CHAPTER 1

INTRODUCTION

1.1 General

In recent years, the study of electromechanical coupling phenomenon of smart materials has been a subjected of interest and an object of numerous investigations due to their useful applications in the fields of science and engineering such as control of flexible structures, pressure transducers, accelerometers, aerospace structures and helicopter control surfaces, etc. (Mason 1981, Crawley 1994, Crawley and de Luis 1991, Rao and Sunar 1994 and Uchino 1998). Several types of smart materials have already been developed, namely, piezoelectric materials, shape memory alloys, electrostrictive materials, magenetostrictive materials, electroactive polymers and electro/magneto-rheological fluids, etc. Among all materials mentioned above, the most widely used smart material in practical applications is a piezoelectric material. A few examples of piezoelectric materials are Barium Titanate ($BaTiO_3$), Lead Zirconate Titanate (PZT) and Polyvinylidene Fluoride (PVDF), etc.

Piezoelectric materials were first discovered by Pierre and Jacques Curie in the 1880's. These materials undergo strain or mechanical deformation in response to an applied electric field (converse piezoelectric effect). They also produce charge when they are subjected to an applied stress (direct piezoelectric effect). The first property makes them suitable as actuators to control structural response, whereas the second property makes them suitable as sensors. Piezoelectric materials thus have the ability to transform mechanical energy to electrical energy and vice versa.

Piezoelectric materials exist primarily in two forms as ceramic and polymer. They need to be polarized to induce the piezoelectric effect. The polarization process as shown in Figure 1.1 is achieved by applying a high DC voltage (>2000 V/mm) across heated piezoelectric materials. After the polarization process, the material has a remnant polarization (see Figure 1.1c) which can be degraded by exceeding the mechanical, thermal and electrical limits of the material.

After being polarized, a voltage of the same polarity as the poling voltage causes a temporary expansion in the poling direction and a contraction in the plane parallel to electrodes (see Figure 1.2a). In both cases, the piezoelectric material returns to its original poled dimensions after the removal of the voltage.

If a compressive force is applied in the poling direction or a tensile force is applied in the plane normal to the poling direction (parallel to the electrodes), a positive voltage is generated. A voltage with the opposite polarity results from a tensile force applied parallel to the poling axis, or a compressive force applied perpendicular to the poling axis as shown in Figure 1.2b.

1.2 Piezoelectric Constitutive Relations

The voltage-strain and charge-stress relationships in a piezoelectric material are usually assumed to be linear. The actuation strain is modeled like a thermal strain. A piezoelectric material can be assumed to behave as a transversely isotropic material, i.e., a composite unidirectional laminate. The constitutive relations are based on the assumption that the total strain in the actuator is the sum of the mechanical strain induced by the stress, the thermal strain due to temperature and the controllable actuation strain due to the electric voltage. A coordinate system has three mutually perpendicular axes, called the x, y, and z or 1, 2 and 3 as illustrated in Figure 1.3. The 3-axis or z-axis is assigned as the direction of the initial polarization of the piezoelectric material. The shear components on the three planes labeled 4, 5 and 6 refer to the plane normal to the 1, 2 and 3 direction, respectively. All piezoelectric material constants are defined using this coordinate system throughout this thesis.

Piezoelectric materials undergo strain or mechanical deformation in response to an applied electric field. They also produce charge when they are under an applied stress. This means that the electrical and mechanical effects are coupled and the constitutive relations therefore take into account the mechanical response together with the electrical effects. Among the equations involving piezoelectric materials, the equations of state are the most employed equations. They can be written in two ways depending upon whether E or D is chosen as the independent variable (Cady, 1962). The following formulation is written with the electric field, E, being the independent variable.

$$\epsilon_i = s_{ij}^E \sigma_j + d_{mi}^T E_m \quad (1.1)$$

$$D_m = d_{mi} \sigma_i + \varepsilon_{mk}^T E_k \quad (1.2)$$

$$\sigma_i = c_{ij}^E \epsilon_j - e_{mi}^T E_k \quad (1.3)$$

$$D_m = e_{mi} \epsilon_i + \varepsilon_{mk}^{\epsilon} E_k \quad (1.4)$$

where

ϵ	=	strain vector	(m/m)
σ	=	stress vector	(N/m^2)
E	=	vector of applied electric field	(V/m)
D	=	electric displacement	(C/m^2)
s	=	vector of compliance coefficient	(m^2/N)
d	=	vector of piezoelectric strain constant	(m/V) or (C/N)
ε	=	dielectric constant	(F/m) or (C/Vm)
c	=	vector of elastic stiffness	(N/m^2)
e	=	piezoelectric coefficient	(C/m^2)

The superscripts ϵ and E denote a piezoelectric material under constant strain and constant electric field, respectively. In addition the superscript T denotes the transpose of a matrix. The equations with electric displacement, D as the independent variable are given in equations (1.5) to (1.8) as follows:

$$\epsilon_{ij} = s_{ij}^D \sigma_j + g_{mi}^T D_m \quad (1.5)$$

$$E_i = -g_{mi} \sigma_i + \beta_{mk}^{\sigma} D_k \quad (1.6)$$

$$\sigma_{ij} = c_{ij}^D \epsilon_j - q_{mi}^T D_m \quad (1.7)$$

$$E_i = -q_{mi} \epsilon_i + \beta_{mk}^{\epsilon} D_k \quad (1.8)$$

The new piezoelectric constants g and q are related to e and d as follows.

$$g_{mh} = s_{ih}^D q_{mi} = \beta_{mn}^T d_{nh} \quad (1.9)$$

$$q_{mh} = c_{ih}^{D} g_{mi} = \beta_{mn}^{\varepsilon} e_{mh} \quad (1.10)$$

where β denotes impermittivity component. In addition, the superscripts σ and D represent a piezoelectric material under constant stress and constant electric displacement, respectively. Some important piezoelectric constants are described in detailed as follows.

- **Piezoelectric Coefficient (d_{ij})**

The coefficient d_{ij} in equation (1.2) represents strain in the j-axis due to a unit electric field in the i-axis, provided that all external stresses are constant. For example, d_{31} denotes a direct strain in the direction 1 due to a unit applied field in the direction 3 (the polarized direction) provided that a piezoelectric material is mechanically free in all directions. The coefficient d_{ij} also denotes the ratio of charge density in direction j due to a unit stress in direction i, provided all other stresses are constant. As an example, d_{33} denotes charge density in direction 3 due to an applied stress in direction 3 when the material is free of external stresses in directions 1 and 2.

- **Elastic Compliance Constants (s_{ij})**

The elastic compliance coefficient s_{ij} represents the strain in the i-direction due to unit stress in the j-direction, provided that there is no change of stress in the other two directions. Direct stresses and strains are denoted by indices from 1 to 3 and shears are denoted by indices from 4 to 6. For example, s_{12} denotes a direct strain in the direction 1 due to a direct stress in the direction 2 with stresses in directions 1 and 3 being unchanged. Similarly, s_{55} denotes a shear strain on the plane perpendicular to the direction 2 due to shear stress on the same plane. If the electric field across the piezoelectric element is held constant, such as the case with short circuiting at the electrodes, the properties are denoted by a superscript E. If the electric charge density is held constant, such as the case with an open circuit at the electrodes, it is denoted by a superscript D.

- **Dielectric Constant or Permittivity (ε_{ij})**

The dielectric coefficient ε_{ij} is defined as a dielectric displacement or charge per unit area in the i-axis due to an applied electric field in the j-axis. If a piezoelectric is in a completely free condition then its dielectric constant is higher than the case when it is restrained. Normally, a superscript is added to the constant to denote this condition. For example, a superscript σ denotes a free boundary condition (constant stress) whereas a superscript $\in$ denotes a completely restrained condition (constant strain).

- **Piezoelectric Constant (g_{ij})**

The piezoelectric coefficient g_{ij} denotes the electric field developed along the i-axis (electrodes perpendicular to the i-axis) due to an applied stress along the j-axis provided that all other external stresses are zero. It also represents the strain developed along the j-axis due to a unit electric charge per unit area of electrodes applied along the i-axis (electrodes perpendicular to the i-axis). For example, g_{33} denotes an electric field developed in the direction 3 due to an applied stress in the direction 3 when other stresses are zero. It also denotes the strain developed in the direction 3 due to a unit charge per unit area of electrodes applied along the direction 3. Similarly, g_{15} denotes a shear strain induced on the plane perpendicular to the direction 2 due to a unit applied charge per unit electrode area with electrodes normal to the direction 1.

1.3 Objectives of the Study

The objectives of this study are the following:

1. To determine analytical solutions for mechanical displacements, electric potential, stresses, electric field and dielectric displacements of a piezoelectric cylinder subjected to axisymmetric surface load and electric field.

2. To study the influence of various parameters, e.g. the elastic stiffness constants, piezoelectric coefficients, dielectric constants etc. on the piezoelectric cylinder.

1.4 Scopes of the Study

The scopes of this study are the following

1. The geometry of the problem considered in this thesis is a solid circular cylinder.
2. The cylinder under consideration is a homogeneous transversely isotropic piezoelectric cylinder.
3. The mechanical loads and electric fields applied on the cylinder end surfaces are axisymmetric with respect to the z-axis, which is assigned as the direction of the initial polarization of the piezoelectric material.

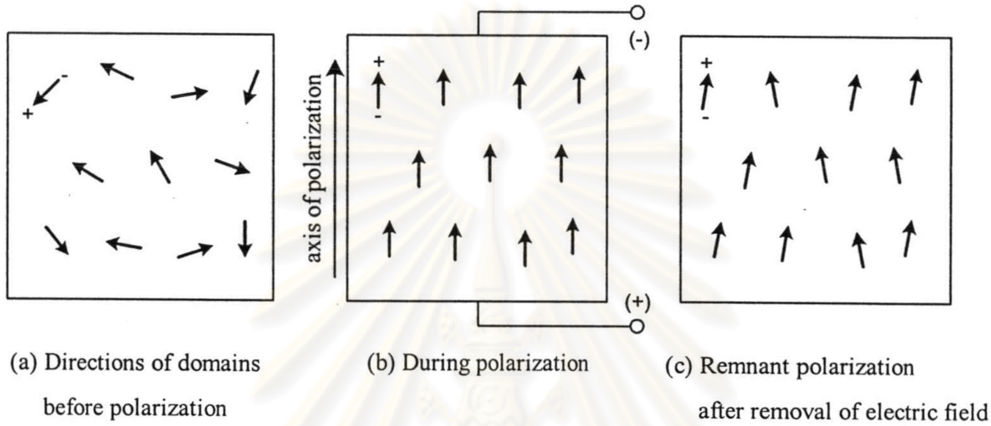
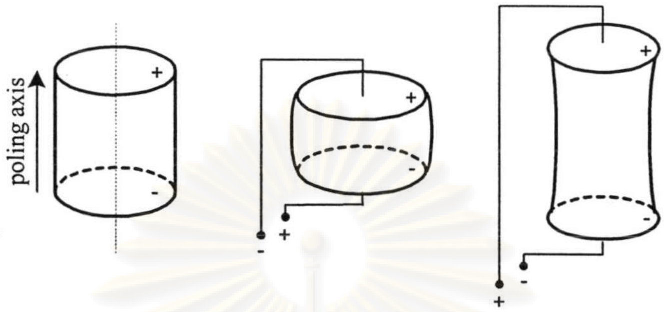
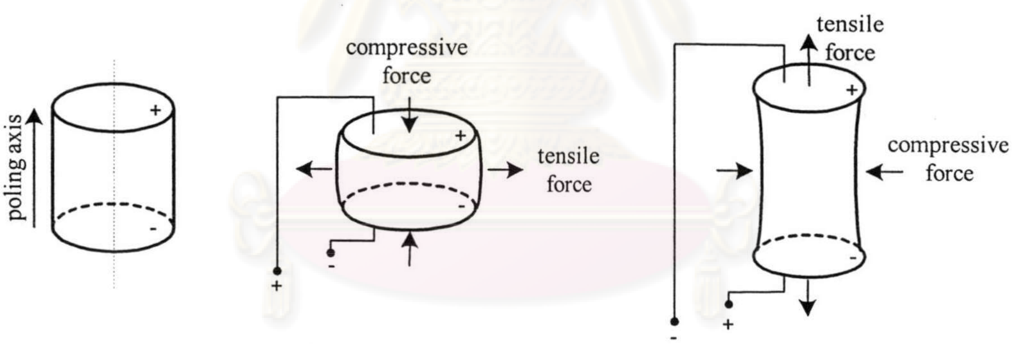


Figure 1.1 Polarizing (poling) of a piezoelectric material.



(a) Mechanical deformation from applied voltages.



(b) Voltages generated from applied forces.

Figure 1.2 Actuator and sensor behaviour of a piezoelectric material.

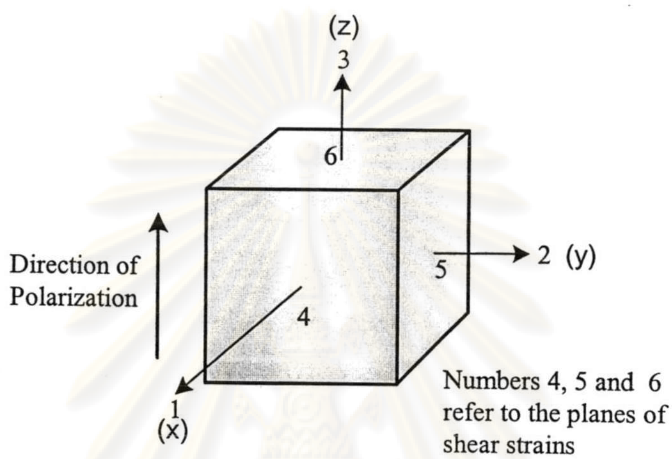


Figure 1.3 The direction of positive polarization and the definition of axes.