CHAPTER III

THE SUPERCONDUCTING PROXIMITY EFFECT

McMillan (McMillan,1968) studied a theoretical model of the proximity effect between superposed normal (N) and superconducting (S) metal films. He imagined a potential barrier between the films and treated an electron transmission through the barrier as a tunneling process. In the McMillan model :

(a) The tunneling Hamiltonian is used to describe the penetration of electron through the barrier. This restricts the transmission probability of the barrier to be much less than one.

(b) Both N and S films are thin compared with the characteristic superconducting length so that the properties of each film are uniform across its thickness.

(c) Tunneling matrix elements $V_{kk'}$ is taken to be of equal magnitude between energy states in S and N .

Mohabir and Nagi (Mohabir and Nagi, 1979) extended the McMillan tunneling model (MTM) of the superconducting proximity effect by treating the tunneling Hamiltonian to all order of self-consistent perturbation theory. They considered the normal metal in MTM as weak superconductor and treated their system as superconductor-superconductor sandwiches. In this review, we will consider the model of Mohabir and Nagi for S_1-S_2 sandwiches.

The Hamiltonian for the sandwich is the sum of the Hamiltonians for S_1 and S_2 slabs and the tunneling Hamiltonain.

$$H = H_{S_1} + H_{S_2} + H_T \tag{3.1}$$

where $H_{S_1}(H_{S_2})$ is the Hamiltonain of the 1st (2nd) superconducting metal and H_T is the tunneling Hamiltonian.

$$H_{S_1} = \sum_{k\sigma} \varepsilon_{k1} a^{+}_{k\sigma} a_{k\sigma} - \Delta_1 \sum_{k} (a^{+}_{k\uparrow} a^{+}_{-k\downarrow} + \text{h.c.}) \tag{3.2.1}$$

$$H_{S_2} = \sum_{k\sigma} \varepsilon_{k2} d^{+}_{k\sigma} d_{k\sigma} - \Delta_2 \sum_{k} (d^{+}_{k\uparrow} d^{+}_{-k\downarrow} + \text{h.c.}) \tag{3.2.2}$$

$$H_T = \sum_{kk'\sigma} [V_{kk'} a^{+}_{k\sigma} d_{k'\sigma} + \text{h.c.}] \tag{3.2.3}$$

where $a^{+}_{k\sigma}$ ($d^{+}_{k\sigma}$) is the creation operator for electron in the $S_1(S_2)$ side, σ is the spin index, and $V_{kk'}$ is the tunneling matrix element, we take $V_{kk'}$ to be independent of k and k'.ε_{k1} (ε_{k2}) is the band energy of the conduction electron in the $S_1(S_2)$ side measured from the Fermi energy. Δ_1 (Δ_2) is the BCS superconductng order parameter in the $S_1(S_2)$ side.

In order to calculate Δ, we introduce the finite-temperature Green's function.

$$G(k, \omega_n) = -\langle T_\tau \psi_k(\tau) \psi^{+}_k(0) \rangle \tag{3.3}$$

where $\psi_k^{+} = (a_{k\uparrow}^{+} \quad a_{-k\downarrow})$ for S_1 side and T_τ is the time ordering operator for the imaginary time $\tau = it$.

We find a single particle Green's function for pure S_1 superconductor $G_1^{\circ}(k, \omega_n)$ as

$$G_1^{\circ}(k, \omega_n) = [i\,\omega_{1\,n} - \varepsilon_{k1}\tau_3 + \Delta_1\,\tau_1]^{-1} \tag{3.4}$$

where τ_i (i=1,2,3) are Pauli matrices.

Treating the electron-phonon interaction and the tunneling Hamiltonian in second order self-consistent perturbation, we find for the Nambu matrix-self-energy of the S_1 side as

$$\Sigma_1(i\omega_n) = V^2\,\tau_3 \sum_k G_1 (k, \omega_n)\;\tau_3 \tag{3.5}$$

where V is the tunneling magnitude.

The self-energy for S_2 side $\Sigma_2 (i\omega_n)$ can be obtained from Eq.(3.5) by interchanging the labels 1 and 2.

We make the ansatz

$$G (k, \omega_n) = [i\,\tilde{\omega}_n - \varepsilon_k\,\tau_3 + \tilde{\Delta}\,\tau_1]^{-1} \tag{3.6.1}$$

here $G(k, \omega_n)$ is the renormalized Green's function which includes the proximity effect. $\tilde{\omega}_n$ and $\tilde{\Delta}$ are renormalized frequency and renormalized order parameter, respectively.

We find the usual form for matrix Green's function

$$G(k, \omega_n) \quad = \quad -[\, i\,\tilde{\omega}_n + \varepsilon_k \tau_3 - \tilde{\Delta}\tau_1] / [\, \tilde{\omega}_n^{2} + \varepsilon_k^{2} + \tilde{\Delta}^2 \,] \qquad (3.6.2)$$

Using the Dyson's equation, the self-energy is related to the Green's functions,we have

$$\Sigma_1(i\omega_n) \quad = \quad i\,(\omega_{1n} - \tilde{\omega}_{1n}) + (\Delta_1 - \tilde{\Delta}_1)\,\tau_1 \qquad (3.7)$$

Substitution of Eq.(3.6.2) into Eq.(3.5) and performing the sum over states, we find

$$\Sigma_1(i\omega_n) \quad = \quad -V^2 A d_2 N_2(0)\pi\, [(\, i\,\tilde{\omega}_{2n} + \tilde{\Delta}_2\, \tau_1)/\sqrt{\tilde{\omega}_{2n}^{\,2} + \tilde{\Delta}_2^{\,2}}\,] \qquad (3.8)$$

where A, d_2, and $N_2(0)$ are the area, thickness and bulk density of states (per unit volume) of the second superconductor.

By comparing Eq.(3.7) and (3.8), we obtain the following equations.

$$\tilde{\omega}_{1n} \quad = \quad \omega_{1n} + \Gamma_1 \tilde{\omega}_{2n}\, [\sqrt{\tilde{\omega}_{2n}^{\,2} + \tilde{\Delta}_2^{\,2}}\,]^{-1} \qquad (3.9.1)$$

$$\tilde{\Delta}_1 \quad = \quad \Delta_1 + \Gamma_1 \tilde{\Delta}_2\, [\sqrt{\tilde{\omega}_{2n}^{\,2} + \tilde{\Delta}_2^{\,2}}\,]^{-1} \qquad (3.9.2)$$

In these equations $\Gamma_1 \ = \ V^2 A d_2 N_2(0)\pi$.

Similarly for the second superconductor, we can also obtain the equations for $\tilde{\omega}_{2n}$ and $\tilde{\Delta}_2$ by changing the labels 1 and 2 , and define $\Gamma_2 \ = \ V^2 A d_1 N_1(0)\pi$.

The solution of the self-energy can be solved by defining a new parameter.

$$U_n \quad = \quad \tilde{\omega}_n / \tilde{\Delta} \tag{3.10}$$

Then, Eqs.(3.9.1) and (3.9.2) can be combined as

$$U_1 \quad = \quad [1/\Delta_1] \; [\; \omega_{1n} + \; \Gamma_1(U_2 - U_1) \; [\sqrt{1+ U_2^2} \;]^{-1} \;] \tag{3.11.1}$$

Similarly , we can show that

$$U_2 \quad = \quad [1/\Delta_2] \; [\; \omega_{2n} + \; \Gamma_2(U_1 - U_2) \; [\sqrt{1+ U_1^2} \;]^{-1} \;] \tag{3.11.2}$$

The self-consistent equation for the order parameter is

$$\Delta_1 \quad = \quad 2\pi\lambda_1 T \sum_{n\geq 0}^{\omega_{D1}/2\pi T} \tilde{\Delta}_1 [\sqrt{\tilde{\omega}_{1n}{}^2 + \tilde{\Delta}_1{}^2} \;]^{-1} \tag{3.12}$$

where $\lambda_1 \ = \ g_1 \, A \, N_1(0) d_1$.

Substitution of Eq.(3.10) into (3.12) and taking the limit that near T_c, Δ_1 and Δ_2 are small and $U_1, U_2 >> 1$, we get

$$\Delta_1 = 2\pi\lambda_1 T \sum_{n\geq 0}^{\omega_{D1}/2\pi T} [U_1]^{-1} \tag{3.13}$$

Further, Eqs.(3.11.1) and (3.11.2) give

$$U_1 = [1/\Delta_1] \{ \omega_n(\Gamma_1+\Gamma_2+\omega_n) [\Gamma_1(\Delta_2/\Delta_1) + \Gamma_2+\omega_n]^{-1} \} \tag{3.14}$$

Eqs.(3.13) and (3.14) give

$$\Delta_1 = \lambda_1 \{ \Delta_1 f_1(T) - \Gamma_1(\Gamma_1+\Gamma_2)^{-1} [\Delta_1-\Delta_2] K_1(\rho) \} \tag{3.15.1}$$

where

$$f_1(T) = 2\pi T \sum_{n\geq 0}^{\omega_{D1}/2\pi T} [\omega_n]^{-1} = \ln(2\gamma\, \omega_{D1}/\pi T) \tag{3.15.2}$$

and

$$K_1(\rho) = 2\pi T \sum_{n\geq 0}^{\omega_{D1}/2\pi T} \{ (\omega_n)^{-1} - (\omega_n+\Gamma_1+\Gamma_2)^{-1} \}$$

$$= \psi(-1/2 + \rho/2) - \psi(-1/2) - \ln(1 - \Gamma/\omega_{D1}) \tag{3.15.3}$$

where $\Gamma = \Gamma_N + \Gamma_S$, $\gamma = 1.781$, $\rho = \Gamma/\pi T$ and ψ is the digamma function (Davis,1965).

The equation for Δ_2 and U_2 are obtained from the above equations by interchanging the labels 1 and 2.

From Eq.(3.15.1) we set up an equation for Δ_2, Eliminating Δ_1 / Δ_2 from these two equations, we finally get

$$\lambda_1\lambda_2 f_1(T) - f_1(T) \{ \lambda_1 + \lambda_2 - \lambda_1\lambda_2\chi + \lambda_1\lambda_2 [\gamma_1 K_1(\rho) + \gamma_2 K_2(\rho)] \}$$

$$+ [1 - \chi\lambda_2] [1 + \lambda_1\gamma_1 K_1(\rho)] + \lambda_2\gamma_2 K_2(\rho) = 0 \qquad (3.16)$$

where $\chi = \ln(\omega_{D2}/\omega_{D1})$, $\gamma_1 = \Gamma_1/\Gamma$, $\gamma_2 = \Gamma_2/\Gamma$.

For a superconductor-normal metal sandwich, we can make the assumption that $\lambda_2 = 0$. Eq.(3.16) gives the transition temperature of an S-N sandwich as

$$\ln(T^{\circ}_{cS}/T_c) = \Gamma_N/\Gamma \; [\; \psi(-1/2 + \Gamma/2\pi T_c) - \psi(-1/2) \;] \qquad (3.17)$$

where T°_{cs} is the transition temperature of the single S slab.

Eq.(3.17) is the McMillan's transition temperature of S-N sandwich (McMillan,1968).

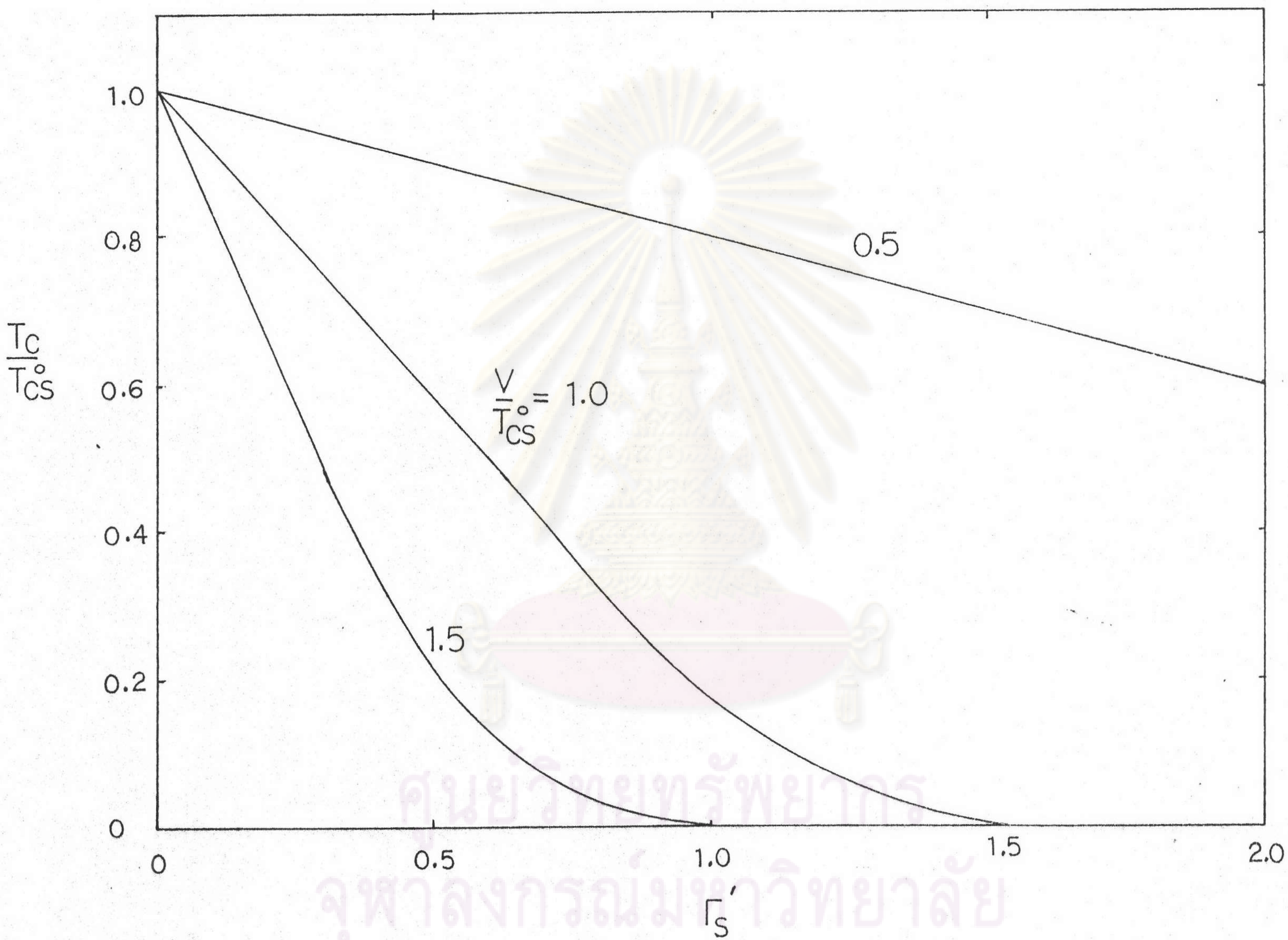

Fig. 3.1 Critical temperature T_c of S-N sandwich vs. Γ_s' ($\Gamma_s'=\pi A d_N N_N(0) T_{cs}^\circ$) for various values of V/T_{cs}°.
The parameter is Γ_N=0.2(Mohabir and Nagi,1979).

In Fig. 3.1, we have introduced dimensionless parameter Γ_s' ($\Gamma_s' = \pi A d_N N_N(0) T^{\circ}_{cs}$) . When $\Gamma_s' = 0$, the ratio of T_c/T°_{cs} is equal 1 for various values of V/T°_{cs} . For fixed values of , the ratio of T_c/T°_{cs} decreases when V/T°_{cs} increases.

The transition temperature of an S_1-S_2 sandwich is given by solving the quadratic Eq.(3.16), we finally get

$$\ln(T^{*}_{c1}/T_c) = (1/2)\left[A + H - \sqrt{A^2 + H^2 + 2AG}\right] \tag{3.18}$$

where

$$A = (\lambda_2)^{-1} - (\lambda_1)^{-1} - \chi$$

$$H(\rho_{c1}) = \gamma_1 K_1(\rho_{c1}) + \gamma_2 K_2(\rho_{c1})$$

$$G(\rho_{c1}) = \gamma_2 K_2(\rho_{c1}) - \gamma_1 K_1(\rho_{c1})$$

Eq.(3.18) is the transition temperature of an S_1-S_2 sandwich as shown by Mohabir and Nagi (Mohabir and Nagi,1979).

In Figs. (3.2) and (3.3) , we show calculated values of the transition temperature of an S_1-S_2 sandwich.

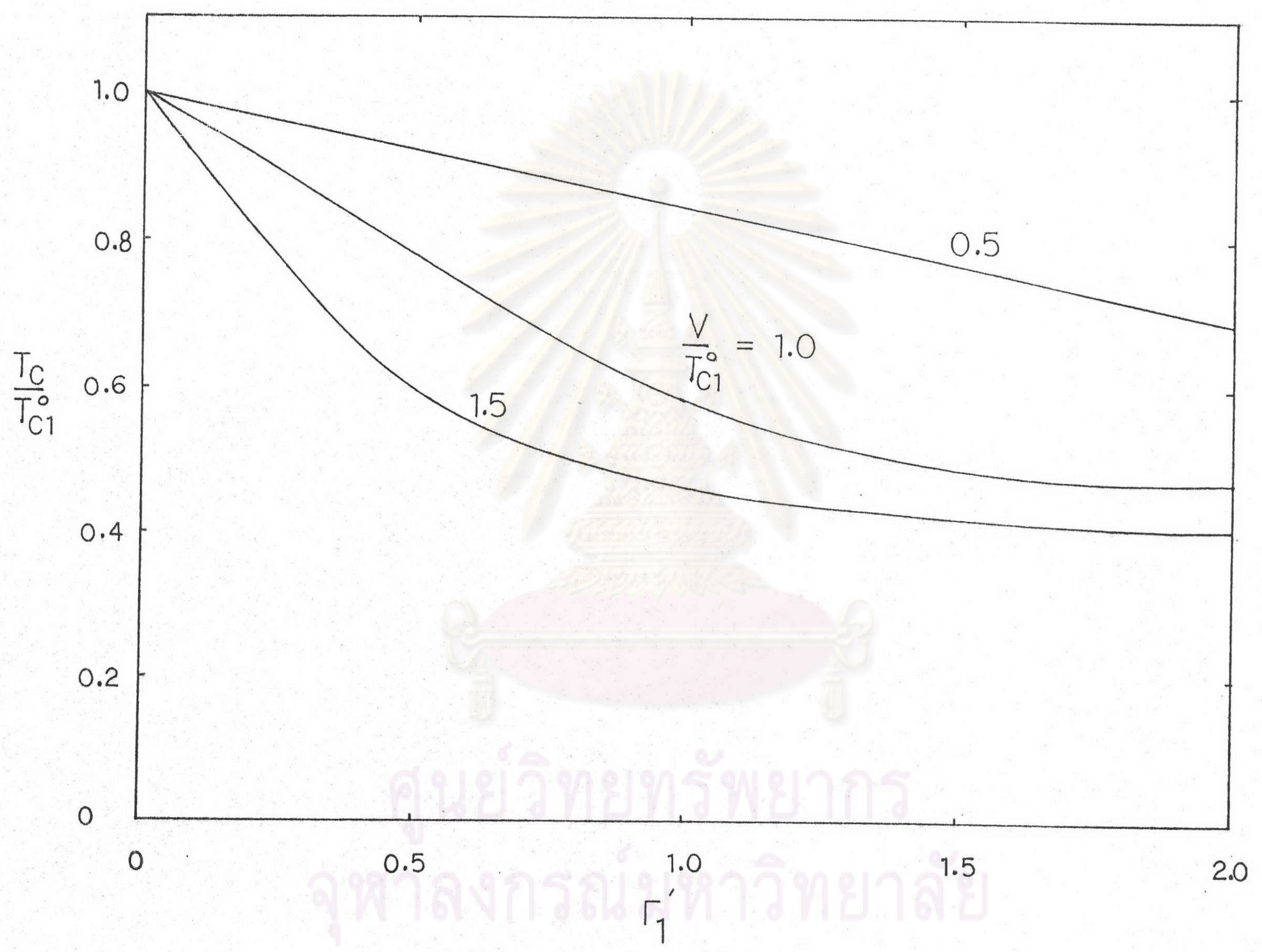

Fig. 3.2 Critical temperature T_c of S_1-S_2 sandwich vs. Γ_1' for various values of V/T_{c1}°. The parameters used are λ_1=0.2447, λ_2=0.1669 ,T_{c1}° =3.72 K (Mohabir and Nagi,1979).

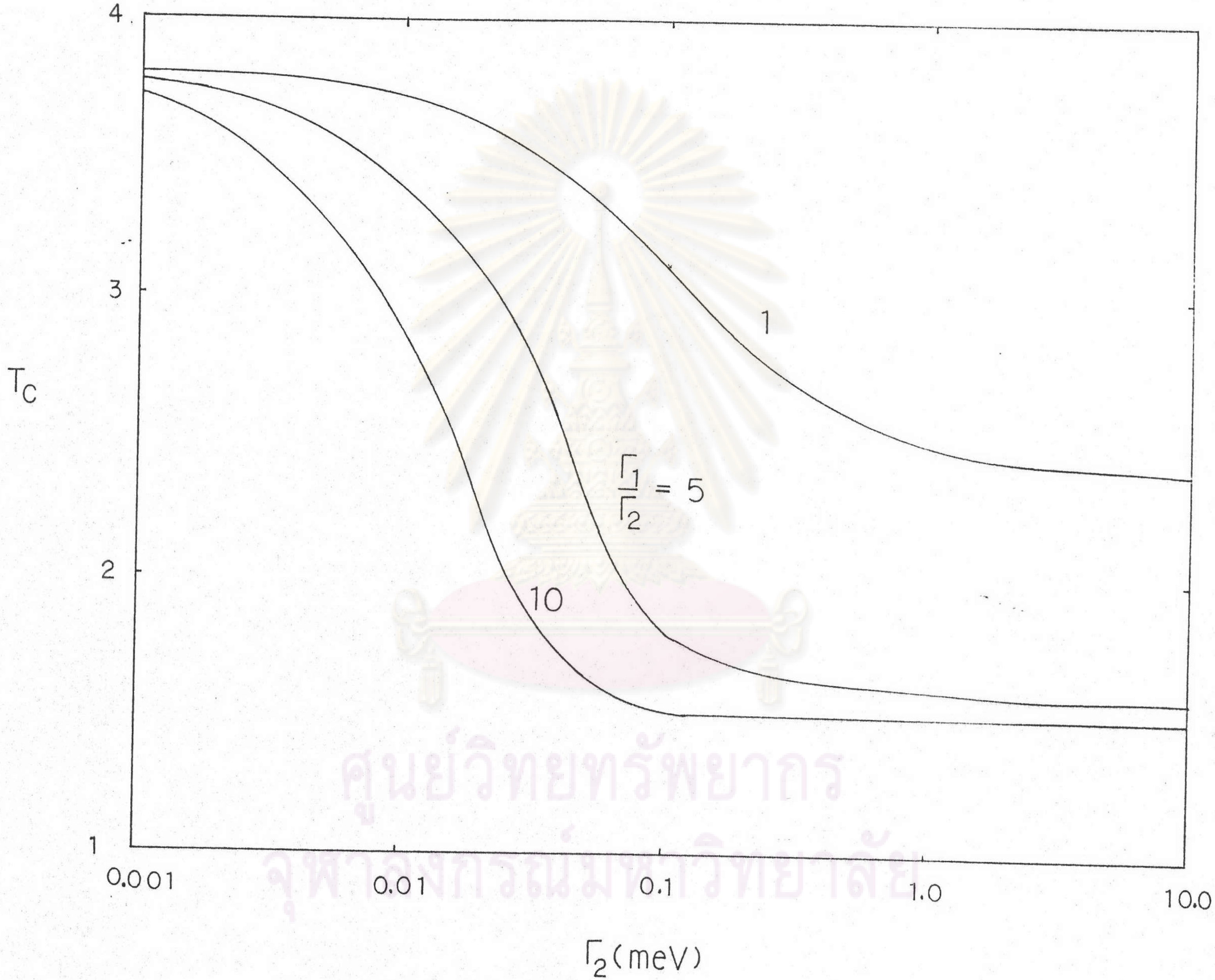

Fig. 3.3 Critical temperature T_c of an S_1-S_2 sandwich vs. Γ_2 for various values of Γ_1 / Γ_2. The parameters used are $\lambda_1 = 0.246$, $\lambda_2 = 0.171$, $\omega_{D1} = 16.78$ meV, $\omega_{D2} = 32.21$ meV (Mohabir and Nagi, 1979).

In Fig. (3.2), we have introduced dimensionless parameter Γ_1' ($\Gamma_1' = \pi A d_2 N_2(0) T^{\circ}_{c1}$) . When $\Gamma_1' = 0$, the ratio of T_c/T°_{c1} is equal to 1 . We found that for fixed value of Γ_1' and V/T°_{c1} , ratio T_c/T°_{c1} of an S_1-S_2 sandwich decreases more slowly than the ratio T_c/T°_{cs} of S-N sandwich.

In Fig. (3.3), we show calculated values of the critical temperature of an S_1-S_2 sandwich as a function of Γ_2 (for various values of Γ_1/Γ_2) . For fixed values of Γ_2, the values of T_c decreases when Γ_1/Γ_2 increases.