CHAPTER 2

THEORY AND SURVEY OF THE LITERATURE

Mentioned previous attempts to develop a theoretical model for the collection efficiency of a cyclone type particulate collector have not met with much success. The experimentally determined grade-efficiency curves for standard gas cyclones are always concave downward and approach 100% efficiency almost asymptotically. Some theoretical models so far proposed give a curve which is S-shaped; others predict a sharp critical value for the smallest particle size which may be collected completely.

The principal reason for these discrepancies seems to be a failure to take into account that turbulence and flow in the gas stream cause a continual back-mixing of the suspended (uncollected particles) in the gas. A new approach, incorporating the concept of back-mixing, is presented herein. Coupled with the determination of an appropriate average residence time of the gas in a cyclone, it leads to a very good agreement with such published experimental data as are adequate for checking. This will be illustrated with reference to conventional cyclone designs using a tangential inlet as shown in Figure 1.2 Certain other refinements in what might be called the classical mathematical model of cyclone collection are also involved in the new approach by Leith, D. and W. Licht (1972).

2.1 Particle motion in cyclone

To begin with it is necessary to have an equation giving the radial component of the trajectory of an individual particle moving in a gas which is spinning under the conditions inside a cyclone. A differential equation describing this motion may be set up by making a force balance on the particle with the following assumptions:

(1) The particle is spherical in shape.

(2) The motion of a particle is not influenced by the presence of neighboring particles.

(3) The drag force radially on the particle is given by Stokes'Law

(4) The radial velocity of the gas is zero.

(5) The tangential velocity component of the particle is the same as that of the gas stream, that is, there is no slip in the tangential direction between the particle and the gas.

(6) The tangential velocity component is related to the radial position by a modified form of the equation for a free vortex in an ideal fluid:

$$u_T R^n = const.$$

In the ideal free vortex law $n = 1$, but experimental observation show that in a cyclone n may range between 0.5 and 0.9 according to the size of the cyclone and the temperature. Alexander (1949) gives the following empirical equations:

$$n = \frac{\left(12D_f\right)^{0.14}}{2.5} = \frac{\left(D_f''\right)^{0.14}}{2.5} \qquad (2.1)$$

$$\frac{1-n_1}{1-n_2} = \left(\frac{T_1}{T_2}\right)^{0.3} \qquad (2.1a)$$

A chart for n prepared by Caplan (1968) based upon these equations is available. This may be used to estimate a value of n which might be expected to arise. No other way of predicting n has been found. However, as will be shown, values of n may be deduced from experimental grade-efficiency data.

Under the above assumptions the force balance yields

$$\frac{d^2R}{dt^2} + \frac{18\mu}{d_p^2\rho_p}\frac{dR}{dt} - \left(u_{T_2}\right)^2 R_2^{2n}\frac{1}{R^{2n+1}} = 0 \qquad (2.2)$$

in which $u_T R^n = u_{T_2} R_2^n = const.$, and where R_2 may be taken as the radius of the cyclone wall. For simplicity if will be assumed that it would be satisfactory to take u_{T_2} as equal to the average velocity of the gas in the inlet duct, that is,

$$u_{T_2} = \frac{Q}{ab}$$

Strictly speaking, the tangential velocity of the vortex at the cyclone wall boundary must be zero. However, the boundary layer, a region in which the captured particles slide down the cyclone wall toward the duct outlet, must be thin. Little error is introduced by setting $R = R_2$ when $u_T = u_{T_2}$ as defined above.

Equation (2.2) is not readily solvable, even for the idealized case where $n =$. An approximate solution can be obtained by arbitrarily neglecting the second-order derivative. This is equivalent to saying that the particle moves radially outward with a constant velocity, which is obviously inconsistent with the resulting approximate equation:

$$\frac{dR}{dt} = \frac{d_p^2 (u_{T_2})^2 \rho_p (R_2)^{2n}}{18\mu} \frac{1}{R^{2n+1}} \tag{2.3}$$

Integration yields

$$t = \frac{9\mu}{\rho_p (n+1)} \left(\frac{R_2}{u_{T_2} d_p} \right)^2 \left[\left(\frac{R}{R_2} \right)^{2n+2} - \left(\frac{R_1}{R_2} \right)^{2n+2} \right] \tag{2.4}$$

if the particle travels from R_1 to R in time t . If the particle just reaches the cyclone wall in time t then $R = R_2$ and

$$t = \frac{9\mu}{\rho_p (n+1)} \left(\frac{R_2}{u_{T_2} d_p} \right)^2 \left[1 - \left(\frac{R_1}{R_2} \right)^{2n+2} \right] \tag{2.5}$$

This equation is more general form of those developed by Davies (1952) and Strauss (1966) and reduces to them if n is taken to be unity. It also becomes equivalent to the equation derived by Umney (1948) as cited by Daniels (1957) if again n is unity, and if in the Umney development no slip is assumed between the particle and the tangential component of the gas velocity. All of these solutions involve the same approximation of neglecting the second order term in the differential equation.

The error involved in making this approximation can only be estimated by performing numerical solutions to the complete differential Equation (2.2). One specified example of this has been worked out by Mehta (1970) in the case where $n = 0.5$. He performed a numerical solution of the complete differential for a typical set of operating conditions [similar to those of Stairmand's tests (1951)], and compared it with the approximation given by Equation (2.4). He found that the two solutions approached each other and gave virtually the same result after t had attained about 1/20 of the time required for the particle to reach R_2. At shorter times the approximate solution tended to overestimate the time of travel. Further investigation of this point for a variety of conditions is highly desirable, although as will be seen below, the approximate equation(2.5) does appear to work out well in the back-mixing model.

2.2 Gas flow in cyclone

Next the pattern of gas flow in a cyclone must be considered. Since there is a core zone of low pressure extending along the axis of the cyclone below the exit duct, the gas must gradually move radially inward. Ter Linden's measurements (1949) show that it does so at a linear velocity which is relatively constant, regardless of radial position or elevation, As a volume of gas passes vertically down the cyclone it is then gradually drawn off into the central core at a constant rate, proportional to the inward radial velocity, Different parcels of gas introduced into the cyclone at the same time may thus have different residence times within the unit, depending upon the level at which they are introduced and upon how soon after entry they are drawn off.

For simplicity it is desirable to determine an average residence time for all of the gas stream, such as will account for the collection obtained. This may be done as follows. The total average residence time may be taken to equal the average time required for gas to descend from the average level of entrance to the level of the bottom of the exit pipe, plus the average time in residence below this point. The average minimum residence time can be calculated by assuming all the gas enters the cyclone at the mid point of the entrance duct. With this assumption.

$$t_{\min_{avg}} = \frac{\pi(S - a/2)\left(D^2 - D_e^2\right)}{4Q} = \frac{V_s}{Q} \qquad (2.6)$$

where

$$V_s = \frac{\pi(S - a/2)\left(D^2 - D_e^2\right)}{4} \qquad (2.7)$$

The additional time of residence will vary from zero to a maximum corresponding to the lowest point of descent of the gas into the conical cyclone body, with the average additional time being taken as one-half of this maximum.

The lowest point of descent of gas is not necessarily the actual bottom of the cyclone as measured by the depth dimension H. Alexander (1949) has observed with very long glass cyclones that the gas vortex will gave a well-defined stable turning point at some distance below the bottom of the exit duct which is less than $(H-S)$. He calls this the natural length l of the cyclone and gives an empirical formula for it as:

$$l = 2.3D_e\left(\frac{D^2}{ab}\right)^{1/3} \tag{2.8}$$

It is noteworthy that this natural length is, according to Alexander (1949), independent of the gas flow rate.

The maximum additional residence time of gas not drawn off until reaching the tip of the vortex will be

$$t_{\max} = \frac{V_{nl}}{Q} \tag{2.9}$$

where V_{nl} is the effective volume of the lower portion of the cyclone at the natural length, where the vortex turns. This effective volume is equal to the total volume of the cyclone from the level of the exit duct down to the level of the natural length, minus the volume of the central core of the cyclone where the gas stream is swept upwards and out of the particle-separating vortex. Taking the diameter of the central core to be equal to that of the gas exit duct gives

$$V_{nl} = \frac{\pi D^2}{4}(h-S) + \frac{\pi}{4}\frac{(l+S-h)D^2}{3}\left[l + \frac{d}{D} + \frac{d^2}{D^2}\right] - \frac{\pi D_e^2 l}{4} \tag{2.10}$$

where

$$d = D-(D-B)\left[\frac{S+l-h}{H-h}\right] \tag{2.10a}$$

However the core diameter, as a fraction of the exit pipe diameter has been variously given as 1/2, 2/3 and 1 by Stairman (1951). Ter Linden (1949) and Barth (1956) respectively.

A practical cyclone should gave a physical length $(H-S)$ near its natural length l If the cyclone body is longer than l, that is $(H-S)>l$, the space at the bottom of the cyclone, below the vortex turning point, will be wasted. If the cyclone body is shorter than l, that is $(H-S)<l$, the full separating potential of the cyclone will not be realized. In this case $t_{\max}$ will be determined simply by the total lower volume of the cyclone minus that of the central core. That is V_{nl} would be replaced by V_H which is

$$V_H = \frac{\pi D^2}{4}(h-S) + \frac{\pi D^2}{4}\frac{(H-h)}{3}\left[l + \frac{B}{D} + \frac{B^2}{D^2}\right] - \frac{\pi D_e^2}{4}(H-S) \qquad (2.10b)$$

The average total residence time of the gas in a cyclone will then be taken as

$$t_{res} = \frac{l}{Q}\left[V_S + \frac{V_{nl}}{2}\right] = \frac{K_C D^3}{Q} \qquad (2.11)$$

where

$$K_C = \frac{(V_S + V_{nl}/2)}{D^3} \qquad (2.12)$$

Here K_C is a dimensionless constant for a given cyclone design. That is it depends only upon the relative proportion of the various dimensions. For cyclones which are geometrically similar in all respects it is independent of the size. The magnitude of K_C is an indication of the relative effective volume a given design provides, in which separation of particles from gas may take place.

The shape of any cyclone of the type under discussion is completely specified by the values of a set of seven basic dimensionless geometric ratios which may be expressed in terms of D as follows:

$$\frac{D_e}{D}, \frac{a}{D}, \frac{b}{D}, \frac{S}{D}, \frac{h}{D}, \frac{H}{D}, \frac{B}{D}$$

with reference to Figure 1.1. The appropriate values in this set also fix ratios l/D and d/D. Hence the value of K_C is determined by these seven design parameters which may in principle all be chosen independently. From experience certain limits are usually placed upon these choices for example $\frac{a}{D} < \frac{S}{D}, \frac{S}{D} < \frac{h}{D}, \frac{h}{D} < \frac{H}{D}$ etc. However within such limitations the designer of a cyclone is free to select values of these ratios such as to accomplish whatever objective he may have in mind. If this be to maximize average residence time, then the ratios should be selected so as to give the largest value of K_C. The effect of this upon collection efficiency will be shown below.

It is now evident that assumption (4) underlying differential Equation (2.2) is not fulfilled for that portion of the cyclone below the bottom of the exit duct. Correction could be made for the effect of the constant inward radial gas flow velocity (u_{rg}) upon the drag force on the particle. This would lead to a modified differential equation:

$$\frac{d^2R}{dt^2}+\frac{18\mu}{\left(d_p\right)^2\rho_p}\frac{dR}{dt}-\left(u_{T_2}\right)^2\left(R_2\right)^{2n}\frac{1}{R^{2n+1}}+\frac{18\mu\,u_{rg}}{\left(d_p\right)^2\rho_p}=0 \tag{2.13}$$

In this, u_{rg} might be estimated as

$$u_{rg}=\frac{Q}{\pi\,D_e\,l}=\frac{u_{T_2}ab}{\pi\,D_e\,l} \tag{2.14}$$

The importance of this correction will depend not only upon the magnitude of the additional constant term in (2.13) but also upon what proportion of the total dust collection takes place below the bottom of the exit duct, that is, upon how much larger is the average residence time than the minimum residence time. The net effect would tend to reduce the calculated collection efficiency. It has been neglected in this new approach because the measured values of u_{rg} ter Linden, A.J. (1949) are low and in fact tend toward zero at the wall where collection take place.

2.3 Particle distribution and collection

No data have been reported on the actual distribution of particles within a cyclone. However, it is clear that three mechanisms, at least will tend to cause back-mixing of uncollected particles: (1) As the gas below the exit duct moves radially inward to be drawn off it will tend to drag particles with it. (2) Turbulence and eddies within the unit will aid in the back-mixing of particles. (3) Also, particles have been observed Jotaki T. (1957) to bounce from the wall of a cyclone back into the gas stream. It will be postulated that drag force, turbulent mixing, and particle bouncing or re-entrainment, are sufficiently prevalent to ensure that a uniform concentration of uncollected dust is maintained in the gas flowing through any horizontal cross section of a cyclone, that is that back-mixing is complete.

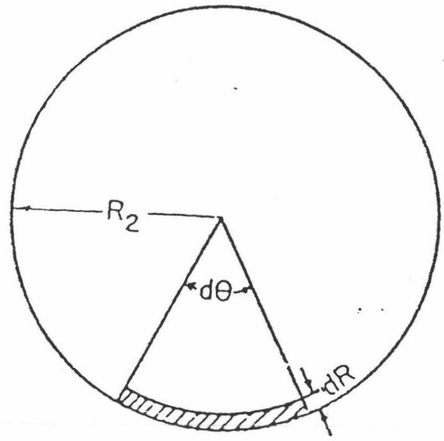

Figure 2.1 Cross section of a cyclone

Source: Leith, D. and W. Licht. The collection efficiency of cyclone type particle collector. AIChE Symposium series. vol 68. No. 126, 1972 p 200.

Consider a horizontal cross section of a cyclone as shown in Figure 2.1. In time dt, all particles a distance dR or less from the cyclone wall will move to the wall and be collected. Meanwhile the particles will travel a distance $Rd\theta$ tangentially and dL vertically. The number of particles removed dn will be

$$-dn' = \frac{d\theta}{2}\left[R_2^2 - \left(R_2 - dR\right)^2\right]cdL \tag{2.15}$$

where c is the number concentration of particles. The total number of particles in the sector from which particles are removed is

$$n' = \frac{d\theta}{2} R_2^2 cdL \tag{2.16}$$

The fraction of particles removed in time dt is therefore

$$-\frac{dn'}{n'} = \frac{2R_2\,dR - (dR)^2}{R_2^2} = \frac{2dR}{R_2} \tag{2.17}$$

In order to relate the fraction of particles collected to the average residence time, it is necessary to express Equation (2.17) in terms of time. This may be done through Equation (2.4) , which was developed to show the relationship between time and the radial position of a single particle in a vortex. A system of uncollected particles, evenly distributed across the cross section of such a vortex, will have its center of mass at the vortex center.

If the system of uncollected particles is instantaneously, continuously, and completely redistributed by back-mixing, as postulated, the center of mass of the uncollected particle system will always remain at the vortex center, even though the total number of uncollected particles is decreasing with increasing gas residence time.

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The rate at which the uncollected particle system moves toward the vortex wall as a function of the time the system has spent within the vortex, may then be obtained by saying that the radial position of the uncollected particle system is at the vortex center $R_1 = 0$ at zero time and differentiating Equation (2.4) to obtain

$$\frac{dR}{dt} = \frac{\rho_p}{18\mu}\left(\frac{d_p u_{T_2}}{R_2}\right)^2 R_2\left[\frac{\rho_p(n+1)}{9\mu}\left(\frac{d_p u_{T_2}}{R_2}\right)^2\right]^{-(2n+1)/(2n+2)} \quad (2.18)$$

Combination Equation (2.17) and (2.18):

$$\int_{n_0}^{n'}\frac{dn'}{n'} = -\int_0^t \frac{\rho_p}{9\mu}\left(\frac{d_p u_{T_2}}{R_2}\right)^2\left[\frac{\rho_p(n+1)}{9\mu}\left(\frac{d_p u_{T_2}}{R_2}\right)^2 t\right]^{-(2n+1)/(2n+2)} dt \quad (2.19)$$

neglecting the second-order differential.

Integrating up to the average residence time $t_{res} = \dfrac{K_C D^3}{Q}$ as given by Equation (2.11) yields

$$\eta = \frac{n'_0 - n'}{n'_0} = 1 - \exp - 2\left[\frac{\rho_{p_i}}{9\mu}\left(\frac{d_{p_i} u_{T_2}}{R_2}\right)^2 (n+1)\frac{K_C D^3}{Q}\right]^{1/(2n+2)} \quad (2.19a)$$

Nothing that $Q = u_{T_2}(ab)$ this Equation may be displayed in a more meaningful form by defining dimensionless parameters as follows:

$$\psi = \frac{\rho_p\, d_p^2\, u_{T_2}}{18\mu D}(n+1)$$

$$K_a = \frac{a}{D} \qquad K_b = \frac{b}{D}$$

a cyclone design number, reflecting the physical shape of the cyclone.

Equation (2.19a) then becomes

$$\eta = 1 - \exp - 2\left[C\psi\right]^{1/(2n+2)} \quad (2.20)$$

or William Licht, (1972). Equation (2.20) can write to

$$\eta = 1 - \exp - 2\left[Md_p{}^N \right] \tag{2.20a}$$

in which

$$M = 2\left[\frac{KQ}{D^3} \frac{\rho_p^{n+1}}{18\mu} \right]^{N/2} \qquad and \qquad N = \frac{1}{n+1}$$

(2.20b)

A 50% cut diameter may be found from

$$d_{cp_{50}} = \left(\frac{0.6931}{M} \right)^{n+1} \tag{2.20c}$$

$$n = 1 - \left[1 - 0.67 D^{0.14} \right] \left(\frac{T_1}{283} \right)^{0.3} \tag{2.20d}$$

where D = diameter of cyclone, m

T_1 = absolute temperature of gas, K

which is the new theoretical Equation for collection efficiency based upon the back-mixing postulate.

2.4 Characteristics of new Equation

Equation (2.20) states that the efficiency of collection is determined primarily by only two dimensionless parameters. One C depends only upon the shape of a cyclone as fixed by the seven size ratios and is independent of its size or of any operating conditions. The other ψ is fixed by the nature of the gas-solid system (particle size, particle density, composition of gas) the temperature and velocity of operation and the size (diameter) of the cyclone. The model thus confirms statements made by Hawksley et al. (1961) to the effect that efficiency is some function of the group here called ψ, and that geometrically similar cyclones (that is having the same value of C) operating with the same value of ψ have the same efficiency.

A grade-efficiency will be found to have the same general shape as the experimentally determined curves illustrated in Figures 2.10 and 2.11. The second derivative of efficiency with respect to particle size $d^2\eta / dd_p^2$ is always negative, so that the curve is everywhere concave downward. It will rise from $\eta = 0$ at $d_p = 0$ to approach $\eta = 1$ asymptotically. It will have no critical particle size.

The grade-efficiency curve may be rectified by plotting $\log[-\ln(1-\eta)]$ vs $\log d_p$. Letting $A = \dfrac{C\psi}{d_p^2}$ in Equation (2.20) the straight-line Equation is

$$\log[-\ln(1-\eta)] = \log 2A^{1/(2n+2)} + \frac{1}{n+1}\log d_p \qquad (2.21)$$

The slope of this log-log plot will depend only upon n, while the ordinate intercept (corresponding to $d_p = 1$) will then be determined by A. Care must be taken with units, for example if d_p is plotted in microns A must be in (microns)$^{-2}$.

The Equation indicated that operation of a given cyclone will be more efficient at higher flow rates or upon particles of greater density and less efficient at higher temperatures. All other factors being held constant a cyclone of larger body diameter will be less efficient. A long thin cyclone will tend to be more efficient than a short fat one at the same operating conditions. All of these effects (which are well-know qualitatively) may readily be predicted quantitatively by calculating the way in which they influence the values of the parameters ψ and C. It is evident that collection efficiency will tend to be maximized by the use of seven design ratios which maximizes the value of C.

It is sometimes desired to compare the size of particles which will be collected with equal efficiency under different operating conditions. This may readily be done through parameter ψ. If ψ and ψ represent two sets of conditions, then for equal efficiency $\psi = \psi$ for the two sets. Hence

$$\frac{d_p}{d_p'} = \left[\frac{\rho_p'}{\rho_p} \times \frac{u_{T_2}'}{u_{T_2}} \times \frac{\mu}{\mu'} \times \frac{D(n'+1)}{D'(n+1)}\right]^{1/2}$$

Thus the particle size, for equal collection efficiency is inversely proportional to the square root of the density of the dust, and of the inlet velocity or flow rate, and directly proportional to the square root of the viscosity of the gas and of the diameter of the cyclone body. These relationship are in agreement with those proposed by Stairmand (1951)

It should be emphasized that if all of the concepts and assumptions used in developing Equation (2.2) are correct, the performance of a cyclone may be predicted a priori, that is, without performing any tests upon it. The important effect of dust-loading (that is, concentration of dust in the inlet air) does not appear explicitly in the Equation. It is known that higher loading produces higher efficiency. This may be due in part to agglomeration of colliding particles particles producing an effectively larger equivalent particle diameter. If so, this effect could be accounted for in the ψ-term. Further, Sproull (1966) has shown that the apparent viscosity of dusty air is substantially lower than that of clean air, as shown in Figure 2.2. This effect too would be accounted for by the ψ-parameter.

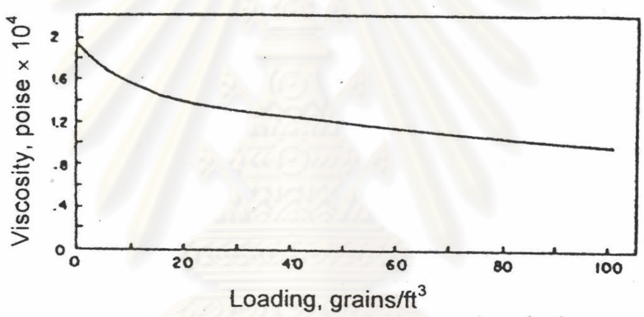

Figure 2.2 apparent viscosity of air as a function of dust loading.

Source: Ibid., p 201.

2.5 Testing the new Equation

Data of sufficient completeness for testing the ability of Equation (2.20) to predict grade-efficiency curves, are very scare for tangential inlet cyclones of the design shown in Figure 1.2. Stairmand (1951) and Peterson and Whitby (1965) have reported laboratory tests performed with dusts of constant density, but varying particle sizes, entrained in air streams feed to cyclones. Fractional efficiency curves were developed from an analysis of the particles separated from the air streams by the cyclones. Stairmand used a typical dust composed of particles with a wide range of sizes. Peterson and Whitby used monodisperse aerosols and determined the cyclone efficiency for each particle size. Details of the test conditions are given in Table 2.1.

TABLE 2.1 SUMMARY OF DATA COMPILED FORM EXPERIMENTAL CYCLONES
Stairmand (1951), Peterson & Whitby (1965), and van Ebbenharst Tengbergen (1965)

Dimension of Cyclones (in.)		Stairmand	Peterson and Whitby	van Ebbenharst Tenghergen	
Term	Description				
D	Body diameter	8	12	11	18.5
a	Inlet height	4	7	9.25	15.5
b	Inlet width	1.6	2.5	2.9	4.7
S	Outlet length	4	7*	11.6	19.5
D_e	Exit diameter	4	6	5.8	9.75
h	Cylinder height	12	16	16.8	28.2
H	Overall height	32	38	31.6	53
B	Dust outlet diameter	3	6	5.8	9.75
Calculate					
l	Natural length	19.8	21.6	22.0	37.6
$H-S$		28	31	20.0	33.5
V_S	cu. in.	75.5	298	473	2290
V_{nl}	cu. in.	556	1470	-	-
V_H	cu. in.	-	-	810	3850
K_C	no units	0.692	0.597	0.660	0.666
Operating Conditions: all taken to be at 20^0C.					
Type particle		Dust	Monodisperce	Clay and Plaster	
Particle density	g/cu. cm.	2.0	1.6	-	-
Loading	g/cu. ft.	-	0.3	10	10
Throughout	cu. ft./min	133	610	-	-

* Assume value

Efficiency predictions calculated for the tests of Stairmand and Peterson and Whitby are compared with their experimental values in Figures 2.3 and 2.4. The agreement is seen to be excellent, within about 4% overall. A more sensitive examination of these data, made by replotting the grade-efficiency curves on log probability paper reveals that such lack of agreement as there is occurs mainly for the very small and the very large particles. This may well be due to re-entrainment of the small particle (less than 1 to 2 microns) and to bouncing from the cyclone wall, such as has been observed by Mori et.al. (1968) of coarser particles. These effects are ofcourse not taken into account in the development of the theoretical Equation.

Data are also available from von Ebbengarst-Tenbergen (1965) who has plotted cyclones efficiency against the quantity $d_p\sqrt{\frac{\rho_p u_{T_2}}{\mu D}}$, which will be called T. As u_{T_2} and ρ_p are unobtainable it is not possible to convert these data to standard grade-efficiency curves. However sufficient information is available the cyclone design number C and ψ is seen to be product of T^2 by $\left(\frac{n+1}{18}\right)$. The theoretical efficiency of these cyclones can be calculated and compared with the experimental data as shown in Figures 2.5 and 2.6 which are for cyclones the same design but different diameters. The scatter in data makes it difficult to give a positive statement regarding the agreement, but it appears to be generally thin about 10% except for the higher end of the range T. In the calculation of C (and K_C) the value of V_{nl} in Equation (2.11) and (2.12) must be replaced by V_H. This results in lower values for C. However the effect is not very great in the cases, as the values of l are not very much larger than those of $(H-S)$.

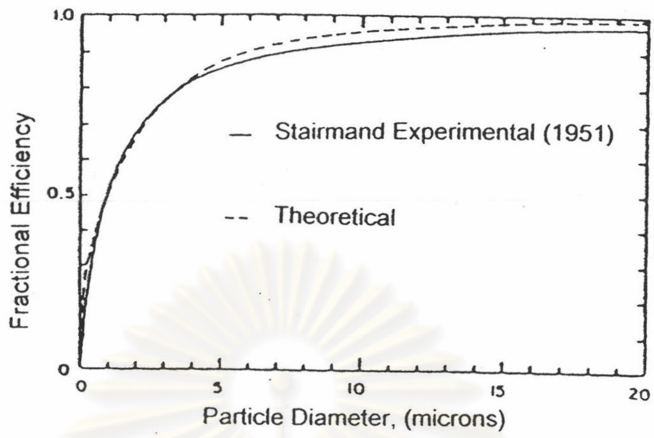

Figure 2.3 Comparison of new theoretical Eff.

Source: Ibid. p 202.

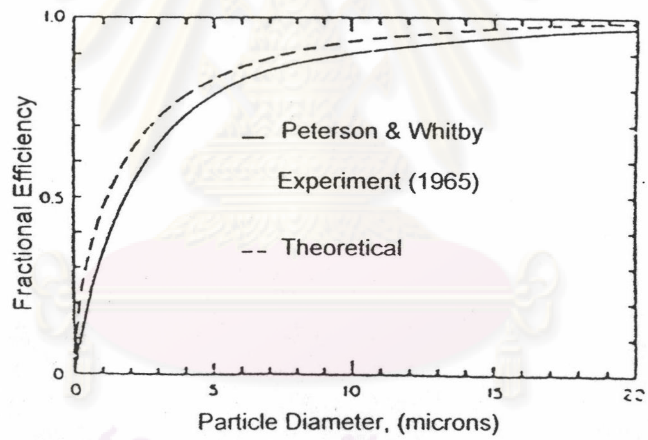

Figure 2.4 Comparison of new theoretical Eff. of Peterson

Source: Ibid. p 202.

All four sets of these data were also tested by means of the log-log plot corresponding to Equation (2.21). The plots are shown in Figures 2.7, 2.8 and 2.9. In the case of von Ebbengorst-Tangbergen data, the plot is modified be replacing $\log d_p$ with $\log T$ the modified equation being

$$\log\left[-\ln(1-\eta)\right] = \log 2\left[\frac{C(n+1)}{18}\right]^{1/(2n+2)} + \frac{1}{n+1}\log T \qquad (2.22)$$

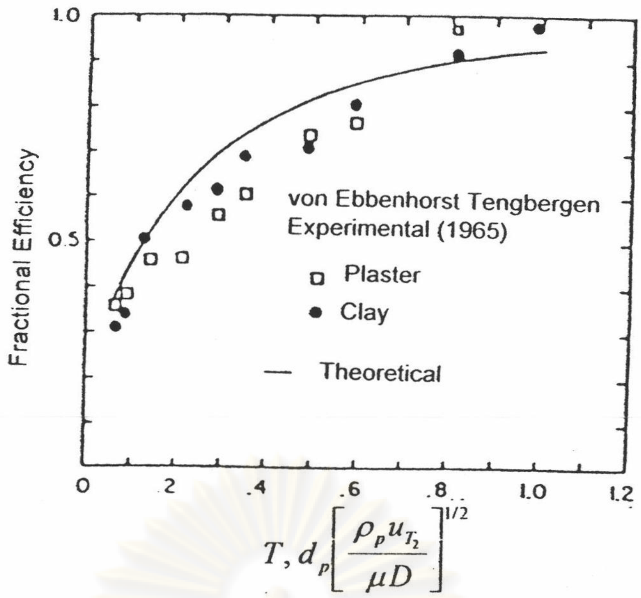

Figure 2.5 Comparison of new theoretical with experiment data von Ebbenhorst Tangbergen

Source: Ibid. p 203.

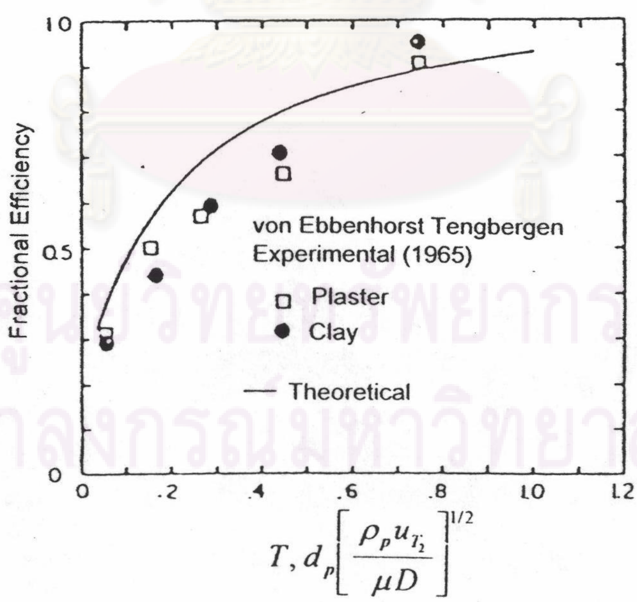

Figure 2.6 Comparison of new theoretical with experiment data von Ebbenhorst Tangbergen

Source: Ibid. p 203.

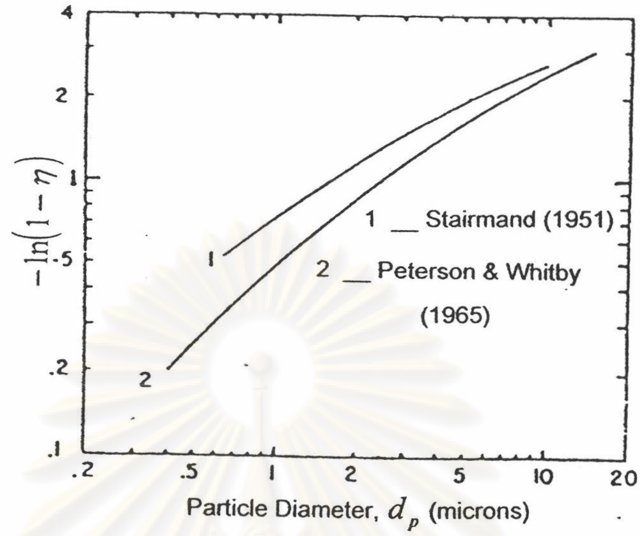

Figure 2.7 Rectified plot of Stairmand data and Peterson & Whitby

Source: Ibid. p 203.

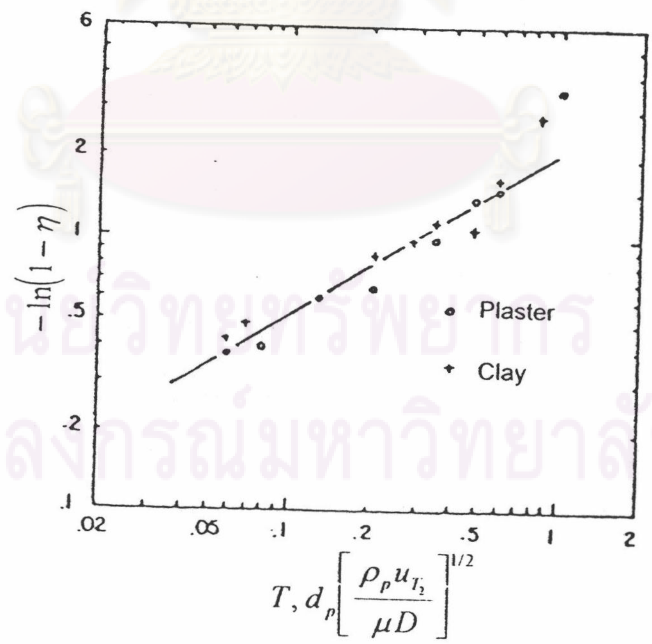

Figure 2.8 Rectified plot of Tengbergen data 11 in. cyclone

Source: Ibid. p 203.

Although there is some curvature to the plots of the Stirmand and the Peterson and Whitby data, reasonable straight lines may be drawn in all four cases. The values of n obtained by measuring the slope of each of these lines are shown below and compared with the value of n calculated from Equation (2.1) which was used in the preparation of the predicted grade-efficiency curves shown in Figures 2.3 to 2.6.

Table 2.2 Comparison between n (Plot) and n [Equation(2.1)]

Source of Data	n (Plot)	n [Equation(2.1)]
Stairmand	0.94	0.54
Peterson and Whitby	0.51	0.58
Tangbergen (11 in.)	0.43	0.56
Tangbergen (18 1/2 in.)	0.64	0.60

The agreement is good except in the case of the Stairmand data.

From the value of the ordinate intercept, that is, where $d_p = 1$ micron or $T = 1$ on each of these log-log plots, a value of C may also be calculated and compared with that obtained from the cyclone dimensions as given by Equation (2.19). These compare as follows:

Table 2.3 Comparison between C (Plot) and C [Equation (2.19)]

Source of Data	C (Plot)	C [Equation (2.19)]
Stairmand	42.8	55.2
Peterson and Whitby	34.9	39.2
Tangbergen (11 in.)	17.6	23.8
Tangbergen (18 1/2 in.)	8.78	23.8

These comparison tend to indicate that the average residence time has been somewhat overestimated.

There are many other published grade-Efficiency curves which may be tested by the log-log plot method, even though insufficient information is given to calculate predicted curves by Equation (2.20). For example those given by Swift (1969) have been found to give good straight lines. However this type of testing is difficult to carry out with precision because most of the published curves are drawn to small scale which is difficult to read and the original raw data are not quoted.

2.6 Comparison with other equations

To indicate how the results obtained with the new equation compare with several other published methods of predicting grade-Efficiency curves, the Stairmand data and the Peterson and Whitby data were also calculated according to the methods of Lapple (1967), Barth (1956) and Sproull (1970). There is insufficient information to do this for the Tengbergen data.

The comparative results are given in Figures 2.10 and 2.11. It is seen that the other methods give curves which are not only further off from the experimental but also show a point of inflection in curvature which is not present in the experimental curves tested here.

Although these comparisons are certainly limited in scope, the new equation does give clear promise of providing a more accurate representation of Grade-Efficiency curves than any other method proposed. Thus is established a preliminary confirmation of the premises upon which it is based, that is the concept of back-mixing of the uncollected particles coupled with the determination of an appropriate average residence time for the gas stream. The model is based upon the fundamental geometry and mechanics of the cyclone with very little recourse to any empiricism.

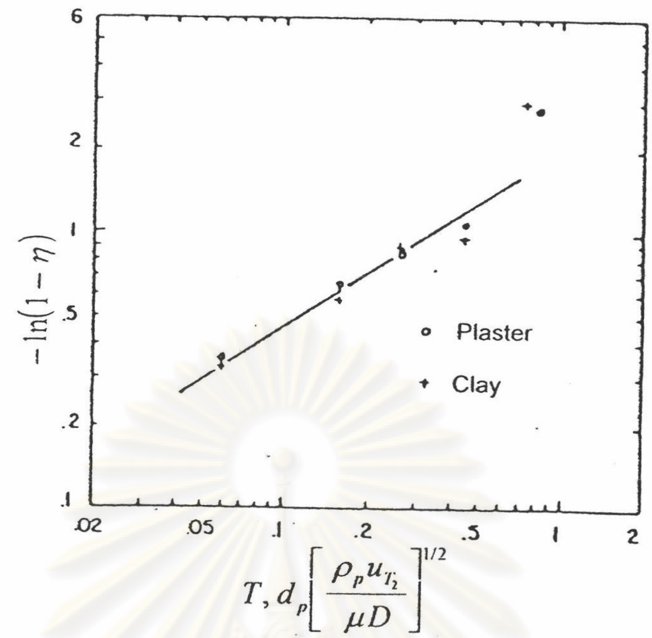

Figure 2.9 Rectified plot of Tengbergen data 18 1/2 in. cyclone

Source: Ibid. p 204.

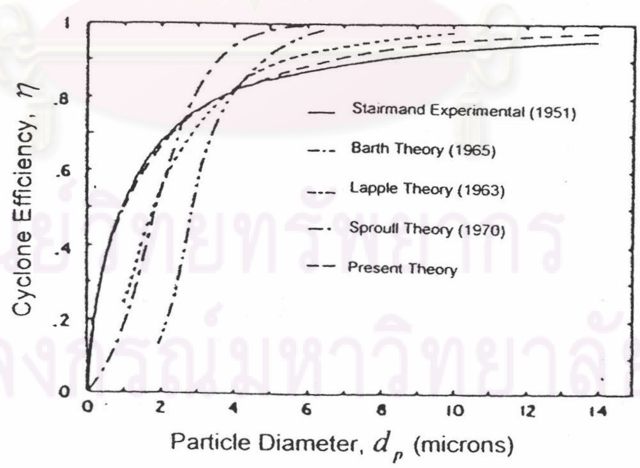

Figure 2.10 Comparison of experimental Stairmand data with theoretical predictions by other methods

Source: Ibid. p 204.

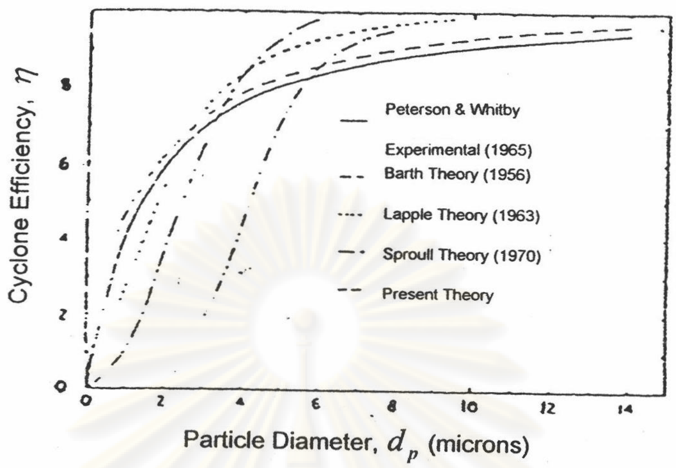

Figure 2.11 Comparison of experimental Peterson & Whitby data with theoretical predictions by other methods

Source: Ibid. p 205.

2.7 Optimizing cyclone design

In an optimization problem one seeks to maximize or minimize a specific quantity, called the objective. These variables may be independent of one another, or they may be related through one or more constraints.

Optimization problem contain three essential categories:

1. At least one objective function to be optimized
2. Equality constraints (Equation)
3. Inequality constraints

There are two types of optimization problems.

1. Linear Programming
2. Non Linear Programming

Linear Programming (LP) is one of the most widely used optimization techniques and one of the most effective, The term linear programming was coined by George Dantzig in 1947 to refer to the procedure of optimization in problems in which both the objective function and the constraints are linear. When stated mathematically, each of these problems potentially involves many variables, many equations, and inequalities. A solution must not only satisfy all of the equations, but also must achieve an extremum of the objective function, such as maximizing profit or minimizing cost.

Linear programming (LP) problems are a type of convex programming problem, where the objective function is convex and the linear constraints form a convex set. This means that a local optimum will be a global optimum. Linear programming problems also exhibit the special characteristic that the optimal solution of the problem must lie on some constraint or at the intersection of several constraints can be satisfied.

The most widely used methods for find Linear Programming solution consists of:

1.1 Graphical method is suitable for 1 to 3 variables cause of x-y-z planes. A graphical illustration of the solution procedure, consisting of three steps:

a) Plot the constraints on the x-y or z plane.

b) Determine the feasible region on x-y or z plane (those values satisfy all of the constraints).

c) Find the point along the boundary of the feasible region that maximizes the objective function $f(x)$ by examining the constraint intersections. The unique maximizing (or minimizing) state for x will always be at a corner of the feasible region for a well-posed problem. For a problem with two variables, the optimum will always occur at the intersection of two or more constraint bounds. An idea can be generalized to n variables; in the n-dimensional case the optimum will lie at the intersection of the bounds of n or more different inequality constraints.

Because of most LP problems will involve more than two variables, a method more versatile than graphical analysis must be employed to obtain the optimum. However, the understanding and analysis of an LP problem on a two-dimensional plot is very helpful in understanding how you ought to treat a higher dimensional problem. In formulating a numerical approach, we shall use the fact that the optimum for a linear programming problem always lies at a corner of the feasible region. An efficient search technique must be used to progress from one corner to the next; in so doing the objective function is continually improved.

1.2 Simplex method the method of the "Sequential Simplex" formulated by Spendley, Hext, and Himsworth (1962) uses a regular geometric figure (a simplex) to select points at the vertices of the simplex at which to evaluate $f(x)$. In two dimensions the figure is an equilateral triangle. In three dimensions this figure becomes a regular tetrahedron, and so on. At each iteration, to minimize $f(x)$, $f(x)$ is evaluated at each of three vertices of the triangle. The direction of search is oriented away from the point with the highest value for the function through the centroid of the simplex. By making the search direction bisect the line between the other two points of the triangle, the direction will go through the centroid. A new point is selected in this reflected direction (examine 6.3), preserving the geometric shape. The objective function is then evaluated at new point, and a new search direction is calculated.

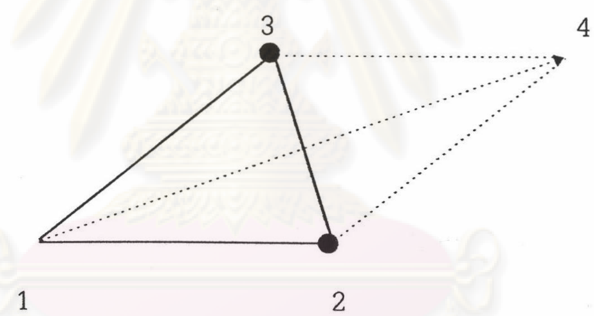

Figure 2.12 Reflection to a new point in the Simplex method.

Source: Edgar and Himmelblau, D.M. Optimization of chemical process. Mc Graw-Hill. p 193, 1989.

Nelder an Mead (1965). Described a more efficient (but more complex) version of the simplex method threat permitted the geometric figures to expand and contract continuously during the search. Their method minimized a function of n variables using $(n+1)$ vertices of a flexible polyhedron. Details of the method together with a computer code to execute the algorithm can be found in Himmelblau (1989).

Non Linear Programming (NLP) treats more difficult problems involving minimization (or maximization) of a nonlinear objective function subject to linear and/or nonlinear constraints:

$$\begin{aligned} &\text{Minimize} && f(x) && x = [x_1\ x_2 \ldots x_n]^T \\ &\text{Subject to} && h_j(x) = 0 && j = 1,2,\ldots,m \\ & && g_j(x) \geq 0 && j = m+1,\ldots,p \end{aligned} \qquad (2.23)$$

The inequality constraints are frequently transformed into equality constraints. One method of handling just one or two linear or nonlinear equality constraints is to solve explicitly for one variable and eliminate that variable from the problem formulation by substitution for it. In all the functions and equations in the problem. In many problems elimination of a single equality constraint will often be a superior method to an approach in which the constraint is retained and some constrained optimization procedure is executed.

In problems in which there are n variables and m equality constraints, we could attempt to eliminate m variables by direct substitution. If all the equality constraints can be removed, and there are no inequality constraints, the objective function can then be differentiated with respect to each of the remaining $(n-m)$ variables and the derivatives set equal to zero. Or, a computer code for unconstrained optimization can be employed to obtain x^*. If the objective function is convex and constraints form a convex region, then there is only one stationary point, which is the global minimum. Unfortunately, very few problems in practice assume this simple form or even permit the elimination of all equality constraints.

Consequently, five major approaches for solving nonlinear programming problems with constraints:

- Lagrange multiplier methods
- Iterative linearization methods
- Iterative quadratic programming methods
- Penalty function methods
- Simulation methods

Simulation permits the evaluation of operating performance prior to the implementation of a system ; it permits the comparison of various operational alternatives without perturbing the real system; it permits time compression so that timely policy decisions so that timely policy decisions can be made. Finally, it can be used by many people because of its readily comprehended structure and the availability of special-purpose computer simulation languages.

However, simulation can be easily misused, and simulation results taken with more confidence than is justified. The proper collection and analysis of data, the use of analytic techniques when they will suffice, the verification and validation of models, and the appropriate design of simulation experiments are all potential pitfall areas.

A simulation is the imitation of the operation of a real-world process or system over time. Whether done by hand or on a computer, simulation involves the operating characteristics of the real system.

The behavior of a system is studied by developing a simulation model. This model usually takes the form of a set of assumptions concerning the operation of the system. These assumptions are expressed in mathematical, logical, and symbolic relationships between the entities, or objects of interest, of the system. Once developed and validated, a model can be used to investigate a wide variety of questions about the real-world system. Potential changes to the system can first be simulated in order to predict their impact on system performance. Simulation can also be used to study systems in the design stage, before such systems are built. Thus, simulation modeling can be used both as an analysis tool for predicting the effect of changes to exiting systems, and as a design tool to predict the performance of new systems under varying sets of variables or circumstances.

In some instances, a model can be developed which is simple enough to be "solved" by mathematical methods. Such solutions may be found by the use of differential calculus, probability theory, algebraic methods, or other mathematical techniques. The solution usually consists of one or more numerical variables which are called measures of performance of the system. However, many real-world systems are so complex that models of these systems are virtually impossible to solve mathematically. In these instances, numerical, compute based simulation can be used to imitate the behavior of the system over time. From the simulation, data are collected as if a real system were being observed. This simulation-generated data is used to estimate the measures of performance of the system.

Simulation can be used for the following purposes:

1. Simulation enables the study of, and experimentation with, the internal interactions of a complex system, or of a subsystem within a complex system.

2. Informational, organizational and environmental changes can be simulated and the effect of these alterations on the model's behavior can be observed.

3. The knowledge gained in designing a simulation model may be of great value toward suggesting improvement in the system under investigation.

4. By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.

5. Simulation can be used as a pedagogical device to reinforce analytic solution methodologies.

6. Simulation can be used to experiment with new designs or policies prior to implementation, so as to prepare for what may happen.

7. Simulation can be used to verify analytic solutions.

2.7.1 System and model

A system is defined as a group of objects that are joined together in some regular interaction or interdependence toward the accomplishment of some purpose. The state of a system is defined to be that collection of variables necessary to describe the system at any time, relative to the objectives of the study.

A model is defined as a representation of a system for the purpose of studying the system. For most studies, it is not necessary to consider all the details of a system; thus, a model is not only a substitute for a system, it is also a simplification of the system Mihram and Mihram, 1974. On the other hand, the model should be sufficiently detailed to permit valid conclusions to be drawn about the real system. Different models of the same system may be required as the purpose of investigation changes. Models can be classified as being mathematical or physical. A mathematical model uses symbolic notation and mathematical equations to represent a system. A simulation model is a particular type of mathematical model of the system.

Simulation models may be further classified as being static or dynamic, deterministic or stochastic, and discrete or continuous. A static simulation model, sometimes called a Monte Carlo simulation, represents a system at a particular point in time. Dynamic simulation models represent systems as they change over time.

Simulation models that contain no random variables are classified as deterministic. Deterministic models have a known of inputs which will result in a unique set of outputs.

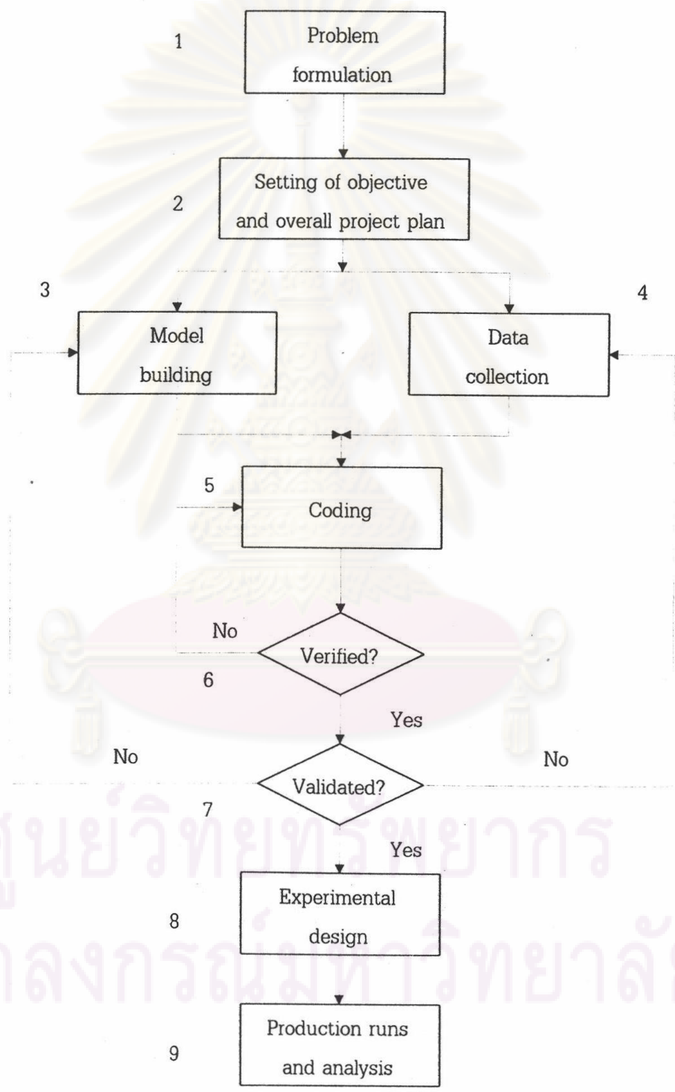

Figure 2.13 Steps in a simulation study

Source: Jerry Banks and John S. Carson, II. Discrete-event system simulation. Prentice-Hall. p 12, 1884

2.7.2 Steps in simulation method

Figure 2.13 shows a set of steps to guide a model builder in a thorough and sound simulation study. Similar figures and discussion of steps can be found in other sources Shannon (1975); Gordon (1978); Law and Kelton (1982). The number beside each symbol in figure 2.13 refers to the more detailed discussion in the text. The steps in a simulation study are as follows;

1. Problem formulation. Every study should begin with a statement of the problem. If the statement is provided by the policymakers, or those that have the problem, the analyst must ensure that the problem being described is clearly understood. If a problem statement is being developed by the analyst, it is important that the policymakers understand and agree with the formulation. Although not shown in Figure 2.13, there are occasions where the problem must be reformulated as the study progresses. In many instances, policymakers and analysts are aware that there is a problem long before the nature of the problem is known.

2. Setting of objectives and overall project plan. The objectives indicate the questions to be answered by simulation. At this point a determination should be made concerning whether simulation is the appropriate methodology for the the problem as formulated and objectives as stated. Assuming that it is decided that simulation is appropriate, the overall project plan should included a statement of the alternative systems to be considered, and a method for evaluating the effectiveness of these alternatives. It should also include the plans for the study in terms of the number of variables involved, the cost of the study, and the number of days required to accomplish each phase of the work with the anticipated results at the end of each stage.

3. Model building. The construction of a model of a system is probably as much art as science. Shannon (1975) provides a lengthy discussion of this step. "Although it is not possible to provide a set of instructions that will lead to building successful and appropriate models in every instance, there are some general guidelines that can be followed" Morris (1967). The art of modeling is enhanced by an ability to abstract the essential features of a problem, to select and modify basic assumptions that characterize the system, and then to enrich and elaborate the model until a useful approximation results.

Thus, it is best to start with a simple model and build toward greater complexity. However, the model complexity need not exceed that required to accomplish the purposes for which the model is intended. Violation of this principle will only add to model building and computer expenses. It is not necessary to have a one-to-one mapping between the model and the real system. Only the essence of the real system is needed.

It is advisable to involve the model user in the model construction. Involving the model user will both enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.

4. Data collection. There is a constant interplay between the construction of the model and the collection of the needed input data Shannon (1975). As the complexity of the model changes, the required data elements may also change. Also, since data collection makes such a large portion of the total time required to perform a simulation, it is necessary to begin it as early as possible, usually together with the early stages of model building.

5. Coding. Since most real-world systems result in models that require a great deal of information storage and computation, the model must be programmed for a digital computer. The modeler must decide whether to program the model in the general-purpose language such as FORTRAN or a special-purpose simulation language such as GPSS, SIMSCRIPT, or SLAM. A general-purpose language requires a much longer development time, but usually executes on the computer much faster than the special-purpose languages. Overall, however, special-purpose language so speed up the coding (and verification) step that more and more model builders are using them.

6. Verified. Verification pertains to the computer program prepared for the simulation model. Is the computer program performing properly? With complex models, it is difficult, if not impossible, to code a model successfully in its entirety without a good deal of debugging. If the input parameters and logical structure of the model are correctly represented in the code, verification has been completed. For the most part, common sense is used in completing this step.

7. Validated? Validation is the determination that a model is an accurate representation of the real system. Validation is usually achieved through the calibration of the model, an iterative process of comparing the model to actual system behavior and use in the discrepancies between the two, and the insights gained, to improve the model.

This process is repeated until model accuracy is judged acceptable. In the example of a bank mentioned above, data were collected concerning the length of waiting lines under current conditions. Does the simulation model replicate this system measure? This is one means of validation.

8. Experimental design. The alternatives that are to be simulated must be determined. Often, the decision concerning which alternatives to simulate may be function of runs that have been completed and analyzed. For each system design that is simulated, decisions need to be made concerning the length of the initialization period, the length of simulation runs, and the number of replications to be made of each run.

9. Production runs and analysis. Production runs, and their subsequent analysis, are used to estimate measures of performance for the system designs that are being simulated.

David Leith and William Licht (1972). Maximum collection efficiency will be obtained by using the largest possible value of the product $(C\psi)$. As was pointed out above C may be maximized independently by appropriate choice of the set of seven geometric ratios which specify the cyclone shape. For a given set of operating condition the size of the cyclone may then be selected so as to maximize ψ independently.

However the pressure drop across the cyclone must also be taken into account. One criterion for optimum operation would be to maximize the ratio of collection efficiency to pressure drop. If a suitable Equation for ΔP may be found then it could be coupled with Equation (2.20) to determine that combination of design and operation which would produce a maximum value of $\eta / \Delta P$. The overall performance of cyclones operated in series may also be predicted by applying the Equation to each cyclone separately with the feed to the second cyclone taken as the outlet stream from the first. Such combination may then be optimized.

It is planned to deal with these topics in detail in further papers as well as to test the applicability of this model to centrifugal separators of different types of design for example scroll inlet or helical inlet cyclones.

2.8 Survey of the literature

William Licht (1972). In order to optimize the design, a parameter may be constructed which is proportional to the benefit cost ratio for a given cyclone configuration.

The "benefit" or accomplishment of collection may be taken to be proportional to the configuration parameter K as used in Equation (2.20b) and given in Table 2.4. Note that this is inversely related to the "cut diameter". Cost is made up mainly of two parts: (a) initial cost of construction, which is essentially proportional to the total surface area of the material of construction used, and (b) cost of operation, which is proportional to power consumption. Power consumption also reflects, roughly, the initial cost of the gas moving system.

A useful optimization parameter (OP) may therefore be conceived as

$$\text{OP} \propto K/(\text{total surface area} \times \text{power})$$

$$power = 0.0981Q \times \Delta P \times W \qquad (2.24)$$

$$\Delta P = 5.12\,\rho_g u_{T_2} N_H \text{ in cm } H_2O\text{, gauge} \qquad (2.25)$$

where

ρ_g = density of gas-particle stream, g/cm^3

u_{T_2} = inlet velocity, m/sec

N_H = number of inlet velocity heads

Total surface area may be calculated by using $(\pi D^2 Surf)$, where "$Surf$" is again a function of configuration only. Value are given in Table 2.4 for the standard configurations, and methods of calculation for other shape are given by Licth. Power is calculated by combining Equation (2.24) and (2.25). For a given set of operating conditions then, let

$$\text{OP} = \frac{K}{(D^2 Surf)(Qu_{T_2} N_H)}$$

dropping the various constants. The group $(D^2 Qu_{T_2}^2)$ may be replaced by $Q^3 / K_a K_b$, resulting in

$$\text{OP} = \frac{K \times K_a K_b}{N_H \times Surf \times Q^3}$$

For a given gas flow, OP depends only on the configuration parameters, K, K_a, K_b, N_H, and *Surf*. Using these as given in Table 2.4, values of OP are as follows:

Table 2.4 Cyclone design configurations (Tangential Entry)

Term	Description	High Efficiency		Shepherd	General Purpose	
		Stairmand	Swift	and Lapple	Swift	Peterson and whitby
D	Body diameter	1.0	1.0	1.0	1.0	1.0
a	Inlet height:	0.5	0.44	0.5	0.5	0.583
b	Inlet width:	0.2	0.21	0.25	0.25	0.208
S	Outlet length:	0.5	0.5	0.625	0.6	0.583
D_e	Outlet diameter:	0.5	0.4	0.5	0.5	0.5
h	Cylinder height:	1.5	1.4	2.0	1.75	1.333
H	Overall height:	4.0	3.9	4.0	3.75	3.17
B	Dust outlet diameter:	0.375	0.4	0.25	0.4	0.5
K	Configuration parameter	551.3	699.2	402.9	381.8	342.3
N_H	Inlet velocity heads	6.40	9.24	8.0	8.0	7.76
Surf	Surface parameter	3.67	3.75	3.78	3.65	3.20

Source: Stairmand, C. J., Trans. Inst. Chem. Engrs. 29, 356 (1951)

Swift, P., Steam Heating Eng. 38, 453 (1969)

Shepherd, G. B., and Lapple, C. E., Ind. Eng. Chem. 31, 972 (1939)

Peterson, C. M., and Whitby, K.T., ASHRAE Journ. 42 (1965)

Table 2.5 Cyclone type and OP value

Configuration	OP
Stairmand	2.35
Swift (high Eff.)	1.96
Lapple	1.66
Swift (general Purpose)	1.64
Peterson and Whitby	1.67

Source: William Licht. "Control of particles by mechanical collectors,". chapter 13. p 328.

J.M.MARTINEZ-BENET and J.CASAL (1983). With high gas volumes, offen a number of cyclones have to be used in parallel, in order to reach a given efficiency maintaining inlet velocity and pressure drop between certain limits.

The total cost per unit time will therefore be a function of the fixed cost of the cyclones, the energy cost of operating the cyclones, and the value of the lost particles:

$$c_t = \text{(fixed cost)} + \text{(energy cost)} + \text{(value of the lost particles)} \quad (2.26)$$

Nevertheless, from a general point of view, it seems more convenient not to include the value of the lost particles in this analysis, as usually other criteria will establish a fixed value for the efficiency of the cyclone: pollution considerations, for example, or the usual practice of setting more than one single stage if high efficiencies are necessary.

The total cost per unit time will therefore be a function of both pressure drop and number of cyclones: an increase in the number of cyclones will produce a reduction in pressure drop, and therefore in power costs; simultaneously, it will produce an increase in capital cost. The total cost of a set of cyclones can then be optimized in terms of the number of cyclones, thus reaching a minimum cost at an optimum number.

In this paper, a new procedure is presented to achieve this optimization. The approach relies in that of Gerranrd and Liddle (1976), and is based on the efficiency calculation method which uses $d_{cp_{50}}$ (considered as the diameter of the smallest particle that will be collected with an efficiency of 50%) and the efficiency curve of the cyclone Licht, W., 1980. Inlet velocity and pressure drop constraints are taken into account, and appropriate actions have been introduced into the computer program whose logic flowchart is also included.

The following equation can be obtained relating the cut diameter d_t and cyclone diameter D:

$$D = \left[\frac{d_t^2 (\rho_p - \rho) \pi n^* Q}{9 K_b^2 K_a \mu N} \right]^{1/3} \quad (2.27)$$

Pressure drop for a cyclone can be calculated by the following correlation:

$$\Delta P = \frac{1}{2} \rho u_{T_2}^2 N_H = \frac{1}{2} \rho \left(\frac{Q}{K_a K_b N} \right)^2 N_H \quad (2.28)$$

The power costs are given by

$$c_{power} = Q\Delta P c_e \qquad (2.29)$$

The capital cost can be expressed in terms of cyclone diameter in the following way Leith. D., and Licht, W., AIChE Sympos. Ser. 68(126), 196 (1972). :

$$c_{capital} = eD^j \qquad (2.30)$$

Where constants e and j can easily be evaluated from the prices of two cyclones of different diameter. The fixed costs per unit time are given by

$$c_{fixed} = \frac{f}{YH} NeD^j \qquad (2.31)$$

Where f is an investment factor[7] to allow for installation, fittings, piping, etc.

The total cost will then be

$$c_t = \frac{\rho Q^3 c_e N_H}{2D^4 K_a^2 K_b^2 N^2} + \frac{f}{YH} NeD^j \qquad (2.32)$$

The operating costs include only the cost of the power, since maintenance is usually negligible; infact, although the economic life will be much shorter, for example 5 years, a cyclone life of 20 years is frequently attainable.

In Equation (2.32) D can be expressed in terms of N. An equation will then be obtained which gives c_t in terms of N:

$$c_t = \frac{\rho Q^3 c_e N_N}{2K_a^2 K_b^2 N^2}\left[\frac{9K_b^2 K_a \mu N}{d_l^2(\rho_p - \rho)\pi n^* Q}\right]^{4/3} + \frac{feN}{YH}\left[\frac{d_l^2(\rho_p - \rho)\pi n^* Q}{9K_b^2 K_a \mu N}\right]^{j/3} \qquad (2.33)$$

This is the general expression for the total cost of a set of any type of cyclones. Equation(2.33) is the objective function, in which the only variable is N. It may be differentiated with respect to N. Setting the resultant relationship equal to zero, an expression will be obtained for the optimum number of cyclones which will correspond to a minimum cost:

$$N_o = Q\left\{\frac{fe(3-j)}{c_e YH\rho}\,\frac{K_a^2 K_b^2}{N_H} \times \left[\frac{d_l^2(\rho_p - \rho)\pi n^*}{9K_b^2 K_a \mu}\right]^{(4+j)/3}\right\}^{3/(j-5)} \qquad (2.34)$$

The number thus obtained must then be rounded off to the nearest integer. The optimum number number of cyclones varies then directly with the total volumetric flow rate, for a given efficiency. Therefore, a change in the total amount of gas to be treated will affect N_o proportionally, without changing the diameter of the cyclones.

There are two important constraints:

- Pressure drop cannot exceed a certain value. The criterion generally accepted is the following:

$$\Delta P \leq 2{,}500 \text{ N/m}^2$$

- Inlet velocity must lie between two values, in order to avoid a decrease in order to avoid a decrease in cyclone efficiency. Usually, it is accepted that

$$15 \text{ m/s} \leq u_{T_2} \leq 30 \text{ m/s}$$

Although inlet velocity for correct operation of cyclones is usually in this range, in some cases another constraint will be necessary, especially for small cyclone diameters. As pointed out in the literature F.A., Zenz (1964), W.H., Koch and W.,Licht (1977) inlet velocity should be less than 1.35 times saltation velocity, in order to avoid reentrainment. A new constraint must therefore be introduced:

$$u_{T_2} \langle v_s$$

saltation velocity being given by W.H. Koch and W. Licht (1977):

$$v_s = 4.91\left(\frac{4g\mu\left(\rho_p - \rho\right)}{3\rho^2}\right)^{1/3} \times \frac{K_b^{0.4}}{1-K_b^{\ 1/3}} D^{0.067} u_{T_2}^{2/3} \qquad (2.35)$$

Using Equation (2.28), these three constraints can be reduced to one. Defining a maximum inlet velocity as that corresponding to maximum allowable pressure drop,

$$u_{T_2\max} = \sqrt{\frac{2 \times 2{,}500}{\rho N_H}} \qquad (2.36)$$

the constraint will be

$$15 \text{ m/s} \leq u_{T_2} \leq \min\left(u_{T_2\max},\ 30 \text{ m/s},\ 1.35 v_s\right)$$

Minimum and maximum values ($N_{\min}$ and $N_{\max}$) will then exit for the number of cyclones, and $N_{\min} \leq N_o \leq N_{\max}$. If the value obtain from Equation (2.34) is not in this range, then the actual optimum will be the value of N_{min} or N_{max} closer to the calculated value. From Equation (2.27) and the above mentioned conditions,

$$N_{max} = \left(\frac{v'}{15}\right)^3 \frac{1}{d_l^4} \tag{2.37a}$$

$$N_{min} = \left(\frac{v'}{u_{T_2\,max}}\right)^3 \frac{1}{d_l^4} + 1 \tag{2.37b}$$

The parameter v being

$$v' = \left(\frac{81QK_b\mu^2}{\pi^2 K_a n^{*2}\left(\rho_s - \rho\right)^2}\right)^{1/3} \tag{2.38}$$

The final optimum value will therefore have to satisfy the following constraints:

$$15 \text{ m/s} \le u_{T_2} \le (u_{T_2\,max}, 30 \text{ m/s}, 1.35\ v_s)$$

$$d_l \le d_{l_{initial}}$$

For high gas volume flow rate two or more cyclones of the same configuration and size may be used in parallel, the flow being divided equally between them. In this case, each smaller cyclone diameter be used than single cyclone diameter. The collection grade efficiency for each cyclone is the same, and is greater than if only one larger one were used. A large number of small cyclones in parallel(called a "multicyclone") may be used to increase efficiency for very large flows of gas.

Table 2.6 Sample results of cyclone optimization program for examples No. 1 and No. 2

Parameter	Data and results	
	Example No. 1	Example No. 2
K_a	0.5	0.5
K_b	0.25	0.25
N_H	6.115	6.155
d_{cp} ; m	10×10^{-6}	10×10^{-6}
Q ; m^3/s	14	1
ρ_p ; kg/m^3	1,800	1,800
ρ ; kg/m^3	1.3	1.3
μ ; kg/(m.s)	22×10^{-6}	22×10^{-6}
j	1.73	1.73
e ; $\$/m^j$	3,900	3,900
f	2.5	2.5
Y ; year	5	5
operating, sec/year	2.16×10^{7}	2.16×10^{7}
$n*$	4	4
c_e , \$/Joule	10^{-8}	10^{-8}
N_o	5	1
c_t ; \$/sec	7.3×10^{-4}	5.7×10^{-5}
d_{I_a} ; m	10×10^{-6}	8.5×10^{-6}
D ; m	1.01	0.64
u_{T_2} ; m/s	22.06	19.44
ΔP ; N/m^2	1,948	1,512