CHAPTER II

EKMAN THEORY ON WIND DRIFT

For the purpose of comparison , the Ekman theory on wind drift shall be briefly reviewed in this chapter.

It was Fridtjof Nansen[11] who first explained, in 1902, that the deviation of a drift current in the ocean from the direction of the wind is a consequence of the earth rotation. On Nansen's suggestion, Walfrid Ekman investigated the problem mathematically[12]. His analytical result, known after as Ekman theory, is widely accepted by physical oceanographers and is considered to be one of the most remarkable theoretical developments in dynamical oceanography[11].

Ekman theory on wind drift is based on the following assumptions.

1. The ocean is homogeneous.

2. The ocean is unbounded in the horizontal direction and indefinitely deep.

3. The wind field over the water is uniform of speed and direction.

4. The considered currents are stationary drift currents.

The drift current predicted by the theory is as given by the following equations[11].

$$u = V_o e^{-(\pi/D)z} \cos(45^\circ - \frac{\pi z}{D}) \qquad (2.1)$$

$$v = V_o e^{-(\pi/D)z} \sin(45^\circ - \frac{\pi z}{D}) \qquad (2.2)$$

where V_o = $\tau/A\ a$ is the velocity of the surface current,

τ = wind stress,

A = $0.1825 \times 10^{-4}\ w^{5/2}$ (w = wind velocity)[13]

a = $(2\Omega \sin\varphi \frac{\rho}{A_e})^{1/2}$

D = $\pi(A_e / \rho\Omega \sin\varphi)^{1/2}$

The drift current structure given by the above equations is true for the oceans in the northern hemisphere. In the southern hemisphere, where the latitude φ is negative, the direction of the y-axis must be reversed in order to obtain a real D[11].

From Eq. (2.1) and Eq. (2.2) it is seen that at the sea surface,

$$u = V_o \cos 45^\circ \qquad (2.3)$$

$$v = V_o \sin 45^\circ \qquad (2.4)$$

Hence the surface current vector points in a direction 45 degrees "cum sole" [2] to the wind diection.

At depth $z = D$, Eq. (2.1) and Eq. (2.2) become

$$u = V_o e^{-\pi} \cos (45^\circ - \pi) \qquad (2.5)$$

$$v = V_o e^{-\pi} \sin (45^\circ - \pi) \qquad (2.6)$$

Eq. (2.5) and Eq. (2.6) indicate that at this depth the current vector has decreased to exp(π) times the surface current vector, or to about 1/23 of the value at the surface. Moreover, the current direction has changed to be opposite to the surface current. The current vector between the surface and deeper layers, according to Eq. (2.1) and Eq. (2.2), turns continuously "cum sole" while the velocity decreases exponentially, such current vectors hence form a logarithmic spiral, the Ekman spiral, from the surface deep down the ocean[7].

Fig. 1-a and Fig. 1-b summarize the characteristics of the drift current in an infinitely deep ocean as predicted by the Ekman theory on wind drift.

In the case of an ocean with a finite depth d, velocity components of the drift current are more complex than Eq. (2.1) and Eq. (2.2). Using a new variable $\xi = d - z$, the velocity functions become[11]

$$u = \alpha \sinh \frac{a\xi}{\sqrt{2}} \cos \frac{a\xi}{\sqrt{2}} - \beta \cosh \frac{a\xi}{\sqrt{2}} \sin \frac{a\xi}{\sqrt{2}} \tag{2.7}$$

$$v = \alpha \cosh \frac{a\xi}{\sqrt{2}} \sin \frac{a\xi}{\sqrt{2}} + \beta \sinh \frac{a\xi}{\sqrt{2}} \cos \frac{a\xi}{\sqrt{2}} \tag{2.8}$$

where

$$\alpha = \frac{\tau D}{A\pi} \frac{\cosh \frac{ad}{\sqrt{2}} \cos \frac{ad}{\sqrt{2}} + \sinh \frac{ad}{\sqrt{2}} \sin \frac{ad}{\sqrt{2}}}{\cosh \frac{2ad}{\sqrt{2}} + \cos \frac{2ad}{\sqrt{2}}} \tag{2.9}$$

$$\beta = \frac{\tau D}{A\pi} \frac{\cosh \frac{ad}{\sqrt{2}} \cos \frac{ad}{\sqrt{2}} - \sinh \frac{ad}{\sqrt{2}} \sin \frac{ad}{\sqrt{2}}}{\cosh \frac{2ad}{\sqrt{2}} + \cos \frac{2ad}{\sqrt{2}}} \tag{2.10}$$

Fig. 2 shows velocity distribution at different levels below the sea surface obtained from Eq. (2.7) and Eq. (2.8). It should be noted that if d/D is close to 1, the deviation from the case of infinite depth is very small.