CHAPTER II

ANALYSIS OF PLANAR STEEL TRUSS STRUCTURES

In the current specifications of structural steel design (e.g. AISC-ASD, LRFD, PD), a whole structure would be analyzed prior to the determination of the member cross-sections. Because of the fact that some of the input data are not known in advance, some parameters, such as the member cross-section, have to be assumed. It is therefore necessary to check for the strength and the stability of the whole structure after the analysis process. This approach does not give an accurate indication of the factor against failure, because it does not consider the interaction of strength and stability between the structural system and its members at the same time (Figure 2.1). The individual member strength equations are not concerned with the system compatibility. There is no verification of the compatibility between the isolated member and the member as part of the frame system. As a result, there is no explicit guarantee that all members will be able to sustain their design loads under the geometrical configuration imposed by the frame system.

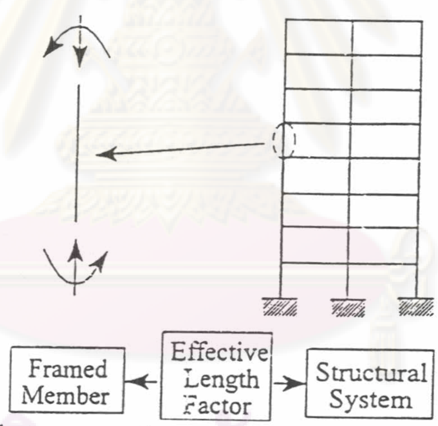

Figure 2.1 Interaction between a structural system and its component members (Chen 2000).

The current design methods also lack certain considerations on the structural behaviors. The stresses and displacements are determined by elastic analysis, while the strength and stability are determined separately by inelastic analysis. This is perhaps the most serious limitation. Not until recently, there has been an increasing awareness of the need for practical analysis/design methods that can account for the compatibility between the member and the system. With the rapid increase in the power of desktop computers and user-friendly software in recent years, the development of an alternative method to the direct design of structural systems has become more attractive and realistic (Figure 2.2).

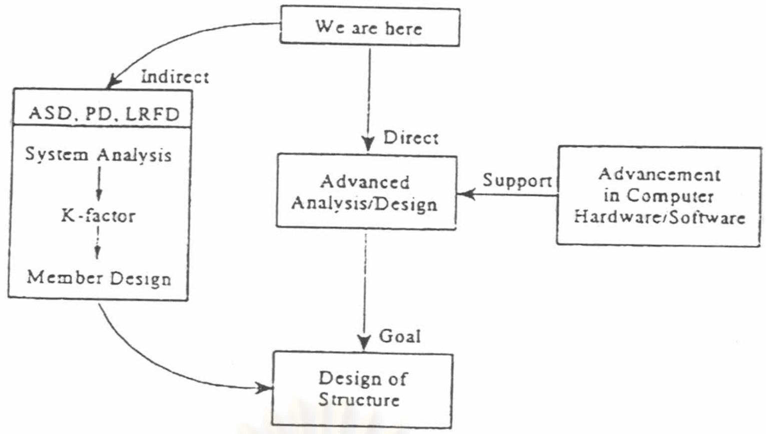

Figure 2.2 Analysis and design methods (Chen 2000).

With the invention of powerful computers, the finite element method using the energy principles and the discretization concept, has received more attention in the past few decades as a more generalized and robust tool for solving structural analysis problems in both linear and nonlinear ranges. Its advantage lies mainly in the flexibility in assigning the structural properties, the geometrical configuration and the boundary and load conditions. Research works are currently in full swing to develop the advanced inelastic analysis methods which can sufficiently represent the primary limit states of the structural members such that the capacity check for separate members is no longer required. Figure 2.3 graphically summarizes some of the various types of non-linear analyses (Chan 2001).

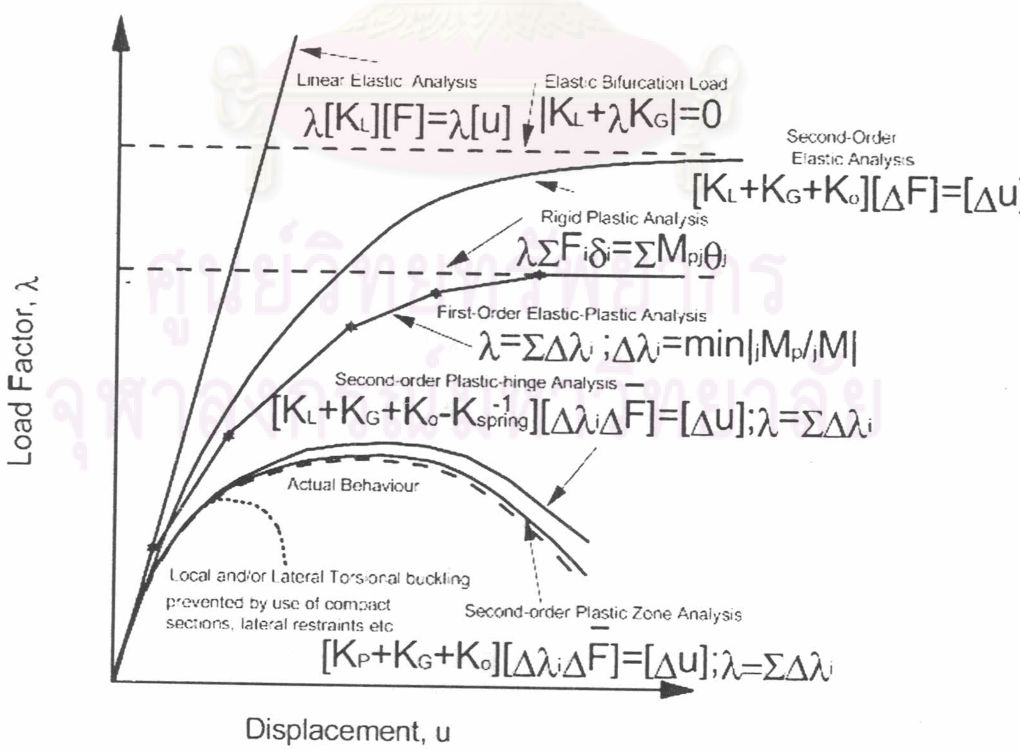

Figure 2.3 General analysis types for framed structures (Chan 2001).

The linear elastic analysis is the most essential and the simplest method. The bifurcation analysis is quite simple, by assuming a sudden interception of a secondary equilibrium path to the primary path and the solution is obtained by solving the characteristic equation

$$\left|\mathbf{K}_L + \lambda \mathbf{K}_G\right| = \mathbf{0} \tag{2.1}$$

where $\mathbf{K}_L$ is the linear stiffness matrix;
λ is the load factor; and
$\mathbf{K}_G$ is the geometric stiffness matrix.

In this eigenvalue analysis, the pre-buckling deformation, initial imperfection and material yielding are ignored. The analysis yields an upper bound solution, which is generally not sufficiently accurate for practical design, even for a very slender structural form like steel scaffolding (Chan *et al.* 1995; Peng *et al.* 1998). However, its solution is simple and can be easily included in the vibration analysis software. Also, it can be used to work out the effective length factor (Kirby 1988).

The rigid plastic analysis method considers only the material yielding and the plastic hinges, ignoring the instability and large deflection. This method can only be used for frames with negligible geometrical nonlinearity, otherwise additional modification is needed.

The plastic zone method assumes yielding to spread across the section and along the element. This concept has been used by numerous researchers including Vogel (1985), Chan (1989), and Clarke (1994). The essence of the method can be stated in the incremental equilibrium equation as follows:

$$\left[\mathbf{K}_P + \mathbf{K}_G + \mathbf{K}_0\right]\left[\Delta\lambda\Delta\bar{\mathbf{K}}\right] = \Delta\mathbf{u} \tag{2.2}$$

where $\mathbf{K}_P$ is the elasto-plastic stiffness matrix for material yielding;
$\mathbf{K}_G$ is the geometric stiffness matrix;
$\mathbf{K}_0$ is the large deflection matrix;
$\Delta\lambda$ is the incremental load factor;
$\Delta\bar{\mathbf{F}}$ and $\Delta\mathbf{u}$ are the incremental external force and displacement vectors.

The above incremental stiffness equation differs from the second-order elastic approach in the use of the elasto-plastic stiffness matrix instead of the linear stiffness matrix, $\mathbf{K}_L$, through a numerical integration approach in sampling element yielded stiffness. It captures the incremental load-versus-deflection response considering the second-order geometrical distortion, and traces the spread of plasticity.

The quasi-plastic zone method (Deierlein 1997) is a compromise between the plastic zone and the elastic plastic hinge methods. In this method, the fully plastic cross-section is calibrated to the plastic zone solution. A simplified residual stress pattern is used and the spread of plasticity is considered by the flexibility coefficients. This method is restricted to two-dimensional problems.

In the plastic hinge method (White and Chen 1993), yielding is concentrated at a small zone modeled by a flexible spring (zero length plastic hinges, no spread of yielding through the cross-section, or along the length). When no yielding occurs, the spring stiffness is infinitely large and, when the plastic moment capacity is reached, the spring stiffness drops to zero. This process can be formulated as an inclusion of a flexible spring stiffness in the incremental equilibrium equation:

$$\left[\mathbf{K}_L + \mathbf{K}_G + \mathbf{K}_0 - \mathbf{K}_{spring}^{-1}\right]\left[\Delta\lambda.\Delta\bar{\mathbf{F}}\right] = \Delta\mathbf{u} \qquad (2.3)$$

where $\mathbf{K}_{spring}$ is the spring stiffness representing the plastic hinges.

Note that when the spring stiffness is infinite, it has no influence on the stiffness computation. When the spring stiffness vanishes due to material yielding, it indicates a smaller or a diminishing stiffness at the associated degree of freedom which, in some case, refers to a plastic collapse mechanism.

It can also be noted that the linear element stiffness by itself does not consider material yielding so that $\mathbf{K}_L$ used is the same as in the second-order elastic analysis. Yielding is considered only at the plastic hinges which are modeled by spring elements $\mathbf{K}_{spring}$. Some researchers (Powell and Chen 1986; King *et al.* 1992; Chen and Chan 1994) have employed the method of tracing the equilibrium path of the structure and found the results to be in good agreement with the plastic zone method in most, but not all, problems. When the spread of plasticity is rather uniform along a member, the concentrated plastic hinge method may not truly reflect the behavior of the member. Nevertheless, it has been suggested that the accuracy of the method is sufficient for practical purposes.

The refined plastic hinge method has been proposed (Liew *et al.* 1993) as a step up from the elastic-plastic model for two dimensions, with the use rotational springs to model the connection flexibility. This method considers inelasticity indirectly by forces rather than strains. The tangent modulus E_t is used to describe the effect of the residual stresses.

The practical refined plastic hinge method (Kim *et al.* 1996) has been proposed to refine the model by calibrating with the AISC\LRFD empirical code equations. In this method, a separate modification of the tangent modulus E_t is imposed to consider the geometrical imperfections and the CRC tangent modulus model is used to allow the residual stresses to be considered separately.

Each of the above methods have their merits and limitations. With the advancement of computer technology, some of these methods are no longer attractive to engineers. However, they still play an important role in the development of the stability theory as well as its application to practical structural analysis problems.

II.1. Practical advanced analysis of planar steel trusses

Among the various types of non-linear analyses, the plastic hinge method appears to have sufficient accuracy for practical purposes. The improvements proposed by Liew *et al.* (1993) and Kim *et al.* (1996) have made the method even more atractive for general applications. The practical refined plastic hinge method incorporates an explicit imperfection modeling, an equivalent notional load modeling, and a reduced tangent modulus modeling. These are the background for a more specified range of structures, the planar trusses, which is implemented in this research. The chosen method can predict the strength of the truss system in addition to the strength of the individual members. Furthermore, the capacity check after the usual analysis step can be neglected and both the material and geometrical nonlinearities can be included in the analysis process.

II.1.1 The virtual work equation

Let us consider a truss element as shown in Figure 2.4. The behavior of the truss is generally nonlinear. However, the approach employed in most cases, due to its simplicity, is the linearization of the problem. In particular, all the pieces of curves can be considered as the pieces of straight lines. Similarly, by using the updated Lagrangian formulation, the curve of strain increments can be considered sufficiently small within each incremental step of the nonlinear analysis. By this manner, the virtual work equation for the truss element can be written as (Kim *et al.* 2001):

$$\int_V C_{ijkl} e_{kl} \delta e_{ij} dV + \int_V \tau_{ij} \delta \eta_{ij} dV + {}^1R = {}^2R \tag{2.4}$$

where C_{ijkl} is the incremental constitutive coefficients;

τ_{ij} is the initial axial stress;

e_{kl} are the linear parts of strain increment ε_{ij};

η_{ij} are the nonlinear parts of strain increment ε_{ij};

${}^1R$ and ${}^2R$ are the virtual works done by the external loads acting on the body at the current configuration ${}^2\mathrm{C}$ and the last calculated configuration ${}^1\mathrm{C}$, respectively.

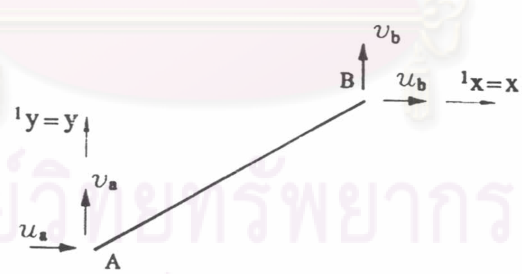

Figure 2.4 The planar truss element in the global coordinates.

II.1.2 The incremental constitutive law

In the field of elasticity, the relation between the axial stress S_{xx} and the axial strain e_{xx} is linear; stated by the Hooke's law $S_{xx} = Ee_{xx}$, where E denotes the modulus of elasticity. Similarly, the incremental constitutive law can be expressed as:

$$\dot{S}_{xx} = E_t \dot{e}_{xx} \tag{2.5}$$

where $\dot{S}_{xx}$ is the axial stress increment;

$\dot{e}_{xx}$ is the axial strain increment; and

E_t is the tangent modulus accounting for gradual yielding due to residual stresses.

By applying the relation in equation (2.5) to (2.4), we obtain:

$$\int_V E_t e_{xx} \delta e_{xx} dV + \int_V \tau_{xx} \delta \eta_{xx} dV + {}^1R = {}^2R \tag{2.6}$$

II.1.3 The derivation of displacements

Suppose the length of a truss element in Figure 2.4 is denoted as L, the displacements of the element at a specified distance x are:

$$u = u_a\left(1 - \frac{x}{L}\right) + u_b \frac{x}{L} \tag{2.7}$$

$$v = v_a\left(1 - \frac{x}{L}\right) + v_b \frac{x}{L} \tag{2.8}$$

where u is the horizontal displacement;
v is the vertical displacement;
u_A, v_A are the displacements at the end A of the element;
u_B, v_B are the displacements at the end B of the element.

As referred to in equation (2.4), the linear and nonlinear parts of the axial strain can be determined by differentiating the displacements:

$$e_{xx} = \frac{\partial u}{\partial x} = \frac{\Delta u}{L} \tag{2.9}$$

$$\eta_{xx} = \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] = \frac{1}{2}\left[\frac{\Delta u^2}{L^2} + \frac{\Delta v^2}{L^2}\right] \tag{2.10}$$

where $\Delta u = u_B - u_A$
$\Delta v = v_B - v_A$

II.1.4 The matrix-form expression

As shown in Figure 2.4, the truss element has four degrees of freedom, which form the nodal displacement vector as follows:

$$\mathbf{u} = \begin{Bmatrix} u_A \\ v_A \\ u_B \\ v_B \end{Bmatrix} \tag{2.11}$$

In the local coordinates, the transverse shear forces ${}^1F_{yA}$ and ${}^1F_{yB}$ are equal to zero while the axial forces ${}^1F_{xA}$ and ${}^1F_{xB}$ are equal in magnitude but opposite in direction. The upper-left superscripts denote the equilibrium status at a specified step. At the step-by-step equilibrium status, the forces vectors are:

$$ {}^{1}\mathbf{f} = \begin{Bmatrix} {}^{1}F_{xA} \\ 0 \\ -{}^{1}F_{xA} \\ 0 \end{Bmatrix} \quad \text{and} \quad {}^{2}\mathbf{f} = \begin{Bmatrix} {}^{2}F_{xA} \\ 0 \\ -{}^{2}F_{xA} \\ 0 \end{Bmatrix} \tag{2.12} $$

It is also noted that the initial axial force ${}^{1}F_{x}$ can be computed as the integration of τ_{xx} over the cross-sectional area A:

$$ {}^{1}F_{x} = \int_{A} \tau_{xx} dA \tag{2.13} $$

Assume that only the point loads are applied on the two ends of the truss element. The self weight is a distributed load that can be replaced by statically equivalent nodal loads. Therefore, we can represent each part of equation (2.6) in matrix form as follows:

$$ \int_{V} E_{t} e_{xx} \delta e_{xx} dV = \int_{V} E_{t} \frac{\Delta u}{L} \delta \frac{\Delta u}{L} dV = \boldsymbol{\delta}\mathbf{u}^{\mathrm{T}} \mathbf{K}_{e} \mathbf{u} \tag{2.14} $$

$$ \int_{V} \tau_{xx} \delta \eta_{xx} dV = \int_{0}^{L} {}^{1}F_{x} \left[\frac{\Delta u}{L} \delta\left(\frac{\Delta u}{L}\right) + \frac{\Delta v}{L} \delta\left(\frac{\Delta v}{L}\right) \right] dx = \boldsymbol{\delta}\mathbf{u}^{\mathrm{T}} \mathbf{K}_{g} \mathbf{u} \tag{2.15} $$

$$ {}^{1}R = \int_{S} t_{i} \delta u_{i} dS = \boldsymbol{\delta}\mathbf{u}^{\mathrm{T}}\, {}^{1}\mathbf{f} \tag{2.16} $$

where $\mathbf{K}_{e}$ and $\mathbf{K}_{g}$ are the inelastic and geometric local stiffness matrices, respectively:

$$ \mathbf{K}_{e} = \begin{bmatrix} \frac{E_t A}{L} & 0 & -\frac{E_t A}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{E_t A}{L} & 0 & \frac{E_t A}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.17} $$

$$ \mathbf{K}_{g} = \begin{bmatrix} \frac{{}^{1}F_{xB}}{L} & 0 & -\frac{{}^{1}F_{xB}}{L} & 0 \\ 0 & \frac{{}^{1}F_{xB}}{L} & 0 & -\frac{{}^{1}F_{xB}}{L} \\ -\frac{{}^{1}F_{xB}}{L} & 0 & \frac{{}^{1}F_{xB}}{L} & 0 \\ 0 & -\frac{{}^{1}F_{xB}}{L} & 0 & \frac{{}^{1}F_{xB}}{L} \end{bmatrix} \tag{2.18} $$

The governing equation for the planar truss element in its local coordinates can be written as:

$$ \left(\mathbf{K}_{e} + \mathbf{K}_{g}\right) \boldsymbol{\delta}\mathbf{u} + {}^{1}\mathbf{f} = {}^{2}\mathbf{f} \tag{2.19} $$

Or equivalently,

$$\left(\mathbf{K}_e + \mathbf{K}_g\right)\delta\mathbf{u} = {}^2\mathbf{f} - {}^1\mathbf{f} = \Delta\mathbf{f} \tag{2.20}$$

where the term ${}^1\mathbf{f}$ represents the initial force acting on the element at the equilibrium status ${}^1C$, the term ${}^2\mathbf{f}$ represents the total force acting on the element at the equilibrium status ${}^2C$, and the term $\Delta\mathbf{f}$ on the left-hand side of the above equation denotes the incremental force between the two configurations.

II.2. LRFD specifications

II.2.1 Loads and load combinations

The required strength of the structure and its elements must be determined from the appropriate critical combination of factored loads. The most critical effect may occur when one or more loads are not present. The following load combinations and the corresponding load factor shall be investigated (Chapter A):

$$\begin{aligned}
&1.4D \\
&1.2D + 1.6L + 0.5(L_r \text{, or } S \text{, or } R) \\
&1.2D + 1.6(L_r \text{, or } S \text{, or } R) + (0.5L \text{ or } 0.8W) \\
&1.2D + 1.3W + 0.5L + 0.5(L_r \text{, or } S \text{, or } R) \\
&1.2D \pm 1.0E + 0.5L + 0.2S \\
&0.9D \pm (1.3W \text{ or } 1.0E)
\end{aligned} \tag{2.21}$$

where D denotes the dead load due to the weight of the structural elements and the permanent features on the structure;
L denotes live load due to occupancy and movable equipment;
L_r is the roof live load;
W is the wind load;
S is the snow load;
E is the earthquake load; and
R is the rainwater/ice load.

For the current study only the dead load (i.e., the self weight of the truss) and the conventional live load are considered. A general combination of these two kinds of loads is proposed as follows:

$$F = \phi_D D + \phi_L L \tag{2.22}$$

where ϕ_D is the self-weight load factor. The default value is 1.2;
ϕ_L is the static point load factor. The default value is 1.6; and
F is the combined factored load.

There are many ways in which the loads can be applied to the structures. For the sake of simplicity, the proportional loading is employed herein. That is, the self weight and the static point loads are applied simultaneously. This scheme does not account for the cases in which the truss is subjected to sequential loading (e.g., the dead load first and the live load after), unloading, etc. The adopted loading scheme is, however, justified for the practical design since the development of the LRFD interaction equations was also based on strength curves subjected to simultaneous loading and the current LRFD elastic analysis uses the proportional loading rather than the sequential loading (Kim *et al.* 2001).

In practice, the process can be incrementally achieved by scaling down the combined factored loads by a number between 20 to 50. The lower bound is generally applied to highly redundant structures. The upper bound is recommended for nearly statically determinate structures, due to their higher tendency to sudden collapse.

II.2.2 Tension members

The design strength of tension members $\phi_t P_n$ should be lower value obtained according to the limit states of yielding in the gross section and fracture in the net section (Chapter D).

For yielding in the gross section:

$$\begin{aligned} \phi_t &= 0.90 \\ P_n &= F_y A_g \end{aligned} \tag{2.23}$$

For fracture in the net section:

$$\begin{aligned} \phi_t &= 0.75 \\ P_n &= F_u A_e \end{aligned} \tag{2.24}$$

where P_n is the nominal axial stress (N);
A_g is the gross area of member (mm^2);
F_y is the specified minimum yield stress (MPa); and
F_u is the specified minimum tensile strength (MPa).

These two values are the upper bounds of axial forces. For the sake of simplicity, only the yielding effect is considered for the tension members. However, with some technical modifications, the advanced analysis method could cover both yielding and fracture without any difficulties.

II.2.3 Compression members

The design strength for flexural buckling of compression members whose width/thickness ratio is less than λ_r from Section B5.1 is $\phi_c P_n$ (Chapter E):

$$\begin{aligned} \phi_c &= 0.85 \\ P_n &= A_g F_{cr} \end{aligned} \tag{2.25}$$

For $\lambda_c \leq 1.5$

$$F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y \tag{2.26}$$

For $\lambda_c > 1.5$

$$F_{cr} = \left(\frac{0.877}{\lambda_c^2}\right) F_y \tag{2.27}$$

where $\lambda_c = \dfrac{L}{\pi r}\sqrt{\dfrac{F_y}{E}}$ is the slenderness parameter;
E is the modulus of elasticity;

L is the lateral unbraced length of the member; and
r is the governing radius of gyration about the axis of buckling.

The equal single angles are very popular in the design of trusses, especially light trusses. This shape is investigated in the current study. Unfortunately, this is the kind of non-compact section in which the yield stress is unable to spread over the entire area of the compression member before buckling. In practice, it is often used in such a manner that rather large eccentricities of load applications are present. Three types of buckling are possible for this section: flexural buckling, local buckling of thin angle legs, and flexural torsional buckling. However, only the flexural buckling is considered in the current study. Adding more cases of buckling means adding more constraints to the problem, which does not significantly affect the mainstream of the approach.

II.3. The tangent modulus for compression members

II.3.1 LRFD tangent modulus

The column tangent modulus E_t can be evaluated based on the inelastic stiffness reduction procedure given in the LRFD manual for the calculation of inelastic column strength. The ratio of the tangent modulus E_t to the elastic modulus E is defined as (Chen 1995):

$$\frac{E_t}{E} = \begin{cases} 1.0 & for\ P_n \le 0.39 P_y \\ -2.7243 \dfrac{P}{P_y} \ln \left[\dfrac{P_n}{P_y} \right] & for\ P_n > 0.39 P_y \end{cases} \tag{2.28}$$

where $P_n = A_g F_{cr}$ and $P_y = A_g F_y$ are the critical load and the yield load, respectively.

Since this E_t model is derived from the LRFD column strength formula, it implicitly includes the effects of residual stresses and initial out-of-straightness in modeling the member effective stiffness.

II.3.2 CRC tangent modulus

The column tangent modulus E_t also can be evaluated based on the Column Research Council (CRC) column equation as follows (Chen and Lui 1992):

$$\frac{E_t}{E} = \begin{cases} 1.0 & for\ P_n \le 0.5 P_y \\ 4 \dfrac{P_n}{P_y} \left[1 - \dfrac{P_n}{P_y} \right] & for\ P_n > 0.5 P_y \end{cases} \tag{2.29}$$

The CRC tangent modulus concept is used to account for gradual yielding (due to residual stresses) along the length of axially loaded members between plastic hinges. The main difference between equations (2.28) and (2.29) is that the latter consider only the residual stress effects in modeling the column effective stiffness, whereas the former is based on LRFD column strength equations that account for the effects of both geometrical imperfections and residual stresses (Chen 1995).

II.3.3 Reduced CRC tangent modulus

To describe geometrical imperfections, three models have been proposed in the literature: explicit imperfection modeling, equivalent notional load procedure, and the reduced tangent stiffness method (Chen and Kim 1997). Since the first two approaches require explicit input of the imperfection, the concept of 'reduced tangent modulus' is proposed as a practical tool. Based on the CRC tangent modulus, a reduction of the tangent modulus E_t is done to account for the degradation of stiffness due to geometrical imperfection (Kim 1996):

$$\frac{E_t}{E} = \begin{cases} \xi & \text{for } P_n \le 0.5P_y \\ 4\xi \dfrac{P_n}{P_y}\left[1 - \dfrac{P_n}{P_y}\right] & \text{for } P_n > 0.5P_y \end{cases} \tag{2.30}$$

where ξ is the reduction factor, often used as 0.85.

Figure 2.5 illustrates the value of the tangent modulus from the three models. The current study employs the LRFD tangent modulus to present the nonlinear behavior of the material in the advanced analysis.

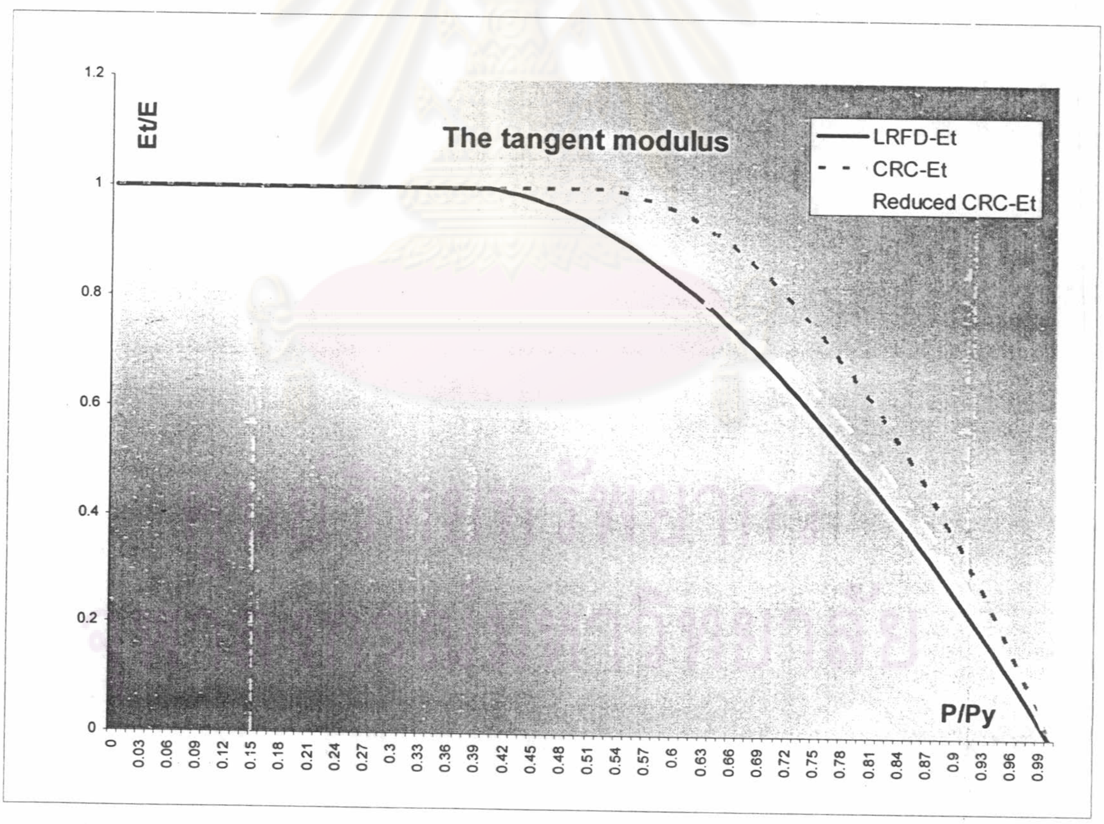

Figure 2.5 The tangent modulus in various models.

II.4. The algorithm

II.4.1 Input parameters

The problem consists of 21 parameters as described below

- The geometry data block includes the number of nodes, the nodal coordinates vector, the number of nodes per element, the number of elements, the nodal connectivity vector, the total system degrees of freedom, the number of degrees of freedom per element, and the element's length vector.

- The boundary condition data block includes the vector of constrained degrees of freedom, and the vector of its values (zeros by default). This scheme allows solving the problem with initial displacements.

- The material data block includes the Young's modulus, the minimum yield stress, and the specified material density.

- The load block data includes the vector of external point loads, the default value for the self weight (dead loads) and the default value for the live loads.

- The constraint data block contains only one parameter: the maximum allowable displacement. In the linear elastic analysis, the constraint of maximum allowable stress is used. However, in the nonlinear elastic-plastic analysis, the load ratios and their history at each step are considered instead.

- The section data block includes the list of available cross-sectional areas, the number of its articles, and the corresponding governing radii of gyration. The last parameter is transformed to a more practical variable, for ease of use when calculating the slenderness ratio.

II.4.2 Outputs

The outputs of the program are:

- The load coefficients for yielding/buckling of the first element and prior to yielding/buckling (the truss collapse) of the last element.
- The nodal displacements
- The history of load – deflection including the stresses after each step of load increment, the displacements after each step of load increment and the load coefficients after each successful step.

II.4.3 Termination criteria

There are two options to terminate the process. First, the analysis is terminated when the maximum displacement exceeds the limit. A default value is $\frac{L}{100}$ in which L is the span of the truss. The displacement limit can be adjusted manually if required. This option is suitable for the optimization process whereas the analysis is repeated many times. The second termination criterion is when the last member of the truss yields or buckles. At this stage, the truss is deemed to collapse and the entire behavior of the truss can be captured, which is one of the advantages of this approach compared with the linear elastic analysis.

II.4.4 The main flowchart

The global database, which consists of 21 parameters (see II.5.1), is registered and then input from a specified data file. The advantage here is that many problems can be stored in separate files that do not impact each other or the main program.

The list of available cross-sectional areas is set as an independent part in the data file. The list can be altered from one problem to another to best fit the various design purposes.

All input data are transformed to a unique style for ease of use. Due to the complexity of the program, the analysis by itself is a part of the optimization process, the data must be organized in such a way that they can be utilized by any sub-routine at any sub-level of the code.

The point loads that are declared in the data file are not the design loads. The program calculates the weight of the truss, and combines different sources of loads proportionally by refering to the dead load and the live load coefficients. The total load is divided into a very small amount to be applied upon the truss at each step of the analysis. By default, the total load is divided into 25 increments which the load ratio increment of 0.04. This value is one of the parameters which must be initially specified.

After each step of the analysis, the configuration of the truss is changed. Some checking must be performed to make sure that the truss is still able to sustain another increasing amount of load. In case the truss fails the checking criteria, the program will redo the analysis by decreasing the load ratio increment by half the value in the previous step. The table below summarizes the possibilities of the truss passing or failing the checking criteria.

Table 2.1 The possibilities of the truss passing or failing the check at each analysis step

Analysis step	Case 1			Case 2			Case 3		
	$\Delta\lambda$	Check	Redo	$\Delta\lambda$	Check	Redo	$\Delta\lambda$	Check	Redo
1	0.04	OK	0	0.04	OK	0	0.04	OK	0
…									
k	0.04	Failed	0	0.04	Failed	0	0.04	Failed	0
k + 1	0.02	Failed	1	0.02	OK	1	0.02	OK	1
k + 2	0.01	Failed	2	0.02	Failed	1	0.02	Failed	1
k + 3	0.005	Failed	3	0.01	Failed	2	0.01	OK	2
k + 4	0.0025			0.005	Failed	3	0.01	Failed	2
k + 5				0.0025			0.005	Failed	3
k + 6							0.0025		

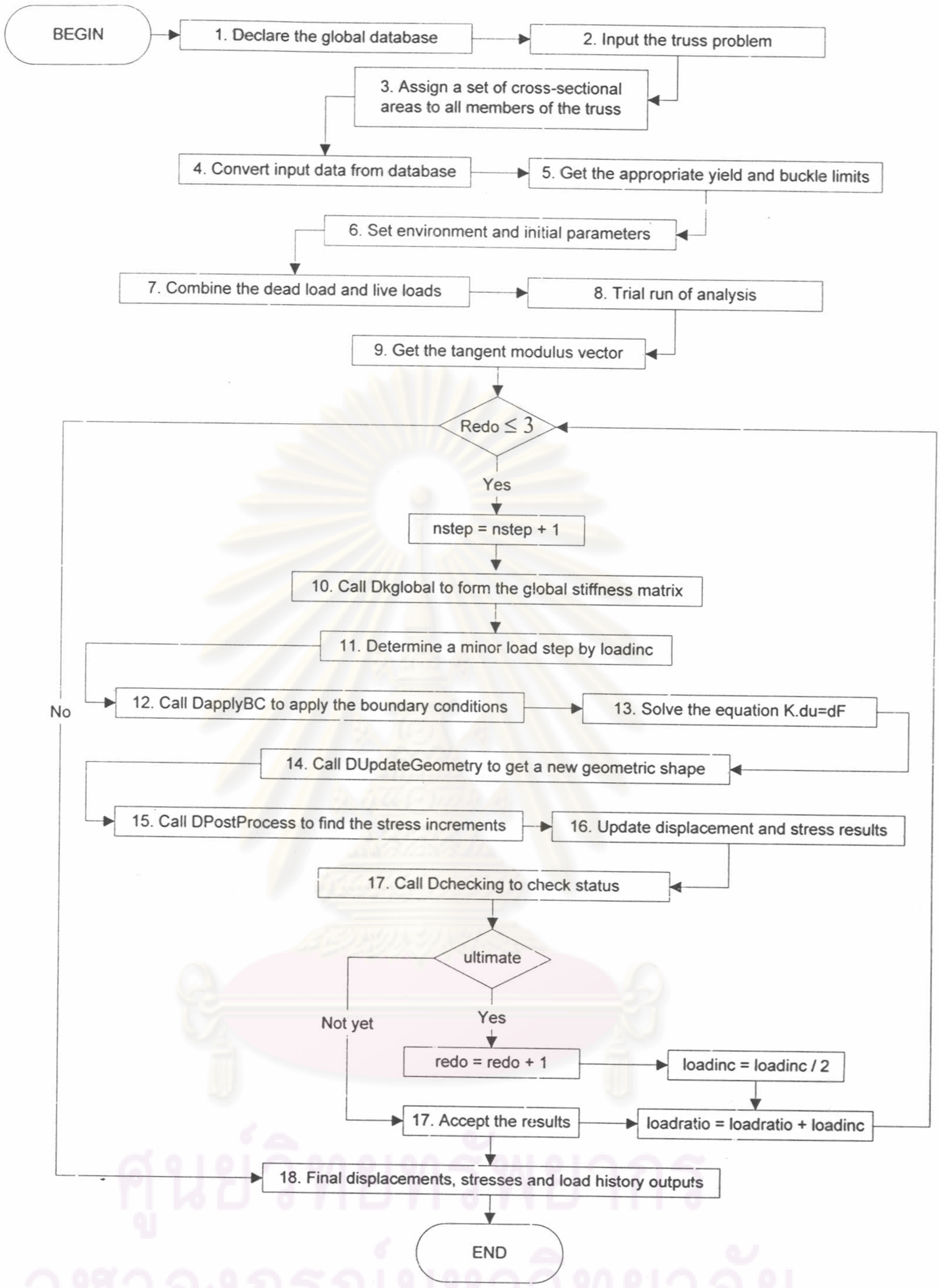

Figure 2.6 The nonlinear structural analysis flowchart

In case 1 of Table 2.1, the truss fails the checking criteria three consecutive times from step (k+1) to step (k+3) and the load ratio increment at step (k+4) is $\frac{1}{16}$ of the initial value. At this step, if the truss passes the checking criteria, one more analysis step would be tried and the final load ratio is $\lambda+\frac{\lambda}{16}$. On the other hand, if the truss fails the checking criteria at step (k+4), the maximum redo number of 4 would be reached and the process would be terminated with the load ratio of λ.

In case 2, the truss passes the checking criteria at step (k+1). At step (k+2) the truss is analyzed using a load ratio increment of 0.02. However, since the truss fails the checking criteria, the analysis is repeated at step (k+3), this time with a smaller load ratio increment of 0.01. For this case, whether the final load ratio is $\lambda+\frac{\lambda}{2}+\frac{\lambda}{16}$ or $\lambda+\frac{\lambda}{2}$ would depend upon the result in the last step.

Case 3 illustrates the scenario in which the truss passes the checking criteria at step (k+3) and thus the load ratio would be $\lambda+\frac{\lambda}{2}+\frac{\lambda}{4}+\frac{\lambda}{16}$ or $\lambda+\frac{\lambda}{2}+\frac{\lambda}{4}$ depending upon the result in the last step. Further, if the truss in case 3 passes the checking criteria at step (k+5), depending upon the final load ratio can either be $\lambda+\frac{\lambda}{2}+\frac{\lambda}{4}+\frac{\lambda}{8}+\frac{\lambda}{16}$ or $\lambda+\frac{\lambda}{2}+\frac{\lambda}{4}+\frac{\lambda}{8}$ depending upon the result in the last step.

In summary, the load ratio increment is divided in half every time the truss fails the checking criteria, with the maximum number of re-analysis steps of 3. The program is able to obtain the critical load ratio to $\frac{\lambda}{16}$ accuracy. In order to obtain a more precise value of the load ratio, one can manually decrease the load ratio increment or increase the number of re-analysis steps.

Each truss element is characterized by two stiffness matrices, the inelastic stiffness $\mathbf{K}_e$ and the geometric stiffness $\mathbf{K}_g$. These matrices must be combined prior to the global stiffness matrix assembly, after which the boundary condition is applied. Subsequently, the governing system of equations is solved to find the displacement increments. The geometrical shape of the truss is then updated and the axial stress increments are computed.

For the current program, a function Dchecking is used to validate the updated displacement and stress results. Dchecking examines a member whether the limit is reached and records the load ratio at which the member fails if necessary. Dchecking also scans all the members to check whether the failure criteria are reached in which case the truss would collapse.

II.4.5. The tangent modulus derivation

For the current study, the tangent modulus is derived using the following procedure.

Step 1: Compute $\lambda_c=\frac{L}{\pi r}\sqrt{\frac{F_y}{E}}$

or
$$\lambda_c=L\left(\frac{1}{\pi r}\sqrt{\frac{F_y}{E}}\right)=L(\lambda_0) \tag{2.31}$$

It is noted that λ_c depends on the geometrical and material parameters, this value is considered unchanged during the analysis.

Step 2: Based upon whether the value of the slenderness ratio is less than or equal to 1.5 or greater, the ratio of the critical load in equations (2.26) and (2.27) to the yield load in equation (2.23) is computed as

$$\frac{P_n}{P_y} = \frac{A_g F_{cr}}{A_g F_y} = \frac{F_{cr}}{F_y} = \begin{cases} 0.658^{\lambda_c^2} & for\ \lambda_c \leq 1.5 \\ \dfrac{0.877}{\lambda_c^2} & for\ \lambda_c > 1.5 \end{cases} \tag{2.32}$$

Step 3: The $\frac{P_n}{P_y}$ ratio is used to compute $\frac{E_t}{E}$ as follows

$$\frac{E_t}{E} = \begin{cases} 1 & for\ \dfrac{P_n}{P_y} \leq 0.39 \\ -2.7243 \dfrac{P_n}{P_y} \ln\left[\dfrac{P_n}{P_y}\right] & for\ \dfrac{P_n}{P_y} > 0.39 \end{cases} \tag{2.33}$$

For tension members, an elastic - perfectly plastic model is employed instead.

$$\frac{E_t}{E} = \begin{cases} 1.0 & for\ \sigma_{xx} < \phi_t F_y \\ 0.0 & for\ \sigma_{xx} \geq \phi_t F_y \end{cases} \tag{2.34}$$

where σ_{xx} is the axial stress of the tension member under consideration $(\sigma_{xx} > 0)$; and $\phi_t F_y$ is the maximum allowable axial stress $(\phi_t = 0.9)$.

In order to identify whether which truss member is in tension and compression, to apply an appropriate value of the tangent modulus, the first run of the analysis is a trial run. Based upon the trial run results, the proper value of E_t can be applied in the subsequent analysis steps.