

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต 9 สาขาวิชาเคมี ภาควิชาเคมี คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

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เพ็ญกันยา อุ่นจิตต์ : การแลกเปลี่ยนของสปีชีส์คอปเปอร์ในโพรงแซดเอสเอ็ม-5 ที่ปริมาณ คอปเปอร์ต่ำ (EXCHANGE OF COPPER SPECIES IN THE CAVITY OF ZSM-5 AT LOW COPPER LOADING) อาจารย์ที่ปรึกษา: รศ. ดร. วุฒิชัย พาราสุข, 92 หน้า, ISBN 974-17-6080-9

ได้มีการคำนวณด้วยทฤษฎีเดนซิตัฟังก์ชันนัล เพื่อศึกษาการแลกเปลี่ยน $\mathrm{Cu}^{2+}$ และ $\mathrm{Cu}^{+}$และ การเกิดออโตรีดักชันของ $\mathrm{Cu}^{2+}$ ไปเป็น $\mathrm{Cu}^{+}$ในโพรง $\mathrm{ZSM}-5$ ที่บริเวณตำแหน่งแลกเปลี่ยน T 6 T 12 , T8T8, T7T12-model1 และ T7T12-model2 การหาโครงสร้างเสถียรทั้งหมดในการศึกษานี้หาด้วย วิธี B3LYP/ $6-31 \mathrm{G}(\mathrm{d}, \mathrm{p})$ เพื่อให้ได้มาซึ่งโครงสร้างของ $\mathrm{ZSM}-5$ ได้มีการทำการคำนวณด้วยวิธีโมเล กุลาร์ไดนามิกซิมุเลชันบนโครงสร้าง MFI ที่มีการแทนที่ Si 2 อะตอมด้วย Al 2 อะตอม และดุล ประจุลบที่เกินมาด้วย $\mathrm{H}^{+} 2$ อนุภาค แบบจำลองคลัสเตอร์ชนิดวง $5-$ และ $6-\mathrm{T}$ สร้างจากโครงสร้าง HZSM-5 ที่ได้จากคำนวณ โดยที่อะตอม O ที่ตำแหน่งปลายแบบจำลองคลัสเตอร์ ให้อิ่มตัวด้วย อะตอม H พารามิเตอร์ทางโครงสร้างทั้งหมดของแบบจำลองคลัสเตอร์จะถูกรักษาให้คงที่ ยกเว้น ตำแหน่ง $\mathrm{H}^{+}$พันธะ $\mathrm{O}-\mathrm{H}$ และ มุมพันธะ $\mathrm{T}-\mathrm{O}-\mathrm{H}(\angle \mathrm{T}-\mathrm{O}-\mathrm{H})$ ผลจากการหาโครงสร้างเสถียรได้ค่า $\mathrm{O}-$ H เป็น $0.96 \AA$ และ $\angle \mathrm{T}-\mathrm{O}-\mathrm{H}$ อยู่ระหว่าง $115.33^{\circ}-118.98^{\circ}$ สำหรับแบบจำลองคลัสเตอร์ของ CuZSM-5 สร้างจากแบบจำลองคลัสเตอร์ของ HZSM-5 โดยแทนที่ $\mathrm{H}^{+}$ด้วย $1 \mathrm{Cu}^{2+}$ หรือ $1 \mathrm{Cu}^{+}$ สำหรับ CuZSM-5 ที่มีการแลกเปลี่ยนด้วยอิออน 1 อิออน หรือ $2 \mathrm{Cu}^{+}$สำหรับ $\mathrm{CuZSM}-5$ ที่มีการ แลกเปลี่ยนด้วยอิออน 2 อิออน มีการหาตำแหน่งที่เสถียรเฉพาะตำแหน่ง $\mathrm{H}^{+}, \mathrm{Cu}^{+}$และ $\mathrm{Cu}^{2+}$ พบว่า ระยะ $\mathrm{O}-\mathrm{H}^{+}, \mathrm{Cu}^{2+}-\mathrm{O}$ และ $\mathrm{Cu}^{+}-\mathrm{O}$ มีระยะอยู่ระหว่าง $0.97-1.00 \AA, 1.92-4.17 \AA$ และ $1.75-5.33 \AA$ ตามลำดับ สำหรับบริเวณการแลกเปลี่ยนทั้งหมด พบว่าสปีชีส์ Cu เกิดทั้ง mono- และ bicoordination กับ $\mathrm{ZSM}-5$ เมื่อพิจารณาถึงพลังงานการแลกเปลี่ยน $\left(\Delta \mathrm{E}_{\text {exch }}\right)$ บริเวณการแลกเปลี่ยน ทั้งหมดมีพลังงานเป็นบวก สปีชีส์ Cu แลกเปลี่ยนที่เสถียรที่สุดคือ $\mathrm{Cu}^{2+}$ และบริเวณการแลกเปลี่ยน ที่ดีที่สุดคือ T 6 T 12 ธดยมีค่า $\Delta \mathrm{E}_{\text {exch }}$ เท่ากับ 9.19 กิโลเเคลอรี่ต่อโมล สำหรับการเกิดออโตรีดักชัน ของ $\mathrm{Cu}^{2+}$ ไปเป็น Cu พลังงานการเกิดออโตรีดักชัน $\left(\Delta \mathrm{E}_{\text {atto }}\right)$ ที่ต่ำสุด คือ 148,93 กิโลแคลอรี่ต่อ โมล โดยเกิดที่บริเวณการแลกเปลี่ยน T T $12-$ model แม่ว่า $\Delta \mathrm{E}_{\text {auto }}$ ของ $\mathrm{Cu}^{2+}$ ในโพ่รง $\mathrm{ZSM}-5$ ต่ำ กว่าในสภาวะสารละลาย แต่ก็ยังมีค่าสูงมาก จากค่า $\Delta \mathrm{E}_{\text {exch }}$ และ $\Delta \mathrm{E}_{\text {aut }}$ ที่ได้ การแลกเปลี่ยนอิออน Cu และ การเกิดออโต้รีดักชันของ $\mathrm{Cu}^{2+}$ ไม่น่าจะเกิดขึ้นได้ที่อุณหภูมิห้อง

ภาควิชา. $\qquad$ คคมี $\qquad$ ลายมือชื่อนิสิต.

สาขาวิชา $\qquad$ เคมี $\qquad$ ลายมือชื่ออาจารย์ที่ปรึกษา. $\qquad$
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KEYWORD: HZSM-5/ CuZSM-5/ Density Functional Theory/ Exchanged Energy/ Autoreduction Energy

PENGUNYA UNJIT: EXCHANGE OF COPPER SPECIES IN THE CAVITY OF ZSM-5 AT LOW COPPER LOADING. THESIS ADVISOR: ASSOC. PROF. VUDHICHAI PARASUK, Ph.D., 92 pp., ISBN 974-17-6080-9.

Density functional theory calculations were performed to examine exchange of $\mathrm{Cu}^{2+}$ and $\mathrm{Cu}^{+}$and autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in the cavity of ZSM-5 at T6T12, T8T8, T7T12model1, and T7T12-model2 exchanged site. All geometry optimizations were carried out using B3LYP/6-31G(d,p). To obtain ZSM-5 structures, Molecular Dynamic (MD) Simulations were performed on the MFI structure, in which 2 Si atoms were replaced by 2 Al atoms. Two excess negative charges were balanced by $2 \mathrm{H}^{+}$. The cluster models of 5 - and 6-T-ring were extracted from the simulated HZSM-5 structures where all terminal oxygen atoms were saturated with H atoms. Geometrical parameters of the cluster were fixed except the position of $\mathrm{H}^{+}$, O-H bond lengths ( $\mathrm{O}-\mathrm{H}$ ), and T-O-H bond angles ( $\angle \mathrm{T}-\mathrm{O}-\mathrm{H}$ ). The results from optimizations yielded O-H distances of $0.96 \AA$ and $\angle \mathrm{T}-\mathrm{O}-\mathrm{H}$ angles of $115.33^{\circ}-118.98^{\circ}$. For the cluster model of CuZSM-5, HZSM-5 structures were taken and $\mathrm{H}^{+}$were replaced either by $1 \mathrm{Cu}^{2+}$, or $1 \mathrm{Cu}^{+}$for one-ion exchanged CuZSM-5 or $2 \mathrm{Cu}^{+}$for two-ion exchanged CuZSM-5. Only positions of $\mathrm{H}^{+}, \mathrm{Cu}^{+}$, and $\mathrm{Cu}^{2+}$ were optimized. For $\mathrm{O}-\mathrm{H}^{+}, \mathrm{Cu}^{2+}-\mathrm{O}$, and $\mathrm{Cu}^{+}-$ O, the distances are between 0.97-1.00 $\AA, 1.92-4.17 \AA$, and $1.75-5.33 \AA$, respectively. For all exchanged sites, Cu species form either mono- or bi- coordination with ZSM-5. When considered exchanged energies ( $\Delta \mathrm{E}_{\text {exch }}$ ), all CuZSM-5 exchanged sites have positive $\Delta \mathrm{E}_{\text {exch }}$. The most stable exchanged Cu species is $\mathrm{Cu}^{2+}$ and the most favorable exchanged site is T6T12 with $\Delta \mathrm{E}_{\text {exch }}$ of $9.19 \mathrm{kcal} / \mathrm{mol}$. For the autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$, lowest autoreduction energy ( $\Delta \mathrm{E}_{\text {auto }}$ ) of $148.93 \mathrm{kcal} / \mathrm{mol}$ is for the conversion at T7T12-model1. Although $\Delta \mathrm{E}_{\text {auto }}$ in ZSM-5 is smaller than that in aqueous solution, still it is very high. From the obtained $\Delta \mathrm{E}_{\text {exch }}$ and $\Delta \mathrm{E}_{\text {auto }}$, the exchange of bare Cu ion and autoreduction from bare $\mathrm{Cu}^{2+}$ would not be preferable at room temperature.
Department......................................... Student's signature.........................................
Field of study...................................... Advisor's signature.........................................
$\qquad$

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5.17Combined $\Delta \mathrm{E}_{\text {exch }}$ and $\Delta \mathrm{E}_{\text {auto }}$ of $\mathrm{Cu}^{2+}$ in ZSM-5 at T6T12, T8T8, T7T12model1, and T7T12-model2 exchanged site.


## CHAPTER 1

## INTRODUCTION

### 1.1 Research Rationale

Zeolites are crystalline aluminosilicate mineral occurring naturally and synthetically. They have unique 3-D structures and possess excellent properties such as molecular sieves, ion exchangers, catalysts, etc. ${ }^{1-3}$

By potential versatility of the zeolites, for many decades, researchers have been inspiring to investigate and develop either aspect of improvement their properties, and of utilities. Recently, zeolites have been developed to use in many industries. For examples; in detergent industry, use of zeolites as builder, instead polyphosphates, to exchange $\mathrm{Ca}^{2+}$ and $\mathrm{Mg}^{2+}$ from washing water to prevent their precipitation by surfactant molecules; based upon a molecular sieving action, nparaffins can be separated from i-paraffins for use as feedstocks for detergent and chemical industry; in petrochemical industry, zeolites are used as catalysts in hydrocracking, isomerization, and alkylation. ${ }^{3}$ Usage of zeolites as catalysts is a major objective of their utilities and many researches in this aspect.

Because their high surface area, they indicate higher efficiency than other catalysts. Not only the zeolites are used as excellent catalysts in petrochemical industry, but also in environmental treatments. ${ }^{3}$ Example is the use of copperexchanged ZSM-5 (CuZSM-5) as catalyst for decomposition of nitrogen oxides $\left(\mathrm{NO}_{\mathrm{x}}\right)$, which was discovered by M. Iwamoto.

## T ${\text { The decomposition of } \mathrm{NO}^{\prime} \text { is very }}_{\sigma}^{\sim}$

The decomposition of $\mathrm{NO}_{\mathrm{x}}$ is very interest reaction. ${ }^{4-17} \mathrm{NO}_{\mathrm{x}}$ exhaust causes environmental damages in form of air pollution and acid rain. If living things receive $\mathrm{NO}_{\mathrm{x}}$ in high quantity and for long time, they can develop lung cancer and brain and nervous system damage.

The oxide forms of $\mathrm{NO}_{\mathrm{x}}\left(\mathrm{NO}, \mathrm{NO}_{2}, \mathrm{~N}_{2} \mathrm{O}\right)$ are the most difficult to eliminated from different combustion sources, such as, factories, power plants, and exhaust pipe of automobiles, etc. Recently, $\mathrm{NO}_{\mathrm{x}}$ can be removed in the selective catalytic reduction (SCR) process using ammonia on $\mathrm{TiO}_{2} / \mathrm{V}_{2} \mathrm{O}_{5}$ catalyst to convert $\mathrm{NO}_{\mathrm{x}}$ to $\mathrm{N}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ in the exhaust. ${ }^{13}$

$$
\begin{gather*}
4 \mathrm{NO}+4 \mathrm{NH}_{3}+\mathrm{O}_{2} \xrightarrow{\text { TiO } / V_{2} \mathrm{O}_{5}} 4 \mathrm{~N}_{2}+6 \mathrm{H}_{2} \mathrm{O}  \tag{2.1}\\
2 \mathrm{NO}_{2}+4 \mathrm{NH}_{3}+\mathrm{O}_{2} \xrightarrow{\mathrm{TiO}_{2} / V_{2} \mathrm{O}_{5}} 3 \mathrm{~N}_{2}+6 \mathrm{H}_{2} \mathrm{O}  \tag{2.2}\\
\mathrm{~N}_{2} \mathrm{O}+2 \mathrm{NH}_{3}+\mathrm{O}_{2} \xrightarrow{\mathrm{TiO}_{2} / \mathrm{V}_{2} \mathrm{O}_{5}} 2 \mathrm{~N}_{2}+3 \mathrm{H}_{2} \mathrm{O} \tag{2.3}
\end{gather*}
$$

These systems are high efficiency, but the key problem is the storage of $\mathrm{NH}_{3}$. Alternatively, CuZSM-5 catalyst can be used to convert $\mathrm{NO}_{\mathrm{x}}$ with the use of more environmentally benign reductant (light hydrocarbon) for indirect decomposition of $\mathrm{NO}_{\mathrm{x}}{ }^{14}$ or without reductant for direct decomposition of $\mathrm{NO}_{\mathrm{x}} .{ }^{15,16}$

CuZSM-5 can be prepared by exchanging with $\mathrm{Cu}^{2+}$ ion from copper chloride $\left(\mathrm{CuCl}_{2}\right)$ or copper acetate $\left(\mathrm{Cu}\left(\mathrm{CH}_{3} \mathrm{COO}\right)_{2}\right)$ or copper propionate $\left(\mathrm{Cu}\left(\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{COO}\right)_{2}\right)$ solutions. ${ }^{7}$ Then calcined at $400^{\circ} \mathrm{C} . \mathrm{Cu}^{+}$ions were found after the calcinations. Experiments also found that activity of CuZSM-5 depends on numbers of $\mathrm{Cu}^{+}$ions in ZSM-5. However, the characterization of copper active sites, the autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in the cavity of $\mathrm{ZSM}-5$ and the mechanism of $\mathrm{NO}_{\mathrm{x}}$ decomposition are still unknown. Goals of this study are to find preferable exchanged Cu species $\left(\mathrm{Cu}\right.$ as $\mathrm{Cu}^{2+}$ and $\mathrm{Cu}^{+}$) and exchanged sites in the cavity of ZSM-5, and the autoreduction process of $\mathrm{Cu}^{2+} \mathrm{ZSM}-5$ to $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at low copper loading.

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## CHAPTER 2

## BACKGROUND

### 2.1 Zeolites

Zeolite minerals were first discovered by Cronstedt in Sweden in $1756 .{ }^{1}$ Since then, about 50 zeolite natural species have been discovered, and more than 100 species have been synthesized in the laboratory. Examples of natural and synthetic zeolites are given in Fig. 2.1.


Fig. 2.1 An example of natural zeolite: (a) Mordenite (MOR)
An example synthetic zeolite: (b) Zeolite A (LTA). ${ }^{2}$
No are aluminosilicate minerals occurring naturally and synthetically. Their well-deefined structures are 3-D framework of microporous inorganic crystalline solids. Generally their framework consists of primary building unit of $\left[\mathrm{SiO}_{4}\right]^{4-}$ and $\left[\mathrm{AlO}_{4}\right]^{5-}$ tetrahedra, as shown in Fig.2.2. All of the oxygen atoms in the tetrahedral are mutually linked to enclose micropores running throughout the framework that lead to an open structures.


Fig. 2.2 The linking of primary building unit of $\left[\mathrm{SiO}_{4}\right]^{4-}$ and $\left[\mathrm{AlO}_{4}\right]^{5-}$ tetrahedra. ${ }^{1}$

The appearance of the micropores may be cages, channels and/or cavities with the diameter is about $1-20 \AA$ in which are filled with exchangable cations, a certain amount of water and/or other molecules.

### 2.1.1 Structures of Zeolites

As mentioned, all zeolites have three-dimensional framework constructed by joining together $\left[\mathrm{SiO}_{4}\right]^{4-}$ and $\left[\mathrm{AlO}_{4}\right]^{5-}$ coordination polyhedra. The substitution of $\mathrm{Al}^{3+}$ into some of $\mathrm{Si}^{4+}$ produces a deficiency in electrical charge that must be locally neutralized by the presence of an additional positive ion (monovalent and divalent cations, such as $\mathrm{Na}^{+}, \mathrm{Cu}^{+}, \mathrm{Ag}^{+}, \mathrm{Mg}^{2+}, \mathrm{Cu}^{2+}$, etc.) within the interstices of the framework. However, according to Lowensteins' rule, the linking between two Altetrahedral units is forbidden because repulsion between the two negative charges of $\equiv \mathrm{Al}-\mathrm{O}^{-}-\mathrm{Si} \equiv$ sites. Different types of zeolites result from difference in the way of linking of tetrahedral and from the types of difference ions that substitute within the interstices.
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The general formula for zeolites can be expressed as:
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where $\left[\left(\mathrm{AlO}_{2}\right)_{\mathrm{x}}^{-}-\left(\mathrm{SiO}_{2}\right)_{\mathrm{y}}\right]$ represents the framework atoms.
$\mathrm{M}^{n+}$ represents the cations M that $n=1,2$.
$x, y, z$ represent the number of Al-, Si-tetrahedral and water molecules, respectively.

The extent and location of the water molecules which held in the interstices of the zeolite framework depends on the size and shape of the cavities and channels present, and the number and nature of the cations in the structure.

The linking of primary building tetrahedral $(\mathrm{Si}, \mathrm{Al}) \mathrm{O}_{4}$ give rise to secondary building units (SBUs) which is important for the classification of zeolite structures. The main SBUs of zeolite are shown in Fig 2.3. These denote only the aluminosilicate skeleton and exclude consideration of the cations and water molecules within the channels and cavities of the framework.

(a)

(b)

(c)

(d)

(e)


Fig. 2.3 The secondary building units (SBUs) recognized in zeolite frameworks; (a) single four ring (S4R), (b) single six ring (S6R), (c) single eight ring (S8R), (d) double four ring (D4R), (e) double six ring (D6R), (f) complex 4-1, (g) complex 5-1,


The joining of the SBUs to create the infinite lattice occur the rings containing 8,10 or 12 linked tetrahedral, which are important to the zeolite structural features. For example, SBUs combines between $\beta$-cages in zeolite A are D4R.

The zeolite framework can be considered in term of large polyhedral building blocks forming characteristic cages, such as the truncated octahedron ( $\beta$-cages). The $\beta$-cages formed has an internal diameter of about $6 \AA$ which sufficient to encapsulate small molecules. Moreover, the combination of $\beta$-cages produce several zeolites (Fig. 2.4) with different size and shape of cavities which utilize to their application in industries.


Fig. 2.4 The combinations of $\beta$-cages produce several zeolites. ${ }^{1}$

### 2.2 Application of Zeolites

By characteristic architecture of zeolites, their three-dimensional framework comprises of a variety of internal channels and cavities with diameter are about 1-20 $\AA$ which enough to locate cation, water and other molecules. Therefore, they have the excellent properties, e.g., molecular sieve, ion exchanger, and catalyst. ${ }^{3}$ Consequently, both synthetic and natural zeolites can be applied into industries. The examples of zeolite applications as follows:

### 2.2.1 Zeolites as Molecular Sieves

The use of zeolites as "molecular sieve" is the first commercial success. The zeolites can selectively take up some molecules into cavities and rejecting others on the basis of their large effective molecular dimensions. Example of this is the use of zeolite A to recover linear hydrocarbons from mixtures with branched and cyclic hydrocarbons as shown in Fig. 2.5.


Fig. 2.5 Recovery of linear hydrocarbons from their mixtures by using zeolite A. ${ }^{3}$

### 2.2.2 Zeolites as Ion Exchangers

The use of zeolites as "ion exchangers" has been taken place to modify the catalytic on molecular sieving action of the parent zeolites. The cations held in their aluminosilicate anion framework are replaced by ion present in external solutions. Examples of this application are; the use of mixtures of zeolite A and zeolite X in the detergent industry as being the most effective builders to compensate the use of harmful chemical Sodium tri-polyphosphate (STPP) which can cause eutrophication of surface water with excessive formation of algae and subsequent oxygen deficiency,
the use of clinoptilolite and synthetic zeolite F in removing ammonium ion from waste water, etc.

### 2.2.3 Zeolites as Catalysts

Zeolites can be made as "catalysts" on the basis of their high surface areas. Since the last 40 years, the major employment of zeolites in this aspect is as acid cracking catalysts with over 99 \% of the world’s petro production from crude oils. In cracking process, zeolite X (fuajasite) is used to breakdown the long chain hydrocarbons where the major products of $\mathrm{C}_{1}-\mathrm{C}_{3}$ gas, gasoline, light cycle oil, heavy gas oil and recycle oil are obtained. The schematic of this process is shown in Fig. 2.6.


Fig. 2.6 Schematic of the cracking process that uses zeolites as catalyst. ${ }^{3}$

Several zeolites are used as shape-selective catalysts (Fig 2.7). Furthermore, ZSM-5 zeolite is used in dewaxing, benzene alkylation, and methanol to gasoline conversion (MTG) processes.
(a)

(b)

(c)


Fig. 2.7 Representation of (a) reactant shape selectivity in channel of zeolite A (rejection of branched chain hydrocarbons), (b) product shape selectivity in ZSM-5 channels (p-xylene diffuse preferentially out of the channels), (c) Transition state shape selectivity in mordenite channels (transition state B can be breakdown and diffuse out as 1, 2, 4-trimethylbenzene, but transition state A cannot). ${ }^{3}$

Besides the application of zeolites in the petrochemical aspect, they are also used as excellent catalyst in environmental treatments. ${ }^{3}$ For instance, exhaust gases (e.g., carbon monoxide (CO), nitrogen oxides $\left(\mathrm{NO}_{\mathrm{x}}\right)$, sulfur dioxide $\left(\mathrm{SO}_{2}\right)$, etc.) that cause of air pollution can be minimized by using zeolites as catalyst.
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### 2.3 ZSM-5

ZSM-5 is an important of the pentasil class of zeolites consists of a 3-D network of $\mathrm{SiO}_{4}$ and $\mathrm{AlO}_{4}$ tetrahedra. ${ }^{4}$ ZSM-5 is a highly siliceous material, one unit cell includes 96 tetrahedra and at most 8 of these can be Al. Tetrahedra are linked in such a way that they enclose channels and/or cavities. Their structure is very open with an internal volume of $0.17 \mathrm{ml} / \mathrm{g}$. or $30 \%$. The lattice density is $1.7 \mathrm{~g} / \mathrm{ml}$. There are two types of channels as the straight channels ( $5.1,5.6 \AA$ ) running perpendicular to sinusoidal channels (5.1, 5.4 $\AA$ ). ${ }^{5,6}$ The illustrations of ZSM-5 structure are shown in Fig 2.8a.


Fig. 2.8 (a) The straight channels and sinusoidal channels of ZSM-5
(b) $\mathrm{Si} / \mathrm{Al}$ tetrahedra comprise as 10 -membered ring. ${ }^{3}$

A channel intersection can be considered as a cavity that can be entered from 4 sides. Each entrance consists of a 10-membered ring. (Fig 2.8b) The compensating negative charges are in the form of cations located near Al.
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### 2.4 CuZSM-5 Zeolite Catalyst

CuZSM-5 is stood for copper ion-exchanged ZSM-5 zeolite. CuZSM-5 can be prepared by exchanging, Sodium ion $\left(\mathrm{Na}^{+}\right)$in the cavities of ZSM-5 (NaZSM-5) are exchanged with Cu from copper chloride $\left(\mathrm{CuCl}_{2}\right)$ or copper acetate $\left(\mathrm{Cu}\left(\mathrm{CH}_{3} \mathrm{COO}\right)_{2}\right)$ or copper propionate $\left(\mathrm{Cu}\left(\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{COO}\right)_{2}\right)$ solutions. ${ }^{7}$ Physical characterization of CuZSM- $5^{8-10}$ suggests that most of the copper is present as isolated cations. In freshly exchanged CuZSM-5, all of Cu is present as $\mathrm{Cu}^{2+}$ species. Presumably as in the framework of $\mathrm{ZSM}-5, \mathrm{Cu}^{2+}$ associated with pairs of $\equiv \mathrm{Al}-\mathrm{O}^{-}-\mathrm{Si} \equiv$ sites located in close proximity. Heating freshly CuZSM-5 results in $\mathrm{Cu}^{2+}$ reduce to $\mathrm{Cu}^{+}$cations. This process referred to as autoreduction. The locations of copper in channels produce active site which suitable as catalytic center for many reactions. ${ }^{11}$

### 2.5 Decomposition of Nitrogen Oxides $\left(\mathrm{NO}_{\mathrm{x}}\right)$ in CuZSM-5 Zeolite Catalyst

The oxide forms of nitrogen oxides ( $\mathrm{NO}, \mathrm{NO}_{2}, \mathrm{~N}_{2} \mathrm{O} \rightarrow \mathrm{NO}_{\mathrm{x}}$ ) are some of the most difficult to eliminated from combustion sources. Catalytic systems have been investigated to address this problem. ${ }^{12}$ Up to now, the most active commercial catalytic system is the selective catalytic reduction (SCR) using ammonia on $\mathrm{TiO}_{2} / \mathrm{V}_{2} \mathrm{O}_{5}$ catalyst to convert $\mathrm{NO}_{\mathrm{x}}$ to $\mathrm{N}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ in the exhaust. ${ }^{13}$

$$
\begin{align*}
& 4 \mathrm{NO}+4 \mathrm{NH}_{3}+\mathrm{O}_{2} \xrightarrow{\mathrm{TiO}_{2} / V_{2} \mathrm{O}_{5}} 4 \mathrm{~N}_{2}+6 \mathrm{H}_{2} \mathrm{O}  \tag{2.1}\\
& 2 \mathrm{NO}_{2}+4 \mathrm{NH}_{3}+\mathrm{O}_{2} \xrightarrow{\mathrm{TiO}_{2} / \mathrm{V}_{2} \mathrm{O}_{5}} 3 \mathrm{~N}_{2}+6 \mathrm{H}_{2} \mathrm{O}  \tag{2.2}\\
& \mathrm{~N}_{2} \mathrm{O}+2 \mathrm{NH}_{3}+\mathrm{O}_{2} \xrightarrow{\mathrm{TiO}_{2} / V_{2} \mathrm{O}_{5}} 2 \mathrm{~N}_{2}+3 \mathrm{H}_{2} \mathrm{O} \tag{2.3}
\end{align*}
$$

These systems are high efficiency, but the key problem is difficult of storing ammonia. In addition, if conversion is not complete, ammonia can be released to the atmosphere, which causes a problem worse than the release of $\mathrm{NO}_{x}$. Consequently, catalysts that will convert $\mathrm{NO}_{x}$ using a more readily available, more environmentally benign reductant are needed.

One of the most promising is a metal-exchanged form of ZSM-5 zeolite catalyst, particularly, CuZSM-5 which shows a significant ability to reduce $\mathrm{NO}_{x}$. It is generally accepted that both $\mathrm{Cu}^{2+}$ and $\mathrm{Cu}^{+}$species play an important role in $\mathrm{NO}_{\mathrm{x}}$ decomposition. ${ }^{8}$ Comparison of the activity of this catalyst with that of Cu exchanged into other zeolites and other metals exchanged into ZSM-5 indicates that the high activity of CuZSM-5 results from the combined properties of the metal cation and the zeolite.

However, CuZSM-5 catalyst has high selectivity for both indirect and direct decomposition of $\mathrm{NO}_{\mathrm{x}}$.

The indirect decomposition of $\mathrm{NO}_{\mathrm{x}}{ }^{14}$ uses light hydrocarbons as reducing agent in CuZSM-5 catalyst to convert $\mathrm{NO}_{\mathrm{x}}$ to $\mathrm{N}_{2}$ and $\mathrm{CO}_{2}$. For example; using CuZSM-5 is active above $300{ }^{\circ} \mathrm{C}$ with $\mathrm{C}_{3} \mathrm{H}_{6}$,

$$
\begin{equation*}
\mathrm{NO}_{\mathrm{x}}+\mathrm{C}_{3} \mathrm{H}_{6} \xrightarrow{\text { CuZSM-5 }} \mathrm{N}_{2}+\mathrm{CO}_{2} \tag{2.4}
\end{equation*}
$$

Direct decomposition of $\mathrm{NO}_{\mathrm{x}}{ }^{15-16}$ uses CuZSM-5 as catalyst to decompose $\mathrm{NO}_{\mathrm{x}}$ to $\mathrm{N}_{2}$ and $\mathrm{O}_{2}$ without any reducing agent. This reaction was discovered by Iwamoto et al. and co-worker in $1986{ }^{17}$ and can be expressed as

$$
\begin{equation*}
\mathrm{NO}_{\mathrm{x}} \xrightarrow{\text { CuZSM }-5} \mathrm{~N}_{2}+\mathrm{O}_{2} \tag{2.5}
\end{equation*}
$$

Although these systems have a high potential, a major disadvantage is that the performance of ZSM-5 under typical conditions for $\mathrm{NO}_{\mathrm{x}}$ removal is inadequate to meet expected requirement. High moisture content in the exhaust streams renders the catalyst inactive. Moreover, thermal instability of ZSM-5, a narrow range of temperatures associated with optimal performance, and the effects of engine operation leading to varying emission of hydrocarbons and nitrogen oxides preclude the ZSM-5 catalytic systems from commercial viability. However, the characterization of copper active sites, the autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in the cavity of ZSM-5 and the mechanism of $\mathrm{NO}_{\mathrm{x}}$ decomposition are still unknown, and should be investigated in order to increase possibility of development CuZSM-5 zeolite catalyst for $\mathrm{NO}_{\mathrm{x}}$ decomposition to commercial viability. However, the grasp of the Cu species in this catalyst should therefore be a first step toward understanding the reaction mechanism.

### 2.6 Literature Review

In experimental studies, presumably as, in freshly exchanged CuZSM-5, active sites are vicinity of $\mathrm{Cu}^{2+}$ species that associated with pairs of $\equiv \mathrm{Al}-\mathrm{O}^{-}-\mathrm{Si} \equiv$ sites located in close proximity. ${ }^{8}$

In theoretical studies, active site structures of CuZSM-5 were investigated by Molecular Dynamic (MD) Simulations and Molecular Orbital (MO) calculations. ${ }^{11}$ For MD simulations, 23 possible pairs of Al substitutions, which satisfy Lowenstein's rule, were determined. The results shown that coordination of $\mathrm{Cu}^{2+}$ at T 8 T 8 site in 6-T ring was the lowest energies of all considered models. For MO Calculations with LANL2MB basis sets were also performed on cluster models based on the simulated T8T8 site. The structure of $\mathrm{Cu}^{2+}$ at T8T8 site was calculated by MD and the cluster model was then optimized by MO calculations are shown in Fig.2.9.


Fig. 2.9 (a) The structure of $\mathrm{Cu}^{2+}$ at T 8 T 8 site was calculated by MD.
(b) The cluster model of $\mathrm{Cu}^{2+}$ at T8T8 site was optimized by MO. ${ }^{11}$

In addition to T8T8 site, other Al substitutions in 6-T ring were also considered as lowest energy group such as T11T11, T7T12 site. Moreover, T5T12, and T6T12 site in 5-T ring was considered as the second lowest energy group following the 6-T ring.

### 2.7 Goals of This study

In the present study, density functional calculation is used to determine the structures and exchanged energy of $\mathrm{Cu}^{2+}$ and $\mathrm{Cu}^{+}$species, which are exchanged in the cavity of ZSM-5 at low copper loading. Moreover, the autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$are investigated and compared between their progresses in aqueous solution and in the cavity of ZSM-5. All calculations have been performed in vicinity of cavity called " T site" that have most chance of attach of Cu species. ${ }^{11}$ These T sites are T6T12, T8T8, and T7T12.


Fig. 2.10 Vicinity of T sites that have most chance of attach of Cu species.

The results of this study may describe more explicitly to the autoreduction of Cu species, which bring to understand in $\mathrm{NO}_{\mathrm{x}}$ decomposition.

## CHAPTER 3

## QUANTUM CHEMICAL CALCULATIONS

### 3.1 Classical and Quantum Mechanic ${ }^{18-21}$

The most molecular behavior can be understood on the basis of one unified concept as "the Schrödinger equation". Energy and many properties of a stationary state of a molecule can be obtained by the solution of the Schrödinger equation.

$$
\begin{equation*}
\hat{H} \Psi=E \Psi \tag{3.1}
\end{equation*}
$$

where the Hamiltonian operator, $\hat{H}$, must be Hermitian. E is the eigenvalue representing the energy of state, and $\Psi$ is the wavefunction.

The classical Hamiltonian or the total energy of a single particle moving in one dimension is the sum of the kinetic energy ( T ) and the potential energy $(\mathrm{V}$ )

$$
\begin{equation*}
\mathrm{H}=\mathrm{E}=\mathrm{T}+\mathrm{V} \tag{3.6}
\end{equation*}
$$

$$
=\frac{p^{2}}{2 m}+V
$$

where p is momentum, m is mass.

In quantum mechanics, the electrons are characterized as wave-particle duality. Wavefunction are brought to better description of electronic behavior instead the classical equation. Energy of a molecule can be obtained by the solution of the Schrödinger wave equation. d 6 han? $9 / \mathrm{c}$ ?

### 3.2 The time-dependent Schrödinger equation ${ }^{18-21}$

In making the transition to a wave equation, physical variables in classical mechanic take the form of "operator", in (3.6), $\mathrm{p} \rightarrow \frac{\hbar}{\mathrm{i}} \frac{\partial}{\partial \mathrm{x}}$ and $\mathrm{x} \rightarrow \mathrm{x}$. Thus, the classical Hamiltonian can be rewritten in three dimensions as the Hamiltonian operator

$$
\begin{equation*}
\hat{\mathrm{H}}=-\frac{\hbar^{2}}{2 \mathrm{~m}}\left(\frac{\partial^{2}}{\partial \mathrm{x}^{2}}+\frac{\partial^{2}}{\partial \mathrm{y}^{2}}+\frac{\partial^{2}}{\partial \mathrm{z}^{2}}\right)+\mathrm{V} \tag{3.7}
\end{equation*}
$$

where $h$ is plank's constant, $\hbar$ is Dirac's constant representing $\mathrm{h} / 2 \pi$.

Thus, "the time-dependent Schrödinger equation" can be given as

$$
\begin{equation*}
\left(-\frac{\hbar^{2}}{2 m}\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}\right)+V\right) \Psi=\frac{\hbar}{i} \frac{\partial \Psi}{\partial t} \tag{3.8}
\end{equation*}
$$

or

$$
\begin{equation*}
\hat{\mathrm{H}} \Psi=\frac{\hbar}{\mathrm{i}} \frac{\partial \Psi}{\partial \mathrm{t}} \tag{3.9}
\end{equation*}
$$

### 3.3 The time-independent Schrödinger equation ${ }^{18-21}$

In the special circumstance where the Hamiltonian is independent of time, the wavefuntion in (3.9) is written as a product of a spatial and temporal part,


$$
\begin{equation*}
\hat{\mathrm{H}}\left(\psi(\mathrm{x}) \mathrm{e}^{-\mathrm{iE} t \hbar}\right)=\mathrm{i} \hbar \frac{\partial}{\partial \mathrm{t}}\left(\psi(\mathrm{x}) \mathrm{e}^{-\mathrm{iE} t \hbar}\right) \tag{3.11}
\end{equation*}
$$

Thus, the time-independent Schrödinger equation is given as

$$
\begin{equation*}
\hat{H} \psi=\mathrm{E} \psi \tag{3.12}
\end{equation*}
$$

For molecular system such as $\mathrm{H}_{2}$, the total Hamiltonian operator consists of kinetic ( $\hat{T}$ ) and potential ( $\hat{V}$ ) energy for all nuclei and electrons.


Fig. 3.1 A molecular coordinate system: i, j = electrons; $\mathrm{A}, \mathrm{B}=$ nuclei.

Thus,

$$
\begin{equation*}
\hat{\mathrm{H}}_{\mathrm{tot}}=\hat{\mathrm{T}}_{\mathrm{N}}+\hat{\mathrm{T}}_{\mathrm{e}}+\hat{\mathrm{V}}_{\mathrm{Ne}}+\hat{\mathrm{V}}_{\mathrm{ee}}+\hat{\mathrm{V}}_{\mathrm{NN}} \tag{3.13}
\end{equation*}
$$

in atomic unit (a.u.);

$$
\begin{equation*}
\hat{\mathrm{H}}_{\text {tot }}=-\sum_{\mathrm{A}=1}^{\mathrm{N}} \frac{1}{2 \mathrm{M}_{\mathrm{A}}} \nabla_{\mathrm{A}}^{2}-\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{2} \nabla_{i}^{2}-\sum_{\mathrm{A}=1}^{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{Z}_{\mathrm{A}}}{\mathrm{r}_{\mathrm{iA}}}+\sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}>\mathrm{i}}^{\mathrm{n}} \frac{1}{\mathrm{r}_{\mathrm{ij}}}+\sum_{\mathrm{A}}^{\mathrm{N}} \sum_{\mathrm{B}>\mathrm{A}}^{\mathrm{N}} \frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}}{\mathrm{R}_{\mathrm{AB}}} \tag{3.14}
\end{equation*}
$$

where the Laplacian operators $\nabla^{2}$ involve differentiation with respect to the coordinates of the particle in the system. $A$ and $B$ refer to nuclei with atomic numbers $Z_{A}$ and $Z_{B}$, respectively. $i$ and $j$ refer to electrons. $M$ is the ratio of the mass of nucleus to the mass of an electron. N is the number of nucleus and n is the number of electrons. The first and second term is the operator for the kinetic energy of the nuclei and electrons, respectively. The third term is the nuclear-electron attraction where $r_{i A}$ being the distance between electron $i$ and nucleus $A$. The fourth term is the electron-electron repulsion where $\mathrm{r}_{\mathrm{ij}}$ being the distance between electrons i and j . The last term is the nuclear-nuclear repulsion where $\mathrm{R}_{\mathrm{AB}}$ being the distance between nuclei A and B .

### 3.4 The Born-Oppenhiemer Approximation ${ }^{18-21}$

The Born-Oppenhiemer Approximation is applied to reduce the complexity of the Schrödinger equation. Since nuclei are much heavier than electrons, they move more slowly. Thus, the electrons in a molecule can be considered as moving in the field of fixed nuclei. The kinetic energy of the nuclei can be neglected and the nuclear-nuclear repulsion can be considered to be constants.

Thus, $\hat{\mathrm{H}}_{\text {tot }}$ in (3.13) is reduced as "electronic Hamiltonian operator", which can be expressed as

$$
\hat{H}^{\mathrm{el}}=\hat{\mathrm{T}}_{e}+\hat{V}_{\mathrm{Ne}}+\hat{\mathrm{V}}_{\mathrm{ee}}
$$

$$
\begin{equation*}
=-\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{2} \nabla_{\mathrm{i}}^{2}-\sum_{\mathrm{A}=1}^{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{Z}_{\mathrm{A}}}{\mathrm{r}_{\mathrm{iA}}}+\sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}>\mathrm{i}}^{\mathrm{n}} \frac{1}{\mathrm{r}_{\mathrm{ij}}} \tag{3.15}
\end{equation*}
$$

and "Electronic Schrödinger equation" can be given as

$$
\begin{equation*}
\hat{\mathrm{H}}^{\mathrm{el}} \Psi^{\mathrm{el}}=\mathrm{E}^{\text {eff }} \Psi^{\mathrm{el}} \tag{3.16}
\end{equation*}
$$

where $\Psi^{\text {el }}$ is the electronic wave function, and $E^{\text {eff }}$ is the effective electronic energy which depends on the relative nuclear coordinates, R.

The constant nuclear-nuclear repulsion has no effect on the operator eigenfunctions and only adds to the operator eigenvalue. They must also be included with electronic energy to obtain the total energy. $\mathbb{\square} \prod \Omega$

The deficiency of the Born-Oppenhiemer Approximation occurs when the energy of over a solution to Schrödinger equation comes close to each other. However, this approximation is usually very good and introduces only very small errors, an accuracy of $10^{-4}$ a.u.

### 3.5 Molecular Orbital Theory ${ }^{18-21}$

Molecular orbital theory is an approach, which uses one-electron functions or orbitals to approximate the full wavefunction. The molecular orbital, $\chi(\mathrm{x}, \mathrm{y}, \mathrm{z})$ is a function of the Cartesian coordinates of a single electron. $|\psi|^{2}$ is interpreted as the probability distribution of the electron in space. To describe the distribution of an electron completely, the dependence on the spin coordinates $(\omega)$ also has to be included. Values of the $\omega$ may be $+1 / 2$ or $-1 / 2$. In the presence of an external magnetic field, the spin aligned along the field corresponding to spin up, $\alpha(\omega)$ and the spin aligned opposite the field corresponding to spin down, $\beta(\omega)$. Thus,

$$
\begin{array}{ll}
\alpha(+1 / 2)=1 & \alpha(-1 / 2)=0 \\
\beta(+1 / 2)=0 & \beta(-1 / 2)=1 \tag{3.19}
\end{array}
$$

The complete wavefunction for a single electron is the product of a molecular orbital and a spin function, $\chi(\mathrm{r}) \alpha(\omega)$ or $\chi(\mathrm{r}) \beta(\omega)$

It is termed a spin orbital; $\chi(\mathrm{r}, \omega)$

### 3.6 The Hartree approximation ${ }^{18-21}$

"The Hartree approximation" proposed the simplest way to approximate an nelectron system, their wayefunction can be described as a product of spin orbitals which is called "Hartree product", $9 \% G \| 5 ?$

$$
\begin{equation*}
2)^{\Psi_{H P}\left(r_{1} s_{1}, r_{2} s_{2}, \ldots r_{n} s_{n}\right)=\frac{\sigma}{\sqrt{N}} \chi_{1}\left(r_{1} s_{1}\right) \chi_{2}\left(r_{2} s_{2}\right) \cdot e^{\chi_{n}}\left(r_{n} s_{n}\right)} \sigma_{6} \tag{3.20}
\end{equation*}
$$

where N is normalization constant.

However, the Hartree approximation does not account for exchange interactions since (3.20) does not satisfy anti symmetry principle.

### 3.7 The Hartree-Fock Approximation ${ }^{18-21}$

The exchange interaction can be rectified by "Hartree-Fock approximation", which the wavefunction is written as an antisymmetrized product of orbitals. Pauli Exclusion Principle state that "It is not possible for a molecular orbital to be occupied by two electrons of the same spin" is required to obtain antisymmetry property of wavefunction. The coordinates of electrons i and $j$ are interchanged, then, the product $\ldots \chi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{i}} \mathrm{s}_{\mathrm{i}}\right) \ldots \chi_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{j}} \mathrm{s}_{\mathrm{j}}\right) \ldots$ become $\ldots \chi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{j}} \mathrm{s}_{\mathrm{j}}\right) \ldots \chi_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{i}} \mathrm{s}_{\mathrm{i}}\right) \ldots$. Thus, the HartreeFock (HF) wavefunction amounts to a linear combination of the terms in (3.20);

$$
\begin{align*}
& \Psi_{\mathrm{HF}}\left(\mathrm{r}_{1} \mathrm{~s}_{1}, \mathrm{r}_{2} \mathrm{~s}_{2}, \ldots \mathrm{r}_{\mathrm{n}} \mathrm{~s}_{\mathrm{n}}\right) \\
& \quad=\frac{1}{\sqrt{\mathrm{~N}!}}\left[\chi_{1}\left(\mathrm{r}_{1} \mathrm{~s}_{1}\right) \chi_{2}\left(\mathrm{r}_{2} \mathrm{~s}_{2}\right) \ldots \chi_{\mathrm{n}}\left(\mathrm{r}_{\mathrm{n}} \mathrm{~s}_{\mathrm{n}}\right)-\chi_{1}\left(\mathrm{r}_{2} \mathrm{~s}_{2}\right) \chi_{2}\left(\mathrm{r}_{1} \mathrm{~s}_{1}\right) \ldots \chi_{\mathrm{n}}\left(\mathrm{r}_{\mathrm{n}} \mathrm{~s}_{\mathrm{n}}\right)+\ldots\right] \tag{3.21}
\end{align*}
$$

or

$$
\begin{equation*}
=\frac{1}{\sqrt{\mathrm{~N}!}} \sum_{p}(-1)^{p} \hat{P}\left[\chi_{1}\left(\mathrm{r}_{1} \mathrm{~s}_{1}\right) \chi_{2}\left(\mathrm{r}_{2} \mathrm{~s}_{2}\right) \ldots \chi_{\mathrm{n}}\left(\mathrm{r}_{\mathrm{n}} \mathrm{~s}_{\mathrm{n}}\right)\right] \tag{3.22}
\end{equation*}
$$

where $\hat{p}$ is a permutation operator, changing the coordinates $1,2, \ldots, n$ according to any of the $n$ ! possible permutations among the $n$ electrons. $(-1)^{p}$ is +1 or -1 for even and odd permutations.

To fulfil (3.22), the HF wavefunction can be efficiently represented as an $n \times n$ determinant, known as Slater determinant; ${ }^{20}$
where the orbitals are subject to the orthonormal constraint,

$$
\begin{equation*}
\int \psi_{\mathrm{i}}^{*}(\mathrm{r}) \psi_{\mathrm{j}}(\mathrm{r}) \mathrm{dr}=\left\langle\psi_{\mathrm{i}} \mid \psi_{\mathrm{j}}\right\rangle=\delta \tag{3.24}
\end{equation*}
$$

Thus, the Slater determinant can also be written as,

$$
\begin{equation*}
\Psi_{\mathrm{HF}}=\frac{1}{\sqrt{\mathrm{~N}!}}\left|\chi_{1}\left(r_{1}, s_{1}\right) \chi_{2}\left(r_{2}, s_{2}\right) \cdots \chi_{n}\left(r_{n}, s_{n}\right)\right\rangle \tag{3.25}
\end{equation*}
$$

The HF energy can be evaluated by taking the expectation value of the Hamiltonian or called Fock operator (3.15) with the Slater determinant,

$$
\begin{align*}
\mathrm{E}_{\mathrm{HF}}= & \left\langle\Psi_{\mathrm{HF}}\right| F\left|\Psi_{\mathrm{HF}}\right\rangle \\
= & \sum_{\mathrm{i}=1}^{\mathrm{n}} \int \chi_{\mathrm{i}}^{*}\left(\mathrm{r}_{\mathrm{i}}\right)\left(-\frac{1}{2} \nabla^{2}+\mathrm{v}_{\text {ext }}\left(\mathrm{r}_{\mathrm{i}}\right)\right) \chi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{i}}\right) \mathrm{dr} \mathrm{r}_{\mathrm{i}}+\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \iint \frac{\left|\chi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{i}}\right)\right|^{2}\left|\chi_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{j}}\right)\right|^{2}}{\left|\mathrm{r}_{\mathrm{i}}-\mathrm{r}_{\mathrm{j}}\right|} \mathrm{dr}_{\mathrm{i}} \mathrm{dr} r_{\mathrm{j}} \\
& -\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \iint \frac{\chi_{\mathrm{i}}^{*}\left(\mathrm{r}_{\mathrm{i}}\right) \chi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{j}}\right) \chi_{\mathrm{j}}^{*}\left(\mathrm{r}_{\mathrm{j}}\right) \chi_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{i}}\right)}{\mathrm{r}_{\mathrm{i}}-\mathrm{r}_{\mathrm{j}} \mid} \delta_{\mathrm{ij}} \mathrm{dr}_{\mathrm{i}} \mathrm{dr} r_{\mathrm{j}} \tag{3.26}
\end{align*}
$$

or

$$
\begin{align*}
\mathrm{E}_{\text {HF }} & \left.=\sum_{\mathrm{i}=1}^{\mathrm{n}}[\mathrm{i}|\mathrm{~h}| \mathrm{i}]+\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}}[\mathrm{ii} \mid \mathrm{j}]\right]-\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}}[\mathrm{i} \mid j \mathrm{j}] \\
& =\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{~h}_{\mathrm{ii}}+\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{~J}_{\mathrm{ij}}-\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{~K}_{\mathrm{ij}} \tag{3.27}
\end{align*}
$$

where $v_{\text {ext }}\left(r_{\mathrm{i}}\right)$ is the HF mean field potential, which two electron interaction is described as an everage one electron interaction, i.e. the interaction experienced by the $\mathrm{i}^{\text {th }}$ electron in the presence of all other electrons, $\mathrm{h}_{\mathrm{ii}}$ is one-electron operator; $\mathrm{J}_{\mathrm{ij}}$ is coulomb operator, and $\mathrm{K}_{\mathrm{ij}}$ is exchange operator.

According to the variational principle, the best spin orbitals are those that give the lowest possible HF energy. 19 g

For closed-shell restricted HF wavefunction, the spin orbitals are constrained to have the same spatial function for $\alpha$ and $\beta$ spin functions and the molecular states are allowed to have only an even number N of electrons, with all electrons paired such that $\mathrm{n}=\mathrm{N} / 2$ spatial orbitals are doubly occupied. Then,

$$
\begin{equation*}
\left|\Psi_{\mathrm{HF}}\right\rangle=\left|\chi_{1} \chi_{2} \chi_{3} \chi_{4} \cdots \chi_{\mathrm{n}-1} \chi_{\mathrm{n}}\right\rangle=\left|\psi_{1} \bar{\psi}_{1} \psi_{2} \bar{\psi}_{2} \cdots \psi_{\mathrm{n} / 2} \bar{\psi}_{\mathrm{n} / 2}\right\rangle \tag{3.28}
\end{equation*}
$$

where $\psi_{\mathrm{i}}$ represents the spatial orbital or molecular orbital (MO) while bar and unbar term represents $\beta$ spin and $\alpha$ spin, respectively.

Thus, the energy $\mathrm{E}_{\mathrm{HF}}$ can be given as

$$
\begin{align*}
\mathrm{E}_{\mathrm{HF}}= & 2 \sum_{\mathrm{i}}^{\mathrm{n} / 2} \int \psi_{\mathrm{i}}^{*}\left(\mathrm{r}_{\mathrm{i}}\right)\left(-\frac{1}{2} \nabla^{2}+\mathrm{v}_{\text {ext }}\left(\mathrm{r}_{\mathrm{i}}\right)\right) \psi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{i}}\right) \mathrm{dr} \mathrm{r}_{\mathrm{i}}+\sum_{\mathrm{i}=1}^{\mathrm{n} / 2} \sum_{\mathrm{j}=1}^{\mathrm{n} / 2} 2 \iint \frac{\left|\psi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{i}}\right)\right|^{2} \mid \psi_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{j}}\right)^{2}}{\left|\mathrm{r}_{\mathrm{i}}-\mathrm{r}_{\mathrm{j}}\right|} \mathrm{dr}_{\mathrm{i}} \mathrm{~d} \mathrm{r}_{\mathrm{j}} \\
& -\sum_{\mathrm{i}=1}^{\mathrm{n} / 2} \sum_{\mathrm{j}=1}^{\mathrm{n} / 2} \iint \frac{\psi_{\mathrm{i}}^{*}\left(\mathrm{r}_{\mathrm{i}}\right) \psi_{\mathrm{i}}\left(\mathrm{r}_{\mathrm{j}}\right) \psi_{\mathrm{j}}^{*}\left(\mathrm{r}_{\mathrm{j}}\right) \psi_{\mathrm{j}}\left(\mathrm{r}_{\mathrm{i}}\right)}{\left|\mathrm{r}_{\mathrm{i}}-\mathrm{r}_{\mathrm{j}}\right|} \delta_{\mathrm{ij}} \mathrm{dr}_{\mathrm{i}} \mathrm{dr} r_{\mathrm{j}} \\
= & 2 \sum_{\mathrm{i}=1}^{\mathrm{n} / 2}(\mathrm{i}|\mathrm{~h}| \mathrm{i})+\sum_{\mathrm{i}=1}^{\mathrm{n} / 2} \sum_{\mathrm{j}=1}^{\mathrm{n} / 2} 2(\mathrm{ii} \mid \mathrm{jj})-(\mathrm{ij} \mid \mathrm{ji}) \\
= & 2 \sum_{\mathrm{i}=1}^{\mathrm{n} / 2} \mathrm{~h}_{\mathrm{ii}}+\sum_{\mathrm{i}=1}^{\mathrm{n} / 2} \sum_{\mathrm{j}=1}^{\mathrm{n} / 2}\left(2 \mathrm{~J}_{\mathrm{ij}}-\mathrm{K}_{\mathrm{ij}}\right) \tag{3.29}
\end{align*}
$$

The last term is of significant interest since it arises from the antisymmetric nature of the HF wavefunction, it vanishes when $\mathrm{i}=\mathrm{j}$, which is an artifact of the Pauli principle. Consequently this term is called the exchange energy, $\mathrm{E}_{\mathrm{xc}}$. It should also be noted that in practice an extra term due to the repulsion energy between the ions must be added to (3.29) in order to obtain the total energy of the system.

In general the HF equations cannot be solved analytically. They are solved using an iterative process known as the self-consistent field procedure. ${ }^{22}$ Since the desired orbitals also make up their one-electron effective potential, the set of orbitals that give rise to the same after solving HF equation are known as the the self-consistent orbitals, and they are the groundstate orbitals for that system within the HF approximation. The self-consistent field procedure ${ }^{22}$ starts with an initial guess for the orbitals, and successive iterations are performed with new orbitals until the selfconsistent condition is achieved.

### 3.8 The Basis Set Approximation and Roothaan-Hall equation ${ }^{18-21}$

The partial molecular orbitals is expanded as linear combination of a set of known one-electron function $\varphi_{\mu}$, known as basis functions.

$$
\begin{equation*}
\psi_{\mathrm{i}}=\sum_{\mu=1}^{\mathrm{k}} \mathrm{C}_{\mu \mathrm{i}} \varphi_{\mu} \tag{3.30}
\end{equation*}
$$

The description of $\psi_{I}$ is improved by increasing the number of basis functions used. If $\varphi_{\mu}$ are taken to the atomic orbitals at the consistent atoms, (3.30) is called a linear combination of atomic orbitals (LCAO). The chosen basis functions should be easy to calculate and have the behavior, which agrees with the physics of the problem.

For small and highly symmetric systems, such as atoms and diatomic molecules, the Hartree-Fock equations may be solved numerically. This approach becomes impractical for large and lowly symmetric systems. Roothaan and Hall suggested the use of the unknown spatial basis functions in solving the complicated HF equation (3.29). The Roothaan-Hall equation is

$$
\begin{equation*}
\sum_{v=1}^{n}\left(F_{p v}-\varepsilon_{i} S_{\mu v}\right) C_{v i}=0 \quad, \mu=1,2, \ldots, n \tag{3.31}
\end{equation*}
$$

integrating the Roothaan-Hall equation (for closed shell system) yields.

$$
\begin{align*}
& \sum_{v} C_{v i} \int d r_{1} \varphi_{\mu}^{*}(1) f(1) \varphi_{v}(1)=\varepsilon_{i} \sum_{v} C_{v i} \int d r_{1} \varphi_{\mu}^{*}(1) \varphi_{v}(1)  \tag{3.32}\\
& \sum_{v} F_{u v} C_{v i}=\varepsilon_{i} \sum_{v} S_{\alpha v} C_{v i}^{c}, i=1,2, \ldots, k \tag{3.33}
\end{align*}
$$

F refers to the Fock matrix, which is the matrix representation of the Fock operator with the set of basis function $\left\{\varphi_{\mu}\right\}$,

$$
\begin{equation*}
F_{\mu v}=\int d r_{1} \varphi_{\mu}^{*}(1) f(1) \varphi_{v}(1) \tag{3.35}
\end{equation*}
$$

S refers to the overlap matrix, which contains the overlap elements between basis functions,

$$
\begin{equation*}
\mathrm{S}_{\mu v}=\int \mathrm{dr}_{1} \varphi_{\mu}^{*}(1) \varphi_{v}(1) \tag{3.36}
\end{equation*}
$$

C is a kx k matrix of the expansion coefficient $\mathrm{C}_{\mu \mathrm{i}}$

$$
C=\left(\begin{array}{cc|cc}
C_{11} & C_{12} & \cdots & C_{1 k}  \tag{3.37}\\
C_{21} & C_{22} & \cdots & C_{2 k} \\
\vdots & \vdots & & \vdots \\
C_{k 1} & C_{k 2} & \cdots & C_{k k}
\end{array}\right)
$$

and $\varepsilon$ is a diagonal matrix of the orbital energies $\varepsilon_{i}$

$$
\varepsilon=\left(\begin{array}{cccc}
\varepsilon_{1} & & & 0  \tag{3.38}\\
& \varepsilon_{2} & & \\
& \cdots & \ddots & \\
0 & & \cdots & \varepsilon_{k}
\end{array}\right)
$$

substitute the Fock operator into (3.35)

$$
\begin{align*}
\mathrm{F}_{\mu v} & =\int \mathrm{dr}_{1} \varphi_{\mu}^{*}(1) \mathrm{h}(1) \varphi_{v}(1)+\sum_{\mathrm{a}}^{\mathrm{N} / 2} \mathrm{dr}_{1} \varphi_{\mu}^{*}(1)\left[2 \mathrm{~J}_{\mathrm{a}}(1)-\mathrm{K}_{\mathrm{a}}(1)\right] \varphi_{v}(1) \\
& =\mathrm{H}_{\mu v}^{\text {core }}+\sum_{\mathrm{a}}^{\mathrm{N} / 2} 2(\mu v \mid \text { aa })-(\mu \mathrm{a} \mid \mathrm{av}) \tag{3.39}
\end{align*}
$$

The first term is a core-Hamiltonian matrix, which is integrals involving the oneelectron operator $\mathrm{h}(1)$ and the second term is the two-electron part. $\frac{e}{6}$

If the linear expansion of molecular orbitals is inserted into the two-electron term, then the Fock matrix is written as

$$
\begin{align*}
\mathrm{F}_{\mu v} & =\mathrm{H}_{\mu v}^{\text {core }}+\sum_{\mathrm{a}}^{\mathrm{N} / 2} \sum_{\lambda \sigma} \mathrm{C}_{\lambda \mathrm{a}} \mathrm{C}_{\sigma \sigma}^{*}[2(\mu 2 \mid \sigma \lambda)-(\mu \mu \mid \sigma v)] \\
& =\mathrm{H}_{\mu v}^{\text {core }}+\sum_{\lambda \sigma} \mathrm{P}_{\lambda \sigma}\left[\left(\mu(\mid \sigma \lambda)-\frac{1}{2}(\mu \mu \mid \sigma v)\right]\right. \\
& =\mathrm{H}_{\mu v}^{\text {core }}+\mathrm{G}_{\mu v} \tag{3.40}
\end{align*}
$$

where $G_{\mu \nu}$ is the two-electron part of the Fock matrix, which depends on the density matrix $P_{\lambda \sigma}$ and a set of two-electron integrals.

$$
\begin{equation*}
P_{\lambda \sigma}=2 \sum_{a}^{N / 2} C_{\lambda a} C_{\sigma a}^{*} \tag{3.41}
\end{equation*}
$$

Since the Fock matrix depends on the density matrix or, equivalently, on the expansion coefficients, the Roothaan-Hall equation is nonlinear and must be solved iteratively. The Roothaan-Hall equation in (3.34) is not the standard eigenvalue equation. To solve this equation, one must first transform to the eigenvalue equation. This can be done by normalizing the overlap matrix.

If $\left\{\varphi_{\mu}^{\prime}\right\}$ is the transformed set of $\left\{\varphi_{\mu}\right\}$ according to

$$
\begin{equation*}
\varphi_{\mu}^{\prime}=\sum_{v} X_{v \mu} \varphi_{v} \tag{3.42}
\end{equation*}
$$

where they form an orthonormal set,
then $\mathrm{S}^{\prime}$ (overlap matrix of the transformed set) is $\curvearrowleft$ then $S_{\text {Mv }}^{\prime}$ (overlap matrix of the transformed set) is 976

$$
\int \mathrm{dr} \varphi_{\mu}^{\prime *}(\mathrm{r}) \varphi_{v}^{\prime}(\mathrm{r})=\int \mathrm{dr}\left[\sum_{\lambda} \mathrm{X}_{\lambda \mu}^{*} \varphi_{\lambda}^{*}(\mathrm{r})\right]\left[\sum_{\sigma} \mathrm{X}_{\sigma v} \varphi_{\sigma}(\mathrm{r})\right]
$$

$$
=\sum_{\lambda} \sum_{\sigma} X_{\lambda \mu}^{*} \int \operatorname{dr} \varphi_{\lambda}^{*}(r) \varphi_{\sigma}(r) X_{\sigma v}
$$

$$
\begin{equation*}
=\sum_{\lambda} \sum_{\sigma} X_{\lambda \mu}^{*} S_{\lambda \sigma} X_{\sigma v}=\delta_{\mu v} \tag{3.44}
\end{equation*}
$$

or

$$
\begin{equation*}
X^{\prime} S X=1=S^{\prime} \tag{3.45}
\end{equation*}
$$

where $\mathbf{X}$ must be nonsingular and posses an inverse $\mathbf{X}^{-1}$.

Then

$$
\begin{equation*}
\mathbf{S}=\mathbf{X} \mathbf{S}^{\prime} \mathbf{X}^{-1}=\mathbf{X} \mathbf{S}^{\prime} \mathbf{X}^{\mathbf{t}} \tag{3.46}
\end{equation*}
$$

substituting eq. (3.46) into eq. (3.34), one obtains

$$
\begin{gather*}
\mathrm{FC}=\mathbf{X S}^{\prime} \mathbf{X}^{\mathbf{t}} \varepsilon \\
\mathbf{X}^{-1} \mathbf{F X}\left(\mathbf{X}^{-1} \mathbf{C}\right)=\mathbf{S}^{\prime}\left(\mathbf{X}^{-1} \mathbf{C}\right) \varepsilon \\
\mathbf{F}^{\prime} \mathbf{C}^{\prime}=\mathbf{C}^{\prime} \varepsilon \tag{3.47}
\end{gather*}
$$

The Roothaan-Hall equation cannot be solved analytically. The illustration of SCF procedure ${ }^{22}$ for solving the Roothaan-Hall equation is given in Fig. 3.2.


### 3.9 Basis Sets ${ }^{18-21}$

One of the approximations in solving the Schrödinger equation is the introduction of a basis set. An unknown molecular orbital (MO) are expanded in a set of known AO functions. A complete basis set contains an infinite number of functions, which is impossible in actual calculations. Finite atom-center basis sets are commonly used. Basis sets should be chosen as large as one is able to afford. However, for a basis set of M functions, about $\mathrm{M}^{4} / 8$ two-electron integrals $\left(\chi_{a} \chi_{b} \mid \chi_{c} \chi_{d}\right)$ are to be calculated and the type of basis functions used also influence the accuracy. Therefore, basis sets should be chosen as small as compatible with the required accuracy.

## Slater and Gaussian Type Orbitals

There are two types of basis functions commonly used in electronic structure calculations; Slater Type Orbitals (STOs) and Gaussian Type Orbitals (GTOs).

Slater type orbitals have the functional form:

$$
\begin{equation*}
\chi_{\zeta, \mathrm{n}, \mathrm{l}, \mathrm{~m}}(\mathrm{r}, \theta, \varphi)=\mathrm{NY} Y_{1, \mathrm{~m}}(\theta, \varphi) \mathrm{r}^{\mathrm{n}-1} \mathrm{e}^{-\zeta r} \tag{3.48}
\end{equation*}
$$

where $Y_{1, m}$ is the spherical harmonic functions,

$$
\begin{equation*}
\mathrm{Y}_{\mathrm{l}, \mathrm{~m}}(\theta, \varphi)=\Theta_{\mathrm{G}, \mathrm{~m}}(\theta) \Phi_{\mathrm{m}}(\varphi)=\sqrt{\frac{(2 l+1)(1-|\mathrm{m}|)!}{2(1+|\mathrm{m}|)!}} \mathrm{P}_{1}^{\mathrm{m} \mid}(\cos \theta) \cdot \frac{1}{\sqrt{2 \pi}} \cdot \mathrm{e}^{\mathrm{im} \mathrm{\varphi}} \tag{3.49}
\end{equation*}
$$

N is normalization constant, n is the principal quantum number and $\zeta$ is exponent of the radial function. The exponential depend on the distance between the nucleus and the electron similar to the exact orbitals for the hydrogen atom. However, two electron integrals of polyatomic molecules are difficult to calculate when three and four atomic centers a, b, c, d are involved. Consequently, STOs are suitable for atomic, diatomic systems and in semi-empirical methods where all three-and fourcenter integrals are neglected.

Gaussian type orbitals (GTOs) can be written in terms of spherical or Cartesian coordinates:

$$
\begin{gather*}
\chi_{\zeta, \mathrm{n}, \mathrm{l}, \mathrm{~m}}(\mathrm{r}, \theta, \varphi)=N Y_{1, \mathrm{~m}}(\theta, \varphi) \mathrm{r}^{(2 \mathrm{n}-2-1)} \mathrm{e}^{-\zeta \mathrm{r}^{2}}  \tag{3.50}\\
\chi_{\zeta, 1_{x}, l_{y, 1, z}}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{NX}^{1_{x}} \mathrm{y}^{1_{y}} \mathrm{z}^{1_{z}} \mathrm{e}^{-\zeta \mathrm{r}^{2}} \tag{3.51}
\end{gather*}
$$

where the type of orbital can be determined by the angular momentum $\left(1=l_{x}+l_{y}+l_{z}\right)$. The spherical GTOs can be formed by linear combination of cartesian GTOs of the same $l$ value. For $\mathrm{l}>1$, the number of spherical and cartesian GTOs differ, i.e. d-type GTOs $(\mathrm{l}=2)$ have five spherical GTOs, but there appear to be six cartesian GTOs ( $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}, \mathrm{xy}$, $\mathrm{xz}, \mathrm{yz}$ ), which may be transformed to the five spherical d-functions and one additional sfunction ( $\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}$ ). Similarly, f-type GTOs, ten cartesian GTOs may be transformed to the seven spherical f-functions and one set of spherical p-functions. The use of only the spherical components reduces the problems of linear dependence for large basis sets.

However, GTOs exhibit the wrong behavior near the nucleus ( $\mathrm{r} \approx 0$ ), the cusp, and for $\mathrm{r} \rightarrow \infty$, where the radial function is vanished too fast (Fig 3.3). Therefore, many more GTOs are required to achieve the same accuracy as with STOs. Because the faster calculation with four-center two-electron integrals GTOs are the most common basis functions used nowadays.

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Fig. 3.3 The approximation of STO by a set of several GTOs.

### 3.9.1 Minimal Basis Set

The minimal basis sets is the representation of a Slater type orbitals with $n$ Gaussian functions (STO-nG), which only enough functions are employed to contain all the electrons of the neutral atoms, one function per atomic orbital, i.e.
hydrogen and helium are represent by a single s-function (1-AO):

$$
\mathrm{H}-\mathrm{He}: 1 \mathrm{~s}
$$

the first row in the periodic table are represent by two s-functions and one set of p-


and the second row in the periodic table are represent by three s-functions and two sets of p-functions (9-AOs):

$$
\mathrm{Na}-\mathrm{Ar}: 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 2 \mathrm{p}_{\mathrm{x}}, 2 \mathrm{p}_{\mathrm{y}}, 2 \mathrm{p}_{\mathrm{z}}, 3 \mathrm{p}_{\mathrm{x}}, 3 \mathrm{p}_{\mathrm{y}}, 3 \mathrm{p}_{\mathrm{z}}
$$

The minimal basis sets are well known to accompany with several deficiencies. The atoms at the end of the period that have more number of electrons are described using
the same number of basis functions as the atoms at the beginning of the period. Furthermore, the radial exponents are not allowed to vary during the calculation. Consequently, the orbital cannot contract in size in accordance with the nature of molecules.

### 3.9.2 Extended Basis Set

To improve the minimal basis sets, the number of all basis functions must be increased. These basis functions are the linear combination of the contracted and diffuse functionals. The size of orbital can be modified during the course of calculation.

The double zeta (DZ) basis is described by double of all STOs minimal basis functions:
$\mathrm{H}-\mathrm{He}: 1 \mathrm{~s}$ and $1 \mathrm{~s}^{\prime}$
Li-Ne : 1 s and $1 \mathrm{~s}^{\prime}, 2 \mathrm{~s}$ and $2 \mathrm{~s}^{\prime}, 2 \mathrm{p}_{\mathrm{x}}, 2 \mathrm{p}_{\mathrm{y}}, 2 \mathrm{p}_{\mathrm{z}}$ and $2 \mathrm{p}_{\mathrm{x}}{ }^{\prime}, 2 \mathrm{py}^{\prime}{ }^{\prime}, 2 \mathrm{p}_{\mathrm{z}}{ }^{\prime}$
$\mathrm{Na}-\mathrm{Ar}: 1 \mathrm{~s}$ and $1 \mathrm{~s}^{\prime}, 2 \mathrm{~s}$ and $2 \mathrm{~s}^{\prime}, 3 \mathrm{~s}$ and $3 \mathrm{~s}^{\prime}, 2 \mathrm{p}_{\mathrm{x}}, 2 \mathrm{p}_{\mathrm{y}}, 2 \mathrm{p}_{\mathrm{z}}$ and $2 \mathrm{p}_{\mathrm{x}}{ }^{\prime}, 2 \mathrm{p}_{\mathrm{y}}{ }^{\prime}, 2 \mathrm{p}_{\mathrm{z}}{ }^{\prime}$, $3 \mathrm{p}_{\mathrm{x}}, 3 \mathrm{p}_{\mathrm{y}}, 3 \mathrm{p}_{\mathrm{z}}$ and $3 \mathrm{p}_{\mathrm{x}}, 3 \mathrm{p}_{\mathrm{y}}{ }^{\prime}, 3 \mathrm{p}_{\mathrm{z}}{ }^{\prime}$

Similarly, the triple zeta (TZ) basis is described by triple of all STOs minimal basis functions:
$\mathrm{H}-\mathrm{He}: 1 \mathrm{~s}, 1 \mathrm{~s}^{\prime}$ and $1 \mathrm{~s}^{\prime \prime}$


Double and triple zeta basis yields a better description of the charge distribution compare to minimal basis. Example of the charge distribution of HCN is given in Fig. 3.4.


Fig. 3.4 Charge distribution of HCN molecule.

Double and triple zeta basis has the flexiblity to describe the charge distribution in both to HCN molecule; C-H $\sigma$-bond consists of the H 1 s orbital and the $\mathrm{C} 2 \mathrm{P}_{\mathrm{z}}$, $\mathrm{C}-\mathrm{N} \pi$-bond is made up of C and $\mathrm{N} 2 \mathrm{P}_{\mathrm{x}}$ and $2 \mathrm{P}_{\mathrm{y}}$ AOs. The optimal AO coefficient (in MO expansion) of the tighter inner $\mathrm{P}_{\mathrm{z}}$ function on C will be larger in the $\mathrm{C}-\mathrm{H}$ bond. Whereas the more diffuse outer $\mathrm{P}_{\mathrm{x}}$ and $\mathrm{P}_{\mathrm{y}}$ function will have larger AO coefficient in the $\pi$-bond. An alternative to the double zeta basis approach is to double the number of functions used to describe the valence electrons but to keep a single function for the inner shells called "split valence double zeta basis".

For example, carbon 6-31G

Examples of other valence double zeta basis sets such as 3-21G, Dunning's DZV, etc. Furthermore, there are still "split valence triple zeta basis" such as 6-311G, Dunning basis TZV, etc.

### 3.9.3 Polarization Basis Set

The most common used polarization function comes from the Pople group. The polarization functions describe atomic orbital distortions due to bonding. In molecular system, it is clear that the influence of the other nucleus will distort or polarize the electron density near the nucleus, consequently, the orbitals have more flexible shapes than the orbital shapes in the free atoms. This is best accomplished by adding in basis functions of higher angular momentum quantum number. Thus, the spherical s-orbital can be distorted by mixing in orbital of p symmetry. The electron distribution along the bond will be different that perpendicular to the bond. The positive lope at one side increases the value of the orbital while the negative lobe at the other side decreases the orbital. Similarly, the p-orbitals can be polarized by mixing in an orbital of d symmetry. For example, Fig. 3.5 shows a polarization of 1 s -orbital is induced by $2 \mathrm{p}_{z}$-function for H atom in HCN.


Fig. $3.52 \mathrm{p}_{\mathrm{z}}$-function induces a polarization of 1 s -orbital for H atom in HCN.
The obtained results from adding polarization functions to these split valence basis sets are a significant improvement, particularly for accurately determination of bond angles. Examples of these basis sets are 6-31G(d), 6-31G(d,p), etc.

### 3.9.4 Diffuse Basis Set

In some cases, the normal basis functions are not adequate for calculation of dipole moment, polarizability, particularly, in excited states and in anions where the electron density is spread out all over the molecule. Thus, the orbitals can be more spread out by adding diffuse functions as single GTOs, not contracting them together. Examples of these basis sets are 6-31G+, 6-31G++(d, p) and 6-311G++, etc.

### 3.10 Density Functional Theory (DFT) ${ }^{23}$

Density functional theory (DFT) is the theory, which describe nature of electron base on the electron density distribution $\rho(\mathrm{r})$ instead of the many-electron wavefunction $\Psi\left(\mathrm{r}_{\mathrm{r}} \mathrm{r}_{2}, \mathrm{r}_{3}, \ldots\right)$. The electron density distribution $\rho(\mathrm{r})$ is the square of the wave function, integrated over $\mathrm{N}-1$ electron coordinates, this only depends on three coordinates, independently of the system size.

$$
\begin{equation*}
\rho(\overrightarrow{\mathrm{r}})=\left.\mathrm{N} \cdot \int \cdots \int \Psi\left(\overline{\mathrm{x}}_{1}, \overrightarrow{\mathrm{x}}_{2}, \ldots . \overrightarrow{\mathrm{x}}_{\mathrm{N}}\right)\right|^{2} \mathrm{ds}_{1} \mathrm{~d} \overrightarrow{\mathrm{x}}_{2} \ldots \mathrm{~d} \overrightarrow{\mathrm{x}}_{\mathrm{N}} \tag{3.52}
\end{equation*}
$$

where

$$
\begin{array}{r}
\rho(\bar{r} \rightarrow \infty)=0 \\
\int \rho(\bar{r}) d \vec{r}=N \tag{3.53}
\end{array}
$$

Since, the probability of finding any of the $N$ electrons within the volume element $d \vec{r}$ (with arbitrary spin) is determined.

DFT can be used to calculate all properties of molecules, clusters and solids, for examples, the ground state density $\rho(n)$, the full $N$-particle ground state wave function $\Psi\left(\mathrm{r}_{1}, \ldots, \mathrm{r}_{\mathrm{N}}\right)$, the electrical polarizability, vibrational force constant, energy, and potential energy surfaces for chemical reactions, etc. The goal of DFT method is to design functionals connecting the electron density with the energy.

Thomas-fermi proposed one of the earliest tractable schemes for solving the many-electron problem. ${ }^{24}$ They state that the electron density $\rho(\mathrm{r})$ is the central variable rather than the wavefunction, and the total energy of a system is written as a functional $E^{\mathrm{TF}}[\rho(\mathrm{r})]$. The thomas-Fermi energy functional is composed of three terms.

$$
\begin{equation*}
E^{T F}[\rho(r)]=A_{k} \int \rho\left(r_{i}\right)^{5 / 3} d r_{i}+\int \rho\left(r_{i}\right) v_{\text {ext }}\left(r_{i}\right) d r_{i}+\frac{1}{2} \iint \frac{\rho\left(r_{i}\right) \rho\left(r_{j}\right)}{\left|r_{i}-r_{j}\right|} d r_{i} d r_{j} \tag{3.54}
\end{equation*}
$$

where the first term is the electronic kinetic energy associated with a system of noninteracting electron in a homogeneous electron gas, which $\mathrm{A}_{\mathrm{k}}=\frac{3}{10}\left(3 \pi^{2}\right)^{2 / 3}$, the second term is the classical electrostatic kinetic energy association between the nuclei and the electrons, which $v_{\text {ext }}(r)$ is the static Coulomb potential arising from the nuclei,

$$
\begin{equation*}
\mathrm{v}_{\text {ext }}(\mathrm{r})=-\sum_{\mathrm{A}=1}^{\mathrm{N}} \frac{\mathrm{Z}_{\mathrm{A}}}{\left|\mathrm{r}-\mathrm{R}_{\mathrm{A}}\right|} \tag{3.55}
\end{equation*}
$$

and the third term represents electron-electron repulsion, known as the Hartree energy.

To obtain the groundstate density and energy of a system, (3.51) must be minimised subject to the constraint that the number of electrons is conserved, which can be performed using the technique of Lagrange multipliers. In general terms, the minimization of a functional, subject to the constraint, leads to the following stationary condition. Minimizing lead to the solution of the Thomas-Fermi equation,

$$
\begin{equation*}
\delta\left\{\mathrm{E}^{\mathrm{TF}}[\rho(\mathrm{r})]-\mu\left(\int \rho(\mathrm{r}) \mathrm{dr}-\mathrm{N}\right)\right\}=0 \tag{3.56}
\end{equation*}
$$

Thus, the so-called Thomas-Fermi equations can be written as

$$
\begin{equation*}
\frac{3}{5} A_{k} \rho\left(r_{i}\right)^{2 / 3}+v_{\text {ext }}\left(r_{i}\right)+\int \frac{\rho\left(r_{j}\right)}{\left|r_{\mathrm{i}}-r_{j}\right|} d r_{j}-\mu=0 \tag{3.57}
\end{equation*}
$$

where $\mu$ is Lagrange multipliers constant

However, Thomas-Fermi theory suffers from many deficiencies, probably the most serious defect is that it does not predict bonding between atoms so molecules and solids cannot form in this theory. The main source of error comes from approximating the kinetic energy in such a crude way. The kinetic energy represents a substantial portion of the total energy of a system and so even small errors prove disastrous. Another shortcoming is the over-simplified description of the electron-electron interactions, which are treated classically and so-do not take account of quantum phenomenon suchas the exchange interaction. d9:

Dirac ${ }^{24}$ developed an approximation for the exchange interaction based on the homogeneous electron gas. ${ }^{24}$ The formula is a local functional of the density,

$$
\begin{equation*}
\mathrm{E}_{\mathrm{x}}[\rho(\mathrm{r})]=-\frac{3}{4}\left(\frac{3}{\pi}\right)^{1 / 3} \int \rho(\mathrm{r})^{4 / 3} \mathrm{dr} \tag{3.58}
\end{equation*}
$$

(3.58) is usually written in terms of the exchange energy density,

$$
\begin{equation*}
\mathrm{E}_{\text {xc }}[\rho(\mathrm{r})]=\int \rho(\mathrm{r}) \varepsilon_{\text {xc }}[\rho(\mathrm{r})] \mathrm{dr} \tag{3.59}
\end{equation*}
$$

where $\left.\varepsilon_{\mathrm{xc}}\left[\alpha_{\mathrm{r}}\right)\right]$ can be given simply in terms of the seitz radius $\mathrm{r}_{\mathrm{s}}$,

$$
\begin{equation*}
\varepsilon_{\mathrm{xc}}[\rho(\mathrm{r})]=-\frac{3}{4}\left({\frac{9}{4 \pi^{2}}}^{1 / 3} \frac{1}{\mathrm{r}_{\mathrm{s}}}\right) \approx-\frac{0.4582}{\mathrm{r}_{\mathrm{s}}} \tag{3.60}
\end{equation*}
$$

The Dirac exchange term was incorporated into Thomas-Fermi theory. However, the inclusion of local exchange did not improve the Thomas-Fermi method.

Hohenberg and Kohn ${ }^{25}$ proved that an exact theory could be based on the density. They formally established the $\rho(\mathrm{r})$ as the central describing electron interactions, so devised the formally exact groundstate method known as density functional theory (DFT). The Hohenberg-Kohn theorems relate to any system consisting of electrons moving under the influence of an external potential, $\mathrm{v}_{\text {ext }}(\overline{\mathrm{r}})$, which the two Hohenberg-Khon theorems are now described.

## The first Hohenberg-Kohn theorem

The external potential, $\mathrm{V}_{\text {ext }}(\overline{\mathrm{r}})$, and hence the total energy, is a unique functional of the electron density $\rho(\mathrm{r})$,

$$
\begin{equation*}
\mathrm{v}_{\mathrm{ext}}(\overrightarrow{\mathrm{r}}) \Rightarrow \rho(\overrightarrow{\mathrm{r}}) \tag{3.61}
\end{equation*}
$$

The energy functional $\mathrm{E}[\rho(\mathrm{r})]$ can be written in terms of the external potential in the following way,

$$
\begin{align*}
& 9 \cap \text { б6 } \\
& \mathrm{E}[\rho(\mathrm{r})]=\int \rho(\mathrm{r}) \mathrm{v}_{\mathrm{ext}}(\mathrm{r}) \mathrm{dr}+\mathrm{F}[\rho(\mathrm{r})] \tag{3.62}
\end{align*}
$$

Where $\mathrm{F}[\rho(\mathrm{r})]$ is an unknown, but otherwise universal functional of the electron density only.

Minimizes the expectation value gives the ground state energy,

$$
\begin{equation*}
\mathrm{E}[\rho(\mathrm{r})]=\langle\Psi| \hat{H}|\Psi\rangle \tag{3.63}
\end{equation*}
$$

Hamiltonian for the system can be written as

$$
\begin{equation*}
\hat{\mathrm{H}}=\hat{\mathrm{F}}+\hat{\mathrm{V}} \tag{3.64}
\end{equation*}
$$

where F is the electronic Hamiltonian

To prove the first theorem, there are two different external potentials $\mathrm{v}_{\text {ext }}(\mathrm{r})$, and $\mathrm{v}_{\text {ext }}^{\prime}(\mathrm{r})$, that give rise to the same density $\rho(\mathrm{r})$. The associated Hamiltonians, H and $\mathrm{H}^{\prime}$, will have different groundstate wavefunctions, $\Psi$, and $\Psi^{\prime}$ that each yields $\rho(\mathrm{r})$.

$$
\begin{array}{ll}
\mathrm{H}=\mathrm{H}_{0}+\mathrm{v}_{\text {ext }}(\mathrm{r}) ; & H \Psi=E \Psi \\
\mathrm{H}^{\prime}=\mathrm{H}_{0}+\mathrm{v}_{\text {ext }}^{\prime}(\mathrm{r}) ; & \mathrm{H}^{\prime} \Psi^{\prime}=E \Psi^{\prime} \tag{3.66}
\end{array}
$$

Using the variational principle one can write the inequality,

$$
\begin{align*}
\mathrm{E}_{0}\left\langle\left\langle\Psi^{\prime}\right| \mathrm{H} \mid \Psi^{\prime}\right\rangle & =\left\langle\Psi^{\prime}\right| \mathrm{H}^{\prime}\left|\Psi^{\prime}\right\rangle+\left\langle\Psi^{\prime}\right| \mathrm{H}-\mathrm{H}^{\prime}\left|\Psi^{\prime}\right\rangle \\
& =\mathrm{E}_{0}^{\prime}+\int \rho(\mathrm{r})\left[\mathrm{v}_{\mathrm{ext}}(\mathrm{r})-\mathrm{v}_{\mathrm{ext}}^{\prime}(\mathrm{r})\right] \mathrm{dr} \tag{3.67}
\end{align*}
$$

It is at this point that the Hohenberg-Kohn theorems apply rigorously to the groundstate only. Adding the interchanged inequality to (3.63) leads to the result,

$$
\begin{align*}
& \text { ลถาบนวทยบรการ } \\
& \mathrm{E}_{0}+\mathrm{E}_{0}^{\prime}<\mathrm{E}_{0}^{\prime}+\mathrm{E}_{0} \tag{3.68}
\end{align*}
$$

It is concluded that there cannot beotwo different external potentials $\mathrm{v}_{\text {ext }}(\mathrm{r})$ and $\mathrm{v}_{\text {ext }}^{\prime}(\mathrm{r})$ that give the same density $\rho(\mathrm{r})$ for their ground states. Since the complete ground state energy $\mathrm{E}_{0}$ is a unique functional of the density $\rho(\mathrm{r})$, so must be its individual parts. One can write.

The second Hohenberg-Kohn theorem

The groundstate energy can be obtained variationally, the trial density that minimizes the total energy is the exact groundstate density,

To prove the second theorem; $\rho(\mathrm{r})$ determines $\mathrm{v}_{\text {ext }}(\mathrm{r})$; N and $\mathrm{v}_{\text {ext }}(\mathrm{r})$ determine H and $\Psi$. Thus, $\Psi$ is a functional of $\rho(\mathrm{r})$ and the expectation value of F is also a functional of $\rho(\mathrm{r})$,

$$
\begin{equation*}
\mathrm{F}[\rho(\mathrm{r})]=\langle\psi| \mathrm{F}|\psi\rangle \tag{3.69}
\end{equation*}
$$

If a density that is the ground-state of some external potential is known as $u$ representable, the $v$-representable energy functional can be defined in which the external potential is unrelated to another density $\rho^{\prime}(\mathrm{r})$,

$$
\begin{equation*}
\mathrm{E}_{v}[\rho(\mathrm{r})]=\int \rho^{\prime}(\mathrm{r}) \mathrm{V}_{\mathrm{ext}}(\mathrm{r}) \mathrm{dr}+\mathrm{F}\left[\rho^{\prime}(\mathrm{r})\right] \tag{3.70}
\end{equation*}
$$

Using the variational principle,

$$
\begin{equation*}
\left\langle\psi^{\prime}\right| \hat{F}\left|\psi^{\prime}\right\rangle+\left\langle\psi^{\prime} \hat{V}_{\text {ext }} \mid \psi^{\prime}\right\rangle>\langle\psi| \hat{F}|\psi\rangle+\langle\psi| \hat{\mathrm{v}}_{\text {ext }}|\psi\rangle \tag{3.71}
\end{equation*}
$$

This lead to,

$$
\begin{equation*}
\int \rho^{\prime}(\mathrm{r}) \mathrm{v}_{\text {ext }}(\mathrm{r}) \mathrm{dr}+\mathrm{F}\left[\rho^{\prime}(\mathrm{r})\right]>\int \rho(\mathrm{r}) \mathrm{v}_{\text {ext }}(\mathrm{r}) \mathrm{dr}+\mathrm{F}[\rho(\mathrm{r})] \tag{3.72}
\end{equation*}
$$

Thus, the second Hohenberg-Kohn theorem is obtained $\prod \int$

${ }^{9}$ Although the Hohenberg-Kohn theorems are extremely powerful, they do not offer a way of computing the groundstate density of a system in practice.

Kohn and Sham ${ }^{26}$ introduced the use of orbital for the Hohenberg-Kohn theorem.

For N -noninteracting system (no e-e repulsion), the Hamiltonian is given as

$$
\begin{align*}
\mathrm{H}_{\mathrm{s}} & =\sum_{\mathrm{i}}^{\mathrm{N}} \mathrm{~h}_{\mathrm{s}}(\mathrm{i})+\mathrm{v}_{\mathrm{s}}(\mathrm{r}) \\
& =\sum_{\mathrm{i}}^{\mathrm{N}}\left[-\frac{1}{2} \nabla_{\mathrm{i}}^{2}\right]+\mathrm{v}_{\mathrm{s}}(\mathrm{r}) \tag{3.74}
\end{align*}
$$

The non-interacting wavefunction $\Psi_{s}$ is

$$
\begin{equation*}
\Psi_{s}=\frac{1}{\sqrt{\mathrm{~N}!}}\left|\psi_{1} \psi_{2} \ldots \psi_{\mathrm{N}}\right\rangle \tag{3.75}
\end{equation*}
$$

Since

$$
\begin{equation*}
E_{0}[\rho]=\left\langle\Psi_{s}\right| \sum_{i}^{N}-\frac{1}{2} \nabla_{i}^{2}+v_{s}(r)\left|\Psi_{s}\right\rangle \tag{3.76}
\end{equation*}
$$

and

$$
=T_{s}[\rho]+V_{\mathrm{Ne}}[\rho]
$$

then for the N -interacting system (3.67) can be modified as
where the last term, $\mathrm{E}_{\mathrm{xc}}[\rho]$, is the exchange correlation functional which is

$$
\begin{equation*}
\mathrm{E}_{\mathrm{xc}}[\rho]=\mathrm{T}[\rho]-\mathrm{T}_{\mathrm{s}}[\rho]+\mathrm{V}_{\mathrm{ee}}[\rho]-\mathrm{J}[\rho] \tag{3.79}
\end{equation*}
$$

and $\mathrm{J}[\rho]$, the coulomb interaction, is

$$
\begin{equation*}
J[\rho]=\int \frac{\rho_{1}\left(\overrightarrow{\mathrm{r}}_{1}\right) \rho_{2}\left(\overrightarrow{\mathrm{r}}_{2}\right)}{\mathrm{r}_{12}} d \overrightarrow{\mathrm{r}}_{1} \mathrm{~d} \overrightarrow{\mathrm{r}}_{2} \tag{3.80}
\end{equation*}
$$

The density can also be written in terms of orbital as

$$
\begin{equation*}
\rho(\mathrm{r})=\sum_{\mathrm{i}}^{\mathrm{N}}\left|\Psi_{i}(r)\right|^{2} \tag{3.81}
\end{equation*}
$$

Similar to Hartree-Fock theory, the variational principle is applied, which lead finally to the self-consistent Kohn-Sham equations.

$$
\begin{equation*}
\hat{\mathrm{f}}^{\mathrm{KS}} \varphi_{\mathrm{i}}=\varepsilon_{i} \varphi_{\mathrm{i}} \tag{3.82}
\end{equation*}
$$

with the Kohn-Sham operator

$$
\begin{equation*}
\hat{\mathrm{f}}^{\mathrm{KS}}=-\frac{1}{2}+\int \frac{\rho\left(\mathrm{r}_{\mathrm{j}}\right)}{\mathrm{r}_{\mathrm{ij}}} d \mathrm{dr}_{\mathrm{j}}+\mathrm{v}_{\mathrm{Xc}}[\rho]+\sum_{\mathrm{A}} \frac{\mathrm{Z}_{\mathrm{A}}}{\mathrm{r}_{\mathrm{iA}}} \tag{3.83}
\end{equation*}
$$

The major problem in DFT is deriving suitable formulas for the exchangecorrelation term. Using the same algorithms as in Hartree-Fock theory, including the usage of basis functions and the self-consistent field (SCF) approach, can solve The Kohn-Sham equation. However, the Kohn-Sham orbitals ( $\varphi_{\mathrm{i}}$ ) are not the same as the Hartree-Fock orbitals ( $\varphi_{i}^{\mathrm{HF}}$ ), which implies that they also lack the physical interpretation of the Hartree-Fock one electron molecular orbitals.

### 3.10.1 The Local Density Approximation

The Local Density approximation (LDA) ${ }^{27,28}$ assumes that the density locally can be treated as a uniform electron gas. The density is a slowly varying function. The exchange correlation energy is given as
where $E_{x}[\rho]$ is accounting for the pure exchange, and $E_{c}[\rho]$ is the correlation part.

Local Spin Density Approximation is used to replace LDA where the spin densities are not the same,

$$
\begin{gather*}
\mathrm{E}_{\mathrm{x}}[\rho]=\mathrm{E}_{\mathrm{x}}^{\alpha}\left[\rho_{\alpha}\right]+\mathrm{E}_{\mathrm{x}}^{\beta}\left[\rho_{\beta}\right]  \tag{3.85}\\
\mathrm{E}_{\mathrm{c}}[\rho]=\mathrm{E}_{\mathrm{c}}^{\alpha \alpha}\left[\rho_{\alpha}\right]+\mathrm{E}_{\mathrm{c}}^{\beta \beta}\left[\rho_{\beta}\right]+\mathrm{E}_{\mathrm{c}}^{\alpha \beta}\left[\rho_{\alpha}, \rho_{\beta}\right] \tag{3.86}
\end{gather*}
$$

This may also be written in terms of the spin polarization, $\zeta$

$$
\begin{equation*}
\zeta=\frac{\rho^{\alpha}-\rho^{\beta}}{\rho^{\alpha}+\rho^{\beta}}, \frac{4}{3} \pi r_{3}^{3}=\rho^{-1} \tag{3.87}
\end{equation*}
$$

For closed-shell systems LSDA is equal to LDA due to the density $\alpha$ and $\beta$ are the same. LSDA is often used interchangeably with LDA.

To include the correlation term, which is determined by Monte Carlo method. Vosko, Wilk and Nusair (VWN) constructed the interpolation formula to use the obtain result in DFT calculation. The functional can be written as

$$
\begin{gather*}
\varepsilon_{\mathrm{c}}^{\mathrm{VWN}}\left(\mathrm{r}_{\mathrm{s}}, \zeta\right)=\varepsilon_{\mathrm{c}}\left(\mathrm{r}_{\mathrm{s}}, 0\right)+\varepsilon_{\mathrm{a}}\left(\mathrm{r}_{\mathrm{s}}\left[\frac{\mathrm{f}(\zeta)}{\mathrm{f}^{\prime \prime}(\zeta)}\right]\left[1-\zeta^{4}\right]+\left[\varepsilon_{\mathrm{c}}\left(\mathrm{r}_{\mathrm{s}}, 1\right)-\left(\varepsilon_{\mathrm{c}}\left(\mathrm{r}_{\mathrm{s}}, 0\right)\right] \mathrm{f}(\zeta) \zeta^{4}\right.\right.  \tag{3.88}\\
\mathrm{f}(\zeta)=\frac{(1+\zeta)^{4 / 3}+(1-\zeta)^{4 / 3}-2}{2\left(2^{1 / 3}-1\right)} \tag{3.89}
\end{gather*}
$$

The $\varepsilon_{\mathrm{c}}\left(\mathrm{r}_{\mathrm{s}}, \zeta\right)$ and $\varepsilon_{\mathrm{a}}\left(\mathrm{r}_{\mathrm{s}}\right)$ functionals are parameterized as

$$
\begin{align*}
& \varepsilon_{c / a}(x)=A\left\{\ln \frac{x^{2}}{X(x)}+\frac{2 b}{Q} \tan ^{-1}\left(\frac{Q}{2 x+b}\right)-\frac{b x_{0}}{X\left(x_{0}\right)}\left[\ln \frac{\left(x-x_{0}\right) 2}{x(x)}+\frac{2\left(b+2 x_{0}\right)}{2} \tan ^{-1}\left(\frac{Q}{2 x+b}\right)\right]\right\}  \tag{3.90}\\
& \text { where } \\
& \begin{array}{l}
\text { Q }
\end{array}=\sqrt{r_{s}}{ }_{9}=\sqrt{4 \mathrm{c}-\mathrm{b}^{2}}
\end{align*}
$$

The parameters $\mathrm{A}, \mathrm{x}_{0}$, b and c are fitting constants, different for $\varepsilon_{\mathrm{c}}\left(\mathrm{r}_{\mathrm{s}}, 0\right), \varepsilon_{\mathrm{c}}\left(\mathrm{r}_{\mathrm{s}}, 1\right)$ and $\varepsilon_{\mathrm{a}}\left(\mathrm{r}_{\mathrm{s}}\right)$.

Perdew and Wang give a modified form for $\varepsilon_{c /{ }_{\mathrm{c}}}\left(\mathrm{r}_{\mathrm{s}}\right)$ obtain the PW91 functional,

$$
\begin{equation*}
\varepsilon_{c / a}^{\text {pw91 }}=-2 a \rho\left(1+\alpha x^{2}\right) \ln \left(1+\frac{1}{2 \alpha\left(\beta_{1} x+\beta_{2} x^{2}+\beta_{3} x^{3}+\beta_{4} x^{4}\right)}\right. \tag{3.91}
\end{equation*}
$$

where a, $\alpha, \beta_{1}, \beta_{2}, \beta_{3}$ and $\beta_{4}$ are suitable constants.

The LSDA approximation underestimates the exchange energy by $\sim 10 \%$, whereas the electron correlation and bond strengths is overestimated. However, LSDA methods are often found to provide results with accuracy similar to that obtained by Hartree-Fock method.

### 3.10.2 The Generalized Gradient Approximation

Generalized Gradient Approximation (GGA) methods ${ }^{29-33}$ are the improvement of LSDA with considering a non-uniform electron gas. The exchange and correlation energies depend not only on the electron density, but also on derivatives of the density.

In 1986, Perdew and Wang (PW86) ${ }^{29}$ proposed modifying the LSDA exchange expression,

$$
\begin{align*}
& \varepsilon_{x}^{\text {PW86 }}=\varepsilon_{x}^{\text {LDA }}\left(1+a x^{2}+b x^{4}+\mathrm{cx}^{6}\right)^{1 / 15}  \tag{3.92}\\
& 66=\frac{\left|\nabla_{p}\right|}{4 / 3} 99 \rho^{4 / 3} 99.9 \text { d? }
\end{align*}
$$

where x is a dimensionless gradient variable, and $\mathrm{a}, \mathrm{b}$ and c are suitable constants.


In 1988, Becke (B88) ${ }^{30}$ modified the LSDA exchange energy, which can be given as

$$
\begin{gather*}
\varepsilon_{\mathrm{x}}^{\mathrm{BB8}}=\varepsilon_{\mathrm{x}}^{\mathrm{LDA}}+\Delta \varepsilon_{\mathrm{x}}^{\mathrm{BB8}} \\
\Delta \varepsilon_{\mathrm{x}}^{\mathrm{BB8}}=-\beta \rho^{1 / 3} \frac{\mathrm{x}^{2}}{1+6 \beta \sinh ^{-1} \mathrm{x}} \tag{3.93}
\end{gather*}
$$

where $\beta$ is parameter that is determined by fitting to known atomic data.

Perdew and Wang (PW91) have proposed an exchange functional similar to B88,

$$
\begin{equation*}
\varepsilon_{\mathrm{x}}^{\mathrm{PW} 91}=\varepsilon_{\mathrm{x}}^{\mathrm{LDA}}\left(\frac{1+\mathrm{xa}_{1} \sinh ^{-1}\left(\mathrm{xa}_{2}\right)+\left(\mathrm{a}_{3}+\mathrm{a}_{4} \mathrm{e}^{-\mathrm{bx}^{2}}\right) \mathrm{x}^{2}}{1+\mathrm{xa}_{1} \sinh ^{-1}\left(\mathrm{xa}_{2}\right)+\mathrm{a}_{5} \mathrm{x}^{2}}\right) \tag{3.94}
\end{equation*}
$$

where $\mathrm{a}_{1}-\mathrm{a}_{5}$ and b are suitable constants and $\mathrm{x}=\frac{\left|\nabla_{\mathrm{p}}\right|}{\rho^{4 / 3}}$.
Lee, Yang and Parr (LYP) ${ }^{31}$ proposed the correlation energy. The functional can be given as

$$
\begin{gather*}
\varepsilon_{\mathrm{c}}^{\mathrm{LYP}}=-\mathrm{a} \frac{\gamma}{\left(1+\mathrm{d} \rho^{-1 / 3}\right)}-a b \frac{\gamma \mathrm{e}^{-\mathrm{c}-1 / 3}}{9\left(1+\mathrm{d} \rho^{-1 / 3}\right) \rho^{8 / 3}}\left[\begin{array}{l}
18\left(2^{2 / 3}\right) \mathrm{C}_{\mathrm{F}}\left(\rho_{\alpha}^{8 / 3}+\rho_{\alpha}^{8 / 3}\right)-18 \rho \mathrm{t}_{\mathrm{w}} \\
+\rho_{\alpha}\left(2 \mathrm{t}_{\mathrm{w}}^{\alpha}+\nabla^{2} \rho_{\alpha}\right)+\rho_{\beta}\left(2 \mathrm{t}_{\mathrm{w}}^{\beta}+\nabla^{2} \rho_{\beta}\right)
\end{array}\right]  \tag{3.95}\\
\mathrm{t}_{\mathrm{w}}^{\sigma}=\frac{1}{8\left(\frac{\left|\nabla \rho_{\sigma}\right|^{2}}{\rho_{\sigma}}\right)-\nabla^{2} \rho_{\sigma}} \\
\gamma=2\left[\frac{\rho_{\alpha}^{2}+\rho_{\beta}^{2}}{\rho^{2}}\right]
\end{gather*}
$$

where the $a, b, c$ and $d$ parameters are determined by fitting to data for the helium atom. The $\mathrm{t}_{\mathrm{w}}^{\sigma}$ functional is the local Weizsacker kinetic energy density and the $\gamma$ factor becomes zero when all the spins are aligned $\left(\rho=\rho_{\alpha}, \rho_{\beta}=0\right)$.

## ลถาบนวทยบรการ

In 1991, Perdew and Wang proposed and modified a gradient correction to the LSDA result that can be written as

$$
\begin{gather*}
\varepsilon_{\mathrm{c}}^{\mathrm{PW} 91}=\varepsilon_{\mathrm{c}}^{\mathrm{LDA}}+\Delta \varepsilon_{\mathrm{c}}^{\mathrm{PW} 91} \\
\Delta \varepsilon_{\mathrm{c}}^{\mathrm{PW} 91}[\rho]=\rho\left(\mathrm{H}_{0}\left(\mathrm{t}, \mathrm{r}_{\mathrm{s}}, \zeta\right)+\mathrm{H}_{1}\left(\mathrm{t}, \mathrm{r}_{\mathrm{s}}, \zeta\right)\right)  \tag{3.96}\\
\mathrm{H}_{0}\left(\mathrm{t}, \mathrm{r}_{\mathrm{s}}, \zeta\right)=\mathrm{b}^{-1} \mathrm{f}(\zeta)^{3} \ln \left[1+\mathrm{a} \frac{\mathrm{t}^{2}+\mathrm{At}^{4}}{1+\mathrm{At}^{2}+\mathrm{A}^{2} \mathrm{t}^{4}}\right]
\end{gather*}
$$

$$
\begin{aligned}
\mathrm{H}_{1}\left(\mathrm{t}, \mathrm{r}_{\mathrm{s}}, \zeta\right) & =\left(\frac{16}{\pi}\right)\left(3 \pi^{2}\right)^{1 / 3}[\mathrm{C}(\rho)-\mathrm{c}] \mathrm{f}(\zeta)^{3} \mathrm{t}^{2} \mathrm{e}^{-\mathrm{dx} / \mathrm{f}(\zeta)^{2}} \\
\mathrm{f}(\zeta) & =\frac{1}{2}\left((1+\zeta)^{2 / 3}+(1-\zeta)^{2 / 3}\right) \\
\mathrm{C}(\rho) & =\ell_{1}+\frac{\ell_{2}+\ell_{3} \mathrm{r}_{\mathrm{s}}+\ell_{4} \mathrm{r}_{\mathrm{s}}^{2}}{1+\ell_{5} \mathrm{r}_{\mathrm{s}}+\ell_{6} \mathrm{r}_{\mathrm{s}}^{2}+\ell_{7} \mathrm{r}_{\mathrm{s}}^{3}} \\
\mathrm{t} & =\left(\frac{192}{\pi^{2}}\right)^{1 / 6} \frac{|\nabla \rho|}{2 \mathrm{f}(\zeta))^{7 / 6}} \\
\mathrm{~A} & =\mathrm{a}\left[\mathrm{e}^{-b \varepsilon_{c}\left(\mathrm{r}_{\mathrm{s}}, \zeta\right) / \mathrm{f}(\zeta)^{3}}-1\right]^{-1}
\end{aligned}
$$

where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d are suitable constants and $\ell_{1}-\ell_{7}$ are numerical constants.
The several proposed functionals have restricted to predict correlation energies for one-electron systems and failed to have the exchange energy cancel the Coulomb selfrepulsion. In 1995, Becke proposed the functionals which does not have these problems,

$$
\begin{align*}
& \varepsilon_{c}^{\mathrm{B} 95}=\varepsilon_{c}^{\alpha \beta}+\varepsilon_{c}^{\alpha \alpha}+\varepsilon_{c}^{\beta \beta} \\
& \varepsilon_{c}^{\alpha \beta}=\left[1+\mathrm{a}\left(\mathrm{x}_{\alpha}^{2}+\mathrm{x}_{\beta}^{2}\right)\right]^{-1} \varepsilon_{\mathrm{c}}^{\mathrm{PW91,} \mathrm{\alpha} \mathrm{\beta}} \\
& \varepsilon_{c}^{\sigma \sigma}=\left[1+\mathrm{bx}_{\sigma}^{2}\right]^{-2} \frac{\mathrm{D}_{\sigma}}{\mathrm{D}_{\sigma}^{\mathrm{LDA}}} \varepsilon_{\mathrm{c}}^{\mathrm{pW91,} \mathrm{\sigma} \mathrm{\sigma}} \tag{3.97}
\end{align*}
$$

$$
\begin{aligned}
& \mathrm{x}=\frac{\left|\nabla_{\mathrm{p}}\right|}{\rho^{4 / 3}}
\end{aligned}
$$

where $\sigma$ is $\alpha$ or $\beta$ spins, a and b are fitting parameters and $\varepsilon_{\mathrm{c}}^{\mathrm{PW91}}$ is the PerdewWang parameterization of the LSDA corelation functional.

Becke proposed the Becke-3 functional for $\mathrm{E}_{\mathrm{xc}}$ which included three semi-empirical parameters, $\mathrm{a}_{\mathrm{o}}, \mathrm{a}_{\mathrm{x}}$, and $\mathrm{a}_{\mathrm{c}}$

$$
\begin{equation*}
\mathrm{E}_{\mathrm{xc}}=\mathrm{E}_{\mathrm{xc}}^{\mathrm{HF}}+\left(1-\mathrm{a}_{0}\right) \mathrm{E}_{\mathrm{x}}^{\mathrm{LDA}}+\mathrm{a}_{\mathrm{x}} \Delta \mathrm{E}_{\mathrm{x}}^{\mathrm{B}}+\left(1-\mathrm{a}_{\mathrm{c}}\right) \mathrm{E}_{\mathrm{c}}^{\mathrm{VWN}} \tag{3.98}
\end{equation*}
$$

### 3.10.3 Hybrid Functional

Hybrids are an interesting class of functionals, which combine exact HartreeFock exchange with conventional GGAs. The general form is

$$
\begin{equation*}
\mathrm{E}_{\mathrm{xC}}^{\mathrm{hybrid}}=\alpha\left(\mathrm{E}_{\mathrm{x}}^{\mathrm{Hf}}-\mathrm{E}_{\mathrm{x}}^{\mathrm{GGA}}\right)+\mathrm{E}_{\mathrm{xc}}^{\mathrm{GGA}} \tag{3.99}
\end{equation*}
$$

Hybrids give significant improvement over GGAs for many molecular properties. Possibly the most widely used hybrid is the B3LYP functional, which the Becke-3 functional is combined with the Lee-Yang-Parr (LYP) correlation functional,

$$
\begin{equation*}
\mathrm{E}_{\mathrm{xc}}=\mathrm{E}_{\mathrm{xc}}^{\mathrm{HF}}+\left(1-\mathrm{a}_{0}\right) \mathrm{E}_{\mathrm{x}}^{\mathrm{LDA}}+\mathrm{a}_{\mathrm{x}} \Delta \mathrm{E}_{\mathrm{x}}^{\mathrm{B}}+\left(1-\mathrm{a}_{\mathrm{c}}\right) \mathrm{E}_{\mathrm{c}}^{\mathrm{VWN}}+\mathrm{a}_{\mathrm{c}} \Delta \mathrm{E}_{\mathrm{c}}^{\mathrm{LYP}} \tag{3.100}
\end{equation*}
$$

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## CHAPTER 4

## CALCULATIONS

Molecular Dynamic (MD) simulation and quantum calculation were employed throughout the work. The MD simulations were applied to obtain ZSM-5 structures. The quantum chemical calculations were applied to gain exchange energies and structures of CuZSM-5.

### 4.1 Molecular Dynamic (MD) Simulation

### 4.1.1 Preparation of ZSM-5 Structure

MD simulations were performed on model of silicalite-1 (MFI) in which 2 Si atoms in the unit cell were replaced by 2 Al atoms at 2 tetrahedral ( T ) sites ( $\mathrm{Si} / \mathrm{Al}=47$ ) and negative charges of the Al-substituted T-site were compensated by $\mathrm{H}^{+}$. There are 23 possible substitutions of this kind ${ }^{11}$ as given in Table 4.1.

Table 4.1 All possible substitutions for 2 Al in MFI.

| Possible substitution of AI sites | Possible substitution of Al sites |
| :---: | :---: |
| T1T1 | T6T6 |
| T1T6 | T6T12 |
| T1T7 | T7T9-model1 |
| $\text { की } \frac{\mathrm{T} 2 \mathrm{~T} 2 \mathrm{~T} 5}{\mathrm{~T}} \mathrm{\sim}$ | $19<{ }_{\text {T7T12-model1 }}^{\text {T7T9-model2 }}$ |
| $99 \cap \sim 13 T 3 \sim \cap 19$ | $8 \cap \text { T7T12-model2 }$ |
| , \| 61 ТЗТ9 ס6 100 | - T7T12-model3 |
| T4T4-model1 | T878 |
| T4T4-model2 | T9T10 |
| T4T10 | T9T12 |
| T5T5 | T11T11 |
| T5T12 |  |

However, only 4 possible substitutions, i.e., T6T12, T8T8, T7T12-model1, and T7T12-model2 were selected. These substitutions from now called "exchanged sites". The sites situate in the cavity of the straight channel and T12 is at the intersection between the straight and sinusoidal channels. Furthermore, previous MD study of $\mathrm{Cu}^{2+}$ exchange on double exchanged CuZSM-5 ${ }^{11}$ showed that favorable exchanges are site for 5-T ring T6T12 and site for 6-T ring T7T12.

To obtain structure for ZSM-5 or HZSM-5, MD simulations were performed. The MD simulations were carried out for all HZSM-5 structures studied using CVFF force field, and NVT ensemble. All structures used runtime of at least 2000 ps, and time step 0.1 fs at 300 K . Structures of HZSM-5 at 4 adsorbed sites were given in Figs. 4.1a-d.

The MD simulations were not performed in this work but the structures were taken from works of Mr.Ukrid Poompub. ${ }^{35}$

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Fig. 4.1 Simulated HZSM-5 structure for Al-substitution at 4 exchanged sites:
19/ (a) HZSM-5(T6T12), (b) HZSM-5(T8T8), (c) HZSM-5(T7T12-model1), and (d) HZSM-5(T7T12-model2).

### 4.2 Quantum Calculation

### 4.2.1 HZSM-5 Cluster Model

Partial geometry optimization using B3LYP method with 6-31G(d,p) basis set were performed on cluster models that were obtained form the MD simulations. According to exchanged sites, two types of cluster models can be obtained. One is 5T ring; HZSM-5(T6T12) exchanged site. And others are 6-T ring; HZSM-5(T8T8), HZSM-5(T7T12-model1), HZSM-5(T7T12-model2) exchanged site. For each obtained cluster model, all O atoms at the edge of ring (T-O-Si) were terminated and saturated with H atoms in replacement of Si atoms. Thus, $\mathrm{H}^{+} \mathrm{T} 6-\mathrm{H}^{+} \mathrm{T} 12$ exchanged site, $\mathrm{H}_{2}^{+}\left[\mathrm{Al}_{2} \mathrm{Si}_{3} \mathrm{O}_{15} \mathrm{H}_{10}\right]$, includes 32 atoms and $\mathrm{H}^{+} \mathrm{T} 8-\mathrm{H}^{+} \mathrm{T} 8, \mathrm{H}^{+} \mathrm{T} 7-\mathrm{H}^{+} \mathrm{T} 12$ (model1), and $\mathrm{H}^{+} \mathrm{T} 7-\mathrm{H}^{+} \mathrm{T} 12$ (model2) exchanged site, $\mathrm{H}_{2}^{+}\left[\mathrm{Al}_{2} \mathrm{Si}_{4} \mathrm{O}_{18} \mathrm{H}_{12}\right]$, consist of 38 atoms each. Geometrical parameters of the cluster were fixed except $\mathrm{H}^{+}, \mathrm{O}-\mathrm{H}$ bond lengths, and T-O-H bond angles ( $\angle \mathrm{T}-\mathrm{O}-\mathrm{H}$ ) were optimized by setting one parameter for all $\mathrm{O}-\mathrm{H}$ and one for $\angle \mathrm{T}-\mathrm{O}-\mathrm{H}$. All calculations were performed at room temperature using the Guassian 98 program. ${ }^{34}$ The structures of the 4 models were shown in Figs. 4.2a-d.

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Fig. 4.2 Cluster models were obtained from the simulated HZSM-5 structure and then adjusted to the quantum calculations; (a) HZSM-5(T6T12),
(b) HZSM-5(T8T8), (c) HZSM-5(T7T12(model1)), and (d) HZSM-5(T7T12(model2)).


### 4.2.2 CuZSM-5 Cluster Model

Cluster models of CuZSM-5 are classified into two groups as one- and two-ion exchanged CuZSM-5. For one-ion exchanged CuZSM-5, $1 \mathrm{H}^{+}$were replaced by $1 \mathrm{Cu}^{+}$or $2 \mathrm{H}^{+}$were replaced by $1 \mathrm{Cu}^{2+}$. For two-ion exchanged CuZSM-5, $2 \mathrm{H}^{+}$were replaced by $2 \mathrm{Cu}^{+}$. Only positions of $\mathrm{H}^{+}, \mathrm{Cu}^{+}$, and $\mathrm{Cu}^{2+}$ were optimized. Totally, $16 \mathrm{CuZSM}-5$ clusters were optimized and list of these clusters was given in Table 4.2.

Table 4.2 CuZSM-5 exchanged sites were optimized partially using B3LYP/6-31G(d,p).


### 4.2.3 Exchanged Energy

Exchanged energy ( $\Delta \mathrm{E}_{\text {exch }}$ ) of the one- and two-ion exchanged CuZSM-5 at T6T12, T8T8, T7T12-model1, and T7T12-model2 exchanged sites were calculated as followed:
(i) Exchange of $1 \mathrm{Cu}^{2+}$ in ZSM-5

$$
\begin{align*}
& \left(\mathrm{H}^{+}\right)_{2} \mathrm{ZSM}-5+\mathrm{Cu}^{2+} \rightarrow \mathrm{Cu}^{2+} \mathrm{ZSM}-5+2 \mathrm{H}^{+}  \tag{4.1}\\
& \quad \Delta \mathrm{E}_{\text {exch }}=\mathrm{E}\left[\mathrm{Cu}^{2+} \mathrm{ZSM}-5\right]-\mathrm{E}\left[\left(\mathrm{H}^{+}\right)_{2} \mathrm{ZSM}-5\right]-\mathrm{E}\left[\mathrm{Cu}^{2+}\right]
\end{align*}
$$

(ii) Exchange of $1 \mathrm{Cu}^{+}$in ZSM-5

$$
\begin{align*}
& \left(\mathrm{H}^{+}\right)_{2} \mathrm{ZSM}-5+\mathrm{Cu}^{+} \rightarrow \mathrm{Cu}^{+} \mathrm{H}^{+} \mathrm{ZSM}-5+\mathrm{H}^{+}  \tag{4.2}\\
& \quad \Delta \mathrm{E}_{\text {exch }}=\mathrm{E}\left[\mathrm{Cu}^{+} \mathrm{H}^{+} \mathrm{ZSM}-5\right]-\mathrm{E}\left[\left(\mathrm{H}^{+}\right)_{2} \mathrm{ZSM}-5\right]-\mathrm{E}\left[\mathrm{Cu}^{+}\right]
\end{align*}
$$

(iii) Exchange of $2 \mathrm{Cu}^{+}$in ZSM-5

$$
\begin{align*}
\left(\mathrm{H}^{+}\right)_{2} \mathrm{ZSM}-5 & +2 \mathrm{Cu}^{+} \rightarrow\left(\mathrm{Cu}^{+}\right)_{2} \mathrm{ZSM}-5+2 \mathrm{H}^{+}  \tag{4.3}\\
\quad \Delta \mathrm{E}_{\text {exch }} & =\mathrm{E}\left[\left(\left(\mathrm{Cu}^{+}\right)_{2} \mathrm{ZSM}-5\right]-\mathrm{E}\left[\left(\mathrm{H}^{+}\right)_{2} \mathrm{ZSM}-5\right]-2 \mathrm{E}\left[\mathrm{Cu}^{+}\right] .\right.
\end{align*}
$$

### 4.2.4 Autoreduction

Autoreduction Energy ( $\Delta \mathrm{E}_{\text {auto }}$ ) of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in ZSM-5 at T6T12, T8T8, T7T12-model1, and T7T12-model2 exchanged sites were determined using:

$$
\begin{gather*}
\mathrm{Cu}^{2+} \mathrm{ZSM}-5+1 / 2 \mathrm{H}_{2} \rightarrow \mathrm{Cu}^{+} \mathrm{H}^{+} \mathrm{ZSM}-5  \tag{4.4}\\
\Delta \mathrm{E}_{\text {auto }}=\mathrm{E}\left[\mathrm{Cu}^{+} \mathrm{H}^{+} \mathrm{ZSM}-5\right]-\mathrm{E}\left[\mathrm{Cu}^{2+} \mathrm{ZSM}-5\right]-1 / 2 \mathrm{E}\left[\mathrm{H}_{2}\right] .
\end{gather*}
$$

## CHAPTER 5

## RESULTS

Structures of HZSM-5, one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5, one- and two-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 at T6T12, T8T8, T7T12-model1 and T7T12-model2 exchanged sites obtained from B3LYP/6-31G(d,p) were presented in Figs. 5.15.20. Important geometrical parameters and total energies were given in the figures for HZSM-5 and in Tables 5.1-5.13 for the one- and two-ion exchanged CuZSM-5.


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### 5.1 HZSM-5

### 5.1.1 Optimized Geometries of HZSM-5

### 5.1.1.1 T6T12

Fig. 5.1 shows optimized structure of HZSM-5(T6T12). The O-H bond distance of $0.96 \AA$ and T-O-H bond angle of $115.33^{\circ}$ were obtained. The $\mathrm{O}-\mathrm{H}^{+}$ distance of $0.99 \AA$ was yielded.

$E=-2489.75624$ a.u.

Fig. 5.1 Optimized structures of HZSM-5(T6T12).



### 5.1.1.2 T8T8

Fig. 5.2 shows optimized structure of HZSM-5(T8T8). The O-H bond distance of $0.96 \AA$ and T-O-H bond angle of $116.00^{\circ}$ were obtained. The $\mathrm{O}-\mathrm{H}^{+}$distance of 0.98 Å was yielded.


$$
\mathrm{E}=-3006.37638 \text { a.u. }
$$

Fig. 5.2 Optimized structures of HZSM-5(T8T8).

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### 5.1.1.3 T7T12-model1

Fig. 5.3 shows optimized structure of HZSM-5(T7T12-model1). The O-H bond distance of $0.96 \AA$ and T-O-H bond angle of $118.47^{\circ}$ were obtained. The $\mathrm{O}-\mathrm{H}^{+}$ distances of 0.97 and $0.98 \AA$ were yielded.


$$
\mathrm{E}=-3006.41297 \text { a.u. }
$$

Fig. 5.3 Optimized structures of HZSM-5(T7T12-model1).
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### 5.1.1.4 T7T12-model2

Fig. 5.4 shows optimized structure of HZSM-5(T7T12-model2). The O-H bond distance of $0.96 \AA$ and T-O-H bond angle of $118.98^{\circ}$ were obtained. The $\mathrm{O}-\mathrm{H}^{+}$ distance of $0.98 \AA$ was yielded.


Fig. 5.4 Optimized structures of HZSM-5(T7T12-model2).

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### 5.2 One-ion Exchanged $\mathrm{Cu}^{2+}$ ZSM-5

### 5.2.1 Optimized Geometries of One-ion Exchanged $\mathrm{Cu}^{2+}$ ZSM-5

### 5.2.1.1 T6T12

The structure of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5 at T6T12 was shown in Fig. 5.5, and the distances between $\mathrm{Cu}^{2+}$ and O in ring of T6T12 are given in Table 5.1. From Table 5.1, $\mathrm{Cu}^{2+}-\mathrm{O}$ distances are in the range between 1.96 and $3.20 \AA$. In this range, only 2 distances, $\mathrm{Cu}-\mathrm{O} 1$ and $\mathrm{Cu}-\mathrm{O} 3$ are less than $2.00 \AA$.


Fig. 5.5 Optimized structures of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T6T12).
Table $5.1 \mathrm{Cu}^{2+}-\mathrm{O}$ distances of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T6T12).

| $6 /$ parameters | distance $(\AA)$ 6) |
| :---: | :---: | :---: |
| $\mathrm{Cu}^{2+}-\mathrm{O} 1$ | 1.96 |
| $\mathrm{Cu}^{2+}-\mathrm{O} 2$ | 3.20 |
| $\mathrm{Cu}^{2+}-\mathrm{O} 3$ | 1.99 |
| $\mathrm{Cu}^{2+}-\mathrm{O} 4$ | 2.13 |
| $\mathrm{Cu}^{2+}-\mathrm{O} 5$ | 2.29 |

### 5.2.1.2 T8T8

The structure of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5 at T8T8 was shown in Fig. 5.6, and the distances between $\mathrm{Cu}^{2+}$ and O in ring of T 8 T 8 are given in Table 5.2. From Table 5.2, $\mathrm{Cu}^{2+}-\mathrm{O}$ distances are in the range between 1.92 and $4.17 \AA$. In this range, only 2 distances, $\mathrm{Cu}-\mathrm{O} 1$ and $\mathrm{Cu}-\mathrm{O} 3$ are less than $2.00 \AA$.

E = -4645.51230 a.u.

Fig. 5.6 Optimized structures of one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5(\mathrm{~T} 8 \mathrm{~T} 8)$.

Table 5.2 $\mathrm{Cu}^{2+}-\mathrm{O}$ distances of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T8T8).

|  | 1 parameters | distance ( $\AA$ ) |
| :---: | :---: | :---: |
|  | $\mathrm{Cu}^{2+}-\mathrm{O} 1$ | 1.93 |
|  | $\mathrm{Cu}^{2+}-\mathrm{O} 3$ | $4 \mathrm{~d}_{1}^{2.44}$ |
| $99$ | $\begin{array}{\|r\|} \hline 8 \mathrm{Cu}^{2+}-\mathrm{O} 4 \\ \mathrm{Cu}^{2+}-\mathrm{O} 5 \end{array}$ |  |
|  | $\mathrm{Cu}^{2+}-\mathrm{O} 6$ | 3.07 |

### 5.2.1.3 T7T12-model1

The structure of one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5$ at T7T12-model1 was shown in Fig. 5.7, and the distances between $\mathrm{Cu}^{2+}$ and O in ring of T7T12-model1 are given in Table 5.3. From Table 5.3, $\mathrm{Cu}^{2+}-\mathrm{O}$ distances are in the range between 1.95 and 3.13 $\AA$. In this range, only 2 distances, $\mathrm{Cu}-\mathrm{O} 1$ and $\mathrm{Cu}-\mathrm{O} 4$ are less than $2.00 \AA$.


Fig. 5.7 Optimized structures of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T7T12-model1).

Table $5.3 \mathrm{Cu}^{2+}-\mathrm{O}$ distances of one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5(\mathrm{~T} 7 \mathrm{~T} 12-m o d e l 1)$.


### 5.2.1.4 T7T12-model2

The structure of one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5$ at T7T12-model2 was shown in Fig. 5.8, and the distances between $\mathrm{Cu}^{2+}$ and O in ring of T 7 T 12 -model2 are given in Table 5.4. From Table 5.4, $\mathrm{Cu}^{2+}$ - O distances are in the range between 1.95 and 3.27 $\AA$. In this range, only 1 distances, $\mathrm{Cu}-\mathrm{O} 3$ is less than $2.00 \AA$.


Fig. 5.8 Optimized structures of one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T7T12-model2).

Table 5.4 $\mathrm{Cu}^{2+}-\mathrm{O}$ distances of one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5(\mathrm{~T} 7 \mathrm{~T} 12-m o d e l 2)$.


### 5.2.2 Exchanged Energies of One-ion Exchanged $\mathrm{Cu}^{2+}$ ZSM-5

Exchanged energies ( $\Delta \mathrm{E}_{\text {exch }}$ ) of $\mathrm{Cu}^{2+}$ on ZSM-5 at various exchanged sites are listed in Table 5.5. All $\Delta \mathrm{E}_{\text {exch }}$ are shown positively. Only exchange of $\mathrm{Cu}^{2+}$ at T 6 T 12 site has the $\Delta \mathrm{E}_{\text {exch }}$ less than $10.00 \mathrm{kcal} / \mathrm{mol}$.

Table 5.5 Exchanged energies of $\mathrm{Cu}^{2+}$ exchanged in T6T12, T8T8, T7T12-model1, and T7T12-model2 exchanged site.

| exchanged sites | $\left.\Delta \mathbf{E}_{\text {exch }} \mathbf{( k c a l} / \mathbf{m o l}\right)$ |
| :---: | :---: |
| T6T12 | 9.19 |
| T8T8 | 48.86 |
| T7T12-model1 | 68.75 |
| T7T12-model2 | 43.77 |

### 5.3 One-ion Exchanged $\mathrm{Cu}^{+}$ZSM-5

### 5.3.1 Optimized Geometries of One-ion Exchanged Cu ${ }^{+}$ZSM-5

### 5.3.1.1 T6T12

The structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at T6T12 were shown in Fig. 5.9, and 5.10, respectively, and distances between $\mathrm{Cu}^{+}$and O in ring of the T6T12 are given in Table 5.6. Form Table 5.6, exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 6, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.89 and $5.33 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+}-\mathrm{O}^{\prime} 1$ is less than 2.00 $\AA$, and exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 12, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.93 and 3.91 $\AA$. In this range, only 1 distance, $\mathrm{Cu}^{+}-\mathrm{O} 4$ is less than $2.00 \AA$.

E = -4129.50105 a.u.

Fig. 5.9 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5(T6T12, $\mathrm{Cu}^{+}$at T6).

$\mathrm{E}=-4129.49735$ a.u.

Fig. 5.10 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 6 \mathrm{~T} 12, \mathrm{Cu}^{+}\right.$at T12).

Table 5.6 Distances of $\mathrm{Cu}^{+}-\mathrm{O}$ and $\mathrm{H}^{+}-\mathrm{O}$ of one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5(T6T12) when $\mathrm{Cu}^{+}$at T 6 and T 12 sites.
$\mathrm{Cu}^{+}$at $\mathbf{T 6}$
$\mathrm{Cu}^{+}$at $\mathbf{T 1 2}$

| parameters | distance $(\mathbf{\AA} \mathbf{)}$ |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O} 3$ | 0.99 |
| $\mathrm{Cu}^{+}-\mathrm{O} 1$ | 2.17 |
| $\mathrm{Cu}^{+}-\mathrm{O} 2$ | 4.45 |
| $\mathrm{Cu}^{+}-\mathrm{O} 3$ | 5.14 |
| $\mathrm{Cu}^{+}-\mathrm{O} 4$ | 5.33 |
| $\mathrm{Cu}^{+}-\mathrm{O} 5$ | 3.32 |
| $\mathrm{Cu}^{+}-\mathrm{O}^{\prime} 1$ | 1.89 |


| parameters | distance $(\boldsymbol{\AA} \mathbf{)}$ |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O} 1$ | 0.98 |
| $\mathrm{Cu}^{+}-\mathrm{O} 1$ | 3.34 |
| $\mathrm{Cu}^{+}-\mathrm{O} 2$ | 3.91 |
| $\mathrm{Cu}^{+}-\mathrm{O} 3$ | 2.03 |
| $\mathrm{Cu}^{+}-\mathrm{O} 4$ | 1.93 |
| $\mathrm{Cu}^{+}-\mathrm{O} 5$ | 3.41 |

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### 5.3.1.2 T8T8

The structures of one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 at T8T8 were shown in Fig. 5.11, and 5.12, respectively, and distances between $\mathrm{Cu}^{+}$and O in ring of the T8T8 are given in Table 5.7. Form Table 5.7, exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 8, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.96 and $3.77 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+}-\mathrm{O} 6$ is less than $2.00 \AA$, and exchange of $\mathrm{Cu}^{+}$at $\mathrm{T}^{\prime} 8, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.75 and $3.85 \AA$. In this range, only 2 distances, $\mathrm{Cu}^{+}-\mathrm{O} 4$ and $\mathrm{Cu}^{+}-\mathrm{O} 4$ are less than $2.00 \AA$.


$$
\mathrm{E}=-4646.10525 \text { a.u. }
$$

Fig. 5.11 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 8 \mathrm{T8}, \mathrm{Cu}^{+}\right.$at T 8$)$.

E = -4646.09282 a.u.

Fig. 5.12 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 8 \mathrm{~T} 8, \mathrm{Cu}^{+}\right.$at $\left.\mathrm{T}^{\prime} 8\right)$.

Table 5.7 Distances of $\mathrm{Cu}^{+}-\mathrm{O}$ and $\mathrm{H}^{+}-\mathrm{O}$ of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5(\mathrm{~T} 8 \mathrm{~T} 8)$ when $\mathrm{Cu}^{+}$at $\mathrm{T8}$ and $\mathrm{T}^{\prime} 8$ sites.

## $\mathrm{Cu}^{+}$at T 8 <br> $\mathrm{Cu}^{+}$at $\mathrm{T}^{\prime} 8$

| parameters | distance $(\mathbf{\AA} \mathbf{)}$ |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O} 3$ | 0.98 |
| $\mathrm{Cu}^{+}-\mathrm{O} 1$ | 2.02 |
| $\mathrm{Cu}^{+}-\mathrm{O} 2$ | 3.77 |
| $\mathrm{Cu}^{+}-\mathrm{O} 3$ | 3.36 |
| $\mathrm{Cu}^{+}-\mathrm{O} 4$ | 3.30 |
| $\mathrm{Cu}^{+}-\mathrm{O} 5$ | 3.25 |
| $\mathrm{Cu}^{+}-\mathrm{O} 6$ | $\mathrm{H}^{+}-\mathrm{O} 1$ |
| $\mathrm{Cu}^{+}-\mathrm{O} 1$ | 0.98 |
|  | $\mathrm{Cu}^{+}-\mathrm{O} 2$ |
| $\mathrm{Cu}^{+}-\mathrm{O} 3$ | 3.56 |
| $\mathrm{Cu}^{+}-\mathrm{O} 4$ | 3.85 |
| $\mathrm{Cu}^{+}-\mathrm{O} 5$ | 1.92 |
| $\mathrm{Cu}^{+}-\mathrm{O} 6$ | 1.75 |

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### 5.3.1.3 T7T12-model1

The structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at T7T12-model1 were shown in Fig. 5.13, and 5.14, respectively, and distances between $\mathrm{Cu}^{+}$and O in ring of the T7T12model1 are given in Table 5.8. Form Table 5.8, exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 7, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.89 and $4.47 \AA$. In this range, only 2 distance, $\mathrm{Cu}^{+}-\mathrm{O}^{\prime} 1$ and $\mathrm{Cu}^{+}-\mathrm{O} 1$ are less than $2.00 \AA$, and exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 12, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.98 and $3.51 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+}-\mathrm{O} 4$ is less than $2.00 \AA$.


Fig. 5.13 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 7 \mathrm{~T} 12-\right.$-model1, $\mathrm{Cu}^{+}$at T 7 ).


Fig. 5.14Optimized structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 7 \mathrm{~T} 12-\right.$ model1, $\mathrm{Cu}^{+}$at T 12$)$.

Table 5.8 Distances of $\mathrm{Cu}^{+}-\mathrm{O}$ and $\mathrm{H}^{+}-\mathrm{O}$ of one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5(T7T12model1) when $\mathrm{Cu}^{+}$at T 7 and T 12 sites.

## $\mathbf{C u}^{+}$at $\mathbf{T 7}$

## $\mathrm{Cu}^{+}$at $\mathbf{T 1 2}$

| parameters | distance (Å) |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O} 4$ | 0.97 |
| $\mathrm{Cu}^{+}-\mathrm{O} 1$ | 1.98 |
| $\mathrm{Cu}^{+}-\mathrm{O} 2$ | 2.23 |
| $\mathrm{Cu}^{+}-\mathrm{O} 3$ | 4.19 |
| $\mathrm{Cu}^{+}-\mathrm{O} 4$ | 4.53 |
| $\mathrm{Cu}^{+}-\mathrm{O} 5$ | 4.47 |
| $\mathrm{Cu}^{+}-\mathrm{O} 6$ | 3.57 |
| $\mathrm{Cu}^{\prime} \mathrm{O}^{\prime} 1$ | 1.89 |


| parameters | distance ( $\AA \mathbf{A})$ |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O}^{\prime} 1$ | 0.98 |
| $\mathrm{Cu}^{+}-\mathrm{O} 1$ | 2.02 |
| $\mathrm{Cu}^{+}-\mathrm{O} 2$ | 2.75 |
| $\mathrm{Cu}^{+}-\mathrm{O} 3$ | 2.40 |
| $\mathrm{Cu}^{+}-\mathrm{O} 4$ | 1.98 |
| $\mathrm{Cu}^{+}-\mathrm{O} 5$ | 2.99 |
| $\mathrm{Cu}^{+}-\mathrm{O} 6$ | 3.51 |

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### 5.3.1.4 T7T12-model2

The structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at T7T12-model2 were shown in Fig. 5.15, and 5.16, respectively, and distances between $\mathrm{Cu}^{+}$and O in ring of the T7T12model2 are given in Table 5.9. Form Table 5.9, exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 7, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.93 and $4.19 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+}-\mathrm{O} 6$ is less than $2.00 \AA$, and exchange of $\mathrm{Cu}^{+}$at $\mathrm{T} 12, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.99 and $3.98 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+}-\mathrm{O} 3$ is less than $2.00 \AA$.


Fig. 5.15 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 7 \mathrm{~T} 12-\right.$ model2, $\mathrm{Cu}^{+}$at T 7 ).


$$
\mathrm{E}=-4646.06599 \text { a.u. }
$$

Fig. 5.16 Optimized structures of one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5(T7T12-model2, $\mathrm{Cu}^{+}$at T12).

Table 5.9 Distances of $\mathrm{Cu}^{+}-\mathrm{O}$ and $\mathrm{H}^{+}-\mathrm{O}$ of one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5(T7T12model2) when $\mathrm{Cu}^{+}$at T 7 and T 12 sites.

## $\mathrm{Cu}^{+}$at $\mathbf{T 7}$

## $\mathrm{Cu}^{+}$at $\mathbf{T 1 2}$

| parameters | distance $(\AA \mathbf{\AA})$ |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O} 6$ | 0.99 |
| $\mathrm{Cu}-\mathrm{O} 1$ | 3.31 |
| $\mathrm{Cu}-\mathrm{O} 2$ | 4.19 |
| $\mathrm{Cu}-\mathrm{O} 3$ | 3.05 |
| $\mathrm{Cu}-\mathrm{O} 4$ | 3.37 |
| $\mathrm{Cu}-\mathrm{O} 5$ | 2.16 |
| $\mathrm{Cu}-\mathrm{O} 6$ | 1.93 |


| parameters | distance ( $\AA$ ) |
| :---: | :---: |
| $\mathrm{H}^{+}-\mathrm{O} 3$ | 1.00 |
| $\mathrm{Cu}-\mathrm{O} 1$ | 2.19 |
| $\mathrm{Cu}-\mathrm{O} 2$ | 2.26 |
| $\mathrm{Cu}-\mathrm{O} 3$ | 1.99 |
| $\mathrm{Cu}-\mathrm{O} 4$ | 3.98 |
| $\mathrm{Cu}-\mathrm{O} 5$ | 3.71 |
| $\mathrm{Cu}-\mathrm{O} 6$ | 2.72 |

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### 5.3.2 Exchanged Energy of One-ion Exchanged Cu ${ }^{+}$ZSM-5

Exchanged energy ( $\Delta \mathrm{E}_{\text {exch }}$ ) of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at various exchanged sites are given in Table 5.10. $\Delta \mathrm{E}_{\text {exch }}$ are in the range of 66.62-105.72 $\mathrm{kcal} / \mathrm{mol}$. The lowest is exchange of $\mathrm{Cu}^{+}$at T12 in T7T12-model1.

Table 5.10 Exchanged energy of one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at T6T12, T8T8, T7T12-model1, and T7T12-model2 exchanged site.

| exchanged sites | $\Delta \mathrm{E}_{\text {exch }}(\mathrm{kcal} / \mathrm{mol})$ |
| :---: | :---: |
| T6T12, $\mathrm{Cu}^{+}$at T6 | 72.85 |
| T6T12, $\mathrm{Cu}^{+}$at T12 | 75.17 |
| T8T8, $\mathrm{Cu}^{+}$at T8 | 82.85 |
| - T8T8, $\mathrm{Cu}^{+}$at T8 ${ }^{\prime}$ | 90.65 |
| T7T12-model1, $\mathrm{Cu}^{+}$at T7 | 105.72 |
| T7T12-model1, $\mathrm{Cu}^{+}$at T12 | 66.62 |
| T7T12-model2, $\mathrm{Cu}^{+}$at T7 | 93.50 |
| T7T12-model2, $\mathrm{Cu}^{+}$at T12 | 97.70 |



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### 5.4 Two-ion Exchanged $\mathrm{Cu}^{+}$ZSM-5

### 5.4.1 Optimized Geometries of Two-ion Exchanged $\mathrm{Cu}^{+}$ZSM-5

### 5.4.1.1 T6T12

The structure of two-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 at T6T12 was shown in Fig. 5.17, and distances between $\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}$, between $\mathrm{Cu}^{+}$and O in ring of the T 6 T 12 are given in Table 5.11. Form Table 5.11, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.96 and $4.85 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+} 2-\mathrm{O} 6$ is less than $2.00 \AA$, and 2 distances, $\mathrm{Cu}^{+} 1-\mathrm{O} 1$ and $\mathrm{Cu}^{+} 1-\mathrm{O} 3$ are equal to $2.00 \AA$.


Fig. 5.17 Optimized structures of two-ion exchanged $\mathrm{Cu}^{+} Z S M-5(T 6 T 12)$.


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Table 5.11 $\mathrm{Cu}^{+}-\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}-\mathrm{O}$ distances of two-ion exchanged $\mathrm{Cu}^{+} Z \mathrm{ZSM}-5(\mathrm{~T} 6 \mathrm{~T} 12)$.

| parameters | distance ( $\AA \mathbf{)}$ |
| :---: | :---: |
| $\mathrm{Cu}^{+} 1-\mathrm{Cu}^{+} 2$ | 2.39 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 1$ | 2.00 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 2$ | 3.17 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 3$ | 2.00 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 4$ | 2.27 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 5$ | 2.50 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 1$ | 2.55 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 2$ | 4.85 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 3$ | 4.37 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 4$ | 3.60 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 5$ | 1.96 |



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### 5.4.1.2 T8T8

The structure of two-ion exchanged $\mathrm{Cu}^{+} Z S M-5$ at T8T8 was shown in Fig. 5.18, and distances between $\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}$, between $\mathrm{Cu}^{+}$and O in ring of the T 8 T 8 are given in Table 5.12. Form Table 5.12, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.94 and $3.90 \AA$. In this range, only 1 distance, $\mathrm{Cu}^{+} 2-\mathrm{O} 6$ is less than $2.00 \AA$.


$$
\mathrm{E}=-6285.83162 \text { a.u. }
$$

Fig. 5.18 Optimized structures of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5(\mathrm{~T} 8 \mathrm{~T} 8)$.

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Table 5.12 $\mathrm{Cu}^{+}-\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}-\mathrm{O}$ distances of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5(\mathrm{~T} 8 \mathrm{~T} 8)$.

| parameters | distance ( $\AA \mathbf{)}$ |
| :---: | :---: |
| $\mathrm{Cu}^{+} 1-\mathrm{Cu}^{+} 2$ | 2.28 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 1$ | 3.22 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 2$ | 3.25 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 3$ | 2.02 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 4$ | 2.02 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 5$ | 3.69 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 6$ | 3.23 |
| $\mathrm{Cu}^{+} 2-\mathrm{O1}$ | 2.10 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 2$ | 3.51 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 3$ | 3.73 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 4$ | 3.72 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 5$ | 3.90 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 6$ | 1.94 |



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### 5.4.1.3 T7T12-model1

The structure of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at T7T12-model1 was shown in Fig. 5.19, and distances between $\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}$, between $\mathrm{Cu}^{+}$and O in ring of the T7T12-model1 are given in Table 5.13. Form Table 5.13, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 1.98 and $4.28 \AA$. In this range, there are 3 distances; $\mathrm{Cu}^{+} 1-\mathrm{O}_{1}, \mathrm{Cu}^{+} 1-$ O 4, and $\mathrm{Cu}^{+} 2-\mathrm{O}^{\prime} 1$ are less than $2.00 \AA$.


$$
\mathrm{E}=-6285.87005 \text { a.u. }
$$

Fig. 5.19 Optimized structures of two-ion exchanged $\mathrm{Cu}^{+} Z \mathrm{ZSM}-5$ (T7T12-model1).

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Table $5.13 \mathrm{Cu}^{+}-\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}-\mathrm{O}$ distances of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ (T7T12-model1).

| parameters | distance $(\AA \mathbf{\AA})$ |
| :---: | :---: |
| $\mathrm{Cu}^{+} 1-\mathrm{Cu}^{+} 2$ | 2.34 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 1$ | 1.98 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 2$ | 2.88 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 3$ | 2.56 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 4$ | 1.99 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 5$ | 2.84 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 6$ | 3.34 |
| $\mathrm{Cu}^{+} 2-\mathrm{O1}$ | 2.67 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 2$ | 4.27 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 3$ | 4.28 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 4$ | 3.65 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 5$ | 3.10 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 6$ | 2.88 |
| $\mathrm{Cu}^{+} 2-\mathrm{O}^{\prime} 1$ | 1.88 |



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### 5.4.1.4 T7T12-model2

The structure of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ at T7T12-model2 was shown in Fig. 5.20, and distances between $\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}$, between $\mathrm{Cu}^{+}$and O in ring of the T7T12-model2 are given in Table 5.14. Form Table 5.14, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range between 2.01 and $4.50 \AA$. In this range, there are 3 distances, $\mathrm{Cu}^{+} 1-\mathrm{O} 3$ is 2.01 $\AA, \mathrm{Cu}^{+} 2-\mathrm{O} 5$, and $\mathrm{Cu}^{+} 2-\mathrm{O} 6$ are $2.02 \AA$.


$$
\mathrm{E}=-6285.81306 \text { a.u. }
$$

Fig. 5.20 Optimized structures of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ (T7T12-model2).

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Table 5.14 $\mathrm{Cu}^{+}-\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}-\mathrm{O}$ distances of two-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ (T7T12-model2).

| parameters | distance $(\AA \mathbf{\AA})$ |
| :---: | :---: |
| $\mathrm{Cu}^{+} 1-\mathrm{Cu}^{+} 2$ | 2.23 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 1$ | 2.16 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 2$ | 2.27 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 3$ | 2.01 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 4$ | 3.99 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 5$ | 3.68 |
| $\mathrm{Cu}^{+} 1-\mathrm{O} 6$ | 2.66 |
| $\mathrm{Cu}^{+} 2-\mathrm{O1}$ | 3.60 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 2$ | 4.50 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 3$ | 3.26 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 4$ | 3.32 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 5$ | 2.02 |
| $\mathrm{Cu}^{+} 2-\mathrm{O} 6$ | 2.02 |



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### 5.3.2 Exchanged Energies of Two-ion Exchanged Cu ${ }^{+}$ZSM-5

Exchanged energies ( $\Delta \mathrm{E}_{\text {exch }}$ ) of $2 \mathrm{Cu}^{+}$on ZSM-5 at various exchanged sites are listed in Table 5.15. Very high $\Delta \mathrm{E}_{\text {exch }}$ were observed from exchange of $\mathrm{Cu}^{+}$in ZSM-5.

Table 5.15 Exchanged energies of $2 \mathrm{Cu}^{+}$exchanged in T6T12, T8T8, T7T12-model1, and T7T12-model2 exchanged site.

| exchanged sites | $\Delta \mathbf{E}_{\text {exch }}(\mathbf{k c a l} / \mathbf{m o l})$ |
| :---: | :---: |
| T 6 T 12 | 143.29 |
| T 8 T 8 | 167.27 |
| T7T12-model1 | 166.12 |
| T7T12-model2 | 164.94 |
|  |  |

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### 5.5 Autoreduction Energy of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in ZSM-5 at T6T12, T8T8, T7T12model1, and T7T12-model2 Exchanged Site

Autoreduction energy ( $\Delta \mathrm{E}_{\text {auto }}$ ) of Cu species resided at various exchanged sites were given in Table 5.16.

Table 5.16 Autoreduction energy of $\mathrm{Cu}^{2+}$ in the cavity of ZSM-5 at T6T12, T8T8, T7T12-model1, and T7T12-model2 exchanged site.

| Autoreductions | $\begin{gathered} \Delta \mathrm{E}_{\text {auto }} \\ (\mathrm{kcal} / \mathrm{mol}) \end{gathered}$ |
| :---: | :---: |
| one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T6T12) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 (T6T12, $\mathrm{Cu}^{+}$at T6) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ (T6T12, $\mathrm{Cu}^{+}$at T12) | $\begin{aligned} & 217.76 \\ & 220.08 \end{aligned}$ |
| one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5$ (T8T8) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ ( $\mathrm{T} 8 \mathrm{~T} 8, \mathrm{Cu}^{+}$at T8) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ (T8T8, $\mathrm{Cu}^{+}$at $\mathrm{T}^{\prime} 8$ ) | $\begin{aligned} & 188.09 \\ & 195.89 \end{aligned}$ |
| $\begin{aligned} & \text { one-ion exchanged } \mathrm{Cu}^{2+} \mathrm{ZSM}-5 \text { (T7T12-model1) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \mathrm{ZSM}-5 \text { (T7T12-model1, } \mathrm{Cu}^{+} \text {at } \mathrm{T} 7 \text { ) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \mathrm{ZSM}-5 \text { (T7T12-model1, } \mathrm{Cu}^{+} \text {at T12) } \end{aligned}$ | $\begin{aligned} & 188.03 \\ & 148.93 \end{aligned}$ |
| $\begin{aligned} & \text { one-ion exchanged } \mathrm{Cu}^{2+} \mathrm{ZSM}-5(\mathrm{~T} 7 \mathrm{~T} 12-\text {-model2) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \mathrm{ZSM}-5 \text { (T7T12-model2, } \mathrm{Cu}^{+} \text {at T7) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \text {ZSM-5 (T7T12-model2, } \mathrm{Cu}^{+} \text {at T12) } \end{aligned}$ | $\begin{aligned} & 208.03 \\ & 203.84 \end{aligned}$ |

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### 5.6 Combined Exchanged and Autoreduction Energy of $\mathrm{Cu}^{2+}$ at T6T12, T8T8,

 T7T12-model1, and T7T12-model2 Exchanged SiteCombined $\Delta \mathrm{E}_{\text {exch }}$ and $\Delta \mathrm{E}_{\text {auto }}$ of $\mathrm{Cu}^{2+}$ resided at various exchanged sites were given in Table 5.17. The combined energies of $\mathrm{Cu}^{2+}$ in ZSM-5 are smaller than $\Delta \mathrm{E}_{\text {auto }}$ of $\mathrm{Cu}^{2+}$ in aqueous solution ( $256.04 \mathrm{kcal} / \mathrm{mol}$ ), except T7T12-model1, Cu at T12.

Table 5.17 Combined $\Delta \mathrm{E}_{\text {exch }}$ and $\Delta \mathrm{E}_{\text {auto }}$ of $\mathrm{Cu}^{2+}$ in ZSM-5 at T6T12, T8T8, T7T12model1, and T7T12-model2 exchanged site.

| Exchanges + Autoreductions | $\begin{gathered} \Delta \mathrm{E}_{\text {exch }}+\Delta \mathrm{E}_{\text {auto }} \\ (\mathrm{kcal} / \mathrm{mol}) \end{gathered}$ |
| :---: | :---: |
| one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T6T12) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 (T6T12, $\mathrm{Cu}^{+}$at T6) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ (T6T12, $\mathrm{Cu}^{+}$at T12) | $\begin{aligned} & 226.95 \\ & 229.77 \end{aligned}$ |
| one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T8T8) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 (T8T8, $\mathrm{Cu}^{+}$at T8) <br> $\rightarrow$ one-ion exchanged $\mathrm{Cu}^{+} \overline{\mathrm{ZSM}}-5$ ( $\mathrm{T} 8 \mathrm{~T} 8, \mathrm{Cu}^{+}$at $\mathrm{T}^{\prime} 8$ ) | $\begin{aligned} & 236.95 \\ & 244.75 \end{aligned}$ |
| $\begin{aligned} & \text { one-ion exchanged } \mathrm{Cu}^{2+} \text { ZSM-5(T7T12-model1) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \text {ZSM-5 (T7T12-model1, } \mathrm{Cu}^{+} \text {at T7) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \text {ZSM-5 (T7T12-model1, } \mathrm{Cu}^{+} \text {at T12) } \end{aligned}$ | $\begin{aligned} & 256.78 \\ & 217.68 \end{aligned}$ |
| $\begin{aligned} & \text { one-ion exchanged } \mathrm{Cu}^{2+} \mathrm{ZSM}-5(\mathrm{~T} 7 \mathrm{~T} 12-\text {-model2) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \text {ZSM-5 (T7T12-model2, } \mathrm{Cu}^{+} \text {at T7) } \\ & \rightarrow \text { one-ion exchanged } \mathrm{Cu}^{+} \text {ZSM-5 (T7T12-model2, } \mathrm{Cu}^{+} \text {at T12) } \end{aligned}$ | 251.80 247.61 |

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## CHAPTER 6

## DISCUSSIONS

### 6.1 Optimized Structures of HZSM-5

According to Figs. 5.1-5.4, the optimized HZSM-5 structures are indicated. For all HZSM-5 cluster models, T6T12, T8T8, T7T12-model1 and T7T12-model2 site show the same O-H bond length of $0.96 \AA$ and show T-O-H bond angles between $115.33^{\circ}$ and $118.98^{\circ}$. For $\mathrm{O}-\mathrm{H}^{+}$distances, they are $0.99 \AA$ for T6T12 site, $0.98 \AA$ for T8T8 and T7T12-model2 site, and 0.97 and $0.98 \AA$ for T7T12-model1 site. This suggests that $\mathrm{H}^{+}$bound to $\mathrm{O}^{-}$in $\equiv \mathrm{Al}-\mathrm{O}^{-}-\mathrm{Si} \equiv$, except T7T12-model1 where $\mathrm{H}^{+}$bound to O at the terminal TOH. All $\mathrm{H}^{+}$were found in the channel of HZSM-5. Since, $\mathrm{O}-\mathrm{H}^{+}$ distances are very short around $0.98 \AA$ similar to OH bond of water, therefore $\mathrm{H}^{+}$ binds to the acidic oxygen very strongly. Among 6-T-ring cluster T7T12-model1 has the lowest energy and, hence, is the most stable site for 6-T-ring cluster. The 5-T-ring cluster T6T12 could not be directly compared. The energies must be compared to evaluate the stability of $\mathrm{H}^{+}$.

### 6.2 Optimized Structures of One-ion Exchanged $\mathrm{Cu}^{2+}$ ZSM-5

According to Figs. 5.5-5.8, and Tables 5.1-5.4, the optimized $\mathrm{Cu}^{2+} \mathrm{ZSM}-5$ structures and their parameters are indicated. The coordination number of $\mathrm{Cu}^{2+}$ could be determined from $\mathrm{Cu}^{2+}-\mathrm{O}$ distance. The coordinate of $\mathrm{Cu}^{2+}$ is established if the $\mathrm{Cu}^{2+}-\mathrm{O}$ distance is around $2 \AA$. Thus, $\mathrm{Cu}^{2+}$ has coordination numbers of 2 for all exchanged sites; T6T12, T8T8, T7T12-model1, and T7T12-model2. For 6-T-ring, the $\mathrm{Cu}^{2+}$ is located near O 1 and O 3 with the distances of 1.93 and $1.92 \AA$ for T8T8 site, near O 1 and O 4 with the distances of 1.96 and $1.95 \AA$ for T7T12-model1, and near O3 and O6 with the distance of 1.95 and $2.02 \AA$ for T7T12-model2. For 5-T-ring, $\mathrm{T} 6 \mathrm{~T} 12, \mathrm{Cu}^{2+}$ is located near O 1 and O 3 with the distances of 1.96 and $1.99 \AA$.

### 6.3 Optimized Structures of One-ion Exchanged $\mathrm{Cu}^{+}$ZSM-5

According to Figs. 5.9-5.16, and Tables 5.6-5.9, the $\mathrm{O}-\mathrm{H}^{+}$distances for all sites are found to be between 0.97 and $1.00 \AA$. From Table 5.6, $\mathrm{Cu}^{+}-\mathrm{O}$ distances of 1.89-5.33 $\AA$ were found for T6T12 site where $\mathrm{Cu}^{+}$at T6, and of 1.93-3.91 $\AA$ for T 6 T 12 site where $\mathrm{Cu}^{+}$at T 12 . For $\mathrm{T} 6 \mathrm{~T} 12, \mathrm{Cu}^{+}$at $\mathrm{T} 6, \mathrm{Cu}^{+}$is located near $\mathrm{O}^{\prime} 1$ with the distance of $1.89 \AA$, and for T6T12, $\mathrm{Cu}^{+}$at $\mathrm{T} 12, \mathrm{Cu}^{+}$is located near O 3 and O 4 with the distances of 2.03 and $1.93 \AA$ respectively,. From Table $5.7, \mathrm{Cu}^{+}-\mathrm{O}$ distances of 1.96-3.77 $\AA$ for one-ion exchanged $\mathrm{Cu}^{+} Z \mathrm{SM}-5\left(\mathrm{~T} 8 \mathrm{~T} 8, \mathrm{Cu}^{+}\right.$at T 8$)$, and of $1.75-3.85 \AA$ for one-ion exchanged $\mathrm{Cu}^{+} Z S M-5\left(\mathrm{~T} 8 \mathrm{~T} 8, \mathrm{Cu}^{+}\right.$at $\left.\mathrm{T} 8^{\prime}\right)$. When $\mathrm{Cu}^{+}$at T 8 , it is located near O 1 and O 6 with the distances of 2.02 and $1.96 \AA$, respectively. When $\mathrm{Cu}^{+}$at $\mathrm{T}^{\prime}$, it is near O3 and O4 with the distances of 1.92 and $1.75 \AA$, respectively. For Table $5.8, \mathrm{Cu}^{+}-\mathrm{O}$ distances of 1.89-4.53 $\AA$ for $\mathrm{Cu}^{+} \mathrm{ZSM}-5\left(\mathrm{~T} 712-m o d e l 1, \mathrm{Cu}^{+}\right.$at T7), and of $1.98-3.51 \AA$ for $\mathrm{Cu}^{+}$ZSM-5(T7T12-model1, $\mathrm{Cu}^{+}$at T12). When $\mathrm{Cu}^{+}$at T 7 , it is located near O 1 and $\mathrm{O}^{\prime} 1$ with the distances of 1.98 and $1.89 \AA$, respectively. And when $\mathrm{Cu}^{+}$at T12, it is located near O 4 with the distances of $1.98 \AA$. For Table 5.9, $\mathrm{Cu}^{+}-\mathrm{O}$ distances of 1.93-4.19 $\AA$ for $\mathrm{Cu}^{+} Z S M-5\left(T 7 T 12-m o d e l 2, \mathrm{Cu}^{+}\right.$at T7), and of 1.99-3.98 $\AA$ for $\mathrm{Cu}^{+} Z S M-5\left(T 7 T 12\right.$-model2, $\mathrm{Cu}^{+}$at T12). When $\mathrm{Cu}^{+}$at T 7 , it is located in the ring plane near O 6 with the distance of $1.93 \AA$, respectively. And when $\mathrm{Cu}^{+}$at T12, it is located near O 3 with the distances of $1.99 \AA$. The $\mathrm{O}-\mathrm{H}^{+}$distances are longer than in HZSM-5 while $\mathrm{Cu}^{+}-\mathrm{O}$ distance is similar to two-ion exchanged $\mathrm{Cu}^{+}$ZSM-5.

### 6.4 Optimized Structures of Two-ion Exchanged $\mathrm{Cu}^{+} \mathbf{Z S M}-5$

According to Figs. 5.17-5.20, and Tables 5.11-5.14, distances between $\mathrm{Cu}^{+}$are in the range of 2.23-2.39 $\AA$. These values are the longest for T6T12 site and shortest for T7T12-model2 site. Distances between $\mathrm{Cu}^{+}$and skeleton O are between 1.88-4.85 $\AA$. For T6T12, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range of $1.96-4.85 \AA$, which one $\mathrm{Cu}^{+}$is near O 1 and O 3 with the distances of $2.00 \AA$ and another $\mathrm{Cu}^{+}$is near O 5 with the distance of $1.96 \AA$. For $\mathrm{T} 8 \mathrm{~T} 8, \mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range of $1.94-3.90 \AA$, which one $\mathrm{Cu}^{+}$ is near O 3 and O 4 with the distances of $2.02 \AA$ and another $\mathrm{Cu}^{+}$is near O 6 with the distance of $1.94 \AA$. For T7T12-model1, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are found to be between 1.88-4.28 $\AA$, which $\mathrm{One} \mathrm{Cu}^{+}$is located near O 1 and O 4 with the distances of 1.98 and
$1.99 \AA$, respectively, and another $\mathrm{Cu}^{+}$is located near $\mathrm{O}^{\prime} 1$ with distance of $1.88 \AA$. For T7T12-model2, $\mathrm{Cu}^{+}-\mathrm{O}$ distances are in the range of 2.01-3.99 $\AA$, which One $\mathrm{Cu}^{+}$is located near O3 with the distances of $2.01 \AA$, and another $\mathrm{Cu}^{+}$is located near O 5 and O6 with distance of $2.02 \AA$. Since, $\mathrm{Cu}^{+}$is near to each other, thus, the complex with 2 $\mathrm{Cu}^{+}$is rather unstable.

### 6.5 Exchanged Energy of Cu Species in ZSM-5 at T6T12, T8T8, T7T12-model1, and T7T12-model2 Exchanged Site at Room Temperature

Exchanged energy ( $\Delta \mathrm{E}_{\text {exch }}$ ) of one- and two-ion exchanged CuZSM-5 were calculated and shown in Table 5.5, 5.10, and 5.15. For bare Cu ion species, these $\Delta \mathrm{E}_{\text {exch }}$ are positive which means that Cu should not exchaged to ZSM-5 as bare Cu ion. For one-ion exchanged $\mathrm{Cu}^{2+} \mathrm{ZSM}-5$, exchange at T6T12 has the lowest energy ( $9.19 \mathrm{kcal} / \mathrm{mol}$ ) while at T7T12-model1 has the highest ( $68.75 \mathrm{kcal} / \mathrm{mol}$ ). Although it has positive $\Delta \mathrm{E}_{\text {exch, }}$, exchange of $\mathrm{Cu}^{2+}$ at T 6 T 12 site is still possible. For two-ion exchanged $\mathrm{Cu}^{+} Z S M-5$, all exchanged sites have the $\Delta \mathrm{E}_{\text {exch }}$ exceeding $100 \mathrm{kcal} / \mathrm{mol}$, T6T12 has the lowest $\Delta \mathrm{E}_{\text {exch }} 143.29 \mathrm{kcal} / \mathrm{mol}$. This suggests that the two-ion exchanged $\mathrm{Cu}^{+} Z S M-5$, exchage is rather impracticle. The large $\Delta \mathrm{E}_{\text {exch }}$ is probably due to electrostatic repulsion between two bare $\mathrm{Cu}^{+}$ions. For single $\mathrm{H}^{+}$exchanged $\mathrm{Cu}^{+} Z S M-5, \Delta \mathrm{E}_{\text {exch }}$ with the range of $66.62-105.72 \mathrm{kcal} / \mathrm{mol}$ was obtained. The site with the lowest $\Delta \mathrm{E}_{\text {exch }}$ is T7T12-model1. Since the $\Delta \mathrm{E}_{\text {exch }}$ is very high, the exchange of $\mathrm{Cu}^{+}$is rather unstable and should not take place. The large $\Delta \mathrm{E}_{\text {exch }}$ again comes form the repulsion between $\mathrm{Cu}^{+}$and $\mathrm{H}^{+}$, this repulsion is less than between two $\mathrm{Cu}^{+}$ions.

### 6.6 Autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in ZSM-5 at T6T12, T8T8, T7T12-model1, and T7T12-model2 Exchanged Site at Room Temperature

Consider the autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in aqueous solution and in the cavity of ZSM-5 at various exchanged sited. In aqueous solution, taken account solvent effects, we found the autoreduction energy ( $\Delta \mathrm{E}_{\text {auto }}$ ) of $256.04 \mathrm{kcal} / \mathrm{mol}$. To compare $\Delta \mathrm{E}_{\text {auto }}$ in aqueous solution and $\Delta \mathrm{E}_{\text {auto }}$ in the exchanged sites that were listed in Table 5.16, it was found that $\Delta \mathrm{E}_{\text {auto }}$ for all exchanged sites; T6T12, T8T8, T7T12model1, and T7T12-model2, are lower than $256.04 \mathrm{kcal} / \mathrm{mol}$. And T7T12-model1 is
the lowest $\Delta \mathrm{E}_{\text {auto }}$ of $148.93 \mathrm{kcal} / \mathrm{mol}$. This suggests that the autoreductions of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$for all exchanged sites can be taken place easier than in aqueous solution.

When consider combined $\Delta \mathrm{E}_{\text {exch }}$ and $\Delta \mathrm{E}_{\text {auto }}$ (Table 5.17), most of the combined energies is smaller than $256.04 \mathrm{kcal} / \mathrm{mol}$. This was confirmed that conversion $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in the cavity of $\mathrm{ZSM}-5$ are easier than in aqueous solution. Still, the conversion at these sites is very low at room temperature.


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## CHAPTER 7

## CONCLUSIONS

### 7.1 Optimized Structures of HZM-5 and CuZSM-5 Exchanged Sites

When all exchanged sites are considered, $\mathrm{H}^{+}, \mathrm{Cu}^{2+}$, and $\mathrm{Cu}^{+}$are located in the cavity of ZSM-5. The distances are varied between $0.97-1.00 \AA$ for $\mathrm{O}-\mathrm{H}^{+}, 1.92-4.17 \AA$ for $\mathrm{Cu}^{2+}-\mathrm{O}$, and 1.75-5.33 $\AA$ for $\mathrm{Cu}^{+}-\mathrm{O}$. For all exchanged sites; T6T12, T8T8, T7T12model1, and T7T12-model2, Cu species form either mono- or bi- coordination.

### 7.2 Exchanged Energy of Cu Species in ZSM-5 at Room Temperature

The exchanged energies ( $\Delta \mathrm{E}_{\text {exch }}$ ) at all exchanged sites show positive values. Thus, it is very unlikely for Cu to be exchanged as bare Cu ion except for $\mathrm{Cu}^{2+}$ at T6T12 site, Exchange of $\mathrm{Cu}^{+}$for two-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 has the largest energy followed by one-ion exchanged $\mathrm{Cu}^{+}$ZSM-5 and one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5 has the lowest $\Delta \mathrm{E}_{\text {exch }}$. The instability of two- and one-ion exchanged $\mathrm{Cu}^{+} \mathrm{ZSM}-5$ is due to the repulsion between charged species, $\mathrm{Cu}^{+}-\mathrm{Cu}^{+}$and $\mathrm{Cu}^{+}-\mathrm{H}^{+}$, respectively. The most stable exchanged species is $\mathrm{Cu}^{2+}$ and the most favorable exchanged site is T6T12 with $\Delta \mathrm{E}_{\text {exch }}$ of $9.19 \mathrm{kcal} / \mathrm{mol}$.


Fig. 7.1 The most favorable exchange site; one-ion exchanged $\mathrm{Cu}^{2+}$ ZSM-5(T6T12).

### 7.3 Autoreduction of $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$in ZSM-5 at T6T12, T8T8, T7T12-model1, and T7T12-model2 Exchanged Site at Room Temperature

Autoreduction energy ( $\Delta \mathrm{E}_{\text {auto }}$ ) of $\mathrm{Cu}^{2+}$ show high positive value for both in aqueous solution ( $256.04 \mathrm{kcal} / \mathrm{mol}$ ) and in the cavity of ZSM-5. Thus, the autoreduction of bare $\mathrm{Cu}^{2+}$ to $\mathrm{Cu}^{+}$should not take place. However, the $\Delta \mathrm{E}_{\text {auto }}$ for all exchanged sites; T6T12, T8T8, T7T12-model1, and T7T12-model2, are lower than $256.04 \mathrm{kcal} / \mathrm{mol}$. This suggests that the autoreduction of $\mathrm{Cu}^{2+}$ in ZSM-5 is more competitive than in aqueous solution. When consider combined energy between $\Delta \mathrm{E}_{\text {exch }}$ and $\Delta \mathrm{E}_{\text {auto }}$, most of the combined energy is still smaller than $256.04 \mathrm{kcal} / \mathrm{mol}$. The exchange of $\mathrm{Cu}^{2+}$ and conversion to $\mathrm{Cu}^{+}$in $\mathrm{ZSM}-5$ is competitive than in aqueous. However, the autoreduction of $\mathrm{Cu}^{2+}$ in $\mathrm{ZSM}-5$ at room temperature, if exists, should take other routes.

## Suggestions for Future Work

For the characterization of copper active sites in the cavity of ZSM-5, exchange of other copper species such as $\left[\mathrm{Cu}\left(\mathrm{H}_{2} \mathrm{O}\right)_{\mathrm{n}}\right]^{2+},[\mathrm{Cu}-\mathrm{O}-\mathrm{Cu}]^{2+}$, etc., should be investigated. Furthermore, the mechanism of $\mathrm{NO}_{\mathrm{x}}$ decomposition by the Cu species should be also investigated.


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## REFERENCES

1. Breck, D. W. Zeolite Molecular Sieves: Structure, Chemistry, and Use. Florida: Robert E. Krieger Publishing Company, 1973.
2. Meimer, W. M.; Olson, D. H. Atlas of Structure Type. London: ButterworthHeinemann, 1992.
3. Dyer, A. An Introduction to Zeolite Molecular Sieves. NewYork: John Wiley \& Sons., 1988.
4. Olson, D. H.; Kokotailo, G. T.; Lawton, S. L.; Meier, W. M. "Crystal Structure and Structure-Related Properties of ZSM-5", J. Phys. Chem., 1981, 85, 2238-2243.
5. Sayle, D. C.; Catlow, C. R. A.; Gale, J. D.; Perrin, M. A.; Nortier, P. "Computer Modeling of the Active-Site Configurations within the NO Decomposition Catalyst Cu-ZSM-5", J. Phys. Chem. A., 1997, 101, 3331-3337.
6. Yongsheng, L; Jinqu, W; Jianlin, S; Xiongfu, Z; Jinming, L; Zhongying, B; Dongsheng, Y. "Synthesis of ZSM-5 zeolite membranes with large area on porous, tubular $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}$ supports", Sep. Purif. Technol., 2003, 32, 397-401.
7. Atsushi, I; Yasushige, K; Mahiko, N. "Elucidation of a preparation method for copper on-exchanged ZSM-5 Samples exhibiting extremely efficient $\mathrm{N}_{2}$-adsorption at room temperature: effect of counter ions in the exchange solution", Micropor. Mesopor. Mater., 2004, 70, 119-126.
8. Trout, B. L.; Chakraborty, A. K.; Bell, A. T. "Local Spin Density Functional Theory Study of Cupper Ion-exchanged ZSM-5", J. Phys. Chem., 1996,100, 4173-4179.
9. $\mathrm{Hu}, \mathrm{S}$.; Reimer, J. A.; Bell, A. T. ${ }^{655} \mathrm{Cu}$ NMR Spectroscopy of Cu -exchanged ZSM-5 Catalysts", J. Phys. Chem. B, 1997, 101, 1869-1871.
10. Larsen, S.C.; Aylor, A.; Bell, A. T.; Reimer, J. A. "Electron Paramagnetic Resonance Studies of Copper Ion-Exchanged ZSM-5", J. Phys. Chem., 1994, 98, 11533-11540
11. Teraishi, K.; Ishida, M.; Irisawa, J.; Kume, M.; Takahashi, Y.; Nakano, T.; Nakamura, H.; Miyamoto, A. "Active Site Structure of Cu/ZSM-5: Computational Study", J. Phys. Chem. B, 1997, 101, 8079-8085.
12. Tonkyn, R. G; Barlow, S. E; Hoard, J. W. " Reduction of $\mathrm{NO}_{\mathrm{x}}$ in synthetic diesel exhaust via two-step plasma-catalyst treatment", Appl. Catal. B: Environ., 2003, 40, 207-217.
13. Shelef, M. "Selective Catalytic Reduction of NOx with N-free Reductants", Chem. Rev., 1995, 95, 209-225.
14. Roberge, D; Raf, A; Kaliaguine, S; Trong On, D; Iwamoto, S. "Selective Catalytic reduction of NO under ambient conditions using ammonia as reducing agent and MFI zeolites as catalyst", Appl. Catal. B: Environ., 1996, 10, L237-L243.
15. Iwamoto, M.; Yahiro, H.; Tanada, K.; Mozino, Y.; Mine, Y.; Kagawa, S. "Removal of Nitrogen Monoxide through a Novel Catalytic Process. 1. Decomposition on Excessively Copper Ion Exchanged ZSM-5 Zeolites", J. Phys. Chem., 1991, 95, 3727-3730.
16. Iwamoto, M.; Yahiro, H.; Mizuno, N.; Zhang, W.; Mine, Y.; Furukawa, H.; Kagawa, S. "Removal of Nitrogen Monoxide through a Novel Catalytk Process. 2. Infrared Study on Surface Reaction of Nltrogen Monoxide Adsorbed on Copper Ion-Exchanged ZSM-5 Zeolltes", J. Phys. Chem., 1992, 96, 9360-9366.
17. Iwamoto, M.; Furukawa, H.; Kagawa, S. In New Developments in Zeolite Science and Technology. Amsterdam: Elsevier, 1986.
18. Warren, J. H; Leo, R; Paul, v.R. S; John, A.P. Ab initio molecular orbital theory. New York: John Wiley \& Sons., 1985.
19. Mark, A. R; Geoge, C. S. Introduction to quantum mechanics in chemistry. Prentice: New Jersey, 2000.
20. Szabo, A.,Ostlund, N. S. "Modern Quantum Chemistry: Introduction to a 9 Advanced Electronic Structure Theory", New York: MacMillan
21. Slater, J. C. "A Simplification of the Hartree-Fock Method", Phys. Rev., 1951, 81, 385-390.
22. Hehre, W. J., Ditchfield, R., Pople, J. A. "Self-Consistent Molecular Orbital Methods. XII. Further Extensions of Gaussian-Type Basis Sets for Use in Molecular Orbital Studies of Organic Molecules", J. Chem. Phys., 1972, 56, 2257-2261.
23. Kohn, W.; Becke, A. D.; Parr, R. G. "Density Functional Theory of Electronic Structure", J. Phys. Chem., 1996, 100, 12974-12980.
24. Dirac, P. A. M. "Note on Exchange Phenomena in the Thomas Atom", Proc. Camb. Phil. Soc., 1930, 26, 376-385.
25. Hohenberg, P; Kohn, W. "Inhomogeneous Electron Gas", Phys. Rev., 1964, 136, B864-B871.
26. Kohn, W.; Sham, L. J. "Self Consistent Equations Including Exchange and Correlation Effects", Phys. Rev., 1965, 140,A1133-A1138.
27. Vosko, S. J., Wilk, L., Nusair, M. "Accurate Spin-Dependent Electron Liquid Correlation Energies for Local Spin Density Calculations: A Critical Analysis", Can. J. Phys., 1980, 58, 1200-1211.
28. Becke, A. D. "Density Functional Calculations of Molecular Bond Energies", J. Chem. Phys., 1986, 84, 4524-4529.
29. Perdew, J. P., Wang, Y. "Accurate and Simple Density Functional for the Electronic Exchange Energy: Generalized Gradient Approximation", Phys. Rev. B, 1986, 33, 8800-8802.
30. Becke, A. D. "A Multicenter Numerical Integration Scheme for Polyatomic Molecules", J. Chem. Phys., 1988, 88, 2547-2553.
31. Lee, C., Yang, W., Parr, R. G. "Development of the Colle-Salvetti CorrelationEnergy Formula into a Functional of the Electron Density", Phys. Rev. B, 1988, 37, 785-789.
32. Perdew, J. P., Wang, Y. "Accurate and Simple Analytic Representation of the Electron Gas Correlation Energy", Phys. Rev. B., 1992, 45, 13244-13249.
33. Perdew, J.P., Burke, K., Ernzerhof, M. "Generalized Gradient Approximation Made Simple", Phys. Rev. Lett., 1996, 77, 3865-3868.
34. Frisch, M. J.; Trucks, G. W.; H. B. Schlegel, G. E.S.; Robb, M. A.; Cheeseman, J. R.; Zakrzewski, V. G.; Montgomery, J. A.; Stratmann, R. E.; Burant, J. C.; Dapprich, S.; Millam, J. M.; Daniels, A. D.; Kudin, K. N.; Strain, M. C.; Farkas, O.; Tomasi, J.; Barone, V.; Cossi, M.; Cammi, R.; Mennucci, B.; Pomelli, C.; Adamo, C.; Clifford, S.; Ochterski, J.; Petersson, G. A.; Ayala, P. Y.; Cui, Q.; Morokuma, K.; Malick, D. K.; Rabuck, A. D.; Raghavachari, K.; Foresman, J. B.; Cioslowski, J.; Ortiz, J. V.; Stefanov, B. B.; Liu, G.; Liashenko, A.; Piskorz, P.;

Komaromi, I.; Gomperts, R.; Martin, R. L.; Fox, D. J.; Keith, T.; Al-
Laham, M. A.; Peng, C. Y.; Nanayakkara, A.; Gonzalez, C.; Challacombe, M.; Gill, P. M. W.; Johnson, B. G.; Chen, W.; Wong, M. W.; Andres, J. L.; Head-Gordon, M.; Replogle, E. S.; Pople, J. A. Gaussian 98 (ReVision A.7) ( Theory and Calculation). Pittsburgh: PA, 1998.
35. Poompub, U. Structures and Acidities of H-ZSM-5 with Si/Al Ratio of 47 and 95.
(Master's Thesis, Petrochemistry and Polymer Science, Faculty of
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