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APPENDIX

<u>Theorem 2.3.</u> Let A be an ideal of an inverse semigroup S and B be an ideal of A. Then B is an ideal of S.

<u>Proof.</u> Let $b \in B$ and $s \in S$. Then $b \in A$ since $B \subseteq A$. Since A is an ideal of S, $b^{-1}bs$ and sbb^{-1} belong to A. Thus $bs = bb^{-1}bs$ and $sb = sbb^{-1}b$ belong to B since B is an ideal of A. Thus B is an ideal of S. #

<u>Theorem 2.4.</u> Let S be a semigroup and $A \subseteq B \subseteq S$ such that A and B are completely prime ideals of S. Then $B \setminus A$ is either a completely prime ideal of $S \setminus A$ or an empty set.

<u>Proof.</u> Assume that $B \setminus A \neq \emptyset$. Let $x \in B \setminus A$ and $y \in S \setminus A$. Then $xy, yx \in B$ since B is an ideal of S. If $xy \in A$ or $yx \in A$ then $x \in A$ or $y \in A$ since A is a completely prime ideal of S, which is a contradiction. Thus $xy, yx \in B \setminus A$. Hence $B \setminus A$ is an ideal of $S \setminus A$.

To show $B \setminus A$ is completely prime, let $x, y \in S \setminus A$ such that $xy \in B \setminus A$. Since B is a completely prime ideal, $x \in B$ or $y \in B$. If $x \in A$ or $y \in A$, then $xy \in A$ since A is an ideal, a contradiction. Thus $x \in B \setminus A$ or $y \in B \setminus A$. Hence $B \setminus A$ is a completely prime ideal of $S \setminus A$. #

<u>Theorem 2.5.</u> For a subset A of a semigroup S, A is a filter of S if and only if $S \setminus A$ is either a completely prime ideal of S or an empty set.

<u>Proof.</u> Assume that A is a filter of S and $S \setminus A \neq \emptyset$. Let $x \in S \setminus A$ and $y \in S$. If $xy \in A$, then $x, y \in A$ since A is a filter of S, which is a contardiction. Therefore $xy \in S \setminus A$. Similarly, we can be show that $yx \in S \setminus A$.

Now we let $x,y \in S$ such that $xy \in S \setminus A$. If $x \in A$ and $y \in A$ then $xy \in A$, since A is a subsemigroup, which is a contradiction. Thus $x \in S \setminus A$ or $y \in S \setminus A$. Hence $S \setminus A$ is a completely prime ideal.

Conversely we assume that $S \setminus A$ is a completely prime ideal of S or an empty set. If $S \setminus A = \emptyset$., then $A = S$ which is a filter of S. Assume $S \setminus A$ is a completely prime ideal of S and $x,y \in S$ such that $xy \in S \setminus A$. If $x,y \in A$, then $xy \in A$ since A is a subsemigroup of S, a contradiction. Hence $x \in S \setminus A$ or $y \in S \setminus A$. #

<u>Theorem 2.6</u>. Let $(S,\circ)$ and $(S', \circ')$ be disjoint semigroup. Then $S \cup S'$ with a binary operation * defined by

$$x * y = x \circ y \quad \text{if } x,y \in S$$

$$x * y = x \circ' y \quad \text{if } x,y \in S'$$

$$x * y = y * x = x \quad \text{if } x \in S \text{ and } y \in S'$$

is a semigroup.

<u>Proof</u>. Let x,y,z belong to $S \cup S'$. We shall show that $(x*y)*z = x*(y*z)$. This is clearly true for the case $x,y,z \in S$ or $x,y,z \in S'$. Since $a * b = b * a$ for all $a \in S$, $b \in S'$, it remains to show that this holds for the case $x,y \in S$, $z \in S'$ and the case $x \in S$, $y,z \in S'$. If $x,y \in S$ $z \in S'$, then

$$(x * y) * z = x * y = x * (y * z).$$

If $x \in S$, $y,z \in S'$, then

$$(x * y) * z = x * z = x ,$$

$$x * (y * z) = x,$$

so

$$(x * y) * z = x * (y * z).$$

#

Lemma 2.7. Let S be an inverse semigroup, F be a field of characteristic different from 2 such that $a^2 \neq -1$ for any a in F. Let h be a homomorphism from S into $C(F)^*$. Then for each x in S, $\frac{h(x) - h(x^{-1})}{2i}$ and $\frac{h(x) + h(x^{-1})}{2}$ belong to F if and only if h(x) belongs to $\Delta(F)$.

Proof. Let Ψ be the automorphism of C(F) fixing all elements of F and taking i into -i.

Assume that $h : S \to C(F)^*$ is a homomorphism such that $\frac{h(x) - h(x^{-1})}{2i}$ and $\frac{h(x) + h(x^{-1})}{2}$ belong to F. Therefore

$$\frac{h(x) + h(x^{-1})}{2} = \Psi\left(\frac{h(x) + h(x^{-1})}{2}\right) = \frac{\Psi(h(x)) + \Psi(h(x^{-1}))}{2}.$$

Thus

$$(1) \qquad h(x) + h(x^{-1}) = \Psi(h(x)) + \Psi(h(x^{-1})).$$

Also,

$$\frac{h(x) - h(x^{-1})}{2i} = \Psi\left(\frac{h(x) - h(x^{-1})}{2i}\right) = \frac{\Psi(h(x)) - \Psi(h(x^{-1}))}{2i}.$$

Thus

$$(2) \qquad h(x) - h(x^{-1}) = \Psi(h(x^{-1})) - \Psi(h(x)).$$

If follows from (1),(2) that $2h(x) = 2\Psi(h(x^{-1}))$. Hence $h(x)\ \Psi(h(x)) = 1$. Therefore h(x) belongs to $\Delta(F)$.

Conversely, assume that h is a homomorphism from S into $\Delta(F)$. Hence $h(x)\ \Psi(h(x)) = 1$ for all x in S. It follows that

$$\Psi(h(x)) = h(x)^{-1} = h(x^{-1}).$$

Let $h(x) = a + bi$ where $a,b \in F$. Therefore,

$$\frac{h(x) + h(x^{-1})}{2} = \frac{h(x) + \Psi(h(x))}{2} = \frac{(a+bi) + (a-bi)}{2} = a$$

and

$$\frac{h(x) - h(x^{-1})}{2i} = \frac{h(x) - \Psi(h(x^{-1}))}{2i} = \frac{(a+bi) - (a-bi)}{2i} = b$$

Hence $\frac{h(x) + h(x^{-1})}{2}$ and $\frac{h(x) - h(x^{-1})}{2i}$ belong to F. #

VITA

Name : Miss Prisana Sayvarin.

Degree : B.S. (Education), Hons, 1981.
Chiangmai University, Chiangmai, Thailand.

Scholarship : University Development Commission (U.D.C.),
Thai Government.