Chapter 1

Introduction

1.1 Prologue

Nowadays, wavelets is a useful tool in many areas, including physics. The analysis using wavelets, the so-called wavelet analysis, is a representation of signal, or function, in both time and frequency (or position space and scale), while this is impossible for Fourier analysis, a well-known analysis with many usages in physics. In addition, the wavelets is a flexible tool; we can choose or construct wavelets that is appropriate for a particular situation of our interest.[1] These attractive properties motivate the physicists to investigate its implementations [5].

In this thesis, we review the works that use wavelets as a statistical analysis tool for detecting the non-Gaussian signatures in the cosmic microwave background (CMB) temperature anisotropy. A map of CMB anisotropy derived from the Wilkinson Microwave Anisotropy Probe satellite (WMAP)[2], the most recent all-sky data, is shown in Fig. 1.1. A linear transformation of a Gaussian random field yields a Gaussian random field in the linear transform space. For this reason, if the CMB anisotropy, as a spherical random field, is Gaussian, its wavelet transform is also Gaussian. On the contrary, non-Gaussian signature would be present in the wavelet space when the CMB anisotropy has non-Gaussianity , which contributes to deviations from Gaussian statistics. The search for non-Gaussian signatures in wavelet space is motivated from the fact that this task is more difficult in the pixel space (the spherical CMB anisotropy itself) or its spherical harmonic

[1]There are many good textbooks about the subject of wavelets: for example, Daubechies (1992) [1], Kaiser (1994) [2], Burrus et al. (1998) [3], and Chui (1992) [4], etc.

[2]The information of this project can be seen in http://map.gsfc.nasa.gov.

Figure 1.1: The all-sky CMB temperature anisotropy map from the Wilkinson Microwave Anisotropy Probe (WMAP), the NASA satellite, foreground-cleaned by Tegmark et al. [6]. The temperature is higher in more red region, the temperature lower in more blue region.

space, due to the central limit theorem of random field. Furthermore, there are several difficulties for the detection of non-Gaussianity in the CMB which will be discussed in this chapter.

This thesis is organized as follows. The following section explains what the CMB is. After that, we point out that its anisotropy is very important in modern cosmology, because it tells us about the initial condition for the formation of the cosmic structure. This information also helps us to discriminate the models of the early universe. Then, we discuss the search for non-Gaussianity, which is a hot subject in current cosmology, including its difficulties.

In Chapter 2, we start with the theoretical background on the cosmological perturbations which are taken to be generated during inflation in modern cosmology. Moreover, the standard model of inflation gives very nearly Gaussian CMB anisotropy .

In Chapter 3, we briefly explain the concepts and basic properties of the continuous wavelet transform and the continuous wavelet transform on the plane.

In Chapter 4, the works of Vielva et al. (2004) and Mukherjee and Wang

(2004) on the pursuits of non-Gaussianity using wavelet based method on the WMAP CMB data are reviewed. Since it is necessary to have the spherical wavelets for the spherical sky, the spherical Mexican hat wavelet is introduced. After the wavelet transformation, we describe some statistical tests on the wavelet space. The results show that some non-Gaussian signals can be extracted using this method. After that, we describe the nonlinear coupling parameter f_{Nl} constraint.

Finally, the Chapter 5 is a summary.

1.2 Cosmic Microwave Background

From the observations, the cosmic microwave background (CMB) is the background of blackbody radiations with the mean temperature T of 2.725 K from all directions in the sky. Its temperature is extremely isotropic; the temperature anisotropy $\Delta T/T$ has the tiny magnitude of 10^{-5}.[3]

We live in the universe where the expansion is of adiabatic type so that its temperature decreases as it expands; the temperature T is inversely proportional to the scale factor a,

$$T \propto \frac{1}{a}, \tag{1.1}$$

where the scale factor a is the factor that characterizes the size of the universe; any proper physical volume can be written as the multiplication of a^3 and the proper volume in comoving frame in which all points on 3-D spatial hypersurface have no relative motion whereas the universe is expanding. The scale factor a is introduced in order to describe the "universal" dynamics of the expanding universe (see appendix A). We define the Hubble expansion rate of the universe H as $H = \dot{a}/a$ where $\dot{a}$ is the derivative of a with respect to the cosmic time t. From the relation (1.1), we can imagine that the universe is hotter and smaller as the time goes backward.

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[3]The dipole anisotropy with the magnitude of order 10^{-3}, which is Doppler-shift effect due to the motion of the observer relative to the other comoving with the expansion of the universe, has been removed.

The CMB photons have travelled from the past when the universe was hotter and smaller. Before the so-called decoupling time, about 379000 years after the Big Bang (about 13.7 billion years ago) [7] when the size of the universe was about 1/1100 of the present size and $T \sim 3000$ K, the universe was hot enough so that most atoms could not be formed because enough photons had the energy more than the ionization energy of hydrogen atom $E_{ionize} = 13.6$ eV ($T \sim 1.58 \times 10^5 K$). This is due to the very high photons to baryons ratio $\sim 10^9$. As a result, the universe was fully occupied by a plasma consisting of free electrons and light nuclei, mostly protons, and the photons were tightly coupled to this plasma, via frequent Thomson scatterings from free electrons while free electrons interacted with protons by Coulomb interaction. Thomson scatterings of the photons with light nuclei were suppressed relative to those with electrons by a factor $m_e/m_p \sim 1/2000$. Consequently, they formed the photon-baryon fluid, where baryon is the term including both free electron and light nuclei.

Since the photon mean-free-path in this fluid was very small, the universe was opaque. The transition of the universe from being opaque to being transparent arose during the decoupling epoch. As the temperature decreased, the population of photons with the energy more than E_{ionize} decreased so that larger number of atoms can be formed. Cosmologists call this event the recombination. Since photons have no interaction with the neutral atoms, the mean-free-path of the photons would be larger and larger as the temperature decreases. When the number density of free electrons was very small due to the combination of them with the light nuclei to form atoms, the mean-free-path would grow to the Hubble radius. The CMB photons that we observe in the present have travelled from the surface of last scattering.

1.3 The CMB Temperature Anisotropy

Since the inhomogeneities both in the density of the photon-baryon fluid and in the gravitational potential, created in the early universe, at the last-scattering surface induced the temperature fluctuations of the released blackbody-photons, we observe them as the temperature anisotropies at the present. Although all photons have redshifted through the travel, the patterns of the inhomogeneities at the surface of last scattering have been preserved. In other words, the CMB temperature anisotropy map is the snapshot of the universe at decoupling. These anisotropies are the primary anisotropies.

The anisotropies of CMB temperature are created by several processes. We can classify them into two types: primary and secondary anisotropies. The primary anisotropies had already been formed at the last-scattering surface, while the secondary anisotropies were formed due to several processes, including gravitational effects from metric distortions and rescattering effects from reionization, during the travel of the CMB photons from the surface of last scattering to us [8]. Since the primary anisotropies were induced by the processes in the early epoch, they are very important in the early universe studies. Mainly, the primary anisotropies were formed by two sources: the primordial gravitational potential and the inhomogeneities resulted from the dynamics in the photon-baryon fluid.

To extract the information about the universe, the cosmologists find that the decomposition of CMB data into different angular scales is appropriate. This is because before the decoupling epoch the perturbations were small enough that their evolution had been well described by linear equation. Consequently, the perturbations in different scales had evolved independently. For a generic perturbation,

$$g(\mathbf{x},t) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\, g_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{1.2}$$

where $\mathbf{x}$ and $\mathbf{k}$ are the coordinates and the wavenumbers in comoving frame[4], and t is the cosmic time. The evolutions in each mode $\mathbf{k}$ can be simply described as [9]

$$g_{\mathbf{k}}(t) = T_g(t,k)\,\varphi_{\mathbf{k}}(t_*), \tag{1.3}$$

where $T_g(t,k)$ is called the transfer function; $k = |\mathbf{k}|$, which is determined by the cosmological model under consideration, and $\varphi_{\mathbf{k}}(t_*)$ is the primordial perturbations, created at an early time t_*, in the mode $\mathbf{k}$ determined by the model of early universe. Notice that the transfer function $T_g(t,k)$ is independent on the directions of $\mathbf{k}$ since the evolution equations are invariant under rotations. Given the

[4]The component i of $\mathbf{k}$: $k_i = 2\pi a/\lambda_i$, where λ_i is the wavelength in component i of perturbations

initial condition $\varphi_{\mathbf{k}}(t_*)$, we can obtain the perturbation with a particular mode $\mathbf{k}$ at time t by multiplying a value of $T_g(t,k)$. While the photons that form CMB today travelled through the space and time, the perturbations in each region of scale had obtained the evolutions from different physical processes. This is the reason why the angular power spectrum, the root-mean-squared amplitude depending on angular scales, is used to test cosmological models and fit cosmological parameters in current cosmology.

As pointed out in Page et al. [10], we may divide the temperature angular power spectrum into three regions of the angular scales depending on the dominant physical processes (see Fig. 1.2). (a) The region of angular scales $\theta > 2^o$ or, equivalently $l < 90$, which is larger than the angular scale ($\approx 2^o$) corresponding to the horizon size at decoupling.[5] In this large angular scale, or low l, the CMB temperature anisotropies imprint the unprocessed primordial perturbations through the Sachs-Wolfe effect [8] [11] which gives the temperature anisotropy in the direction in the sky $\hat{n}$

$$\left(\frac{\Delta T}{T}\right)_{\theta>2^o}(\hat{n}) = -\frac{1}{3}\,\Phi(\hat{n}), \tag{1.4}$$

where $\Phi(\hat{n})$ is the gravitational potential at last-scattering surface, which is induced from $\varphi(t_*)$. From Fig. 1.2, we can see the low-l anisotropy have the nearly scale-invariant power spectrum, the amplitude of temperature in different scales are nearly the same, which is predicted in the inflationary paradigm. (b) The region of $0.2^o < \theta < 2^o$, or $90 < l < 900$, where the characteristic of the angular power spectrum is the sequence of peaks, so-called the acoustic peaks (see Fig. 1.3). This can be explained by the behavior of the photon-baryon fluid under the presence of the gravitational potential before decoupling, more precisely, the propagation of sound wave in the fluid. The acoustic peaks is very useful for fitting several cosmological parameters. By fitting the curve using the CMBFAST [12] with the angular power spectrum from the observation of WMAP, for example, we obtain the contents of the universe at the present: 4 % of atoms, 23 % of cold dark matter, and 73 % of dark energy [7]. (c) The region of $\theta < 0.2^o$, the so-called Silk damping tail [8] [13] which is due to the imperfections of the photon-baryon fluid corresponding to shear viscosity and heat conduction in the fluid.

[5] The discussion of the horizon will be described in chapter 2.

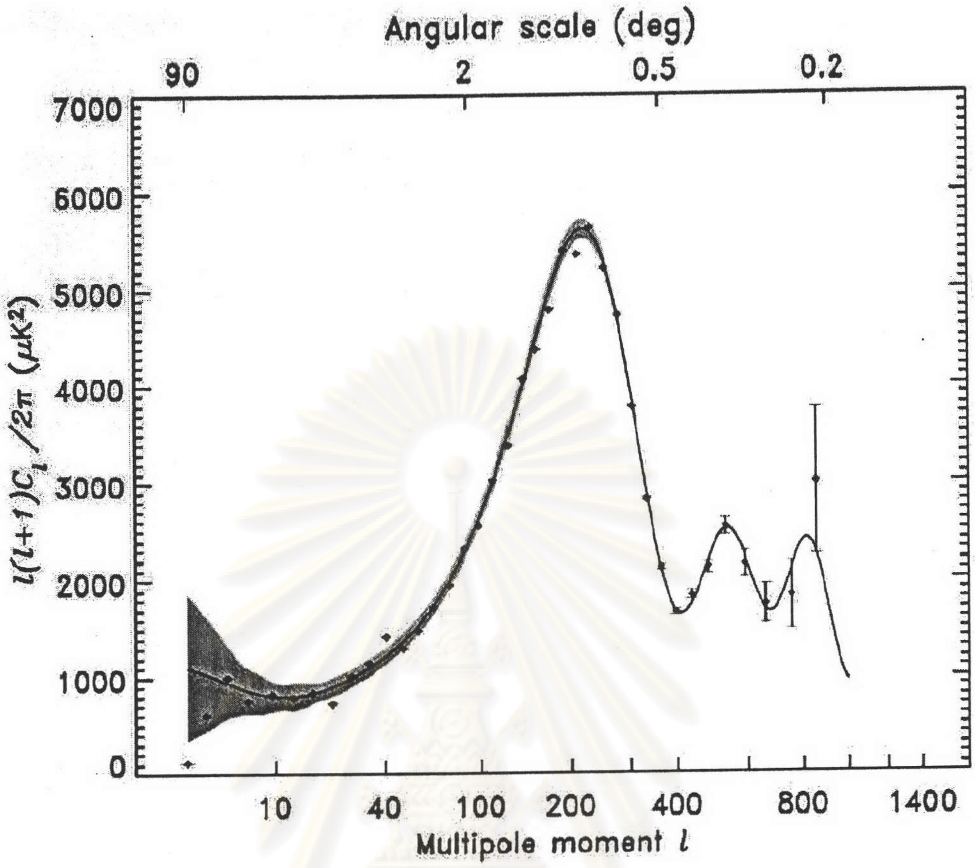

Figure 1.2: The CMB angular power spectrum, $l(l+1)C_l/2\pi$, measured on the WMAP first-year data. It is the quantity that represents the r.m.s. amplitude of CMB temperature anisotropies at a given angular scale, represented by the multipole moment l [14].

Hence, the CMB anisotropy is one of the most powerful tools in cosmology, in spite of its smallness. In particular, the analyses of the CMB anisotropy and large scale structure data have enabled us to fit the cosmological parameters with high accuracy. In addition, because it imprints the primordial perturbations, the "seeds" for cosmic structures such as galaxies, clusters of galaxy, and superclusters, etc., it is very useful for testing the models of the early universe. In modern cosmology, inflation has been established to be the reasonable model of the early universe. While all models of inflation suggest that the primordial perturbations were generated from quantum fluctuations during inflationary epoch, when the universe expanded accelerately, the different models predict different features of the perturbations. Furthermore, there are the models other than the inflationary paradigms possible to be either the alternatives for the creation of the cosmological perturbations or the mechanisms that partly contribute to the perturbations. Therefore, one purpose of analyzing CMB anisotropy data is to discriminate between these models for the creation of the perturbations.

1.4 The Search for Non-Gaussianity in CMB

There are a few ways to discriminate the models of the early universe using CMB anisotropy. According to the discussion in Bartolo et al. [15], the primordial non-Gaussianity, the deviation from a pure Gaussian statistics of the primordial perturbations that retains its signature on the CMB, is an observable being expected to be the discriminator between some models that cannot be disentangled by using only the other methods: namely, the detection of the B-mode of the CMB polarization which is a signal of the gravitational waves created during slow-roll inflation, and the measurement of the spectral index of comoving curvature perturbations. More precisely, different models predict different degrees of non-Gaussianity that can be used as a complement to other observables in the discrimination. Moreover, the cosmological perturbation theory up to second order predicts a possible additional, but only weak, non-Gaussianity. Although the non-Gaussianity studies, especially in theory, have just been paid more attention during about the last two years, it becomes a hope for uncovering more information about our universe.

For CMB anisotropy field $f(\hat{n}) \equiv (\Delta T/T)(\hat{n})$, the angular n-point correlation function is defined as $\langle f(\hat{n}_1)f(\hat{n}_2)\dots f(\hat{n}_n)\rangle$, where the bracket denotes the ensemble average.[6] If a perturbation field is Gaussian, its joint probability distribution is a multivariate Gaussian distribution so that it can be completely characterized by the two-point correlation function, i.e. any even higher correlation can be expressed as a sum of products of two-point correlations while all odd higher correlations are zero (see appendix B). In contrast, if it has a non-Gaussian signature, we find the presence of the extra terms in some higher correlations,

[6] The ensemble average is obtained by averaging over many universes. Obviously, it is impossible for us to obtain it because we have only one universe as one realization of the ensemble. We have to assume that the anisotropy field is ergodic so that the ensemble average can be reasonably replaced by the average over the whole sky.

i.e. we need higher-order correlation functions to specify the statistical properties. Hence, we need to have a complement to the angular power spectrum, the spherical harmonic transform of the angular two-point correlation function, of the CMB anisotropy for describing the statistical properties of the CMB sky. Note that the angular power spectrum is an observable of the CMB anisotropy which has been widely used in a variety of tasks in cosmology.

The pursuit of the primordial non-Gaussianity is a challenging work in current cosmology because of many reasons. One reason is that the non-Gaussianity seems very weak for detection in the CMB. Many observational tests conclude that the CMB anisotropy is consistent with being Gaussian to a high level. In addition, many models for the creation of primordial perturbations predict that the non-Gaussian signature is small [15]. Moreover, there are the contaminations from other sources of non-Gaussianity such as the foreground contamination, instrumental noise and secondary sources of anisotropies. The search for non-Gaussianity is difficult since the presence of non-Gaussian signature is possible in many ways and then a non-Gaussian signature may prefer to be present in one statistical method than in the others. This makes the conclusion of a Gaussian test complicated. The consistency with Gaussianity using one statistical method does not imply that CMB is really Gaussian. Moreover, some non-Gaussian signatures are due to the statistical fluctuations, called cosmic variance, because of the uniqueness of the observed CMB sky, i.e we have only one universe.