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APPENDIX A

NOETHER'S THEOREM

The continuous symmetries of classical field theories along with the equations of motion for the fields imply the existence of conserved currents from which one can construct conserved charges. This is usually called Noether's theorem, which in both its classical and operator forms is very important for classifying the general physical characteristics of quantum field theories(Cheng and Li, 1984). A brief review of the theorem given here is summarized from the papers by Karatas and Kowalski (1990), and AL-Kuwari and Taha (1991).

Let us consider a classical field theory characterized by a Lagrangian density $L\left[\phi_a(X^\mu),\ \partial_\mu\phi_a(X^\mu)\right]$ involving the fields ϕ_a at the spacetime point $X^\mu = (X^0,\ \mathbf{x})$ and their first-order spacetime derivatives. For the sake of simplicity, we ignore any explicit dependence of L on X^μ. Here, the index *a* enumerates the different field types including reference to their transformation properties with respect to the Lorentz group (scalar, spinor, vector, etc.). Also, $\partial_\mu = \partial/\partial X^\mu$, where $\mu = 0,1,2,3$, and we employ the diag $(1,-1,-1,-1)$ to raise and lower the vector index μ.

A signature of the symmetry of a classical field theory is the invariance of the action integral

$$S_{21}[\phi] = \int_1^2 d^4X^\mu \; L\left[\phi_a(X^\mu),\ \partial_\mu\phi_a(X^\mu)\right], \qquad \text{(A-1)}$$

taken between two spacelike surfaces under the associated transformations of the fields. Hamilton's principle then implies that the equations of motion are also invariant under these transformations. Noether's theorem refers to the local implications of a symmetry and these are determined by exploring the consequences of the invariant

$$\delta S_{21}[\phi] = 0, \qquad (A-2)$$

under the infinitesimal transformations

$$\phi_a(X^\mu) \rightarrow \phi_a(X^\mu) + \delta\phi_a(X^\mu). \qquad (A-3)$$

The variations are assumed to vanish on the boundary surfaces 1 and 2.

Corresponding to Eq.(A-3) we have the variation in the Lagrangian density

$$\delta L = \frac{\partial L}{\partial\phi_a(X^\mu)}\delta\phi_a(X^\mu) + \frac{\partial L}{\partial[\partial_\mu\phi_a(X^\mu)]}\delta[\partial_\mu\phi_a(X^\mu)], \qquad (A-4)$$

where summation over any repeated index is implied. If we suppose that we can interchange the δ and operation ∂_μ, Eqs.(A-2) and (A-4), together with equations of motion

$$\partial_\mu \frac{\partial L}{\partial[\partial_\mu\phi_a(X^\mu)]} - \frac{\partial L}{\partial\phi_a(X^\mu)} = 0, \qquad (A-5)$$

imply that

$$0 = \int_V d^4X^\mu\, \partial_\mu \frac{\partial L}{\partial[\partial_\mu\phi_a(X^\mu)]}\delta\phi_a(X^\mu). \qquad (A-6)$$

Since the integrand of Eq.(A-6) vanishes on the boundary surfaces and involves otherwise arbitrary variations of the fields induced by the symmetry transformations, it follows that

$$\partial_\mu J^\mu = 0, \qquad (A-7)$$

where

$$J^{\mu} = \frac{\partial L}{\partial[\partial_{\mu}\phi_a(X^{\mu})]} \delta\phi_a(X^{\mu}), \qquad (A-8)$$

is the conserved Noether current. Evidently we can always define a Noether current corresponding to Eq.(A-3) whether or not Eq.(A-2), and therefore Eq.(A-7), is realized.

Actually, since arbitrary variations of the fields are involved, Eq.(A-8) defines an entire family of currents as well as the charges

$$Q(X^0) \equiv \int d^4X^{\mu} \; J^0(X^{\mu}), \qquad (A-9)$$

which are also conserved,

$$\frac{dQ(X^0)}{dX^0} = 0, \qquad (A-10)$$

as a consequence of Eq.(A-7) provided vanishes sufficiently quickly in spacelike directions at infinity.

From Eq.(A-8), we find that although J^{μ} is conserved but it is not unique. When this is applied to scalar electrodynamics,

$$L = -(1/4)F^{\mu\nu}F_{\mu\nu} + (\partial_{\mu}-ieA_{\mu})\phi^*(\partial_{\mu}+ieA_{\mu})\phi + m^2\phi^*\phi, \qquad (A-11)$$

one obtains form invariance under the local gauge transformation

$$\phi(X^{\mu}) \to e^{-ie\chi(X^{\mu})}\phi(X^{\mu}), \quad A_{\mu}(X^{\mu}) \to A_{\mu}(X^{\mu}) + \partial_{\mu}\chi(X^{\mu}), \qquad (A-12)$$

the conserved Noether current

$$J_\mu(X^\mu) = ie(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^* + 2ieA_\mu\phi^*)\chi(X^\mu) - F_{\mu\nu}(X^\mu)\partial^\mu\chi(X^\mu). \qquad (A-13)$$

Since J_μ turns out to be explicitly dependent on the gauge transformation function $\chi(X^\mu)$, Karatas and Kowalski then define an intrinsic Noether's current $J^N{}_\mu$ given by the coefficient of $\chi(X^\mu)$ in Eq.(A-13),

$$J^N{}_\mu = ie(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^* + 2ieA_\mu\phi^*). \qquad (A-14)$$

This intrinsic current is also conserved and is the source of the electromagnetic field

$$\partial_\mu F^{\mu\nu}(X^\mu) = J^N{}_\mu(X^\mu). \qquad (A-15)$$

It is thus implied that local gauge invariance enlarges the class of conserved Noether currents and a choice of what is physical relevant must be made.

The conditions of local gauge invariance under a general non-Abelian group are also discussed by Al-Kuwari and Taha. They show that no extra conserved currents associated with local gauge invariance, such as $J^N{}_\mu$ is needed but it is necessary to introduce the gauge fields in the Lagrangian or $L \equiv L(\phi_a, A_g{}^\mu)$. The consequences of local gauge invariance include the field equations for the gauge fields as well as the existence of the usual conserved Noether currents. Their discussion conclusively proves that the conserved Noether currents that arise in the case of local gauge invariance are exactly those that follow from global gauge invariance.

APPENDIX B

ADDITION VELOCITY LAW

We will show here how we can directly obtain the addition law of velocity described in Eq.(3.16) from the two initial conditions I and II. The inertial transformation is derived directly from this result. This approach was first proposed by Pisistha Ratanavararaksa in 1989 (Ratanavararaksa,1989).

We consider for the relative motion of two inertial frames S and S$'$ along the xx$'$-axis with constant relative velocity v. We assume first that the addition law for velocities must be satisfied two initial postulates:

I. The postulate of inertial motion. If the particle is at rest in frame S$'$ then it will have velocity v in frame S.

II. The postulate of universal velocity. If the particle has the velocity k (-k) in frame S$'$ then it will also have velocity k (-k) in frame S.

We immediatly find that the conventional Galilean addition law,

$$u_x' = u_x - v, \tag{B-1}$$

is not satisfied the second postulate therefore it has to be modified. At first, we are suggested that its proper form should be

$$u_x' = u_x - v + vF(u_x, u_x'), \tag{B-2}$$

where $F(u_x,u_x')$ is the correction term to be determined. From the above two postulates we find that $F(u_x,u_x')$ must be corresponded to the initial conditions,

$$F(v,0) = F(0,-v) = 0, \qquad F(k,k) = F(-k,-k) = 1. \qquad \text{(B-3)}$$

From the second condition in Eq.(B-3), we suggest that $F(u_x,u_x')$ be appeared as $u_x u_x'$, or u_x/u_x', or u_x'/u_x, but finally the second and third forms are omitted to preserve the first condition. Thus, the modified addition velocity law in Eq.(B-2) should be written as

$$u_x' = u_x - v + v(cu_x u_x'), \qquad \text{(B-4)}$$

where c is the proper constant. If we make use of the condition $F(k,k) = 1$, we find that the constant c equals $1/k^2$ then the modified addition velocity law in Eq.(B-2) becomes

$$u_x' = u_x - v + v(u_x u_x'/k^2),$$

or, after rearranging,

$$u_x' = \frac{(u_x-v).}{(1-vu_x/k^2)} \qquad \text{(B-5)}$$

This equation is the same as the addition law for velocities we have already developed in Eq.(3.16). From this, we can easily obtain the inertial transformation in Eq.(3.23a) by rewriting Eq.(B-5) as

$$u_x' = (dx'/dt') = \frac{\left[(dx/dt)-v\right],}{\left[1-v(dx/dt)/k^2\right]}$$

or, after rearranging,

$$dx' = \beta(dx - vdt), \qquad \text{(B-6a)}$$

and

$$dt' = \beta(dt - vdx/k^2), \qquad \text{(B-6b)}$$

where β is the proper constant to be determined. If we integrate Eq.(B-6), and also assume that when $(x',t')=(0,0)$, then $(x,t)=(0,0)$, too, we will obtain the linear homogeneous coordinate transformation:

$$x' = \beta(x - vt), \qquad \text{(B-6a)}$$

and

$$t' = \beta(t - vx/k^2). \qquad \text{(B-6b)}$$

The inverse of these transformation laws are given by

$$x = \beta(x' + vt'), \qquad \text{(B-7a)}$$

and

$$t = \beta(t' + vx'/k^2). \qquad \text{(B-7b)}$$

We now substitute x and t into Eq.(B-6) to get

$$\beta^2 = \frac{1}{(1 - v^2/k^2)}. \qquad \text{(B-8)}$$

To satisfy the initial condition, only the positive value of β is valid then we obtain the complete coordinate transformation laws:

$$x' = \beta(x-vt), \tag{B-9a}$$

$$y' = y, \tag{B-9b}$$

$$z' = z, \tag{B-9c}$$

$$t' = \beta(t-vx/k^2), \tag{B-9d}$$

where we assume that the x-component and y-component are not disturbed by the motion along xx'-axis and $\beta = 1/(1-v^2/k^2)^{1/2}$. The complete set of Eq.(B-9) is in fact the inertial transformation we have already developed in Chapter III. As usual, its inverse is

$$x = \beta(x'+vt'), \tag{B-10a}$$

$$y = y', \tag{B-10b}$$

$$z = z', \tag{B-10c}$$

$$t = \beta(t'+vx'/k^2). \qquad \text{(B-10b)}$$

By using Eq.(B-10), the additional law for other components can be shown as follows:

$$u_y' = \frac{u_y}{\beta(1 - vu_y/k^2)}, \tag{B-11}$$

and

$$u_z' = \frac{u_z}{\beta(1 - vu_z/k^2)}. \tag{B-12}$$

APPENDIX C

HELMHOLTZ'S THEOREM

Helmholtz's theorem provides the basis for a complete investication of the sources of a vector field. A lucidly presented proof of the theorem we paraphrase here is given by Griffths (1991), Miller (1984) and Kobe (1986).

Helmholtz theorem is commonly stated in two parts as (1) and (2) below.

(1) A continuous vector field is uniquely determined by its divergence and its curl within a region and its normal component over the boundaries.

(2) For any continuous vector function $\mathbf{F}(\mathbf{r})$ defined within a volumetric region V which is bounded, or may be infinite under the conditions that in the limit as r becomes infinite $\nabla\cdot\mathbf{F}$ and $\nabla\times\mathbf{F}$ vanish faster than $1/r$ which may then require a lower boundary, we define

$$U(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\nabla'\cdot\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, dV', \qquad \text{(C-1)}$$

$$W(\mathbf{r}) = \frac{1}{4\pi}\int_V \frac{\nabla'\times\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, dV', \qquad \text{(C-2)}$$

in which $\mathbf{r}'$ specifies a variable source point whose coordinates are the variables of integration and all the primed elements are refered to these coordinates, and $\mathbf{r}$ specifies a fixed field point. It follows:

$$\mathbf{F}(\mathbf{r}) = -\nabla U + \nabla \times \mathbf{W}. \qquad (C\text{-}3)$$

A proof of Helmholtz theorem for three-vectors, which is shorter and slightly more general than the usual one, is given by Kobe as follows. Helmholtz theorem states that a vector field $\mathbf{F}(\mathbf{r})$ which vanishes at the boundaries may be written as the sum of two terms, one of which is irrotational and the other solenoidal. Consider the Laplacian of a vector field $\mathbf{V}(\mathbf{r})$:

$$-\nabla^2\mathbf{V} = -\nabla(\nabla\cdot\mathbf{V}) + \nabla\times\nabla\times\mathbf{V}. \qquad (C\text{-}4)$$

If the vector field $\mathbf{F}$ is taken to be

$$\nabla^2\mathbf{V} = -\mathbf{F}, \qquad (C\text{-}5)$$

then it follows from Eq.(C-1) that

$$\mathbf{F}(\mathbf{r}) = -\nabla U + \nabla\times\mathbf{W}. \qquad (C\text{-}6)$$

The scalar field U is

$$U = \nabla\cdot\mathbf{V}, \qquad (C\text{-}7)$$

and the vector field $\mathbf{W}$ is

$$\mathbf{W} = \nabla\times\mathbf{V}. \qquad (C\text{-}8)$$

Equation (C-6) is Helmholtz theorem, where $-\nabla U$ is irrotational and $\nabla\times\mathbf{W}$ solenoidal.

A corollary of Helmholtz theorem is that the vector field which vanishes at the boundaries is determined by its divergence and curl. The uniqueness is proved by Arfken (1970). From the above equations this corollary can be proved. The divergence of Eq.(C-6) is Poisson equation

$$\nabla^2 U = -\nabla\cdot\mathbf{F}. \qquad (C\text{-}9)$$

The solution to Poission equation for a function which vanishes at the boundaries of the volume V is

$$U(\mathbf{r}) = \int_V G(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{F}(\mathbf{r}')\, dV'. \qquad (C-10)$$

The Green's function is

$$G(\mathbf{r},\mathbf{r}') = G_1(\mathbf{r},\mathbf{r}') + G_0(\mathbf{r},\mathbf{r}'), \qquad (C-11)$$

where $G(\mathbf{r},\mathbf{r}') = (4\pi|\mathbf{r}-\mathbf{r}'|)^{-1}$ and $G_0(\mathbf{r},\mathbf{r}')$ is a solution to Laplace's equation as determined from the boundary condition that G=0 on the boundary of V. The curl of Eq.(C-6) gives

$$\nabla\times\mathbf{F}(\mathbf{r}) = \nabla\times\nabla\times\mathbf{W} = \nabla(\nabla\cdot\mathbf{W})-\nabla^2\mathbf{W} \qquad (C-12)$$

from Eq.(C-1). Because of Eq.(C-8), $\nabla\cdot\mathbf{W}=0$, and Eq.(C-12) is Poisson equation. The solution to the equation is

$$\mathbf{W}(\mathbf{r}) = \frac{1}{4\pi}\int_V G(\mathbf{r},\mathbf{r}')\nabla'\times\mathbf{F}(\mathbf{r}')\, dV'. \qquad (C-13)$$

Therefore **F** can be determined by its divergence and curl when Eqs.(C-10) and (C-13) are used in Eq.(C-6). If only the green function G_1 in equation (C-11) is used in Eqs.(C-10) and (C-13), then Helmholtz theorem in Eq.(C-6) should have a harmonic term $\mathbf{F}_0$ added to it which satisfies Laplace's equation in order to satisfy the boundary conditions.

Helmholtz theorem has been employed in electromagnetism texts in connection with the magnetic vector potential to justify the form used to specify the potential gauge (Jackson,1975), and it has been extended, by Kobe (1984), to an antisymmetric second-rank tensor field. An antisymmetric second-rank tensor $F^{\mu\nu}$ can be expressed as (Hauser,1970),

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) - {}^*(\partial_\mu C_\nu - \partial_\nu C_\mu) + F^{(o)}{}_{\mu\nu}, \qquad \text{(C-14)}$$

where $A_\mu(X^\mu)$ and $C_\mu(X^\mu)$ are non-singular four-vectors. The dual, denote by an asterisk, of an antisymmetric second-rank tensor $F_{\mu\nu}$ is defined as

$$^*F_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}, \qquad \text{(C-15)}$$

where $\varepsilon_{\mu\nu\alpha\beta}$ is totally the antisymmetric Levi-Cavita tensor with $\varepsilon_{0123}=-1$. The first and second terms on the right-hand side of Eq.(C-14) are analogous to the corresponding terms in Eq.(C-6). The third term $F^{(o)}{}_{\mu\nu}$ on the right-hand side of Eq.(C-14) is a solution to the homogeneous wave equation, which is added to satisfy the boundary conditions. It is analogous to a solution of Laplace's equation, which can be added to Eq.(C-6) to satisfy the boundary conditions.

CURRICULUM VITAE

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