# CHAPTER II

## LITERATURE REVIEWS

In this chapter, literatures involving in this study, the use of Monte Carlo simulation for oil reserve calculation, statistical relationship between input variables, and correlation of data of an input variable in space will be presented.

Monte Carlo method has been developed for solving several probabilistic and deterministic problems for a long time. Hammersley and Handscomb (1964) stated that Monte Carlo method was developed in about 1944 and the real use of Monte Carlo method as a research tool stems from work on the atomic bomb during the second world war. This work involved a direct simulation of the probabilistic problems concerned with random neutron diffusion in fissile material. For deterministic problems the possibility of applying Monte Carlo methods to this subject was noticed by Fermi, von Neumann, and Ulam and popularized by them in the immediate post-war years (Hammersley and Handscomb, 1964). About 1948 Fermi, Metropolis, and Ulam obtained Monte Carlo estimates for the eigenvalues of the Schrodinger equation. The intensive study of Monte Carlo methods was done in the 1950s, particularly in the United State of America. During the last few years Monte Carlo methods have come back into favor. This is mainly due to better recognition of problems in which it is the best, and sometimes the only, available solving technique.

The oil reserve estimation using Monte Carlo simulation has been developed more than 20 years. Several investigators especially in Mining and Petroleum engineering field have presented articles and publishing text books on this topic showing its principle and the procedure for assessing uncertainty of oil reserve using Monte Carlo simulation. Literatures involved Monte Carlo method for oil reserve calculation are presented as follows.

Walstrom, Mueller and McFarlane(1967) demonstrated the method to evaluate uncertainty in petroleum engineering using Monte Carlo simulation. They stated that the uncertainty of an input variable may be resulted from difficulty in directly and accurately measuring the quantity. The uncertainty in the value of a variable may be indicated by a probabilistic description accomplished by expressing the quantity by a probability distribution. The Monte Carlo simulation technique consists of mathematical simulating and experiment to determine the probability distribution for a complicated expression involving one or more input variables, each of which has its associated uncertainty. The experiment uses a random sampling of the input variables distributions involved in the expression being studied. The sampling procedure operates in the following way for expressions involving only independent random variables. A value of each input random variable appearing in the expression is selected from its respective probability distribution. This set of values is then substituted into the expression being studied, and an output value is computed. Subsequent values of output are obtained by repeating the simulation process with additional sets of randomly sampled values of the input variables.

In addition to the evaluation of uncertainty in oil reserve calculation from the volumetric method, they also presented examples to illustrate the evaluation of uncertainty in other engineering calculations such as uncertainty in the calculated value of water saturation from log data, uncertainty in a recovery factor determination using a material balance calculation.



Other investigators also presented the use of Monte Carlo method for oil reserve calculation and for other aspects in the same manner as

Walstrom et al.'s as follows.

Pritchard (1970) demonstrated a technique, including Monte Carlo simulation and history matching, for evaluating hydrocarbon pore volume in an environment of uncertainty through a case history study of a specific Rainbow-Zama pool in Alberta.

McCray (1975) presented the Monte Carlo method in his text book. He stated that there are several types of solutions in mining and reservoir engineering that have been obtained by simulation such as estimates probability distributions of rates of return, probability distributions of the ultimate recovery from an oil reservoir, determination of the most profitable number of parallel facilities, and so on. He also presented examples of calculations using Monte Carlo method such as calculating reservoir porosity and water saturation from well logs, calculating a recovery factor, petroleum recovery from a reservoir, and rate of return from a business venture.

From above literatures, the oil reserve estimation using Monte Carlo method was presented in the same manner, the input variables will be selected randomly by random numbers and substituted into the volumetric expression for oil reserve calculation. The calculation of oil reserve will be performed repeatedly to constructing probability density function of calculated oil reserve.

However, block system has not been introduced into the calculation and presented in any literatures. There was no consideration about division of a reservoir into several blocks. In this study consideration of block system will be taken into account so that all available information can be efficiently utilized. This is an improvement of oil reserve calculation using Monte Carlo simulation.

The second improvment is the consideration of statistical relationship between input variables. In some case there may be statistical relationship between input variables. So, the value of an input variable should not be assigned randomly by a random number, but should be corresponding to the one it is related to. Many works were done to investigate statistical relationship between input variables.

Walstrom et al. (1967) presented a procedure to incorporate statistical relationship between any of the variables in the problem by assigning three curves as shown in Figure 2.1. These three curves indicate the minimum, the most likely, and the maximum relationships between the area and thickness (whereas area is an independent variable and thickness is a dependent variable of the statistical relationship in this example) that are believed to exist in the problem. If the statistical relationship between these two variables were known with greater precision, the three curves would be closer together. Certainty in this statistical relationship would be represented by a single curve.

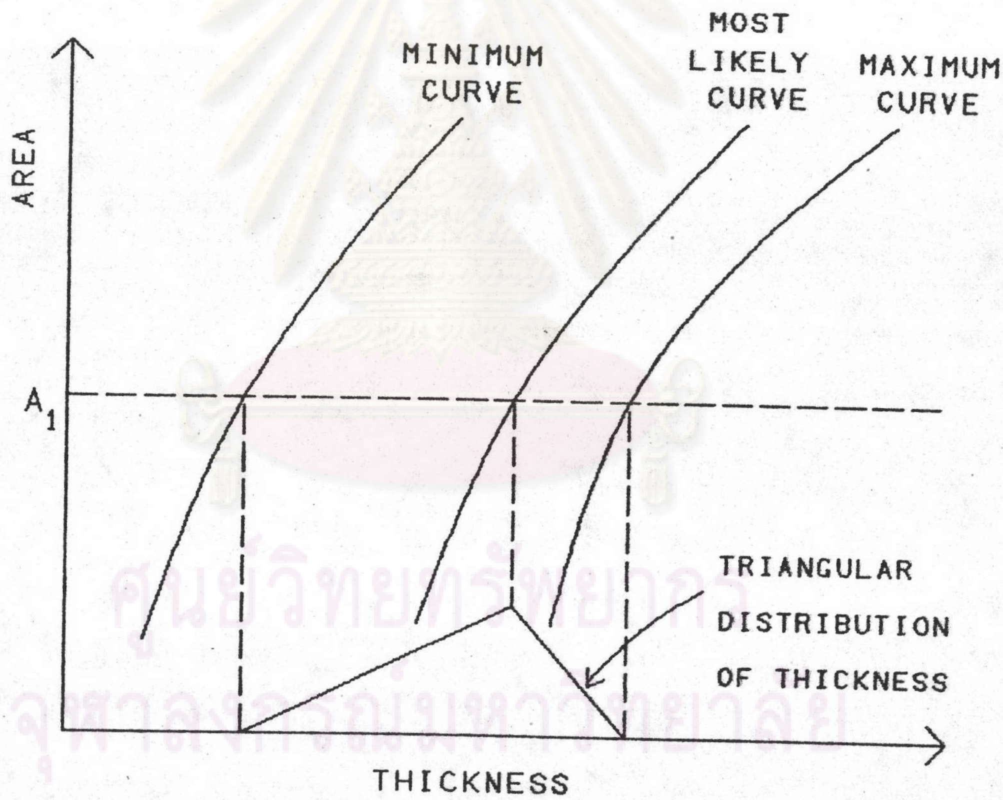




Figure 2.1. Example of dependency of thickness on area.

In the simulation procedure a value of area is sampled at random, and labeled $A_1$ (in Figure 2.1). $A_1$ defines a range of values of thickness for a one case. A value of thickness is then chosen at random from the constructed distribution (being triangular distribution in this case).

McCray (1975) also presented this statistical relationship in his textbook in the same manner as that of Walstrom et al. (1967). To simulate this statistical relationship, it was neccesary to generate two random numbers. The first was used to select the choice of area and the second was use to select the choice of thickness.

In this study beside development of a computer program for evaluating uncertainty of oil reserve using Monte Carlo simulation technique, the incorporation of statistical relationship between two input variables, porosity and water saturation will also be performed in different manner from that of Walstrom et al. (1967). In addition, the studies of effect of this statistical relationship on obtained oil reserve will be also investigated.

In addition to the incorporation of block system and statistical relationship between input variables, statistical relationship of values of an input variable in space called spatial correlation will also be considered in this study. The spatial correlation has already been wellknown in rainfall-runoff and other hydrologic models. To the best of the author's knowledge incorporation of the spatial correlation in the oil reserve calculation has not been considered elsewhere. Incorporation of the spatial correlation in reserve calculation will be an improvement over previous studies.

The spatial correlation of an input variable can be taken into account in any problem by using various techniques. In this study turning bands method will be used to simulate values of an input variable due to less computational cost. The development of turning bands method and spectral turning bands method has been presented in several literatures and is repeated here as follows.

The turning bands method for simulation of stationary random fields was originated by Matheron (Journel and Huijbregts, 1978). The turning bands method is one such algorithm which generates multi-dimensional fields by combining values found from a series of one-dimensional simulations along lines radiating outward from a coordinate origin.

Shinozuka and Jan (1972) presented efficient methods for digital simulation of a general homogeneous process (multi-dimensional or multivariate or multivariate-multidimensional) as a series of cosine functions with weighted amplitudes, almost evenly spaced frequencies, and random phase angles. In this work Shinozuka and Jan stated that a random process (random variable) can be simulated by a series of cosine functions with random frequency (or wave number).

Mejia and Rodriguez-Iturbe (1974) presented a procedure for the synthesis of processes (or random variables) corresponding their covariance function. The method involves the addition of harmonics of random frequencies that are sampled from the spectral density function or the radial spectral density function.

Journel and Huijbregts (1978) developed the turning bands method which allows a three-dimensional data set with specified covariance to be obtained by the simulation of several one-dimensional realizations which have an intermediate covariance.

Mantoglou and Wilson (1982) presented the turning bands method for the simulation of multi-dimensional random field based on a spectral approach. The general turning bands method equations for two- and three-dimensional field are derived with particular emphasis on the more complicated two-dimensional case. For stationary two-dimensional fields the uni-dimensional line process is generated by a simple spectral method, a technique which can be generally applied to any two-dimensional covariance function. Guidelines are presented for the selection of model parameters which will be helpful in the design of simulation experiments. The turning bands method was compared to other methods in terms of cost and accuracy, demonstrating that the turning bands method is as accurate as and much less expensive than multi-dimensional spectral techniques and more accurate than the most expensive approaches which use matrix inversion, such as the nearest neighbor approach.

Mantoglou (1987) presented digital simulation of multivariate two- and three-dimensional stochastic processes with a spectral turning bands method. His paper extends the spectral turning bands method even further for simulation of much more general classes of multi-dimensional stochastic processes. Particular extensions include: simulation of three-dimensional processes, simulation of anisotropic two- or three-dimensional stochastic processes, simulation of multivariate stochastic processes, and simulation of spatial averaged (integrated) processes.

Krajewski and Duffy (1988) presented a simulation study to estimate correlation of structure for a homogeneous isotropic random field using spectral turning bands method. They investigated the effect of sampling density on the estimation of the covariance and semivariogram for homogeneous, isotropic, random fields.

Tompson, Ababou, and Gelhar (1989) presented implementation of the three-dimensional turning bands random field generator. They investigated about line process generation and suggested appropriate values of harmonic number and frequency range involving spectral turning bands method.

In this study spectral turning bands method will be developed to generate values of random variables with desired covariance function (exponential model) for oil reserve calculation using Monte Carlo simulation. The effects of spatial correlation of an input variable on calculated oil reserve will be also studied.