Chapter 5

Generalization of Newton-Raphson Load-Flow by Diakoptics

5.1. Introduction

The main focus of this Chapter is to present the mathematical models which are embeded Diakoptics in NRLF and DLF. The equations of NRLF and DLF are extended from linearlization technique and non-linear resistor (or reactor) theory to use in linear Diakoptics. Also, the assumptions of FDLF are applied; the equations of DLF are satisfied by the B' and B"-network models.

5.2. Notations

P_d = Active Power Demand

Q_d = Reactive Power Demand

E = Complex Bus Voltage

V = Voltage Magnitude

δ = Electrical Angle

I = Electrical Current

$*$ = Complex Conjugate

Y_{km} = Element of [Y-Bus] at Position km

= $G_{km} + j B_{km}$

$$S = P + jQ$$

$$= E\, I^{*}$$

Re = Real Part of Complex Number

Im = Imaginary Part of Complex Number

P_g = Active Power Generation

Q_g = Reactive Power Generation

5.3. Review of Mathematical Formula of NRLF

From Kirchhoff's current law; KCL,

$$[\,I\,] = [\,\text{Y-Bus}\,]\,[\,E\,] \tag{5.1}$$

Then, from the definition of complex power, the equation can be written as

$$S = E\, I^{*} \tag{5.2}$$

For bus k, the current is

$$I_k = \sum_{m=1}^{n} Y_{km} E_m \tag{5.3}$$

where n is total bus in system.

Such that, the complex power is

$$S_k = E_k \left(\sum_{m=1}^{n} Y_{km} E_m \right)^{*} \tag{5.4}$$

Conjugate the equation (5.4), thus

$$S_k^{*} = E_k^{*} \left(\sum_{m=1}^{n} Y_{km} E_m \right)$$

$$= \sum_{m=1}^{n} (G_{km} + j B_{km}) V_k V_m \angle -\delta_{km} \qquad (5.5)$$

where $\delta_{km} = \delta_k - \delta_m$

$$\dot{S}_k = \sum_{m=1}^{n} V_k V_m (G_{km} + j B_{km}) (\cos \delta_{km} - j \sin \delta_{km})$$

$$= \sum_{m=1}^{n} V_k V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km})$$

$$- j \sum_{m=1}^{n} V_k V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}) \qquad (5.6)$$

and

$$\dot{S}_k = P_k - j Q_k$$

Then, take Re and Im of $\dot{S}_k$, it yields

$$P_k = G_{kk} V_k^2 + \sum_{m=1}^{n} V_k V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}) \quad , m \neq k \qquad (5.7)$$

$$Q_k = -B_{kk} V_k^2 + \sum_{m=1}^{n} V_k V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}) \quad , m \neq k \qquad (5.8)$$

From the definition of net-power

$$P_{net\,k} = P_{g\,k} - P_{d\,k}$$

$$Q_{net\,k} = Q_{g\,k} - Q_{d\,k}$$

The mismatch terms, in eq. (2.1), are

$$\Delta P_{k} = P_{net\ k} - P_{k} \qquad (5.9)$$

$$\Delta Q_{k} = Q_{net\ k} - Q_{k} \qquad (5.10)$$

From eq. (5.7), real power flow from bus k to bus m is

$$P_{km} = G_{km} V_{k}^{2} + V_{k} V_{m} (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}) \qquad (5.11)$$

reactive power flow from bus k to bus m is

$$Q_{km} = -B_{km} V_{k}^{2} + V_{k} V_{m} (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}) \qquad (5.12)$$

The elements of the Jacobian matrix; [H], [N], [J], and [L], can be calculated as follow:

$$H_{km} = \frac{\partial P_{k}}{\partial \delta_{m}}$$

$$= V_{k} V_{m} (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km})$$

$$= Q_{km} + B_{km} V_{k}^{2} \quad , k \neq m \qquad (5.13)$$

$$H_{kk} = \frac{\partial P_{k}}{\partial \delta_{k}}$$

$$= V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(B_{km} \cos \delta_{km} - G_{km} \sin \delta_{km} \right)$$

$$= - Q_k - B_{kk} V_k^2 \qquad (5.14)$$

$$N_{km} = \frac{\partial P_k}{\partial V_m}$$

$$= V_k \left(G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km} \right)$$

$$= \frac{P_{km}}{V_m} - G_{km} V_k \qquad , k \neq m \qquad (5.15)$$

$$N_{kk} = \frac{\partial P_k}{\partial V_k}$$

$$= 2 G_{kk} V_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km} \right)$$

$$= \frac{P_k}{V_k} + G_{kk} V_k \qquad (5.16)$$

$$J_{km} = \frac{\partial Q_k}{\partial \delta_m}$$

$$= V_k V_m \left(-G_{km} \text{COS}\,\delta_{km} - B_{km} \text{SIN}\,\delta_{km}\right)$$

$$= -P_{km} + G_{km} V_k^2 \quad ,k \neq m \qquad (5.17)$$

$$J_{kk} = \frac{\partial Q_k}{\partial \delta_k}$$

$$= V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} \left(G_{km} \text{COS}\,\delta_{km} + B_{km} \text{SIN}\,\delta_{km} \right)$$

$$= -P_k - G_{kk} V_k^2 \qquad (5.18)$$

$$L_{km} = \frac{\partial Q_k}{\partial V_m}$$

$$= V_k \left(G_{km} \text{SIN}\,\delta_{km} - B_{km} \text{COS}\,\delta_{km} \right)$$

$$= \frac{Q_{km}}{V_m} + \frac{B_{km} V_k^2}{V_m} \quad ,k \neq m \qquad (5.19)$$

$$L_{kk} = \frac{\partial Q_k}{\partial V_k}$$

$$= -2 B_{kk} V_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \text{SIN}\,\delta_{km} - B_{km} \text{COS}\,\delta_{km} \right)$$

$$= \frac{Q_k}{V_k} - B_{kk} V_k \qquad (5.20)$$

5.4. Diakoptics in DLF

From tearing, the mismatch is calculated by,

$$\Delta P_k = P_{net\,k} - P_k - P_{tie\,k} \qquad (5.21)$$

$$\Delta Q_k = Q_{net\,k} - Q_k - Q_{tie\,k} \qquad (5.22)$$

The tie-line's power at bus k is,

$$S_k = E_k \sum_{i=1}^{t} I^{*}_{tie\,k-i} \qquad (5.23)$$

$$I_{tie\,k-i} = (E_k - E_i)\, Y_{tie\,k-i} \qquad (5.24)$$

$$Y_{tie\,k-i} = G_{tie\,k-i} + j\, B_{tie\,k-i}$$

where t is total tie-line in system

$Y_{tie\,k-i}$ is tie-line admittance

The tie-line's power is

$$P_{tie\,k} = V_k^2 \sum_{i=1}^{t} G_{k-i} - V_k \sum_{i=1}^{t} V_i \,(G_{k-i} \cos \delta_{ki} +$$

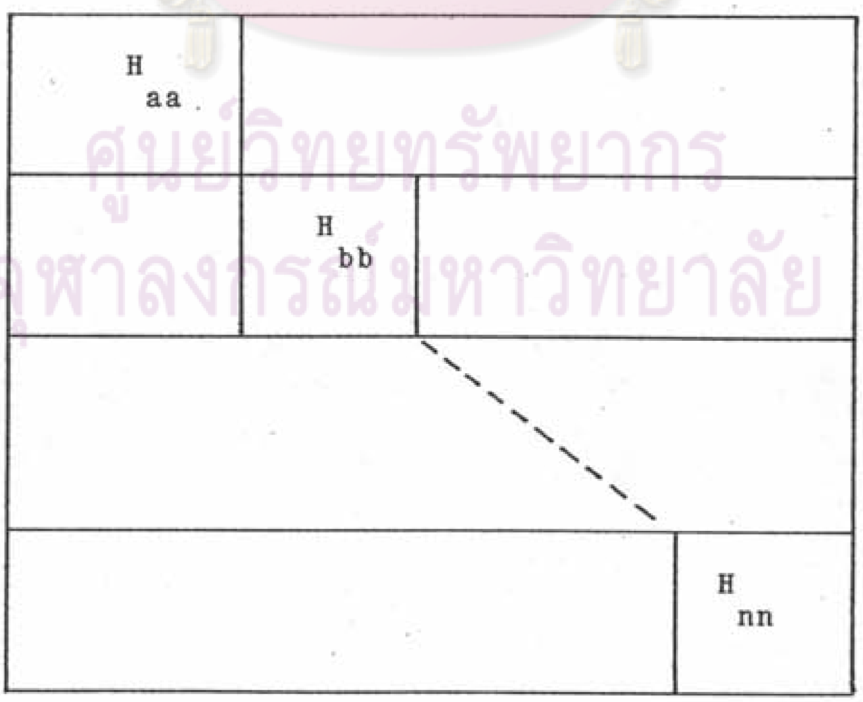
$$B_{k-i} \; SIN\, \delta_{ki} \,) \qquad (5.25)$$

$$Q_{tie\,k} = -V_k^2 \sum_{i=1}^{t} B_{k-i} - V_k \sum_{i=1}^{t} V_i \,(G_{k-i} \; SIN\, \delta_{ki} - B_{k-i} \; COS\, \delta_{ki} \,) \qquad (5.26)$$

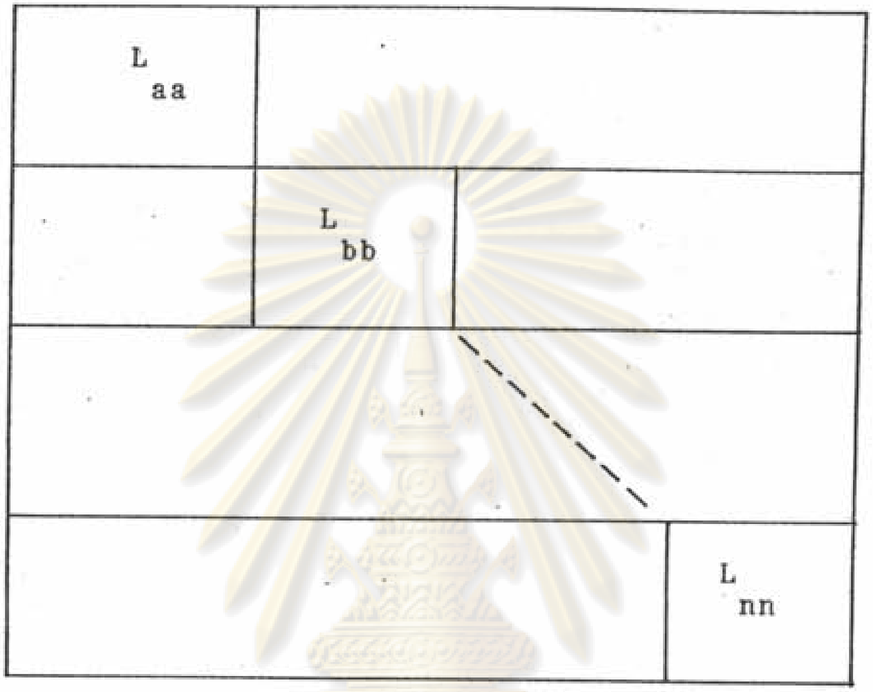
$$P_{tie\,k-i} = V_k^2 G_{k-i} - V_k V_i \,(G_{k-i} \; COS\, \delta_{ki} + B_{k-i} \; SIN\, \delta_{ki} \,) \qquad (5.27)$$

$$Q_{tie\,k-i} = -V_k^2 B_{k-i} - V_k V_i \,(G_{k-i} \; SIN\, \delta_{ki} - B_{k-i} \; COS\, \delta_{ki} \,) \qquad (5.28)$$

The block diagonal of Jacobian matrices are,



and



For [H] or H-model, in block diagonal, they can be calculated;

$$H_{km} = 0$$

, k and m are not in the same area (5.29a)

$$= \frac{\partial P_k}{\partial \delta_m} + \frac{\partial P_{tie\ k}}{\partial \delta_m}$$

, k and m are in the same area (5.29b)

The term $\frac{\partial P_{tie\ k}}{\partial \delta_k}$ has effect only on diagonal elements, because the tie-line's power is directly injected to bus k. Then,

$$\frac{\partial P_{tie\ k}}{\partial \delta_m} = 0 \quad , m \neq k \qquad (5.30a)$$

$$\frac{\partial P_{tie\ k}}{\partial \delta_k} = - V_k^2 \sum_{m=1}^{t} B_{k-i} - Q_{tie\ k} \qquad (5.30b)$$

H_{km} is:

$$H_{km} = 0$$

, k and m are not in the same area (5.31a)

$$= Q_{km} + B_{km} V_k^2$$

, k ≠ m, and k and m are in the same area (5.31b)

$$= - Q_k - B_{kk} V_k^2 - Q_{tie\ k} - V_k^2 \sum_{i=1}^{t} B_{k-i}$$

, k = m (5.31c)

By the principle of Diakoptics, the tie-line shall be used in $[Z_4]$. Then, eq.(5.31c), the susceptance, B_{k-i}, must be taken, so eq.(5.31c) is become,

$$H_{kk} = - Q_k - B_{kk} V_k^2 - Q_{tie\ k} \qquad (5.32)$$

For [L] or L-model, in block diagonal, they can be calculated;

$$L_{km} = 0$$

, k and m are not in the same area

$$= \frac{\partial Q_k}{\partial V_m} + \frac{\partial Q_{tie\,k}}{\partial V_m}$$

, k and m are in the same area

The term $\frac{\partial Q_{tie\,k}}{\partial V_k}$ has effect only diagonal element, because the tie-line's power is directly injected to bus k. Then,

$$\frac{\partial Q_{tie\,k}}{\partial V_m} = 0 \qquad (5.33a)$$

$$\frac{\partial Q_{tie\,k}}{\partial V_k} = -2 V_k \sum_{i=1}^{t} B_{k-i} - \sum_{i=1}^{t} V_i (G_{k-i} \, SIN \, \delta_{ki} - B_{k-i} \, COS \, \delta_{ki})$$

$$= \frac{Q_{tie}}{V_k} - V_k \sum_{i=1}^{t} B_{k-i} \qquad (5.33b)$$

L_{km} is:

$$L_{km} = 0$$

, k and m are not in the same area (5.34a)

$$= \frac{Q_{km}}{V_m} + \frac{B_{km} V_k^2}{V_m}$$

, $k \neq m$, and k and m are in the same area (5.34b)

$$= \frac{Q_k}{V_k} - B_{kk} V_k + \frac{Q_{tie\ k}}{V_k} - V_k^2 \sum_{i=1}^{t} B_{k-i}$$

, $k = m$ (5.34c)

By the principle of Diakoptics, the tie-line shall be used in $[\ Z_4\ ]$. Then, eq.(5.34c), the susceptance, B_{k-i}, must be taken, so eq.(5.34c) is become,

$$L_{kk} = \frac{Q_k + Q_{tie\ k} - V_k B_{kk}}{V_k} \qquad (5.32)$$

5.5. <u>Embeded Diakoptics in DLF</u>

The network of H-model and L-model are nonlinear. Because they are from partial differential. Thus, before linear Diakoptics is used, the linearlization technique of nonlinear resistor theory must be applied. If current is in a function of voltage;

$$I = f(V) \qquad (5.36)$$

then, the admittance is defined by (49):

$$Y = \frac{\partial I}{\partial V}$$

$$= \frac{\partial f(V)}{\partial V} \qquad (5.37)$$

For P-δ problem, consider eq. (2.2); $\Delta\delta$ as voltage, ΔP as current, and [H] or H-model as Bus Admittance Matrix ; the swing bus is treated as shorted-circuit. The effect of tie-line, which is connected to the swing bus, shall be included in the bus at the position on the other side of swing bus and it is never used again. The other tie-lines are used to construct [Z_4], the H-model of non-linear resistor is;

$$Y_{tie\ k-i} = \frac{\partial P_{tie\ k-i}}{\partial \delta_k}$$

$$= V_k V_i (G_{k-i} SIN\ \delta_{ki} - B_{k-i} COS\ \delta_{ki})$$

$$= - Q_{tie\ k-i} - V_k^2 B_{k-i} \qquad (5.38)$$

and

$$Z_{tie\ k-i} = \frac{1}{Y_{tie\ k-i}} \qquad (5.39)$$

$Z_{tie\ k-i}$ is the primitive impedance of tie-line k-i in P-δ problem. Note that, $Z_{tie\ k-i}$ is not bilateral (its characteristic of V-I is a curve that is symmetric with respect to the origin).

$$\frac{\partial P_{tie\ k-i}}{\partial \delta_i} \neq \frac{\partial P_{tie\ k-i}}{\partial \delta_k}$$

and

$$\frac{\partial P_{tie\ k-i}}{\partial \delta_k} \neq \frac{\partial P_{tie\ i-k}}{\partial \delta_k}$$

For Q-V problem, consider eq. (2.3); ΔV as voltage, ΔQ as current, and [L] or L-model as Bus Admittance Matrix ; the PV and swing bus are treated as shorted-circuit. The effect of tie-line, which is connecting the buses, shall be included in the bus at the position on the other side of PV and swing bus and the line is never used again. The other tie-lines are used to constructed $[Z_4]$, the L-model of non-linear resistor is;

$$Y_{tie\ k-i} = \frac{\partial Q_{tie\ k-i}}{\partial V_k}$$

$$= -2 V_k B_{k-i} - V_i (G_{k-i} \ SIN\ \delta_{ki} - B_{k-i} \ COS\ \delta_{ki})$$

$$= \frac{Q_{tie\ k-i}}{V_k} - V_k B_{k-i} \qquad (5.40)$$

and

$$Z_{tie\ k-i} = \frac{1}{Y_{tie\ k-i}}$$

$Z_{tie\ k-i}$ is the primitive impedance of tie-line k-i in Q-V problem. Note that, $Z_{tie\ k-i}$ is not bilateral.

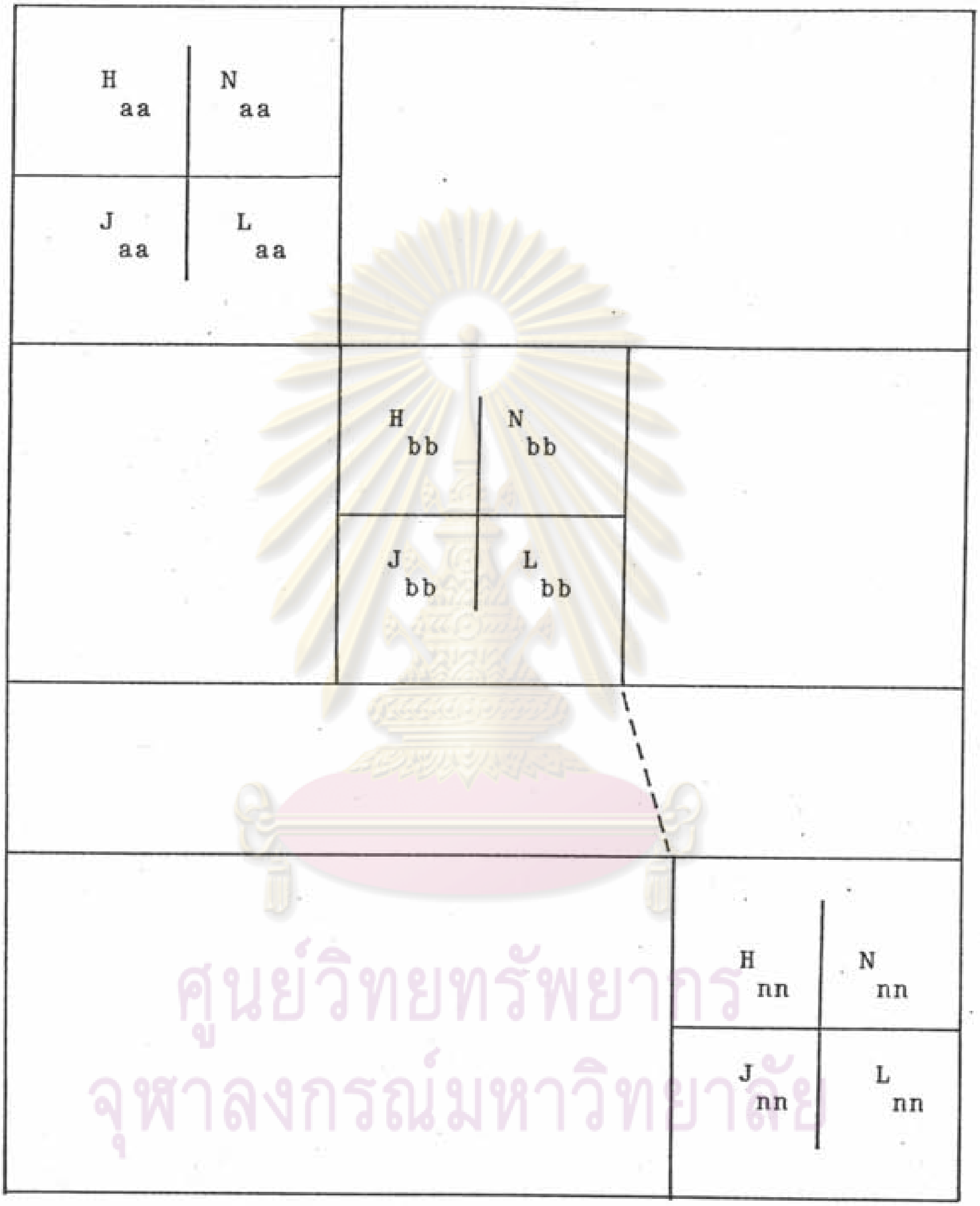
$$\frac{\partial Q_{tie\ k-i}}{\partial V_i} \neq \frac{\partial Q_{tie\ k-i}}{\partial V_k}$$

and

$$\frac{\partial Q_{tie\ k-i}}{\partial V_k} \neq \frac{\partial Q_{tie\ i-k}}{\partial V_k}$$

5.6. Embeded Diakoptics in NRLF

As in DLF, eq.(2.1) can be considered as voltage, current, and Bus Admittance Matrix in KCL. But the Jacobian matrix is nonlinear admittance coupling between H-model and L-model, and the Jacobian is a block diagonal matrix:



For [N] or N-model, it can be shown that

$$N_{km} = 0$$

,k and m are not in the same area

$$= \frac{\partial P_k}{\partial V_m} + \frac{\partial P_{tie\,k}}{\partial V_m}$$

, k and m are in the same area

The term $\frac{\partial P_{tie\,k}}{\partial V_m}$ has effect only on the diagonal elements of [N], then.

$$\frac{\partial P_{tie\,k}}{\partial V_k} = 0 \quad , m \neq k$$

$$\frac{\partial P_{tie\,k}}{\partial V_k} = 2 V_k \sum_{i=1}^{t} G_{k-i} - \sum_{i=1}^{t} V_i (G_{k-i} \cos \delta_{ki} + B_{k-i} \sin \delta_{ki})$$

$$= \frac{P_{tie\,k}}{V_k} + V_k \sum_{i=1}^{t} G_{k-i} \qquad (5.41)$$

Then N_{km} is;

$$N_{km} = 0$$

, k and m are not in the same area (5.42a)

$$= \frac{P_{km} - G_{km} V_k}{V_m}$$

,k $\neq$ m, k and m are in the same area (5.42b)

$$= P_k + P_{tie\ k} + G_{kk} V_k + V_k \sum_{i=1}^{t} G_{k-i} \quad ,k = m \qquad (5.42c)$$

The nonlinear resistor model of N-model is

$$Y_{tie\ k-i} = \frac{\partial P_{tie\ k-i}}{\partial V_k}$$

$$= 2 V_k G_{k-i} - V_i (G_{k-i} \cos \delta_{ki} + B_{k-i} \sin \delta_{ki})$$

$$= P_{tie\ k-i} + V_k G_{k-i} \qquad (5.43)$$

The primitive of N-model is not bilateral.

For [J] or J-model, it can be shown that

$$J_{km} = 0$$

, k and m are not in the same area

$$= \frac{\partial Q_k}{\partial \delta_m} + \frac{\partial Q_{tie\ k}}{\partial \delta_m}$$

, k and m are in the same area

The term $\frac{\partial Q_{tie\ k}}{\partial \delta_k}$ has effect only on the diagonal elements of [L], then

$$\frac{\partial Q_{tie\ k}}{\partial \delta_m} = 0 \quad , k \neq m$$

$$\frac{\partial Q_{tie\ k}}{\partial \delta_k} = V_k \sum_{i=1}^{t} V_i \left(G_{k-i} \cos \delta_{ki} + B_{k-i} \sin \delta_{ki} \right)$$

$$= P_{tie\ k} - V_k^2 \sum_{i=1}^{t} G_{k-i} \qquad (5.44)$$

Then J_{km} is;

$$J_{km} = 0$$

, k and m are not in the same area (5.45a)

$$= - P_{km} + G_{km} V_k$$

,$k \neq m$, k and m are in the same area (5.45b)

$$= P_k + P_{tie\ k} - G_{kk} V_k^2 - V_k^2 \sum_{i=1}^{t} G_{k-i}$$

,$k = m$ (5.45c)

The nonlinear resistor model of J-model is

$$Y_{tie\ k-i} = \frac{\partial Q_{tie\ k-i}}{\partial \delta_k}$$

$$= - V_k V_i \left(G_{k-i} \cos \delta_{ki} + B_{k-i} \sin \delta_{ki} \right)$$

$$= P_{tie\ k-i} - V_k^2 G_{k-i} \qquad (5.46)$$

The primitive of L-model is not bilateral. Clearly, the last terms in eq. (5.42c) and (5.45c) are excluded.

For complete primitive model of nonlinear resistor in NRLF, with coupling of tie-line, in matrix form is

$$Y_{tie\ k-i} = \begin{bmatrix} \dfrac{\partial P_{tie\ k-i}}{\partial \delta_k} & \dfrac{\partial P_{tie\ k-i}}{\partial V_k} \\ \dfrac{\partial Q_{tie\ k-i}}{\partial \delta_k} & \dfrac{\partial Q_{tie\ k-i}}{\partial V_k} \end{bmatrix}$$

The primitive impedance for a tie-line is

$$[\,Z_{tie\ k-i}\,] = [\,Y_{tie\ k-i}\,]^{-1}$$

Then, for total tie-lines, the primitive impedance can be written as a block diagonal matrix:

$$\begin{bmatrix} Z_{(tie\ k-i)_1} & & & \\ & Z_{(tie\ k-i)_2} & & \\ & & \ddots & \\ & & & Z_{(tie\ k-i)_t} \end{bmatrix}$$

Note that, the tie-line, which is connected to the PV-bus has only $\dfrac{\partial P_{tie\ k-i}}{\partial \delta_k}$ term.

In electrical network, the network model for NRLF is shown in Fig. 5.1.

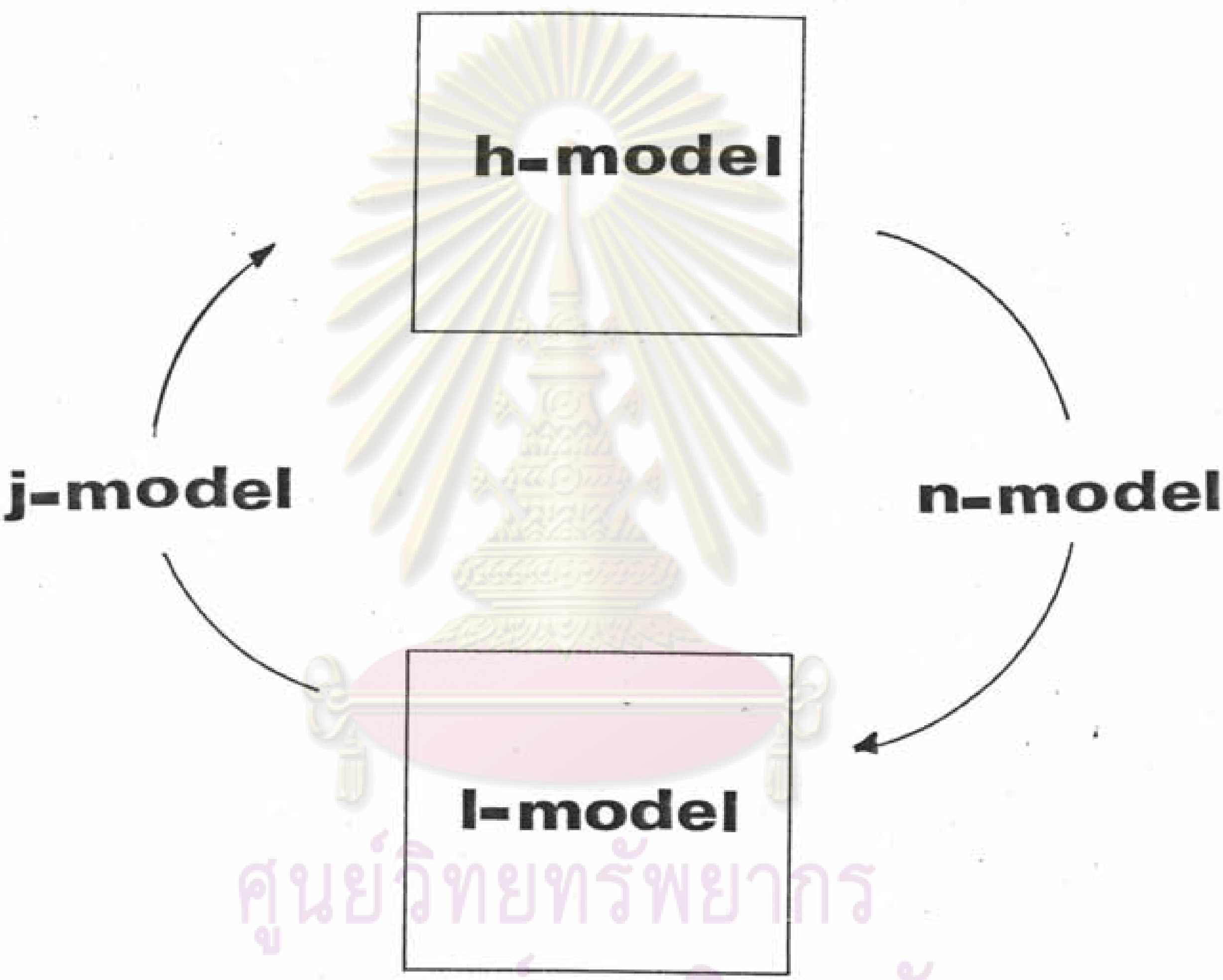


Fig. 5.1 Network Model for NRLF