



## CHAPTER I

### INTRODUCTION

The ground state energy of liquid  ${}^4\text{He}$  ( $T = 0 \text{ K}$ ) has been considered by Verlet(1) and McMillan(2). The Jastrow form(3), a product of pair function, is used as a variational wave function for the quantum system of interacting boson ( ${}^4\text{He}$  atom). A parameterized form for the pair function is chosen, and the values of the parameter are determined by minimizing the ground state energy. The resulting wave function gave zero-temperature properties of liquid helium generally closed to the experimental value.

In the earlier discussion(4), it was postulated that the behavior of the condensate is governed by a single wave function  $\psi$  of coherent phase  $s$ ,

$$\psi(\vec{r}, t) = \psi_0(\vec{r}, t) e^{is(\vec{r}, t)} \quad (1-1)$$

The assumption was made later in such a way that  $\psi(\vec{r}, t)$  is a solution of a Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2 \nabla^2 \psi}{2m} + \bar{V}(\vec{r}) \psi \quad (1-2)$$

where  $\bar{V}(\vec{r})$  is an average potential energy.  $\bar{V}(\vec{r})$  was taken to be the chemical potential  $\mu$  of the whole fluid at rest. The chemical potential acts by the potential energy source per particle of the superfluid, when superfluid moves from a region of high  $\mu$  to one of low it is accelerated.

Under this assumption, we have been obtained the expression for

the ground state energy involving the strong interacting potential or the interatomic potential given by Visoottiviseth(5). We have calculated the numerical value of the ground state energy of liquid  $^4\text{He II}$ , using the work of McMillan(2) and Puoskari et.al(6). The interatomic potential of liquid  $^4\text{He II}$ , which have been proposed are the L-J 12-6 potential(7), the Morse- $V_{DD}$  potential(8), and the HFDHE-2 potential(9).

The unique properties of superfluid  $^4\text{He}$  are believed(10) to result from a Bose-Einstein condensation which is characterized by the macroscopic occupation of the zero momentum state. The fraction of atoms in this state is called the condensate fraction. It is well known that the condensate accounts for only a small fraction (in  $^4\text{He II}$  about ten percent) of the total density even at zero temperature and recent theoretical calculation of condensate fraction at temperature  $T = 0$  K give values for condensate fraction(11) of 11.3% and 9.0% depending on the interatomic-force model. In this thesis, we will be used the method of extraction the condensate fraction in liquid  $^4\text{He II}$  proposed by Visoottiviseth (12). We have calculated the numerical value of condensate fraction in  $^4\text{He II}$  as shown in Table II. We compare our results to the experimental work and the other theoretical works(2,13,14).

In chapter II some basic properties of liquid  $^4\text{He}$  are presented. The flow properties of liquid  $^4\text{He}$  below the  $\lambda$ -point and thermomechanical effect show clearly that heat transfer and mass transfer in  $\text{He II}$  are inseparable. In the last section, the two fluid model will be defined.

In chapter III, the reduced density matrices are introduced in Schrödinger, Heisenberg and Second Quantization representations. We will show how ODLRO can be incorporated into the reduced density matrices.

In chapter IV, the microscopic theory of  $^4\text{He II}$  is developed. The Bose-Einstein condensation is indicated by off-diagonal long-range order in the reduced density matrices. The derivation of a closed set thermo-hydrodynamic equation, for the condensate and the depletion of He II in the bulk system, are given. Second quantization has been used in the derivation of the chemical potential at the ground state. In the last section of this chapter, we present a simple method for the determination of the condensate fraction based on the relationship between the condensate density and the superfluid density proposed by Visoottiviseth(12).

In chapter V, we show how can be determined the screening factor,  $\xi(\vec{r})$ , the function  $h(\vec{r})$ . These two quantities will be very useful in our work for calculating the numerical values of the chemical potential and the condensate fraction for liquid helium II at 0 K. In our calculation, we used a computer (number 25-558, BASIS(MAD-FLY) , 130 columns) for calculating  $\xi(\vec{r})$ ,  $h(\vec{r})$  and the areas under curve of the function  $I_a$  and  $I_b$ , in which the areas under curve of the function  $I_a$  and  $I_b$  have been obtained by Simpson's rule method. In the last section, our results are compared with the experimental work and the other theoretical works.

In chapter VI, the discussion and conclusion are given and comparison with other works are made compared the other works, re-considered the relationship between the condensate density and superfluid density. We find that the screening factor  $\xi(\vec{r})$  and the function  $h(\vec{r})$  must depend on the temperature.



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