\frac{1}{2}I_{\text{S}k}, \frac{1}{2}I_{\text{SD+}k\text{D}}\right\} < x\right\}. \tag{4.42}$$

Then, solving x^* such that $F(x^*) = \varepsilon$ yields the outage capacity. Since we already knew the outage probability, it is not necessary to calculate the cumulative distribution function of the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links between the relay node and the destination node and between the source node and the destination node, and the optimizer is the transmission rate R such that

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) = \epsilon. \tag{4.43}$$

From Theorem 3,

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R,\text{SNR}) = 1 + \sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l \frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^\alpha}{d_{\text{SD}}^\alpha}\right)\exp\left(-\left(1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^\alpha}{d_{\text{SD}}^\alpha}\right)\right)\frac{2^{2R}-1}{\text{SNR}}\right) - \exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^\alpha}{d_{\text{SD}}^\alpha}+\frac{d_{k\text{D}}^\alpha}{d_{\text{SD}}^\alpha}\right)\frac{2^{2R}-1}{\text{SNR}}\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^\alpha}{d_{\text{SD}}^\alpha}\right)-1} \tag{4.44}$$

By equating (4.44) to ϵ, that is,

$$1+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\exp\left(-\left(1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\right)\frac{2^{2R}-1}{\mathrm{SNR}}\right)-\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\mathrm{SNR}}\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}=\epsilon \tag{4.45}$$

Then, R becomes $C_{\mathrm{out}}^{\text{FSDF-direct}}(\varepsilon,\mathrm{SNR})$.

$$1+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\left[\begin{array}{l}\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\exp\left(-\left(1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\right)\frac{2^{2C_{\mathrm{out}}^{\text{FSDF-direct}}(\varepsilon,\mathrm{SNR})}-1}{\mathrm{SNR}}\right)\\-\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\frac{2^{2C_{\mathrm{out}}^{\text{FSDF-direct}}(\varepsilon,\mathrm{SNR})}-1}{\mathrm{SNR}}\right)\end{array}\right]}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}=\epsilon \tag{4.46}$$

Unfortunately, the expression cannot be manipulated further to solve for $C_{\mathrm{out}}^{\text{FSDF-direct}}(\epsilon,\mathrm{SNR})$. With careful inspection, we propose a transformation

$$w^{\text{FSDF-direct}}=\exp\left(-\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)\right), \tag{4.47}$$

to apply to the left-hand-side expression in (4.45), denoted as the function $f(R,\mathrm{SNR})$.

The intuitive reasons behind the proposed transformation are as follows. First, the domain of the transformed function is bounded between 0 and 1. Therefore, efficient solving is promising. Second, the transformation maps both R and SNR into a single domain. Hence, writing R as a function of SNR by knowing a single parameter can be expected. Third, as will be shown later, the transformed function is continuous and strictly decreasing. So, the equation solving can be done.

By applying the transformation to the function $f(R,\mathrm{SNR})$, we obtain

$$1+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\exp\left(-\left(1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\right)\frac{2^{2R}-1}{\mathrm{SNR}}\right)-\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\mathrm{SNR}}\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}$$

$$\leftrightarrow 1+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)w^{\text{FSDF-direct}^{1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}-w^{\text{FSDF-direct}^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}$$

(4.48)

The transformed function, denoted by $F\left(w^{\text{FSDF-direct}}\right)$, is now in $w^{\text{FSDF-direct}}$-domain. Then, we show that $F\left(w^{\text{FSDF-direct}}\right)$ is continuous. Since $\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)$ and $\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)$ is a constant, $w^{\text{FSDF-direct}^{1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}$ and $w^{\text{FSDF-direct}^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}$ are parabolic functions, which are continuous. The numerator

$$\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)w^{\text{FSDF-direct}^{1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}-w^{\text{FSDF-direct}^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}$$

is the linear combination of those two parabolic functions, so it is continuous. The denominator is a constant. The linear combination of all terms is continuous, so $F\left(w^{\text{FSDF-direct}}\right)$ is continuous.

From Lemma 3, $F\left(w^{\text{FSDF-direct}}\right)$ is strictly decreasing. Hence, we can efficiently solve for the unique $w_{\varepsilon}^{\text{FSDF-direct}}\in(0,1]$ in

$$F\left(w_{\varepsilon}^{\text{FSDF-direct}}\right)=\epsilon, \tag{4.49}$$

that is,

$$1+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)w^{\text{FSDF-direct}^{1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}-w^{\text{FSDF-direct}^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}}}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}=\epsilon. \tag{4.50}$$

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After solving for $w_{\varepsilon}^{\text{FSDF-direct}}$, by taking inverse transformation to

$$w^{\text{FSDF-direct}} = w_{\varepsilon}^{\text{FSDF-direct}}, \tag{4.51}$$

we obtain

$$\exp\left(-\left(\frac{2^{2R}-1}{\text{SNR}}\right)\right) = w_{\varepsilon}^{\text{FSDF-direct}}, \tag{4.52}$$

which means

$$\exp\left(-\left(\frac{2^{2C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\text{SNR})}-1}{\text{SNR}}\right)\right) = w_{\varepsilon}^{\text{FSDF-direct}}. \tag{4.53}$$

Therefore, at a given ε, $C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\text{SNR})$ can be written as a function of SNR via a single parameter, which is $w_{\varepsilon}^{\text{FSDF-direct}}$. This indicates that the characteristic of the wireless relay network, parameterized by $\left\{d_{\text{SD}}^{\alpha}, d_{\text{S}k}^{\alpha}, d_{k\text{D}}^{\alpha}; \forall k \in \mathcal{K}\right\}$, is lumped into the single parameter. By taking logarithms on both sides,

$$-\left(\frac{2^{2C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\text{SNR})}-1}{\text{SNR}}\right) = \ln\left(w_{\varepsilon}^{\text{FSDF-direct}}\right), \tag{4.54}$$

which can be manipulated further to

$$2^{2C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\text{SNR})} = 1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right). \tag{4.55}$$

Taking logarithms on both sides again and dividing both sides by 2 yield (4.38). Q.E.D.

Lemma 3 : The function $F\left(w^{\text{FSDF-direct}}\right):(0,1]\rightarrow[0,1)$, which is

$$F\left(w^{\text{FSDF-direct}}\right) \triangleq 1+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\ |\mathcal{S}|=l}}(-1)^{l}\frac{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)w^{\text{FSDF-direct}^{1+\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)}}-w^{\text{FSDF-direct}^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)}}}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1} \tag{4.56}$$

is strictly decreasing, and is bounded between 0 and 1.

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Proof : From Theorem 3, it can be shown that

$$F\left(w^{\text{FSDF-direct}}\right)=\int_{0}^{\frac{\left(2^{2R}-1\right)}{d_{\text{SD}}^{\alpha}\text{SNR}}}\prod_{k=1}^{K}\left[1-w^{\text{FSDF-direct}\,\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right)\right]d_{\text{SD}}^{\alpha}\exp\left(-d_{\text{SD}}^{\alpha}x\right)dx$$
$$+\int_{\frac{\left(2^{2R}-1\right)}{d_{\text{SD}}^{\alpha}\text{SNR}}}^{\infty}\prod_{k=1}^{K}\left(1-w^{\text{FSDF-direct}\,\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\right)d_{\text{SD}}^{\alpha}\exp\left(-d_{\text{SD}}^{\alpha}x\right)dx. \tag{4.57}$$

Let $w_2 > w_1$. Then,

$$w_2^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}} > w_1^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}, \quad \forall k \in \mathcal{K}. \tag{4.58}$$

For $x > 0$,

$$w_2^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right) > w_1^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right). \tag{4.59}$$

By multiplying -1 thoroughly,

$$-w_2^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right) < -w_1^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right) \tag{4.60}$$

By adding 1 thoroughly,

$$1-w_2^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right) < 1-w_1^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right) \tag{4.61}$$

Using mathematical induction,

$$\prod_{k=1}^{K}\left(1-w_2^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right)\right) < \prod_{k=1}^{K}\left(1-w_1^{\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\exp\left(d_{k\text{D}}^{\alpha}x\right)\right) \tag{4.62}$$

Since $d_{\text{SD}}^{\alpha}\exp\left(-d_{\text{SD}}^{\alpha}x\right) > 0$ for any x,

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$$\prod_{k=1}^{K}\left(1-w_2^{\frac{d_{Sk}^{\alpha}}{d_{SD}^{\alpha}}+\frac{d_{kD}^{\alpha}}{d_{SD}^{\alpha}}}\exp\left(d_{kD}^{\alpha}x\right)\right)d_{SD}^{\alpha}\exp\left(-d_{SD}^{\alpha}x\right)<\prod_{k=1}^{K}\left(1-w_1^{\frac{d_{Sk}^{\alpha}}{d_{SD}^{\alpha}}+\frac{d_{kD}^{\alpha}}{d_{SD}^{\alpha}}}\exp\left(d_{kD}^{\alpha}x\right)\right)d_{SD}^{\alpha}\exp\left(-d_{SD}^{\alpha}x\right) \tag{4.63}$$

Define two functions $f_{y_1}(x)$ and $f_{y_2}(x)$, where

$$f_{y_1}(x)<f_{y_2}(x),\quad \forall x\in I \tag{4.64}$$

where I is an interval, and

$$f_{y_2}(x)-f_{y_1}(x)>0,\quad \forall x\in I'\subset I \tag{4.65}$$

where I' is any interval such that $I'\subset I$. Then, then the integral of $f_{y_1}(x)$ over I will be less than the integral of $f_{y_2}(x)$ over I, that is,

$$\int_I f_{y_1}(x)\,dx<\int_I f_{y_2}(x)\,dx. \tag{4.66}$$

It can be shown easily that the expressions on both sides of the inequality in (4.63) are continuous. Thus, the inequality holds for $x\in[0,\delta]$ for $\delta>0$. Therefore, if we consider that $I=[0,\delta]$, $y_1=w_1$, and $y_2=w_2$, then

$$\begin{aligned}&\int_0^{\frac{\left(2^{2R}-1\right)}{d_{SD}^{\alpha}\mathrm{SNR}}}\prod_{k=1}^{K}\left[1-w_2^{\text{FSDF-direct}\,\frac{d_{Sk}^{\alpha}}{d_{SD}^{\alpha}}+\frac{d_{kD}^{\alpha}}{d_{SD}^{\alpha}}}\exp\left(d_{kD}^{\alpha}x\right)\right]d_{SD}^{\alpha}\exp\left(-d_{SD}^{\alpha}x\right)dx<\\&\int_0^{\frac{\left(2^{2R}-1\right)}{d_{SD}^{\alpha}\mathrm{SNR}}}\prod_{k=1}^{K}\left[1-w_1^{\text{FSDF-direct}\,\frac{d_{Sk}^{\alpha}}{d_{SD}^{\alpha}}+\frac{d_{kD}^{\alpha}}{d_{SD}^{\alpha}}}\exp\left(d_{kD}^{\alpha}x\right)\right]d_{SD}^{\alpha}\exp\left(-d_{SD}^{\alpha}x\right)dx.\end{aligned} \tag{4.67}$$

With the same approach, we obtain

$$\int_{\frac{\left(2^{2R}-1\right)}{d_{SD}^{\alpha}\mathrm{SNR}}}^{\infty}\prod_{k=1}^{K}\left(1-w_2^{\text{FSDF-direct}\,\frac{d_{Sk}^{\alpha}}{d_{SD}^{\alpha}}}\right)d_{SD}^{\alpha}\exp\left(-d_{SD}^{\alpha}x\right)dx<\int_{\frac{\left(2^{2R}-1\right)}{d_{SD}^{\alpha}\mathrm{SNR}}}^{\infty}\prod_{k=1}^{K}\left(1-w_1^{\text{FSDF-direct}\,\frac{d_{Sk}^{\alpha}}{d_{SD}^{\alpha}}}\right)d_{SD}^{\alpha}\exp\left(-d_{SD}^{\alpha}x\right)dx. \tag{4.68}$$

Since we defined $w_1<w_2$, $F\left(w^{\text{FSDF-direct}}\right)$ is strictly decreasing. Q.E.D.

4.4 Smart Selective Decode-and-forward Scheme

The formula for calculating the smart selective decode-and-forward scheme does not exist in standard materials, due to the complication, which is more than finding the average capacity. Without an analytical result, the outage capacity can only be studied by computer simulations. In this thesis, we propose a formula that can calculate the value analytically.

For doing comparisons among different schemes later, the outage capacity is still a function of an acceptable outage probability ϵ and an average signal-to-noise ratio SNR, where the average signal-to-noise ratio in this case is the average signal-to-noise ratio of the link between the source node and the destination node. The average signal-to-noise ratio of the link between other pair of nodes is the function of the average signal-to-noise ratio of the link between the source node and the destination node. The formula is provided in Theorem 7.

Theorem 7 : The outage capacity, which is obtained from employing the cooperative diversity with the smart selective decode-and-forward scheme in the wireless relay network having K relay nodes, at an acceptable outage probability of ϵ and at an average signal-to-noise ratio of the link between the source node and the destination node of SNR is given by [28]

$$C_{\text{out}}^{\text{SSDF}}(\varepsilon,\mathsf{SNR})=\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon,\mathsf{SNR}}^{\text{SSDF}}}\right)\right), \tag{4.69}$$

in which $w_{\varepsilon,\mathsf{SNR}}^{\text{SSDF}}$ is solved from w^{SSDF} in the equation

$$\begin{aligned}\epsilon &= 1-\exp\left(-\frac{2^R-1}{\mathsf{SNR}}\right)\\ &+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^l\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\mathsf{SNR}}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1\right)\frac{2^R-1}{\mathsf{SNR}}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1},\end{aligned} \tag{4.70}$$

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after applying the transformation

$$w^{\mathrm{SSDF}} = \exp\left(-\left(\frac{2^R - 1}{\mathsf{SNR}}\right)\right), \tag{4.71}$$

where $\mathcal{K} \in \{1, 2, 3, \ldots, K\}$ is the set of the indices of all relay nodes, $\mathcal{S}$ is a subset of $\mathcal{K}$, $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$, $C_{\mathrm{out}}^{\mathrm{SSDF}}$ denotes the outage capacity of the smart selective decode-and-forward scheme, d_{SD} is the distance between the source node and the destination node, $d_{\mathrm{S}k}$ is the distance between the source node and the k th relay node, $d_{k\mathrm{D}}$ is the distance between the k th relay and the destination node, and α is the path loss exponent.

Proof : From the definition, the outage capacity is the maximum end-to-end transmission rate, trough the better flow between the flow from the source node to the destination node without transmission time dividing and the flow from the link between the source node and the selected relay node to the combined links between the selected relay node and the destination node and between the source node and the destination node, that is guaranteed to be supported if outages are allowed to occur within a determined probability.

$$\begin{aligned} C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon, \mathsf{SNR}) &= \max R \\ &\qquad \text{subject to } \mathbb{P}_{\mathrm{out}}^{\mathrm{SSDF}}(R, \mathsf{SNR}) \le \epsilon \end{aligned} \tag{4.72}$$

which can be solved by first finding the cumulative distribution function of the higher between the instantaneous mutual information of the link from the source node to the destination node and the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links between the relay node and the destination node and between the source node and the destination node, that is,

$$F(x) = \Pr\left\{\max\left\{I_{\mathrm{SD}}, \max_k \min\left\{\frac{1}{2} I_{\mathrm{S}k}, \frac{1}{2} I_{\mathrm{SD}+k\mathrm{D}}\right\}\right\} < x\right\}. \tag{4.73}$$

Then, solving x^* such that $F\left(x^*\right)=\varepsilon$ yields the outage capacity. Since we already knew the outage probability, it is not necessary to calculate the cumulative distribution function of the higher between the instantaneous mutual information of the link from the source node to the destination node and the maximum among all relay nodes of the minimum between the instantaneous mutual information of the link from the source node to the relay node and the instantaneous mutual information of the combined links between the relay node and the destination node and between the source node and the destination node, and the optimizer is the transmission rate R such that

$$\mathbb{P}_{\text{out}}^{\text{SSDF}}\left(R,\text{SNR}\right)=\epsilon. \tag{4.74}$$

From Theorem 4,

$$\mathbb{P}_{\text{out}}^{\text{SSDF}}(R,\text{SNR})=1-\exp\left(-\frac{2^{R}-1}{\text{SNR}}\right) + \sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\text{SNR}}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1\right)\frac{2^{R}-1}{\text{SNR}}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1} \tag{4.75}$$

By equating (4.75) to ϵ, that is,

$$1-\exp\left(-\frac{2^{R}-1}{\text{SNR}}\right)+ \sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\text{S}k}^{\alpha}}{d_{\text{SD}}^{\alpha}}+\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\text{SNR}}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1\right)\frac{2^{R}-1}{\text{SNR}}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}\right)-1}=\epsilon \tag{4.76}$$

Then, R becomes $C_{\text{out}}^{\text{SSDF}}\left(\varepsilon,\text{SNR}\right)$.

$$\epsilon=1-\exp\left(-\frac{2^{C_{\text{out}}^{\text{SSDF}}(\varepsilon,\text{SNR})}-1}{\text{SNR}}\right)+$$

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$$\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\frac{2^{2C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1\right)\frac{2^{C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}$$

(4.77)

Unfortunately, the expression cannot be manipulated further to solve for $C_{\mathrm{out}}^{\mathrm{SSDF}}(\epsilon,\mathsf{SNR})$. With careful inspection, we propose a transformation

$$w^{\mathrm{SSDF}}=\exp\left(-\left(\frac{2^{R}-1}{\mathsf{SNR}}\right)\right), \tag{4.78}$$

to apply to the left-hand-side expression in (4.76), denoted as the function $f(R,\mathsf{SNR})$.

The intuitive reasons behind the proposed transformation are as follows. First, the domain of the transformed function is bounded between 0 and 1. Therefore, efficient solving is promising. Second, the transformation maps both R and SNR into a single domain and a residual SNR. Hence, writing R as a function of SNR by knowing a single parameter at each SNR can be expected. Third, as will be shown later, the transformed function is continuous and strictly decreasing. So, the equation solving can be done.

By applying the transformation to the function $f(R,\mathsf{SNR})$, we obtain

$$1-\exp\left(-\frac{2^{R}-1}{\mathsf{SNR}}\right)+$$

$$\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(-\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)\frac{2^{2R}-1}{\mathsf{SNR}}\right)\left(\exp\left(\left(\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1\right)\frac{2^{R}-1}{\mathsf{SNR}}\right)-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}\leftrightarrow$$

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$$1-w^{\mathrm{SSDF}}+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(2\ln\left(w^{\mathrm{SSDF}}\right)-\left(\ln w^{\mathrm{SSDF}}\right)^{2}\mathsf{SNR}\right)^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}\left({w^{\mathrm{SSDF}}}^{1-\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}$$

(4.79)

The transformed function, denoted by $F\left(w^{\mathrm{SSDF}}\right)$, is now in w^{SSDF}-domain. Then, we show that $F\left(w^{\mathrm{SSDF}}\right)$ is continuous, when an SNR is given. Since $\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)$ and $\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)$ is a constant, ${w^{\mathrm{SSDF}}}^{1-\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}$ is parabolic function, which is continuous. Since a logarithmic function is continuous, the polynomial $2\ln\left(w^{\mathrm{SSDF}}\right)-\left(\ln w^{\mathrm{SSDF}}\right)^{2}\mathsf{SNR}$ is continuous. Since an exponential function is continuous, the composite function between the exponential function and the parabolic function is continuous. Hence,

$$\exp\left(2\ln\left(w^{\mathrm{SSDF}}\right)-\left(\ln w^{\mathrm{SSDF}}\right)^{2}\mathsf{SNR}\right)^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}$$

is continuous. Due to multiplicative rule, the numerator

$$\exp\left(2\ln\left(w^{\mathrm{SSDF}}\right)-\left(\ln w^{\mathrm{SSDF}}\right)^{2}\mathsf{SNR}\right)^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}\left({w^{\mathrm{SSDF}}}^{1-\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}-1\right)$$

is continuous. The denominator is a constant. The linear combination of all terms is continuous, so $F\left(w^{\mathrm{SSDF}}\right)$ is continuous.

From Lemma 4, the function $F\left(w^{\mathrm{SSDF}}\right)$ is strictly decreasing on $w^{\mathrm{SSDF}}\in(0,1]$. Therefore, we can efficiently solve for the unique $w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}\in\left(0,1\right]$ in

$$F\left(w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}\right)=\epsilon, \quad (4.80)$$

that is,

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$$1-w^{\mathrm{SSDF}}+\sum_{l=0}^{K}\sum_{\substack{\mathcal{S}\subseteq\mathcal{K}\\|\mathcal{S}|=l}}(-1)^{l}\frac{\exp\left(2\ln\left(w^{\mathrm{SSDF}}\right)-\left(\ln w^{\mathrm{SSDF}}\right)^{2}\mathsf{SNR}\right)^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}+\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}\left({w^{\mathrm{SSDF}}}^{1-\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)}-1\right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)-1}=\epsilon. \tag{4.81}$$

After solving for $w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}$, by taking inverse transformation to

$$w^{\mathrm{SSDF}}=w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}, \tag{4.82}$$

we obtain

$$\exp\left(-\left(\frac{2^{R}-1}{\mathsf{SNR}}\right)\right)=w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}, \tag{4.83}$$

which means

$$\exp\left(-\left(\frac{2^{C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)\right)=w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}. \tag{4.84}$$

Therefore, at a given ε and SNR, $C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon,\mathsf{SNR})$ can be found via a single parameter, which is $w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}$. This indicates that the characteristic of the wireless relay network, parameterized by $\{d_{\mathrm{SD}}^{\alpha},d_{\mathrm{S}k}^{\alpha},d_{k\mathrm{D}}^{\alpha};\forall k\in\mathcal{K}\}$, is lumped into the single parameter. By taking logarithms on both sides,

$$-\left(\frac{2^{C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon,\mathsf{SNR})}-1}{\mathsf{SNR}}\right)=\ln\left(w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}\right), \tag{4.85}$$

which can be manipulated further to

$$2^{C_{\mathrm{out}}^{\mathrm{SSDF}}(\varepsilon,\mathsf{SNR})}=1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon,\mathsf{SNR}}^{\mathrm{SSDF}}}\right). \tag{4.86}$$

Taking logarithms on both sides again yields (4.69). Q.E.D.

Lemma 4 : The function $F\left(w^{\mathrm{SSDF}}\right)$ defined by

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$$F\left(w^{\mathrm{SSDF}}\right) \triangleq 1 - w^{\mathrm{SSDF}} + \sum_{l=0}^{K} \sum_{\substack{\mathcal{S} \subseteq \mathcal{K} \\ |\mathcal{S}|=l}} (-1)^l \frac{\exp\left(2\ln\left(w^{\mathrm{SSDF}}\right) - \left(\ln w^{\mathrm{SSDF}}\right)^2 \mathrm{SNR}\right)^{\sum_{k\in\mathcal{S}}\left(\frac{d_{\mathrm{S}k}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}} + \frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)} \left({w^{\mathrm{SSDF}}}^{1-\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right)} - 1 \right)}{\sum_{k\in\mathcal{S}}\left(\frac{d_{k\mathrm{D}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}}\right) - 1} \quad (4.87)$$

is strictly decreasing.

Proof : Let $g:[0,\infty) \to [0,\infty)$ denote a function

$$g(a) \triangleq 2a + \mathrm{SNR}\, a^2. \quad (4.88)$$

Function g is strictly increasing because

$$\frac{dg(a)}{da} > 0. \quad (4.89)$$

Hence, g is a bijection, and g^{-1} exists and is strictly increasing. By fixing $\mathrm{SNR} > 0$, we show that function $\tilde{f}:[0,\infty) \to [0,1]$ below is strictly increasing:

$$\tilde{f}(y) \triangleq \mathbb{P}\left\{\max\left\{\max_{k\in\mathcal{K}} \min\left\{|h_{\mathrm{S}k}|^2, |h_{\mathrm{SD}}|^2 + |h_{k\mathrm{D}}|^2\right\},\ g\left(|h_{\mathrm{SD}}|^2\right)\right\} \le y\right\}. \quad (4.90)$$

Let $0 \le y_1 < y_2$ be given. We want to show that $\tilde{f}(y_1) < \tilde{f}(y_2)$. Let

$$a_1 \triangleq g^{-1}(y_1)$$
$$a_2 \triangleq g^{-1}(y_2)$$
$$\beta \triangleq \frac{a_2}{y_2}.$$

Since a_2 and y_2 are positive,

$$0 < \beta < 1,$$

which gives the left-most inequality, and since

$$y_2 < g(y_2),$$

we obtain

$$g^{-1}(y_2) < y_2,$$

which gives the right-most inequality. Now, consider

$$\tilde{f}(y_2) - \tilde{f}(y_1) = \mathbb{P}\left\{ y_1 < \max\left\{ \max_{k\in\mathcal{K}} \min\left\{ |h_{\mathrm{S}k}|^2, |h_{\mathrm{SD}}|^2 + |h_{k\mathrm{D}}|^2 \right\},\ g\left(|h_{\mathrm{SD}}|^2\right) \right\} \le y_2 \right\}.$$

Since one event is a subset of the others,

$$\begin{aligned}
&\mathbb{P}\left\{ y_1 < \max\left\{ \max_{k\in\mathcal{K}} \min\left\{ |h_{\mathrm{S}k}|^2, |h_{\mathrm{SD}}|^2 + |h_{k\mathrm{D}}|^2 \right\},\ g\left(|h_{\mathrm{SD}}|^2\right) \right\} \le y_2 \right\} \\
&\ge \mathbb{P}\left\{ \begin{array}{l} \forall k \in \mathcal{K}, y_1 < |h_{\mathrm{S}k}|^2 \le y_2 \wedge \forall k \in \mathcal{K}, (1-\beta) y_2 < |h_{k\mathrm{D}}|^2 \le (1-\beta) y_2 \\ \wedge \beta y_1 < |h_{\mathrm{SD}}|^2 \le \beta y_2 \wedge y_1 < g\left(|h_{\mathrm{SD}}|^2\right) \le y_2 \end{array} \right\}.
\end{aligned}$$

Due to independence and the definition of β, we can take the inverse g^{-1} in the inequalities to obtain

$$\begin{aligned}
&\mathbb{P}\left\{ \begin{array}{l} \forall k \in \mathcal{K}, y_1 < |h_{\mathrm{S}k}|^2 \le y_2 \wedge \forall k \in \mathcal{K}, (1-\beta) y_2 < |h_{k\mathrm{D}}|^2 \le (1-\beta) y_2 \\ \wedge \beta y_1 < |h_{\mathrm{SD}}|^2 \le \beta y_2 \wedge y_1 < g\left(|h_{\mathrm{SD}}|^2\right) \le y_2 \end{array} \right\} = \\
&\left[\prod_{k\in\mathcal{K}} \mathbb{P}\left\{ y_1 < |h_{\mathrm{S}k}|^2 \le y_2 \right\} \right] \times \left[\prod_{k\in\mathcal{K}} \mathbb{P}\left\{ (1-\beta) y_1 < |h_{\mathrm{S}k}|^2 \le (1-\beta) y_2 \right\} \right] \\
&\qquad \times \mathbb{P}\left\{ \beta y_1 < |h_{\mathrm{SD}}|^2 \le a_2 \wedge a_1 < |h_{\mathrm{SD}}|^2 \le a_2 \right\}.
\end{aligned}$$

Since each probability term is positive,

$$\begin{aligned}
&\left[\prod_{k\in\mathcal{K}} \mathbb{P}\left\{ y_1 < |h_{\mathrm{S}k}|^2 \le y_2 \right\} \right] \times \left[\prod_{k\in\mathcal{K}} \mathbb{P}\left\{ (1-\beta) y_1 < |h_{\mathrm{S}k}|^2 \le (1-\beta) y_2 \right\} \right] \\
&\qquad \times \mathbb{P}\left\{ \beta y_1 < |h_{\mathrm{SD}}|^2 \le a_2 \wedge a_1 < |h_{\mathrm{SD}}|^2 \le a_2 \right\} > 0,
\end{aligned}$$

which gives

$$\tilde{f}(y_2) - \tilde{f}(y_1) > 0,$$

and so $\tilde{f}(y_1) < \tilde{f}(y_2)$.

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Considering $F\left(w^{\text{SSDF}}\right)$, it can be verified that

$$F\left(w^{\text{SSDF}}\right) = \tilde{f}\left(g\left(\ln\frac{1}{w^{\text{SSDF}}}\right)\right), \quad 0 < w^{\text{SSDF}} \le 1,$$

and hence is strictly decreasing on $w^{\text{SSDF}} \in (0,1]$. Q.E.D.

4.5 Follow-up Theories

The analyzed outage capacities lead to insights with analytical proofs as follows. First, we loosely compare the outage capacities of the considered schemes in Lemma 5.

Lemma 5 : The outage capacities of the three considered schemes satisfy

$$C_{\text{out}}^{\text{FSDF-nodirect}}\left(\epsilon, \text{SNR}\right) \le C_{\text{out}}^{\text{FSDF-direct}}\left(\epsilon, \text{SNR}\right) \le C_{\text{out}}^{\text{SSDF}}\left(\epsilon, \text{SNR}\right) \qquad (4.91)$$

for any ϵ and SNR.

Proof : Since

$$R_{\text{SSDF}} = \max\{R_{\text{FSDF-direct}}, R_{\text{DC}}\}, \qquad (4.92)$$

we obtain

$$R_{\text{FSDF-direct}} \le R_{\text{SSDF}}. \qquad (4.93)$$

From

$$R_{\text{MRC}}(k) = \frac{1}{2}\log_2\left(1 + SNR_{\text{S}k} + SNR_{k\text{D}}\right), \qquad (4.94)$$

and

$$R_{k\text{D}} = \frac{1}{2}\log_2\left(1 + SNR_{k\text{D}}\right), \qquad (4.95)$$

we know that

$$R_{\text{MRC}}(k) > R_{k\text{D}}, \quad \forall k \qquad (4.96)$$

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because $SNR_{Sk} > 0$ and $\log_2(\cdot)$ is a strictly increasing function. Therefore, from

$$R_{\text{FSDF-direct}} = \max_{k\in\mathcal{K}} \min\{R_{Sk}, R_{\text{MRC}}(k)\}, \tag{4.97}$$

and

$$R_{\text{FSDF-nodirect}} = \max_{k\in\mathcal{K}} \min\{R_{Sk}, R_{kD}\}, \tag{4.98}$$

we obtain

$$R_{\text{FSDF-nodirect}} \le R_{\text{FSDF-direct}}. \tag{4.99}$$

Then, it can be verified that

$$R_{\text{FSDF-nodirect}} \le R_{\text{FSDF-direct}} \le R_{\text{SSDF}}, \tag{4.100}$$

which gives (4.91) almost surely. Q.E.D.

At first sight, the cooperative diversity seems to unconditionally provide advantage over the direct communication, and the results on outage probabilities support that conjecture. On the other hand, when we concern about outage capacity, the using cooperative diversity is not always beneficial. We show the limitation of the considered cooperative diversity schemes in Propositions 5, 6, and 7. Also, we show that it is possible to use cooperative diversity schemes only when they are beneficial in Corollary 1 and Corollary 2.

Proposition 5 : In the high signal-to-noise ratio regime, we have

$$\lim_{\text{SNR}\to\infty} \frac{C_{\text{out}}^{\text{FSDF-nodirect}}(\epsilon, \text{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon, \text{SNR})} = \frac{1}{2}, \tag{4.101}$$

and in the low signal-to-noise ratio regime, we have

$$\lim_{\text{SNR}\to 0} \frac{C_{\text{out}}^{\text{FSDF-nodirect}}(\epsilon, \text{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon, \text{SNR})} = \frac{1}{2} \frac{\ln\left(w_{\varepsilon}^{\text{FSDF-nodirect}}\right)}{\ln(1-\epsilon)}. \tag{4.102}$$

Proof : Let's consider the high signal-to-noise ratio regime. Since

$$C_{\text{out}}^{\text{FSDF-nodirect}}\left(\varepsilon,\mathsf{SNR}\right)=\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right), \tag{4.103}$$

and

$$C_{\text{out}}^{\text{DC}}\left(\varepsilon,\mathsf{SNR}\right)=\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right), \tag{4.104}$$

the limit of the ratio can be written as

$$\lim_{\mathsf{SNR}\to\infty}\frac{C_{\text{out}}^{\text{FSDF-nodirect}}\left(\epsilon,\mathsf{SNR}\right)}{C_{\text{out}}^{\text{DC}}\left(\epsilon,\mathsf{SNR}\right)}=\lim_{\mathsf{SNR}\to\infty}\frac{\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}{\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}. \tag{4.105}$$

By the l'Hospital's rule, we take differentiation for both the numerator and the denominator with respect to SNR.

$$\lim_{\mathsf{SNR}\to\infty}\frac{\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}{\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}=\lim_{\mathsf{SNR}\to\infty}\frac{\frac{d\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}{d\mathsf{SNR}}}{\frac{d\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathsf{SNR}}}, \tag{4.106}$$

where

$$\frac{d\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}{d\mathsf{SNR}}=\frac{d\frac{1}{2\ln 2}\ln\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}{d\mathsf{SNR}}$$

$$=\frac{1}{2\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}\frac{d1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)}{d\mathsf{SNR}} \quad \text{(chain rule)}$$

$$=\frac{1}{2\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right),$$

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and

$$\frac{d\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathsf{SNR}}=\frac{1}{\ln 2}\frac{d\ln\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathsf{SNR}}$$

$$=\frac{1}{\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}\frac{d1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)}{d\mathsf{SNR}}\quad\text{(chain rule)}$$

$$=\frac{1}{\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}\ln\left(\frac{1}{1-\varepsilon}\right).$$

(4.107)

Now, we obtain

$$\lim_{\mathsf{SNR}\to\infty}\frac{C_{\text{out}}^{\text{FSDF-nodirect}}\left(\epsilon,\mathsf{SNR}\right)}{C_{\text{out}}^{\text{DC}}\left(\epsilon,\mathsf{SNR}\right)}=\lim_{\mathsf{SNR}\to\infty}\frac{\frac{1}{2\ln 2}\frac{\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)}}{\frac{1}{\ln 2}\frac{\ln\left(\frac{1}{1-\varepsilon}\right)}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}}$$

$$=\lim_{\mathsf{SNR}\to\infty}\frac{\frac{1}{2}\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}.$$

(4.108)

By the l'Hospital's rule, we take another differentiation for both the numerator and the denominator with respect to SNR.

$$\lim_{\mathsf{SNR}\to\infty}\frac{\frac{1}{2}\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}=$$

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$$\lim_{\text{SNR}\to\infty} \frac{\dfrac{d\,\frac{1}{2}\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)}{d\text{SNR}}}{\dfrac{d\left(1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}{d\text{SNR}}}, \tag{4.109}$$

where

$$\begin{aligned} \frac{d\,\frac{1}{2}\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)}{d\text{SNR}} &= \frac{1}{2}\frac{d\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)}{d\text{SNR}} \\ &= \frac{1}{2}\ln\left(\frac{1}{1-\varepsilon}\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right), \end{aligned} \tag{4.110}$$

and

$$\frac{d\left(1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}{d\text{SNR}} = \ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\ln\left(\frac{1}{1-\varepsilon}\right). \tag{4.111}$$

Then, we obtain

$$\begin{aligned} \lim_{\text{SNR}\to\infty}\frac{C_{\text{out}}^{\text{FSDF-nodirect}}(\epsilon,\text{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon,\text{SNR})} &= \lim_{\text{SNR}\to\infty}\frac{\frac{1}{2}\ln\left(\frac{1}{1-\varepsilon}\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)}{\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\ln\left(\frac{1}{1-\varepsilon}\right)} \\ &= \frac{1}{2}\lim_{\text{SNR}\to\infty}\frac{\ln\left(\frac{1}{1-\varepsilon}\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)}{\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\ln\left(\frac{1}{1-\varepsilon}\right)}. \end{aligned} \tag{4.112}$$

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which gives (4.101).

Now, we consider the low signal-to-noise ratio regime. The limit of the ratio can be written as

$$\lim_{\mathrm{SNR}\to 0}\frac{C_{\mathrm{out}}^{\mathrm{FSDF\text{-}nodirect}}\left(\epsilon,\mathrm{SNR}\right)}{C_{\mathrm{out}}^{\mathrm{DC}}\left(\epsilon,\mathrm{SNR}\right)}=\lim_{\mathrm{SNR}\to 0}\frac{\frac{1}{2}\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}{\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}. \quad (4.113)$$

By the l'Hospital's rule, we take differentiation for both the numerator and the denominator with respect to SNR .

$$\lim_{\mathrm{SNR}\to 0}\frac{\frac{1}{2}\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}{\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}=\lim_{\mathrm{SNR}\to 0}\frac{\dfrac{d\,\frac{1}{2}\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}{d\mathrm{SNR}}}{\dfrac{d\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathrm{SNR}}},$$

(4.114)

where

$$\frac{d\,\frac{1}{2}\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}{d\mathrm{SNR}}=\frac{d\,\frac{1}{2\ln 2}\ln\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}{d\mathrm{SNR}}$$

$$=\frac{1}{2\ln 2}\frac{1}{\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}\frac{d1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)}{d\mathrm{SNR}} \quad \text{(chain rule)} \quad (4.115)$$

$$=\frac{1}{2\ln 2}\frac{1}{\left(1+\mathrm{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right)\right)}\ln\left(\frac{1}{w_{\varepsilon}^{\mathrm{FSDF\text{-}nodirect}}}\right),$$

and

$$\frac{d\log_2\left(1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathrm{SNR}}=\frac{1}{\ln 2}\frac{d\ln\left(1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathrm{SNR}}$$

$$=\frac{1}{\ln 2}\frac{1}{\left(1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}\frac{d1+\mathrm{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)}{d\mathrm{SNR}}$$

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$$= \frac{1}{\ln 2} \frac{1}{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{1-\varepsilon}\right)\right)} \frac{d1 + \mathsf{SNR} \ln\left(\frac{1}{1-\varepsilon}\right)}{d\mathsf{SNR}}$$

$$= \frac{1}{\ln 2} \frac{1}{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{1-\varepsilon}\right)\right)} \ln\left(\frac{1}{1-\varepsilon}\right). \tag{4.116}$$

Now, we obtain

$$\lim_{\mathsf{SNR}\to 0} \frac{C_{\text{out}}^{\text{FSDF-nodirect}}(\epsilon, \mathsf{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon, \mathsf{SNR})} = \lim_{\mathsf{SNR}\to 0} \frac{\frac{1}{2\ln 2} \frac{\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)}{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)}}{\frac{1}{\ln 2} \frac{\ln\left(\frac{1}{1-\varepsilon}\right)}{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{1-\varepsilon}\right)\right)}}$$

$$= \lim_{\mathsf{SNR}\to 0} \frac{\frac{1}{2}\left(1 + \mathsf{SNR} \ln\left(\frac{1}{1-\varepsilon}\right)\right) \ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)}{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right) \ln\left(\frac{1}{1-\varepsilon}\right)}$$

$$= \frac{1}{2} \frac{-\ln\left(w_\varepsilon^{\text{FSDF-nodirect}}\right)}{-\ln(1-\varepsilon)} \lim_{\mathsf{SNR}\to 0} \frac{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{1-\varepsilon}\right)\right)}{\left(1 + \mathsf{SNR} \ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)}, \tag{4.117}$$

which gives (4.102). Q.E.D.

In the high signal-to-noise ratio regime, the outage capacity indicates that the fixed selective decode-and-forward without direct-link combining scheme is always worse than direct communication. In particular, direct communication offers double outage capacity, irrespective of $w_\varepsilon^{\text{FSDF-nodirect}}$. This means that the loss in degrees of freedom due to signal repetition is detrimental compared to the gain in diversity using cooperation. In the low signal-to-noise ratio regime, the topology of the relay network, which determines $w_{\varepsilon}^{\text{FSDF-nodirect}}$, can make the ratio to be greater or less than 1. If it is greater than 1, i.e.,

$$w_{\varepsilon}^{\text{FSDF-nodirect}} < (1-\epsilon)^2, \tag{4.118}$$

then the fixed selective decode-and-forward without direct-link combining scheme will provide higher outage capacity than direct communication (in the low signal-to-noise ratio regime). On the other hand, if the topology of the relay network makes

$$w_{\varepsilon}^{\text{FSDF-nodirect}} \geq (1-\epsilon)^2,$$

then the fixed selective decode-and-forward without direct-link combining scheme will provide lower outage capacity than direct communication in both high and low signal-to-noise ratio regimes. Intuitively, supposing all relay nodes are very far from the source and destination nodes, it is likely that the loss in degrees of freedom due to dividing the channel into two time slots dominates the improvement of the received signal strength from maximum ratio combining.

Suppose that the topology of the relay nodes yields a performance gain to the fixed selective decode-and-forward without direct-link combining scheme. The performance gain will become smaller as a function of signal-to-noise ratio, and vanish eventually. Then, the performance gain turns out to be a performance loss. Thus, we expect that there exists an SNR threshold at which both the fixed selective decode-and-forward without direct-link combining scheme and direct communication provide equal outage capacity, resulting in Corollary 1.

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Corollary 1 : For a given outage probability ϵ, the SNR at which the fixed selective decode-and-forward without direct-link combining scheme and direct communication have the same outage capacity is given by

$$\mathsf{SNR}_{\text{threshold}} = \frac{2\ln(1-\varepsilon) - \ln\left(w_{\varepsilon}^{\text{FSDF-nodirect}}\right)}{\left(\ln(1-\varepsilon)\right)^2}. \tag{4.119}$$

Proof : Since $\mathsf{SNR}_{\text{threshold}}$ is the SNR such that

$$C_{\text{out}}^{\text{DC}}\left(\varepsilon,\text{SNR}\right)=C_{\text{out}}^{\text{FSDF-nodirect}}\left(\varepsilon,\text{SNR}\right), \tag{4.120}$$

we equate (4.1) and (4.10), that is,

$$\log_2\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)=\frac{1}{2}\log_2\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right). \tag{4.121}$$

By multiplying both sides with $2\ln 2$, we obtain

$$2\ln 2\log_2\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)=\ln 2\log_2\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right). \tag{4.122}$$

Changing the base of the logarithm and using the properties of logarithm function, the equation can be manipulated as

$$\ln\left(\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2\right)=\ln\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right). \tag{4.123}$$

By taking anti-logarithm on both sides, we obtain

$$\left(1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2=1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right), \tag{4.124}$$

which means

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$$1+2\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)+\text{SNR}_{\text{threshold}}^2\left(\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2=1+\text{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right). \tag{4.125}$$

By subtracting 1 and dividing $\text{SNR}_{\text{threshold}}$ on both sides, we obtain

$$2\ln\left(\frac{1}{1-\varepsilon}\right)+\text{SNR}_{\text{threshold}}\left(\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2=\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right), \tag{4.126}$$

or equivalently,

$$-2\ln\left(1-\varepsilon\right)+\text{SNR}_{\text{threshold}}\left(-\ln\left(1-\varepsilon\right)\right)^2=-\ln\left(w_\varepsilon^{\text{FSDF-nodirect}}\right), \tag{4.127}$$

which can be manipulated further to give (4.119). Q.E.D.

The intuition is made precise as follows. We can improve the overall performance by adaptively switching between the fixed selective decode-and-forward without direct-link combining scheme and direct communication depending on the operating signal-to-noise ratio. When SNR is below $\text{SNR}_{\text{threshold}}$, we use the fixed selective decode-and-forward without direct-link combining scheme, which provides higher outage capacity. When SNR is above $\text{SNR}_{\text{threshold}}$, we switch to use direct communication, which provides higher outage capacity.

Proposition 6 : In the high signal-to-noise ratio regime, we have

$$\lim_{\text{SNR}\to\infty}\frac{C_{\text{out}}^{\text{FSDF-direct}}\left(\epsilon,\text{SNR}\right)}{C_{\text{out}}^{\text{DC}}\left(\epsilon,\text{SNR}\right)}=\frac{1}{2}, \tag{4.128}$$

and in the low signal-to-noise ratio regime, we have

$$\lim_{\text{SNR}\to 0}\frac{C_{\text{out}}^{\text{FSDF-direct}}\left(\epsilon,\text{SNR}\right)}{C_{\text{out}}^{\text{DC}}\left(\epsilon,\text{SNR}\right)}=\frac{1}{2}\frac{\ln\left(w_\varepsilon^{\text{FSDF-direct}}\right)}{\ln\left(1-\epsilon\right)}. \tag{4.129}$$

Proof : Let's consider the high signal-to-noise ratio regime. Since

$$C_{\text{out}}^{\text{FSDF-direct}}\left(\varepsilon,\text{SNR}\right)=\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right), \tag{4.130}$$

and

$$C_{\text{out}}^{\text{DC}}\left(\varepsilon,\text{SNR}\right)=\log_2\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right), \tag{4.131}$$

the limit of the ratio can be written as

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\text{out}}^{\text{FSDF-direct}}(\epsilon,\mathsf{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon,\mathsf{SNR})} = \lim_{\mathsf{SNR}\to\infty} \frac{\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}. \quad (4.132)$$

By the l'Hospital's rule, we take differentiation for both the numerator and the denominator with respect to SNR.

$$\lim_{\mathsf{SNR}\to\infty} \frac{\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)} = \lim_{\mathsf{SNR}\to\infty} \frac{\dfrac{d\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{d\mathsf{SNR}}}{\dfrac{d\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathsf{SNR}}}, \quad (4.133)$$

where

$$\frac{d\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)}{d\mathsf{SNR}} = \frac{d\frac{1}{2\ln 2}\ln\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{d\mathsf{SNR}}$$

$$= \frac{1}{2\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}\frac{d1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)}{d\mathsf{SNR}} \text{ (chain rule)} \quad (4.134)$$

$$= \frac{1}{2\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right),$$

and

$$\frac{d\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathsf{SNR}} = \frac{1}{\ln 2}\frac{d\ln\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\mathsf{SNR}}$$

$$= \frac{1}{\ln 2}\frac{1}{\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}\frac{d1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)}{d\mathsf{SNR}}$$

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$$= \frac{1}{\ln 2} \frac{1}{\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)} \ln\left(\frac{1}{1-\varepsilon}\right). \tag{4.135}$$

Now, we obtain

$$\lim_{\text{SNR}\to\infty} \frac{C_{\text{out}}^{\text{FSDF-direct}}(\epsilon,\text{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon,\text{SNR})} = \lim_{\text{SNR}\to\infty} \frac{\frac{1}{2\ln 2}\frac{\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\left(1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)}}{\frac{1}{\ln 2}\frac{\ln\left(\frac{1}{1-\varepsilon}\right)}{\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}}$$

$$= \lim_{\text{SNR}\to\infty} \frac{\frac{1}{2}\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\left(1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}. \tag{4.136}$$

By the l'Hospital's rule, we take another differentiation for both the numerator and the denominator with respect to SNR.

$$\lim_{\text{SNR}\to\infty} \frac{\frac{1}{2}\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\left(1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)} = \lim_{\text{SNR}\to\infty} \frac{\frac{d\,\frac{1}{2}\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{d\text{SNR}}}{\frac{d\left(1+\text{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}{d\text{SNR}}}, \tag{4.137}$$

where

$$\frac{d\,\frac{1}{2}\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{d\text{SNR}} = \frac{1}{2}\frac{d\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{d\text{SNR}}$$

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$$= \frac{1}{2}\ln\left(\frac{1}{1-\varepsilon}\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right), \tag{4.138}$$

and

$$\frac{d\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)\ln\left(\frac{1}{1-\varepsilon}\right)}{d\mathsf{SNR}} = \ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\ln\left(\frac{1}{1-\varepsilon}\right). \tag{4.139}$$

Then, we obtain

$$\begin{aligned}
\lim_{\mathsf{SNR}\to\infty}\frac{C_{\text{out}}^{\text{FSDF-direct}}(\epsilon,\mathsf{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon,\mathsf{SNR})} &= \lim_{\mathsf{SNR}\to\infty}\frac{\frac{1}{2}\ln\left(\frac{1}{1-\varepsilon}\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\ln\left(\frac{1}{1-\varepsilon}\right)} \\
&= \frac{1}{2}\lim_{\mathsf{SNR}\to\infty}\frac{\ln\left(\frac{1}{1-\varepsilon}\right)\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\ln\left(\frac{1}{1-\varepsilon}\right)}.
\end{aligned} \tag{4.140}$$

which gives (4.128).

Now, we consider the low signal-to-noise ratio regime. The limit of the ratio can be written as

$$\lim_{\mathsf{SNR}\to 0}\frac{C_{\text{out}}^{\text{FSDF-direct}}(\epsilon,\mathsf{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon,\mathsf{SNR})} = \lim_{\mathsf{SNR}\to 0}\frac{\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)}{\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}. \tag{4.141}$$

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By the l'Hospital's rule, we take differentiation for both the numerator and the denominator with respect to SNR.

$$\lim_{\text{SNR}\to 0} \frac{\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{\log_2\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)} = \lim_{\text{SNR}\to 0} \frac{\frac{d\,\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{d\text{SNR}}}{\frac{d\log_2\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\text{SNR}}}, \tag{4.142}$$

where

$$\begin{aligned}
\frac{d\,\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{d\text{SNR}} &= \frac{d\,\frac{1}{2\ln 2}\ln\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}{d\text{SNR}} \\
&= \frac{1}{2\ln 2}\frac{1}{\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}\frac{d1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)}{d\text{SNR}} \quad \text{(chain rule)} \\
&= \frac{1}{2\ln 2}\frac{1}{\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right)}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right),
\end{aligned} \tag{4.143}$$

and

$$\begin{aligned}
\frac{d\log_2\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\text{SNR}} &= \frac{1}{\ln 2}\frac{d\ln\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}{d\text{SNR}} \\
&= \frac{1}{\ln 2}\frac{1}{\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}\frac{d1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)}{d\text{SNR}} \quad \text{(chain rule)} \\
&= \frac{1}{\ln 2}\frac{1}{\left(1+\text{SNR}\ln\left(\frac{1}{1-\varepsilon}\right)\right)}\ln\left(\frac{1}{1-\varepsilon}\right).
\end{aligned} \tag{4.144}$$

Now, we obtain

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$$\lim_{\mathrm{SNR}\to 0}\frac{C_{\mathrm{out}}^{\text{FSDF-direct}}(\epsilon,\mathrm{SNR})}{C_{\mathrm{out}}^{\mathrm{DC}}(\epsilon,\mathrm{SNR})}=\lim_{\mathrm{SNR}\to 0}\frac{\dfrac{1}{2\ln 2}\dfrac{\ln\left(\dfrac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\left(1+\mathrm{SNR}\ln\left(\dfrac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)}}{\dfrac{1}{\ln 2}\dfrac{\ln\left(\dfrac{1}{1-\varepsilon}\right)}{\left(1+\mathrm{SNR}\ln\left(\dfrac{1}{1-\varepsilon}\right)\right)}}$$

$$=\lim_{\mathrm{SNR}\to 0}\frac{\dfrac{1}{2}\left(1+\mathrm{SNR}\ln\left(\dfrac{1}{1-\varepsilon}\right)\right)\ln\left(\dfrac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)}{\left(1+\mathrm{SNR}\ln\left(\dfrac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)\ln\left(\dfrac{1}{1-\varepsilon}\right)} \tag{4.145}$$

$$=\frac{1}{2}\frac{-\ln\left(w_{\varepsilon}^{\text{FSDF-direct}}\right)}{-\ln(1-\varepsilon)}\lim_{\mathrm{SNR}\to 0}\frac{\left(1+\mathrm{SNR}\ln\left(\dfrac{1}{1-\varepsilon}\right)\right)}{\left(1+\mathrm{SNR}\ln\left(\dfrac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)},$$

which gives (4.131). Q.E.D.

From Proposition 5 and Proposition 6, the outage capacity of the fixed selective decode-and-forward without direct-link combining scheme converges to that of the fixed selective decode-and-forward with direct-link combining scheme in both high and low signal-to-noise ratio regimes. Therefore, choosing not to do maximum ratio combining at the destination node is harmful to the obtained outage capacity only in the medium signal-to-noise ratio regime.

In the high signal-to-noise ratio regime, the outage capacity indicates that the fixed selective decode-and-forward with direct-link combining scheme is always worse than direct communication. In particular, direct communication offers double outage capacity, irrespective of $w_{\varepsilon}^{\text{FSDF-direct}}$. This means that the loss in degrees of freedom due to signal repetition is detrimental compared to the gain in diversity using cooperation. In the low signal-to-noise ratio regime, the topology of the relay network, which determines $w_{\varepsilon}^{\text{FSDF-direct}}$, can make the ratio to be greater or less than 1. If it is greater than 1, i.e.,

$$w_{\varepsilon}^{\text{FSDF-direct}} < (1-\epsilon)^2, \tag{4.146}$$

then the fixed selective decode-and-forward with direct-link combining scheme will provide higher outage capacity than direct communication (in the low signal-to-noise ratio regime). On the other hand, if the topology of the relay network makes

$$w_{\varepsilon}^{\text{FSDF-direct}} \geq (1-\epsilon)^2,$$

then the fixed selective decode-and-forward with direct-link combining scheme will provide lower outage capacity than direct communication in both high and low signal-to-noise ratio regimes. Intuitively, supposing all relay nodes are very far from the source and destination nodes, it is likely that the loss in degrees of freedom due to dividing the channel into two time slots dominates the improvement of the received signal strength from maximum ratio combining.

Suppose that the topology of the relay nodes yields a performance gain to the fixed selective decode-and-forward with direct-link combining scheme. The performance gain will become smaller as a function of signal-to-noise ratio, and vanish eventually. Then, the performance gain turns out to be a performance loss. Thus, we expect that there exists an SNR threshold at which both the fixed selective decode-and-forward with direct-link combining scheme and direct communication provide equal outage capacity, resulting in Corollary 2.

Corollary 2 : For a given outage probability ϵ, the SNR at which the fixed selective decode-and-forward with direct-link combining scheme and direct communication have the same outage capacity is given by

$$\mathsf{SNR}_{\text{threshold}} = \frac{2\ln(1-\varepsilon) - \ln\left(w_{\varepsilon}^{\text{FSDF-direct}}\right)}{\left(\ln(1-\varepsilon)\right)^2}. \tag{4.147}$$

Proof : Since $\mathsf{SNR}_{\text{threshold}}$ is the SNR such that

$$C_{\text{out}}^{\text{DC}}\left(\varepsilon, \mathsf{SNR}\right) = C_{\text{out}}^{\text{FSDF-direct}}\left(\varepsilon, \mathsf{SNR}\right), \tag{4.148}$$

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we equate (4.1) and (4.38), that is,

$$\log_2\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)=\frac{1}{2}\log_2\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right). \tag{4.149}$$

By multiplying both sides with $2\ln 2$, we obtain

$$2\ln 2\log_2\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)=\ln 2\log_2\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right). \tag{4.150}$$

Changing the base of the logarithm and using the properties of logarithm function, the equation can be manipulated as

$$\ln\left(\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2\right)=\ln\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right). \tag{4.151}$$

By taking anti-logarithm on both sides, we obtain

$$\left(1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2=1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right), \tag{4.152}$$

which means

$$1+2\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{1-\varepsilon}\right)+\mathsf{SNR}_{\text{threshold}}^2\left(\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2=1+\mathsf{SNR}_{\text{threshold}}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right). \tag{4.153}$$

By subtracting 1 and dividing $\mathsf{SNR}_{\text{threshold}}$ on both sides, we obtain

$$2\ln\left(\frac{1}{1-\varepsilon}\right)+\mathsf{SNR}_{\text{threshold}}\left(\ln\left(\frac{1}{1-\varepsilon}\right)\right)^2=\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right), \tag{4.154}$$

or equivalently,

$$-2\ln(1-\varepsilon)+\mathsf{SNR}_{\text{threshold}}\left(-\ln(1-\varepsilon)\right)^2=-\ln\left(w_{\varepsilon}^{\text{FSDF-direct}}\right), \tag{4.155}$$

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which can be manipulated further to give (4.147). Q.E.D.

The intuition is made precise as follows. We can improve the overall performance by adaptively switching between the fixed selective decode-and-forward with direct-link combining scheme and direct communication depending on the operating signal-to-noise ratio. When SNR is below $\mathsf{SNR}_{\text{threshold}}$, we use the fixed selective decode-and-forward with direct-link combining scheme, which provides higher outage capacity. When SNR is above $\mathsf{SNR}_{\text{threshold}}$, we switch to use direct communication, which provides higher outage capacity. Also, the fixed selective decode-and-forward without direct-link combining scheme can be used instead if the performance penalty in the medium signal-to-noise ratio regime is acceptable in the considered topology.

Proposition 7 : In the high signal-to-noise ratio regime, we have

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\text{out}}^{\text{SSDF}}\left(\epsilon, \mathsf{SNR}\right)}{C_{\text{out}}^{\text{DC}}\left(\epsilon, \mathsf{SNR}\right)} = 1. \tag{4.156}$$

Proof : By definition, $R_{\text{DC}} \le R_{\text{SSDF}}$, implying that

$$C_{\text{out}}^{\text{DC}}(\epsilon, \mathsf{SNR}) \le C_{\text{out}}^{\text{SSDF}}(\epsilon, \mathsf{SNR}), \tag{4.157}$$

and hence,

$$1 \le \liminf_{\text{SNR}\to\infty} \frac{C_{\text{out}}^{\text{SSDF}}(\epsilon, \mathsf{SNR})}{C_{\text{out}}^{\text{DC}}(\epsilon, \mathsf{SNR})}. \tag{4.158}$$

(We take the limit infimum because we are not sure yet whether the limit exists.) Next, fix k and consider the following bound:

$$\min\left\{ R_{Sk}, R_{\text{MRC}}(k) \right\} \le 2R_{\text{MRC}}(k). \tag{4.159}$$

Taking a maximum over $k \in \mathcal{K}$ and over R_{DC} gives

$$R_{\text{SSDF}} \le \bar{\bar{R}}_{\text{SSDF}}, \tag{4.160}$$

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where

$$\bar{\bar{R}}_{\text{SSDF}} \triangleq \max\left\{ \max_{k\in\mathcal{K}}\{2R_{\text{MRC}}(k)\}, R_{\text{DC}} \right\}. \tag{4.161}$$

Since

$$R_{\text{MRC}}(k) > R_{\text{SD}}, \quad \forall k, \tag{4.162}$$

we obtain

$$\begin{aligned} \max\left\{ \max_{k\in\mathcal{K}}\{2R_{\text{MRC}}(k)\}, R_{\text{DC}} \right\} &= \max_{k\in\mathcal{K}} 2R_{\text{MRC}}(k) \\ &= \max_{k\in\mathcal{K}} 2\frac{1}{2} I_{\text{SD}+k\text{D}}\left(SNR_{k\text{D}} + SNR_{\text{SD}}\right). \end{aligned} \tag{4.163}$$

Since the mutual information is a concave function,

$$\max_{k\in\mathcal{K}} I_{\text{SD}+k\text{D}}\left(SNR_{k\text{D}} + SNR_{\text{SD}}\right) = \log_2\left[1 + X\,\text{SNR}\right], \tag{4.164}$$

for the random variable

$$X \triangleq \left|h_{\text{SD}}\right|^2 + \max_{k\in\mathcal{K}}\left|h_{k\text{D}}\right|^2. \tag{4.165}$$

The inequality (4.160) implies that

$$C_{\text{out}}^{\text{SSDF}}(\epsilon, \text{SNR}) \le \bar{\bar{C}}_{\text{out}}^{\text{SSDF}}(\epsilon, \text{SNR}), \tag{4.166}$$

where $\bar{\bar{C}}_{\text{out}}^{\text{SSDF}}(\epsilon, \text{SNR})$ is the outage capacity of a transmission scheme that has the maximum instantaneous end-to-end mutual information of $\bar{\bar{R}}_{\text{SSDF}}$:

$$\bar{\bar{C}}_{\text{out}}^{\text{SSDF}}(\epsilon, \text{SNR}) = \log_2\left[1 + \text{SNR}\, F_X^{-1}(\varepsilon)\right], \tag{4.167}$$

where F_X is the cumulative density function of X. (The fact that $F_X : [0,\infty) \to [0,1)$ is strictly increasing and is a bijection can be shown by using a similar argument shown in Lemma 4.)

Dividing the outage capacities by $C_{\text{out}}^{\text{DC}}(\epsilon, \text{SNR})$ and taking the limit supremum give

$$\limsup_{\mathrm{SNR}\to\infty} \frac{C_{\mathrm{out}}^{\mathrm{SSDF}}(\epsilon,\mathsf{SNR})}{C_{\mathrm{out}}^{\mathrm{DC}}(\epsilon,\mathsf{SNR})} \le \limsup_{\mathrm{SNR}\to\infty} \frac{\overline{\overline{C}}_{\mathrm{out}}^{\mathrm{SSDF}}(\epsilon,\mathsf{SNR})}{C_{\mathrm{out}}^{\mathrm{DC}}(\epsilon,\mathsf{SNR})}. \tag{4.168}$$

By the l'Hospital's rule,

$$\limsup_{\mathrm{SNR}\to\infty} \frac{\overline{\overline{C}}_{\mathrm{out}}^{\mathrm{SSDF}}(\epsilon,\mathsf{SNR})}{C_{\mathrm{out}}^{\mathrm{DC}}(\epsilon,\mathsf{SNR})} = 1, \tag{4.169}$$

which implies that

$$\limsup_{\mathrm{SNR}\to\infty} \frac{C_{\mathrm{out}}^{\mathrm{SSDF}}(\epsilon,\mathsf{SNR})}{C_{\mathrm{out}}^{\mathrm{DC}}(\epsilon,\mathsf{SNR})} \le 1. \tag{4.170}$$

The inequalities (4.158) and (4.170) imply that the limit supremum and the limit infimum equal 1, and hence the existence of the limit, as given in the statement of the proposition. Q.E.D.

In the high signal-to-noise ratio regime, we see that the smart selective decode-and-forward scheme converges to direct communication, regardless of the relay network topology. This means that if the channel between the source and destination nodes is strong, the increased loss in bandwidth from dividing the channel into two time slots to do relaying dominates the increased received signal strength from combining the regenerated signals from any relay. In the low signal-to-noise ratio regime, the smart selective decode-and-forward scheme performs better than or, in the worst case, as good as direct communication with the help from relay nodes. This means that if no relay can improve the performance, the smart selective decode-and-forward scheme will simply use direct communication. In any case, the smart selective decode-and-forward scheme does not perform worse than direct communication for any signal-to-noise ratio. This is because the smart selective decode-and-forward scheme decides not to use cooperative diversity when direct communication offers a higher rate.

Naturally, increasing the number of relay nodes in the cooperative diversity schemes with relay selection is beneficial in terms of performance. Hence, the network that has larger number of relay nodes provides greater outage capacity than that of the network with smaller number of relay nodes at any signal-to-noise ratio. From Proposition 7, it is clear that the outage capacity of the smart selective decode-and-forward scheme in the high signal-to-noise ratio regime does not increase by adding more relay nodes. This can be attributed to the fact that the smart selective decode-and-forward scheme converges to direct communication when the signal-to-noise ratio increases. On the other hand, It is different for the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme. In Corollary 3 and Corollary 4, we investigate the performance gap when we plot the outage capacities of the network with different number of relay nodes as a function of signal-to-noise ratio for both schemes.

Corollary 3 : In the high signal-to-noise ratio, the curve gaps among the outage capacity plot of the fixed selective decode-and-forward without direct link combining scheme as a function of signal-to-noise ratio are constant. Hence, adding more relays still increases the outage capacity in the high signal-to-noise ratio regime.

Proof : From Theorem 5,

$$C_{\text{out}}^{\text{FSDF-nodirect}}\left(\varepsilon,\text{SNR}\right)=\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right). \tag{4.171}$$

As the signal-to-noise ratio becomes high,

$$\begin{aligned}\lim_{\text{SNR}\to\infty} C_{\text{out}}^{\text{FSDF-nodirect}}\left(\varepsilon,\text{SNR}\right)&=\lim_{\text{SNR}\to\infty}\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)\\&=\frac{1}{2}\log_2\left(\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\lim_{\text{SNR}\to\infty}\text{SNR}\right),\end{aligned} \tag{4.172}$$

where the logarithm can be split, thus we obtain

$$\lim_{\text{SNR}\to\infty} C_{\text{out}}^{\text{FSDF-nodirect}}\left(\varepsilon,\text{SNR}\right)=\frac{1}{2}\log_2\left(\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-nodirect}}}\right)\right)+\frac{1}{2}\log_2\left(\lim_{\text{SNR}\to\infty}\text{SNR}\right). \tag{4.173}$$

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The second term makes the curve of outage capacity linear as a function of SNR in decibels, and the first term, which is a constant, introduces the vertical offset of the curve. Hence, two networks with different number of relay nodes have equal second term and different constant terms. This means the gap between two curves is constant.

Now, we show that adding more relays still increases the outage capacity even in the high signal-to-noise ratio regime. Considering two networks with unequal number of relay nodes, suppose the corresponding $w_{\varepsilon}^{\text{FSDF-nodirect}}$'s are x and y, respectively. The corresponding outage capacities are

$$\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{x}\right)\right),$$

and

$$\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{y}\right)\right),$$

respectively. As the signal-to-noise ratio becomes high,

$$\begin{aligned}
\lim_{\text{SNR}\to\infty}\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{x}\right)\right)-\frac{1}{2}\log_2\left(1+\text{SNR}\ln\left(\frac{1}{y}\right)\right) &= \lim_{\text{SNR}\to\infty}\frac{1}{2}\log_2\left(\frac{1+\text{SNR}\ln\left(\frac{1}{x}\right)}{1+\text{SNR}\ln\left(\frac{1}{y}\right)}\right)\\
&= \frac{1}{2}\log_2\left(\frac{\ln\left(\frac{1}{x}\right)\lim_{\text{SNR}\to\infty}\text{SNR}}{\ln\left(\frac{1}{y}\right)\lim_{\text{SNR}\to\infty}\text{SNR}}\right)\\
&= \frac{1}{2}\log_2\left(\frac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{y}\right)}\right),
\end{aligned}$$

where the first line uses the logarithm's property. Since $x \neq y$,

$$\frac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{y}\right)} \neq 1, \tag{4.174}$$

and so

$$\frac{1}{2}\log_2\left(\frac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{y}\right)}\right) \neq 0, \tag{4.175}$$

which means

$$\left|\lim_{\mathsf{SNR}\to\infty} \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{x}\right)\right) - \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{y}\right)\right)\right| > 0. \tag{4.176}$$

Since two networks must have different outage capacities and having larger number of relay nodes is beneficial, adding more relays still increases the outage capacity in the high signal-to-noise ratio regime. Q.E.D.

Corollary 4 : In the high signal-to-noise ratio, the curve gaps among the outage capacity plot of the fixed selective decode-and-forward with direct link combining scheme as a function of signal-to-noise ratio are constant. Hence, adding more relays still increases the outage capacity in the high signal-to-noise ratio regime.

Proof : From Theorem 6,

$$C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\mathsf{SNR}) = \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right). \tag{4.177}$$

As the signal-to-noise ratio becomes high,

$$\begin{aligned}\lim_{\mathsf{SNR}\to\infty} C_{\text{out}}^{\text{FSDF-direct}}(\varepsilon,\mathsf{SNR}) &= \lim_{\mathsf{SNR}\to\infty}\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\right) \\ &= \frac{1}{2}\log_2\left(\ln\left(\frac{1}{w_\varepsilon^{\text{FSDF-direct}}}\right)\lim_{\mathsf{SNR}\to\infty}\mathsf{SNR}\right),\end{aligned} \tag{4.178}$$

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where the logarithm can be split, thus we obtain

$$\lim_{\mathsf{SNR}\to\infty} C_{\text{out}}^{\text{FSDF-direct}}\left(\varepsilon,\mathsf{SNR}\right)=\frac{1}{2}\log_2\left(\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right)+\frac{1}{2}\log_2\left(\lim_{\mathsf{SNR}\to\infty}\mathsf{SNR}\right). \qquad (4.179)$$

The second term makes the curve of outage capacity linear as a function of SNR in decibels, and the first term, which is a constant, introduces the vertical offset of the curve. Hence, two networks with different number of relay nodes have equal second term and different constant terms. This means the gap between two curves is constant.

Now, we show that adding more relays still increases the outage capacity even in the high signal-to-noise ratio regime. Considering two networks with unequal number of relay nodes, suppose the corresponding $w_{\varepsilon}^{\text{FSDF-direct}}$'s are x and y, respectively. The corresponding outage capacities are

$$\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{x}\right)\right),$$

and

$$\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{y}\right)\right),$$

respectively. As the signal-to-noise ratio becomes high,

$$\begin{aligned}
&\lim_{\mathsf{SNR}\to\infty}\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{x}\right)\right)-\frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{y}\right)\right)\\
&=\lim_{\mathsf{SNR}\to\infty}\frac{1}{2}\log_2\left(\frac{1+\mathsf{SNR}\ln\left(\frac{1}{x}\right)}{1+\mathsf{SNR}\ln\left(\frac{1}{y}\right)}\right)\\
&=\frac{1}{2}\log_2\left(\frac{\ln\left(\frac{1}{x}\right)\lim_{\mathsf{SNR}\to\infty}\mathsf{SNR}}{\ln\left(\frac{1}{y}\right)\lim_{\mathsf{SNR}\to\infty}\mathsf{SNR}}\right)
\end{aligned}$$

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$$= \frac{1}{2}\log_2\left(\frac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{y}\right)}\right), \tag{4.180}$$

where the first line uses the logarithm's property. Since $x \neq y$,

$$\frac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{y}\right)} \neq 1, \tag{4.181}$$

and so

$$\frac{1}{2}\log_2\left(\frac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{y}\right)}\right) \neq 0, \tag{4.182}$$

which means

$$\left|\lim_{\mathsf{SNR}\to\infty} \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{x}\right)\right) - \frac{1}{2}\log_2\left(1+\mathsf{SNR}\ln\left(\frac{1}{y}\right)\right)\right| > 0. \tag{4.183}$$

Since two networks must have different outage capacities and having larger number of relay nodes is beneficial, adding more relays still increases the outage capacity in the high signal-to-noise ratio regime. Q.E.D.

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It is interesting to examine whether improving the outage capacity by keep adding more relay nodes for the cooperative diversity schemes with relay selection leads to a saturation. Intuitively, it is likely that all schemes have the saturations because only the best relay node is utilized regardless of the number of relay nodes. Surprisingly, the proof in Theorem 8 indicates the answer in the other way around for all schemes.

Theorem 8 : By keep adding more relay nodes, the fixed selective decode-and-forward without direct link combining scheme, the fixed selective decode-and-forward with direct link combining scheme, and the smart selective decode-and-forward scheme are unbounded.

Proof : For the fixed selective decode-and-forward without direct link combining scheme with K relay nodes, we solve for $w_{\epsilon}^{\text{FSDF-nodirect}}$ in

$$\prod_{k=1}^{K}\left[1-w_{\epsilon}^{\text{FSDF-nodirect}\frac{d_{\text{S}k}^{\alpha}+d_{k\text{D}}^{\alpha}}{d_{\text{SD}}^{\alpha}}}\right]=\epsilon,\quad \epsilon>0, w_{\epsilon}^{\text{FSDF-nodirect}}<1. \tag{4.184}$$

Based on the proof by contradiction, we claim that the outage capacity of the fixed selective decode-and-forward without direct link combining scheme is bounded, that is,

$$C_{\text{out}}^{\text{FSDF-nodirect}}\left(\epsilon,\text{SNR}\right)<\infty. \tag{4.185}$$

To find a contradiction, let

$$d_{\text{S}k}=d_{k\text{D}}=\frac{d_{\text{SD}}}{2} \tag{4.186}$$

for every k. We obtain

$$\begin{aligned}\epsilon&=\prod_{k=1}^{K}\left[1-w_{\epsilon}^{\text{FSDF-nodirect}\frac{(d_{\text{SD}}/2)^{\alpha}+(d_{\text{SD}}/2)^{\alpha}}{d_{\text{SD}}^{\alpha}}}\right]\\&=\left[1-w_{\epsilon}^{\text{FSDF-nodirect}\frac{2(d_{\text{SD}}/2)^{\alpha}}{d_{\text{SD}}^{\alpha}}}\right]^{K}\\&=\left[1-w_{\epsilon}^{\text{FSDF-nodirect}\,2^{1-\alpha}}\right]^{K}.\end{aligned} \tag{4.187}$$

Since

$$\lim_{K\to\infty}\sqrt[K]{\epsilon}=1, \tag{4.188}$$

we obtain

$$\left|\left[1-w_{\epsilon}^{\text{FSDF-nodirect}\,2^{1-\alpha}}\right]-1\right|<\delta \tag{4.189}$$

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for any $\delta > 0$. Thus, there exists K such that

$$w_{\epsilon}^{\text{FSDF-nodirect}\,2^{\alpha-1}} < \delta^{2^{\alpha-1}}. \tag{4.190}$$

Since $\alpha > 1$, $2^{1-\alpha} < 1$. Therefore,

$$w_{\epsilon}^{\text{FSDF-nodirect}} < \delta, \tag{4.191}$$

which means that we can keep increasing K to make $w_{\epsilon}^{\text{FSDF-nodirect}}$ arbitrarily small, and as $\delta \to 0$

$$\begin{aligned} C_{\text{out}}^{\text{FSDF-nodirect}}(\epsilon, \mathsf{SNR}) &= \frac{1}{2}\log_2\left(1 + \mathsf{SNR}\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right) \\ &\not< \infty. \end{aligned} \tag{4.192}$$

From Lemma 5,

$$C_{\text{out}}^{\text{FSDF-nodirect}}(\epsilon, \mathsf{SNR}) \le C_{\text{out}}^{\text{FSDF-direct}}(\epsilon, \mathsf{SNR}) \le C_{\text{out}}^{\text{SSDF}}(\epsilon, \mathsf{SNR}) \tag{4.193}$$

for any ϵ and SNR. It follows that

$$C_{\text{out}}^{\text{FSDF-direct}}(\epsilon, \mathsf{SNR}) \not< \infty, \tag{4.194}$$

and

$$C_{\text{out}}^{\text{SSDF}}(\epsilon, \mathsf{SNR}) \not< \infty \tag{4.195}$$

as well. Q.E.D.

4.6 Verification with Computer Simulations

In this section, we verify the analytical outage capacities with Monte Carlo simulations. To cover several cases, we vary the number of relay nodes at 1, 4, or 9, and the acceptable outage probabilities at 0.1 and 0.01. The relay nodes are arranged in grid topology between the source node and the destination node, between which the distance is 1000 m. All curves are plotted as a function of SNR, which is the average signal-to-noise ratio between the source node and the destination node.

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In Figure 25 – Figure 30, we plot the analytical and simulated outage capacities of the fixed selective decode-and-forward without direct link combining scheme, the fixed selective decode-and-forward with direct link combining scheme, and the smart selective decode-and-forward scheme with the number of relay nodes $K = \{1, 4, 9\}$ and at an outage probability $\epsilon = \{0.1, 0.01\}$, as a function of the SNR. It can be observed that the analytical results are in good agreement with the simulation results, showing the validity of our analytical expressions in Theorem 5 – Theorem 7.

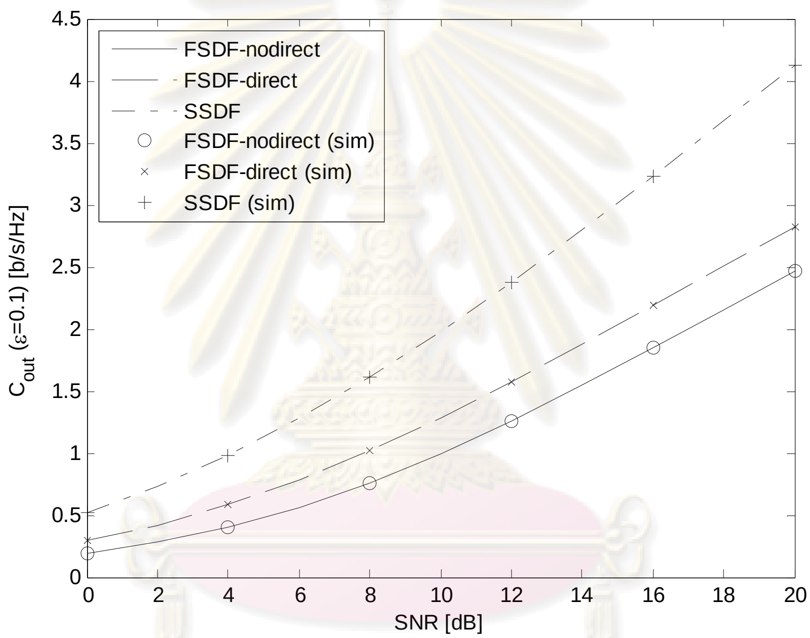

Figure 25 Verification of the outage capacities of all analyzed cooperative diversity schemes at an outage probability of 0.1 for 1-relay network

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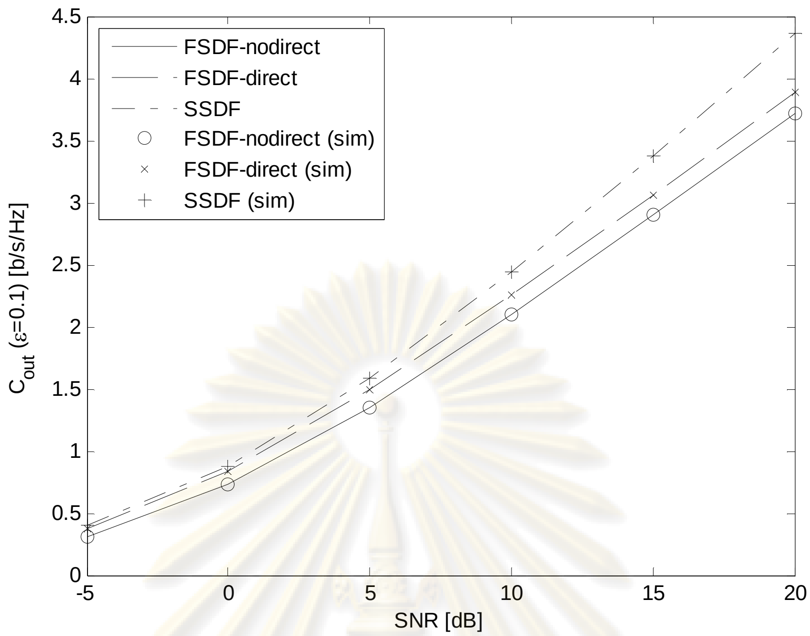

Figure 26 Verification of the outage capacities of all analyzed cooperative diversity schemes at an outage probability of 0.1 for 4-relay network

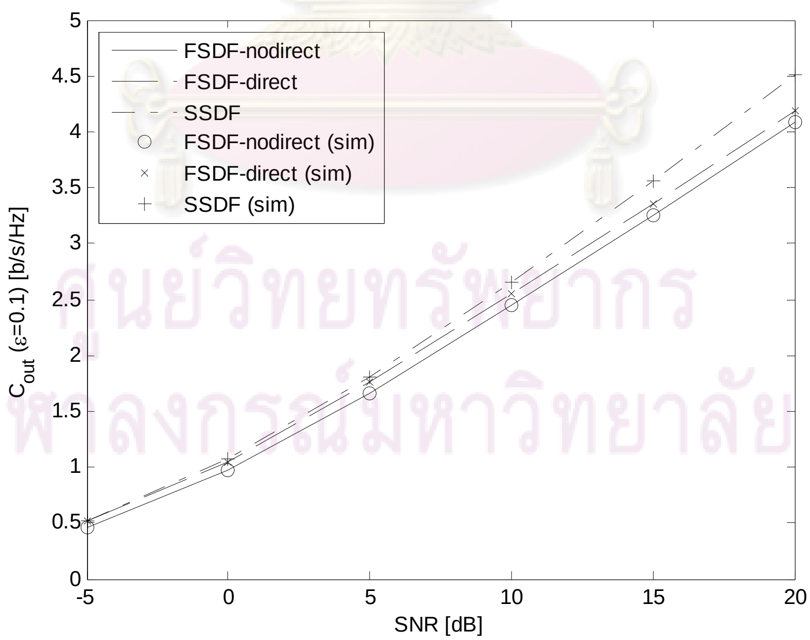

Figure 27 Verification of the outage capacities of all analyzed cooperative diversity schemes at an outage probability of 0.1 for 9-relay network

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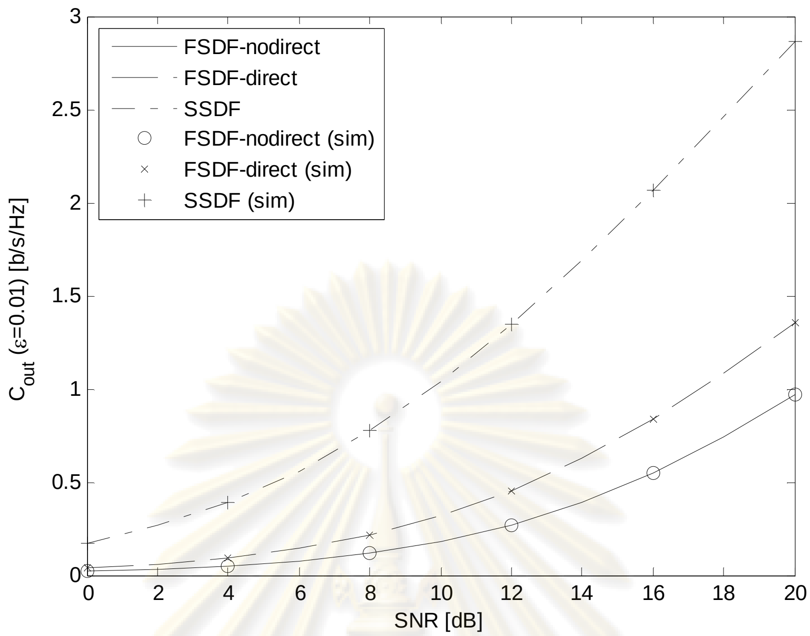

Figure 28 Verification of the outage capacities of all analyzed cooperative diversity schemes at an outage probability of 0.01 for 1-relay network

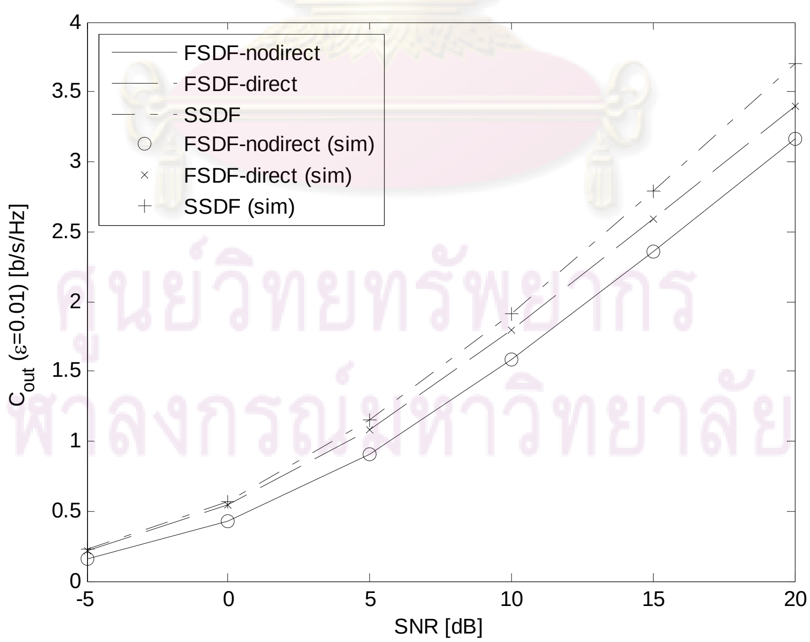

Figure 29 Verification of the outage capacities of all analyzed cooperative diversity schemes at an outage probability of 0.01 for 4-relay network

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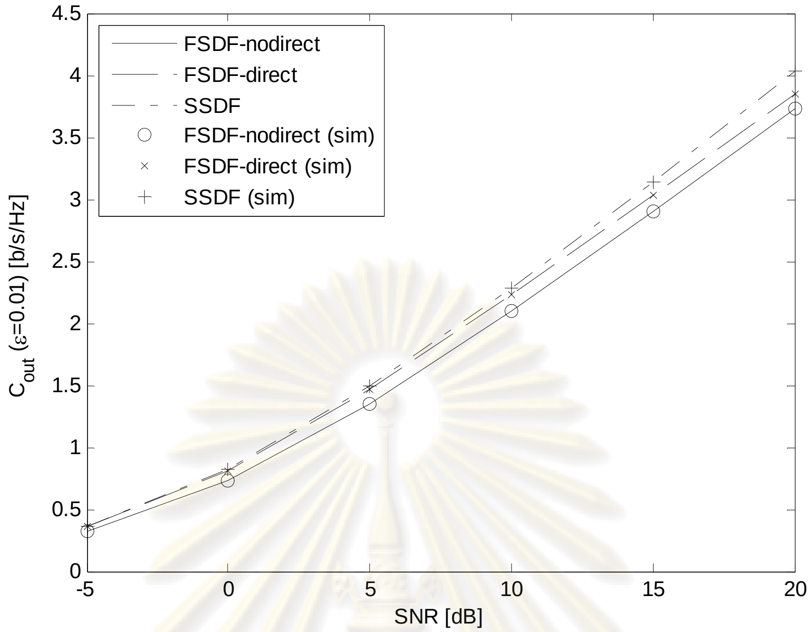

Figure 30 Verification of the outage capacities of all analyzed cooperative diversity schemes at an outage probability of 0.01 for 9-relay network

4.7 Results and Discussions

We examine the performance gain of using cooperative diversity schemes over direct communication in terms of outage capacity with various parameters. The outage capacity of direct communication is provided in Proposition 4. In Figure 31, the acceptable outage probability is 0.01, and the number of relay nodes is 4 with grid topology. Compared to direct communication, all cooperative diversity schemes offer significant performance gain. Among the cooperative diversity schemes, the fixed selective decode-and-forward with direct link combining scheme is superior to the fixed selective decode-and-forward without direct link combining scheme, and the smart selective decode-and-forward scheme is in turn superior to the fixed selective decode-and-forward with direct link combining scheme as expected in Lemma 5. As we increase the number of relay nodes, the performance gain is larger for all cooperative diversity schemes as shown in Figure 32, where we increase the number of relay nodes to 9 relay nodes. Also, the performance gaps among cooperative diversity schemes become smaller. Therefore, the ability, to decide to cooperate or not cooperate, introduced in the smart selective decode-and-forward scheme or the ability, to combine the signals from the direct link, introduced in the fixed selective decode-and-forward scheme with direct link combining scheme can be overlooked in favor of the simpler fixed selective decode-and-forward without direct link combining scheme when the number of relay nodes is large enough. Then, we examine in the reverse way by decreasing the number of relay nodes as shown in Figure 33, where we decrease the number of relay nodes to 1 relay node. It can be observed that the gap among the cooperative diversity schemes become wide, and the performance gain obtained from the fixed selective decode-and-forward without direct link combining scheme over direct communication becomes marginal.

We predict in Corollary 1 and Corollary 2 that the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme have the signal-to-noise ratio threshold that the outage capacity becomes lower than that of direct communication, and also predict the value of that signal-to-noise ratio threshold. Therefore, we compare the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme thresholds to direct communication at higher signal-to-noise ratio in Figure 34 and Figure 35, respectively. It can be observed that the threshold really exists for both schemes, and the predicted value is exact.

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Proposition 5 proves that the ratio between the outage capacities obtained by the fixed selective decode-and-forward without direct link combining scheme and by direct communication converges to a half at very high signal-to-noise ratio. In Figure 36, it can be observed that the ratio between two curves converges to a half as the signal-to-noise ratio rises. Likewise, Proposition 6 proves that the ratio between the outage capacities obtained by the fixed selective decode-and-forward with direct link combining scheme and by direct communication converges to a half at very high signal-to-noise ratio, and Figure 37 illustrates that. Also, Proposition 7 proves that the ratio between the outage capacities obtained by the smart selective decode-and-forward scheme and by direct communication converges to one at very high signal-to-noise ratio, and that is illustrated in Figure 38. Therefore, it is not necessary to use cooperative diversity scheme when the signal-to-noise ratio is very high, and it is not harmful to use the smart selective decode-and-forward scheme because this scheme will tend to use direct communication automatically.

Corollary 3 and Corollary 4 prove that the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme have the linear outage capacity curve in the medium and high signal-to-noise ratio regimes. Also, when we compare the outage capacity curves using different number of relay nodes, the gaps among the curves are constant, and so increasing the number of relay nodes still improves the outage capacity even in high signal-to-noise ratio. These are shown in Figure 39 and Figure 40 for the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme, respectively.

We examine the effect of increasing the number of relay nodes in Figure 41, where we compare the outage capacities of the fixed selective decode-and-forward without direct link combining scheme with various number of relay nodes. By observing the curves alone, it can be conjectured that the saturation in improvement exists, that is, we cannot improve the outage capacity by simply increasing the number of relay nodes. However, Theorem 8 proves that the outage capacities of all considered cooperative diversity schemes are unbounded, and so the saturation does not exist. Therefore, we fix the signal-to-noise ratio at 20 dB and plot the outage capacity as a function of number of relay nodes in Figure 42. It can be observed that the curve crosses the horizontal line even at very large number of relay nodes. Hence, the curve does not become constant and the saturation does not exist. From both figures, we can show that increasing the number of relay nodes always improves the outage capacity but the improvement keeps being smaller.

We examine the effect of varying topology. We fix the signal-to-noise ratio at 20 dB, and generate 10,000 random topologies. Hence, the outage capacity of the network is now a random variable, depending on the specific network topology, and we consider the cumulative distribution function of the outage capacity. In Figure 43, we set the number of relay nodes at 4. The vertical line that crosses each cumulative distribution function curve marks the outage capacity (on the x-axis) of deterministic grid topology. About 35% of random network realizations provide better performance than the grid topology for all relaying schemes. Hence, the placement of relay nodes optimization offers benefit if it is possible. The performance differences among the schemes are reduced when the number of relays increases from 4 to 9 as shown in Figure 44. This supports the idea that the improvement from increasing the number of relay nodes keeps being smaller as the number of relay nodes becomes large. Also, it can be observed that when the number of relay nodes is 9, the vertical lines mark the probabilities of approximately 1 on the y-axis, implying that almost none of random network realizations provide better outage capacities than the grid topology provides. Hence, when the number of relay nodes becomes large, the grid topology is a good topology to use in deploying the relays, and the placement of relay nodes optimization is not necessary. On the other hand, when the number of relays is reduced to 1, there are different effects among schemes as shown in Figure 45. The fixed selective decode-and-forward without direct link combining only relies on the two-hop relaying via a single relay node. The grid topology, that is, placing the relay node exactly in the middle, balances and maximizes the mutual information of both hops. Hence, the vertical line marks the probability of 1. The fixed selective decode-and-forward with direct link combining has a stronger second hop, and the balance of the mutual information can occur by placing the relay node closer to the source node. Hence, the vertical line marks the probability lower than 1.

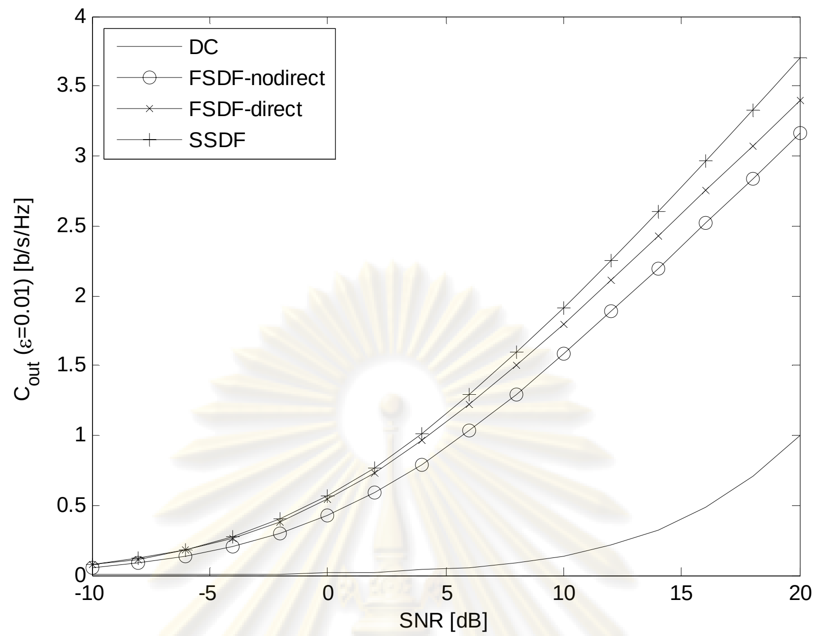

Figure 31 Outage capacity comparisons of all considered cooperative diversity schemes at an outage probability of 0.01 for 4-relay network with direct communication

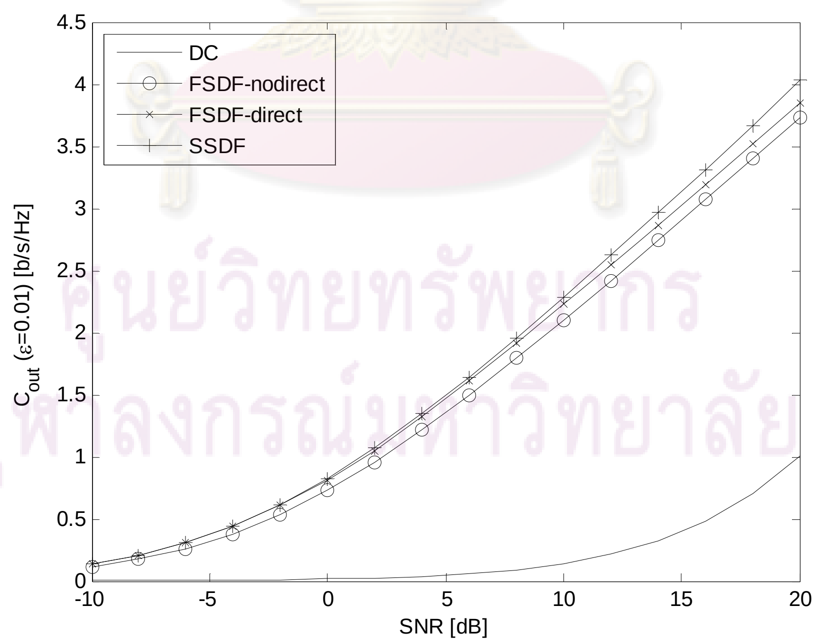

Figure 32 Outage capacity comparisons of all considered cooperative diversity schemes at an outage probability of 0.01 for 9-relay network with direct communication

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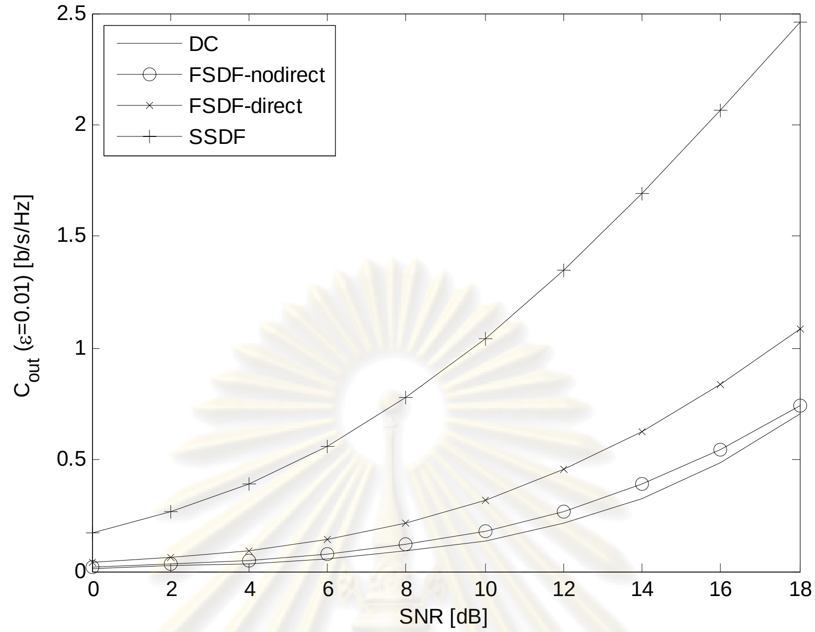

Figure 33 Outage capacity comparisons of all considered cooperative diversity schemes at an outage probability of 0.01 for 1-relay network with direct communication

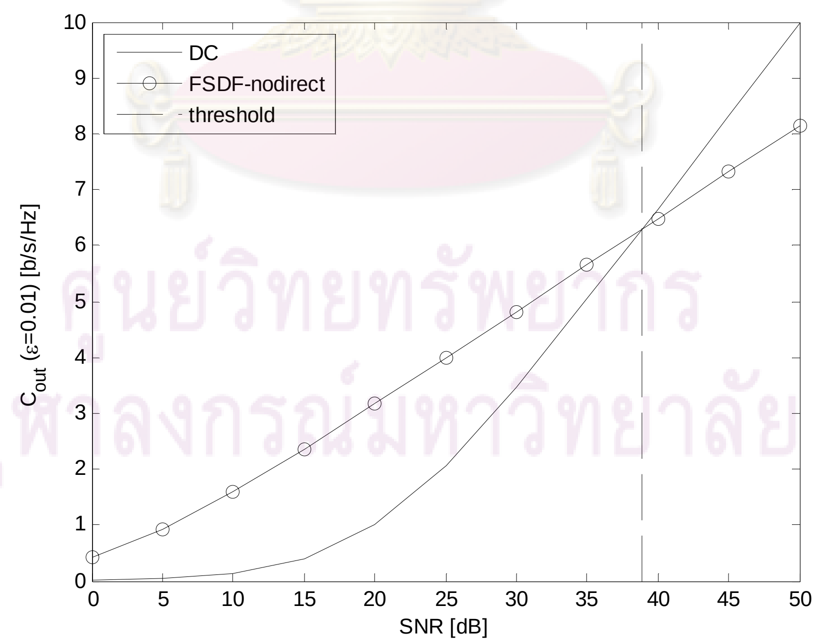

Figure 34 The threshold in the fixed selective decode-and-forward without direct link combining scheme at an outage probability of 0.01 for 4-relay network

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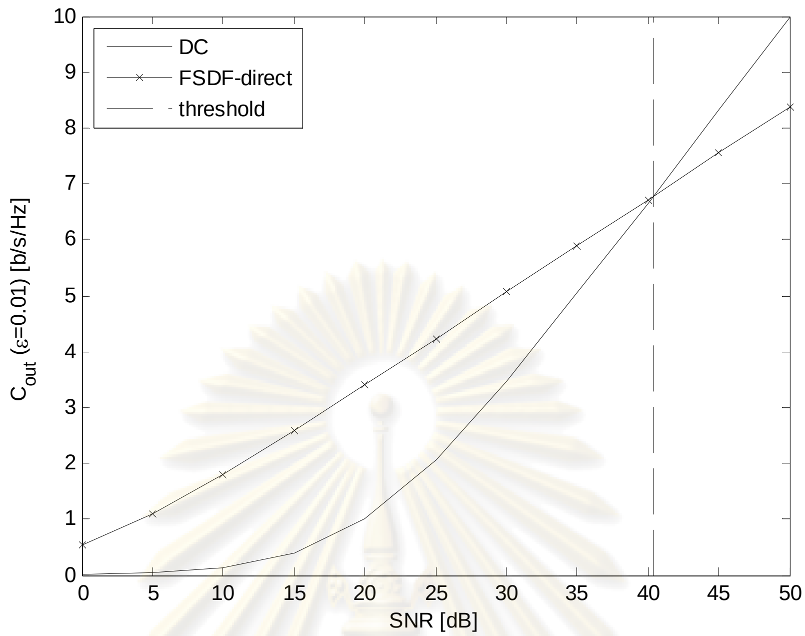

Figure 35 The threshold in the fixed selective decode-and-forward with direct link combining scheme at an outage probability of 0.01 for 4-relay network

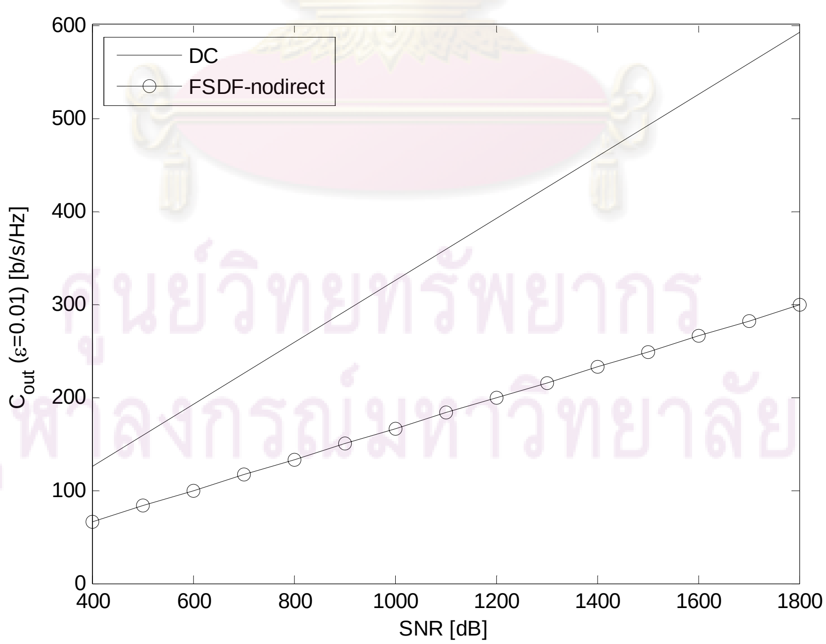

Figure 36 The fixed selective decode-and-forward without direct link combining scheme in high SNR regime at an outage probability of 0.01 for 4-relay network

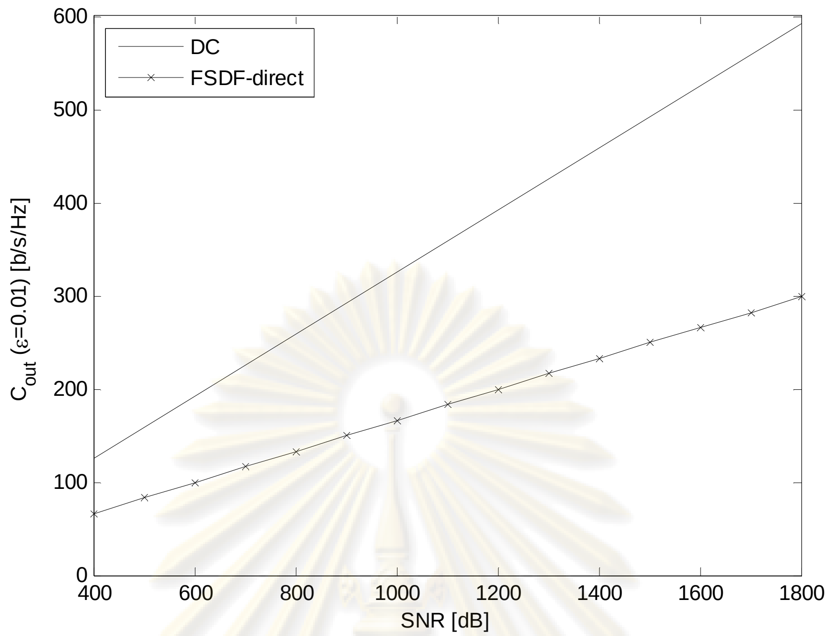

Figure 37 The fixed selective decode-and-forward with direct link combining scheme in high SNR regime at an outage probability of 0.01 for 4-relay network

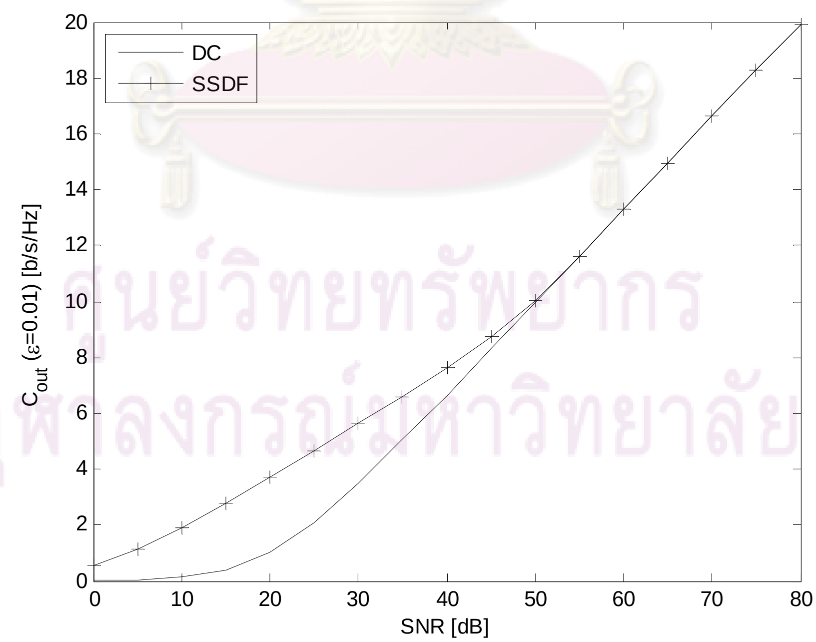

Figure 38 The smart selective decode-and-forward scheme in high SNR regime at an outage probability of 0.01 for 4-relay network

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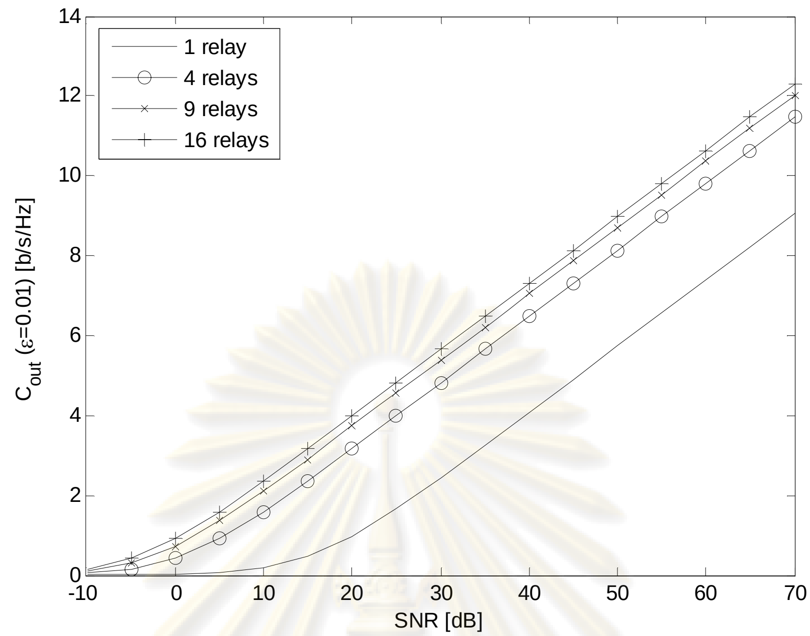

Figure 39 The fixed selective decode-and-forward without direct link combining scheme with varying number of relay nodes at an outage probability of 0.01

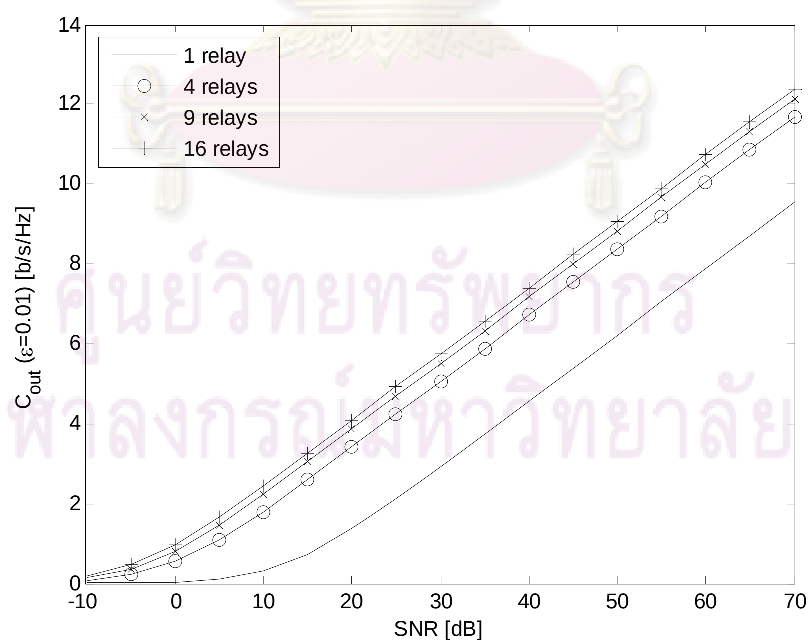

Figure 40 The fixed selective decode-and-forward with direct link combining scheme with varying number of relay nodes at an outage probability of 0.01

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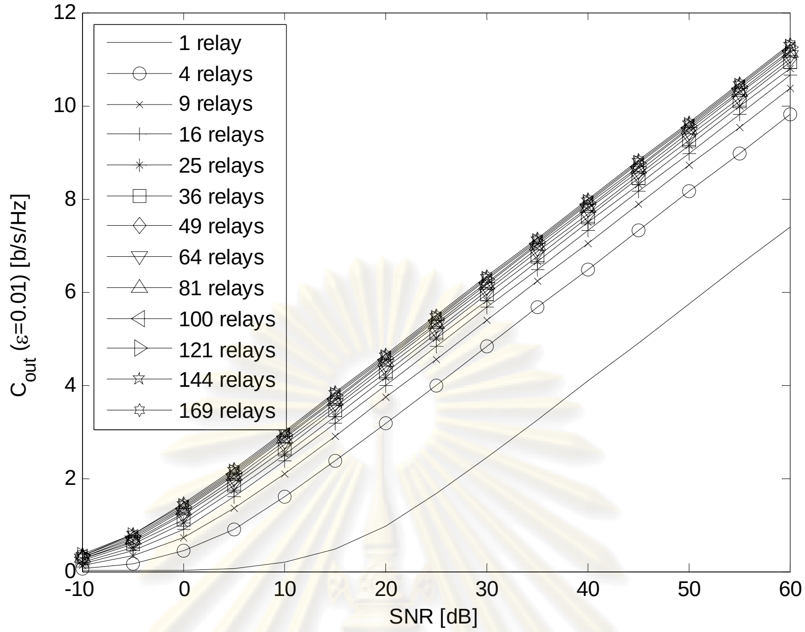

Figure 41 The fixed selective decode-and-forward without direct link combining scheme with increasing number of relay nodes at an outage probability of 0.01

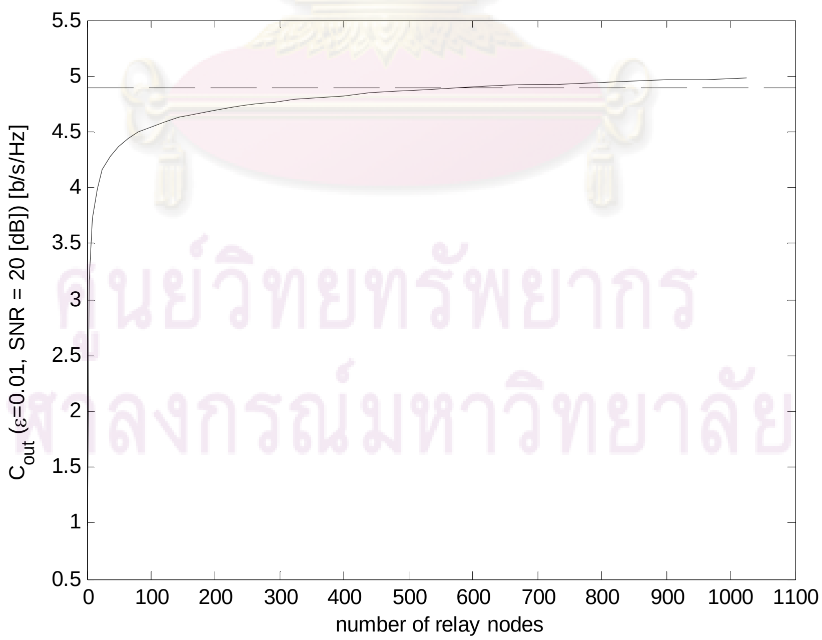

Figure 42 The fixed selective decode-and-forward without direct link combining scheme with large number of relay nodes at an outage probability of 0.01

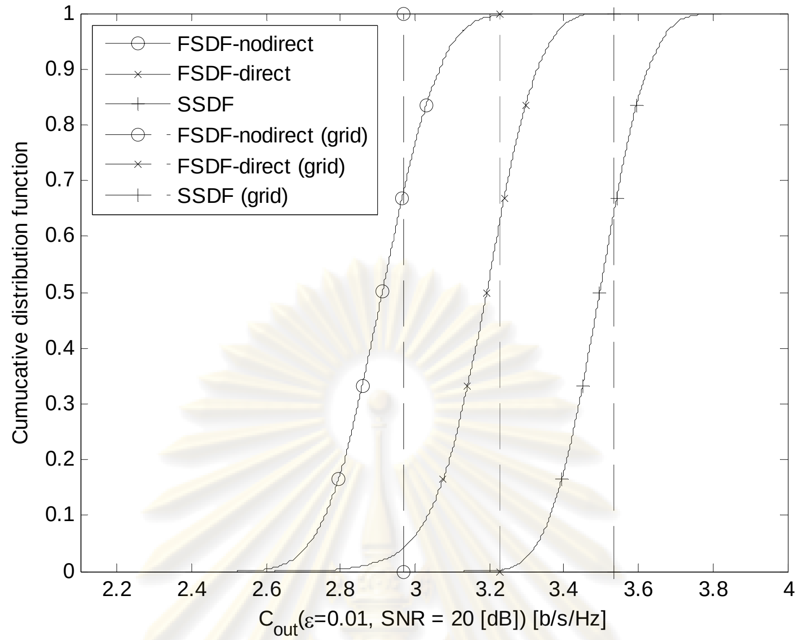

Figure 43 Cumulative distribution functions of outage capacities at signal-to-noise ratio of 20 dB at an outage probability of 0.01 for random topologies with 4-relay network

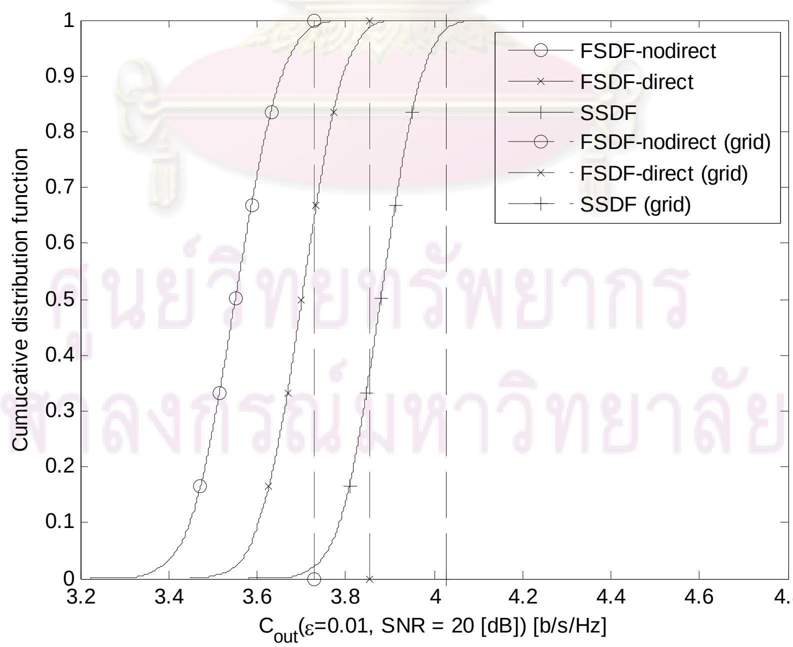

Figure 44 Cumulative distribution functions of outage capacities at signal-to-noise ratio of 20 dB at an outage probability of 0.01 for random topologies with 9-relay network

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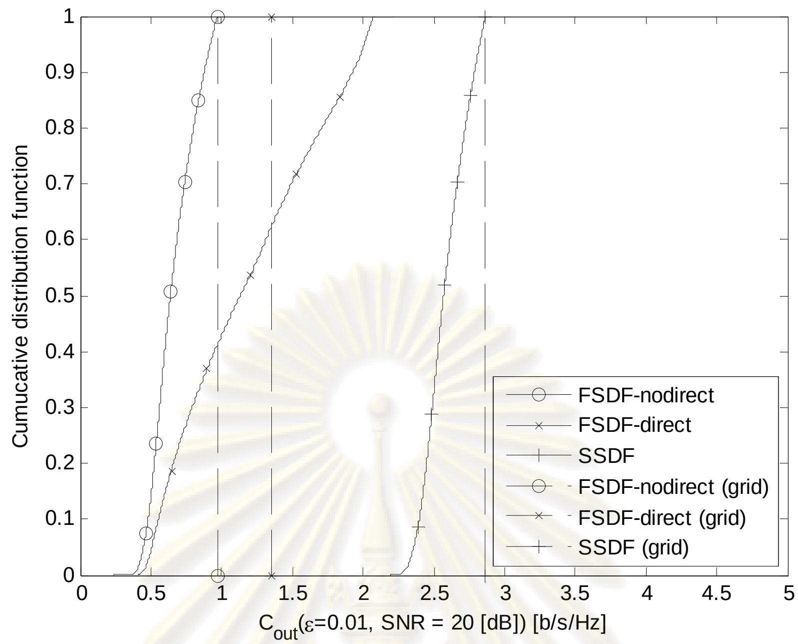

Figure 45 Cumulative distribution functions of outage capacities at signal-to-noise ratio of 20 dB at an outage probability of 0.01 for random topologies with 1-relay network

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CHAPTER V
CONCLUSIONS

This dissertation presents the research work on wireless communications using cooperative diversity schemes. Our focus is on the information-theoretic studies, which would pave the way to practical design. We derive the exact expressions for outage probabilities and outage capacities for several decode-and-forward cooperative diversity schemes with relay selection in the systems using multiple relay nodes. The derived expressions are simple, and applicable for arbitrary network topologies and signal-to-noise ratios. Also, these expressions give the important insights. First, the fixed selective decode-and-forward without direct link combining scheme improves the outage capacity compared to direct communication only when the signal-to-noise ratio is below a certain threshold. That also occurs for the fixed selective decode-and-forward with direct link combining scheme. Then, we characterize the signal-to-noise ratio region for which relaying is beneficial. Second, in the high signal-to-noise ratio regime, the outage capacity of fixed selective decode-and-forward without direct link combining converges to half of that provided by direct communication. That also occurs for the fixed selective decode-and-forward with direct link combining scheme. Third, the outage capacity of smart selective decode-and-forward scheme (which is the best relaying scheme under consideration, but requires every relay to know the partial channel state information between the source and destination) converges to that of direct communication in the high signal-to-noise ratio regime. These results can guide the practical wireless communication network design, such as, the optimal relay node placement.

In addition, we prove several follow-up theories and illustrate them in figures. First, we prove and illustrate the order of performance gain obtained from the considered cooperative diversity schemes. Second, we prove and illustrate the signal-to-noise ratio thresholds of the fixed selective decode-and-forward without direct link combining and the fixed selective decode-and-forward with direct link combining scheme. Third, we prove and illustrate the convergences of the considered cooperative diversity schemes at very high signal-to-noise ratio. Forth we prove and illustrate the linearity and the constant gap among the outage capacity curves of the fixed selective decode-and-forward without direct link combining scheme and the fixed selective decode-and-forward with direct link combining scheme using different number of relay nodes. Fifth, we prove and illustrate that the saturation in improvement by increasing the number of relay nodes does not exist, but the improvement keeps being smaller as the number of relay nodes become large. Last, we show the effect of the topology, which entangles with the number of relay nodes.

5.1 Scheme Choosing

In the situation that the destination node is not far from the source node and the radio environment is not in an urban area, the average signal-to-noise ratio is high and using cooperative diversity does not provide any performance gain. Therefore, it is not necessary to employ the cooperative diversity scheme. We can tell exactly whether the average signal-to-noise ratio is high enough to ignore the use of cooperative diversity by checking with our analytical results.

In the situation that the destination node is not far from the source node and the radio environment is in an urban area, the average signal-to-noise ratio is high but can largely drop when the direct link is shadowed by an obstruction. Therefore, the smart selective decode-and-forward scheme is recommended. When the average signal-to-noise ratio is high, the smart selective decode-and-forward scheme performs as good as direct communication. When the average signal-to-noise ratio drops due to shadowing, the smart selective decode-and-forward scheme can greatly improve the performance.

In the situation that the destination node is quite far from the source node, the average signal-to-noise ratio is low but not completely blind. The destination node can still receive signals from the source node via the direct link even though the signals are weak. Therefore, the fixed selective decode-and-forward with direct link combining scheme is recommended. This scheme can use the signals from the direct link to strengthen the signals from the relay to gain the better performance without using additional channel resource.

In the situation that the destination node is very far from the source node, the average signal-to-noise ratio is so low that the direct link is completely blind. The destination node cannot receive signals from the source node via the direct link. Therefore, the fixed selective decode-and-forward without direct link combining scheme is recommended. This scheme improves the performance with the simplest protocol among schemes, while the further improvement by combining the signals from the direct link is negligible. We can tell exactly whether the average signal-to-noise ratio is low enough to ignore the use of other more complicated schemes by checking with our analytical results.

The number of relay nodes should be sufficiently dense to obtain the good performance, but should not be too dense because it is not worth due to the nonlinear improvement. As a rule of thumb, observed from our results, we recommend one-tenth of the coverage. For example, if the source node is designed to cover the destination node at 1 km away, then 10 relay nodes should be employed. Employing more relay nodes yields better performance but not significant.

To choose the topology of employing the relay nodes, the grid topology is recommended because most of the other topologies do not perform better than the grid topology, especially when the number of relay nodes is dense enough. When the number of relay nodes is small, the relay node placement should be optimized. The optimization can be done by using our analytical formula as the objective function and the locations of relay nodes as the optimizer. This optimization can be done with reasonable computation time because the number of relay nodes is small.

5.2 Future Work

The cooperative diversity schemes, especially the smart selective decode-and-forward scheme, are promising. The fixed selective decode-and-forward without direct link combining and the fixed selective decode-and-forward with direct link combining schemes have a trade-off, where the loss in degree of freedoms due to the duplex may offset the benefit both in terms of the outage probability and in terms of the outage capacity. Hence, it should be used selectively, depending on the condition of the channels. The extension to the multi-hop case is not likely to be analyzed in closed-form, and is troublesome to do computer simulations. The optimal relay nodes placement can be conducted based on the provided results.

The future work in this line of research is given as follows. Instead of information-theoretic study, the error rate of cooperative diversity scheme is analyzed approximately in [33]. The idea of cooperative multiple access is proposed in [34], and the idea of cooperative spectrum sharing protocol with secondary user selection is proposed in [35]. Due to the trade-off incurred by the half-duplex constraint in conventional cooperative diversity schemes, it is interesting to combine the ARQ with the cooperative diversity schemes to save the bandwidth [36,37,38,39,40], as well as the effect of imperfect channel state information on ARQ scheme [41]. The interference occurs in cooperative diversity scheme is an interesting issue to take into account. Several studies have been explored in the literature [42,43,44,45,46]. Using the compress-and-forward protocol, the cooperative diversity schemes with backhaul link are studied in [47,48,49,50].

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Vitae

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