REFERENCES

[1] JPL Mars Pathfinder [online]. Available from: http://mars.jpl.nasa.gov/MPF, (February 2003).

[2] Patrick F.Muir, Modeling and Control of Wheeled Mobile Robots, Ph.D. Dissertation, Department of Electrical and Computer Engineering, The Robotics Institute, Carnegie Mellon University, 1988.

[3] J.Chottiner, Simulation of a Six-Wheeled Martian Rover Called the Rocker-Bogie, M.S.Thesis, The Ohio State University, Columbus, OH, 1992.

[4] M.Tarokh, G.McDermott, S.Hayati and J.Hung, Kinematic Modeling of a High Mobility Mars Rover, Proceedings of 1999 IEEE International Conference on Robotics and Automation, 2 (1999): 992-998.

[5] D.B.Reister, M.A.Unseren, Position and Constraint force Control of a Vehicle with Two or More Steerable Drive Wheels, IEEE Transaction on Robotics and Automation, 9 (1993): 723-731.

[6] S.Sreenivasan, B.Wilcox, Stability and Traction control of an Actively Actuated Micro-Rover, Journal of Robotic Systems, (1994).

[7] H.Hacot, Analysis and Traction Control of a Rocker-Bogie Planetary Rover, M.S. Thesis, Massachusetts Institute of Technology, Cambridge, MA, 1998.

[8] K.Iagnemma, S.Dubowsky, Mobile Robot Rough-Terrain Control (RTC) for Planetary Exploration, Proceedings of the 26th ASME Biennial Mechanisms and Robotics Conference, (September10-13, 2000).

[9] K.Yoshida, H.Hamano, Motion Dynamics of a Rover with Slip-Based Traction Model, Proceeding of 2002 IEEE International Conference on Robotics and Automation, (2002)

[10] Bickler, B., A New Family of JPL Planetary Surface Vehicles, Missions, Technologies and Design of Planetary Mobile Vehicle, pages 301-306, Toulouse, France, September 28-30, 1992.

[11] S.Hayati, R.Volpe, P.Backes, J.Balaram, R.Welch, R.Ivlev, G.Tharp, S.Peters, T.Ohm, R.Petras, S.Laubach, The Rocky 7 Rover: A Mars Sciencecraft Prototype, Proceeding of 1997 IEEE International Conference on Robotics

and Automation, 3 (1997): 2458-2464.

[12] David P. Miller & Tze-Liang Lee, High-Speed Traversal of Rough Terrain Using a Rocker-Bogie Mobility System, School of Aerospace & Mechanical Engineering, University of Oklahoma.

[13] Y.Fuke, D.Apostolopoulos, E.Rollins, J.Silberman, W.Whittaker, A Prototype Locomotion Concept for a Lunar Robotic Explorer, Proceeding of the Intelligent Vehicles'95 Symposium, (1995): 382-387.

[14] M.Thianwiboon, V.Sangveraphunsiri, R.Chanchareon, Rocker-Bogie Suspension Performance, Proceeding of the 11th International Pacific Conference on Automotive Engineering (IPC-11), (November 2001).

[15] National Semiconductor, LM628/629 User Guide, Application Note 706, (October 1993).

[16] Analog Device, Inc., ADXL311, ±2g, Dual Axis Accelerometer. Available from: http://www.analog.com/en/prod/0%2C2877%2CADXL311%2C00.html

[17] Silicon Sensing Systems Japan, Ltd. Rate Gyroscope CRS03. Available from: http://spp.co.jp/sssj/sirikon-e.html

[18] US Digital Corporation, T4 Incremental Inclinometer [online]. Available from: http://www.usdigital.com/products/t4/, (January 2005)

[19] John J. Craig, Introduction to Robotics Mechanics and Control, Second Edition, Addison-Wesley Publishing, 1989.

APPENDICES

APPENDIX A

WHEEL-GROUND CONTACT ANGLE ESTIMATION

A.1 Left Side

A.1.1 Left Bogie

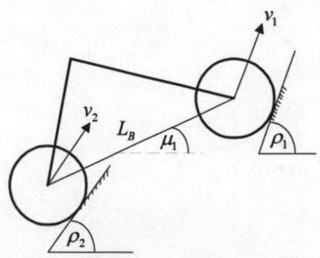


Figure A.1: Left Bogie on uneven terrain

$$v_1 \cos(\rho_1 - \mu_1) = v_2 \cos(\rho_2 - \mu_1) \tag{A.1}$$

$$v_1 \sin(\rho_1 - \mu_1) - v_2 \sin(\rho_2 - \mu_1) = L_B \dot{\mu}_1 \tag{A.2}$$

From (A.1)

$$v_2 = v_1 \frac{\cos(\rho_1 - \mu_1)}{\cos(\rho_2 - \mu_1)}$$

From (A.2)

$$v_1 \sin(\rho_1 - \mu_1) - \frac{v_1 \cos(\rho_1 - \mu_1)\sin(\rho_2 - \mu_1)}{\cos(\rho_2 - \mu_1)} = L_B \dot{\mu}_1$$

$$\sin(\rho_1 - \mu_1)\cos(\rho_2 - \mu_1) - \cos(\rho_1 - \mu_1)\sin(\rho_2 - \mu_1) = \frac{L_B \dot{\mu}_1}{v_1}\cos(\rho_2 - \mu_1)$$

$$\sin[(\rho_1 - \mu_1) - (\rho_2 - \mu_1)] = \frac{L_B \dot{\mu}_1}{v_1}\cos(\rho_2 - \mu_1)$$

$$\sin[(\rho_1 - \mu_1) + (\mu_1 - \rho_2)] = \frac{L_B \dot{\mu}_1}{v_1}\cos(\mu_1 - \rho_2) \tag{A.3}$$

Define

$$\delta_1 = \rho_1 - \mu_1 \qquad \varepsilon_1 = \mu_1 - \rho_2$$

$$a_1 = \frac{L_B \dot{\mu}_1}{v_1} \qquad b_1 = \frac{v_2}{v_1}$$

From (A.1)

$$\cos\delta_1 = b_1 \cos\varepsilon_1 \tag{A.4}$$

From (A.3)

$$\sin(\delta_1 + \varepsilon_1) = a_1 \cos\varepsilon_1$$

$$\sin\delta_1 \cos\varepsilon_1 + \cos\delta_1 \sin\varepsilon_1 = a_1 \cos\varepsilon_1$$

Substitute into (A.4)

$$\sin\delta_1\cos\varepsilon_1 + b_1\cos\varepsilon_1\sin\varepsilon_1 = a_1\cos\varepsilon_1$$

$$(\sin\delta_1 + b_1\sin\varepsilon_1)\cos\varepsilon_1 = a_1\cos\varepsilon_1 \tag{A.5}$$

$$\sin\delta_1 + b_1\sin\varepsilon_1 = a_1$$

$$b_1\sin\varepsilon_1 = a_1 - \sin\delta_1$$

$$b_1^2\sin^2\varepsilon_1 = a_1^2 - 2a_1\sin\delta_1 + \sin^2\delta_1 \tag{A.6}$$

From (A.4)

$$\cos\varepsilon_1 = \frac{\cos\delta_1}{b_1}$$

$$1 - \sin^2\varepsilon_1 = \frac{\cos^2\delta_1}{b_1^2}$$

$$\sin^2\varepsilon_1 = 1 - \frac{\cos^2\delta_1}{b_1^2} \tag{A.7}$$

Substitute (A.7) into (A.6)

$$b_1^2\left(1 - \frac{\cos^2\delta_1}{b_1^2}\right) = a_1^2 - 2a_1\sin\delta_1 + \sin^2\delta_1$$

$$b_1^2 - \cos^2\delta_1 = a_1^2 - 2a_1\sin\delta_1 + \sin^2\delta_1$$

$$b_1^2 = a_1^2 - 2a_1\sin\delta_1$$

$$\sin\delta_1 = \frac{a_1^2 - b_1^2}{2a_1}$$

$$\sin(\rho_1 - \mu_1) = \frac{a_1^2 - b_1^2}{2a_1}$$

Estimated contact angle of front left wheel

$$\rho_1 = \mu_1 + \arcsin\left(\frac{a_1^2 - b_1^2}{2a_1}\right)$$

From (A.4)

$$\cos\delta_1 = b_1\cos\varepsilon_1$$

$$1 - \sin^2\delta_1 = b_1^2\cos^2\varepsilon_1$$

$$\sin^2\delta_1 = 1 - b_1^2\cos^2\varepsilon_1 \tag{A.8}$$

From (A.6)

$$\sin^2\delta_1 = b_1^2\sin^2\varepsilon_1 + 2a_1\sin\delta_1 - a_1^2$$

Substitute by (A.8)

$$1 - b_1^2\cos^2\varepsilon_1 = b_1^2\sin^2\varepsilon_1 + 2a_1\sin\delta_1 - a_1^2$$

$$1 = b_1^2 + 2a_1\sin\delta_1 - a_1^2$$

$$\sin\delta_1 = \frac{1 + a_1^2 - b_1^2}{2a_1}$$

$$\sin(\rho_2 - \mu_1) = \frac{1 + a_1^2 - b_1^2}{2a_1}$$

Estimated contact angle of middle left wheel

$$\rho_2 = \mu_1 + \arcsin\left(\frac{1 + a_1^2 - b_1^2}{2a_1}\right)$$

A.1.2 Left Bogie joint

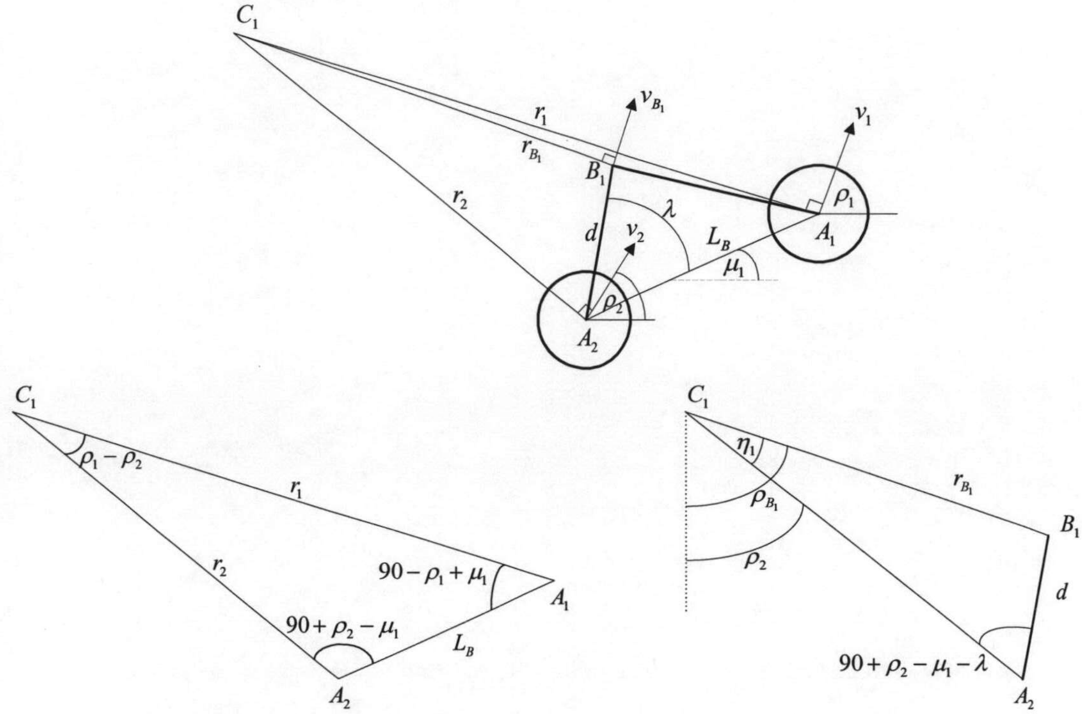


Figure A.2: Instantaneous center of rotation of the left bogie

$$\frac{L_B}{\sin(\rho_1 - \rho_2)} = \frac{r_1}{\sin(90 + \rho_2 - \mu_1)} = \frac{r_2}{\sin(90 - \rho_1 + \mu_1)}$$

$$r_1 = \frac{L_B \sin(90 + \rho_2 - \mu_1)}{\sin(\rho_1 - \rho_2)}$$

$$r_2 = \frac{L_B \sin(90 - \rho_1 + \mu_1)}{\sin(\rho_1 - \rho_2)}$$

From robot geometry

$$\lambda = \arctan(\frac{85}{127}) = 33.8$$

$$d = 152.82 \text{ mm}$$

$$r_{B_1}^2 = r_2^2 + d^2 - 2r_2 d \cos(90 + \rho_2 - \mu_1 - \lambda)$$

$$r_{B_1} = \sqrt{r_2^2 + d^2 - 2r_2 d \cos(90 + \rho_2 - \mu_1 - \lambda)}$$

$$\frac{r_{B_1}}{\sin(90 + \rho_2 - \mu_1 - \lambda)} = \frac{d}{\sin \eta_1}$$

$$\eta_1 = \arcsin\left[\frac{d \sin(90 + \rho_2 - \mu_1 - \lambda)}{r_{B_1}}\right]$$

$$\rho_{B_1} = \rho_2 + \eta_1$$

Estimated bogie joint velocity

$$v_{B_1} = r_{B_1}\dot{\mu}_1$$

A.1.3 Left Rocker

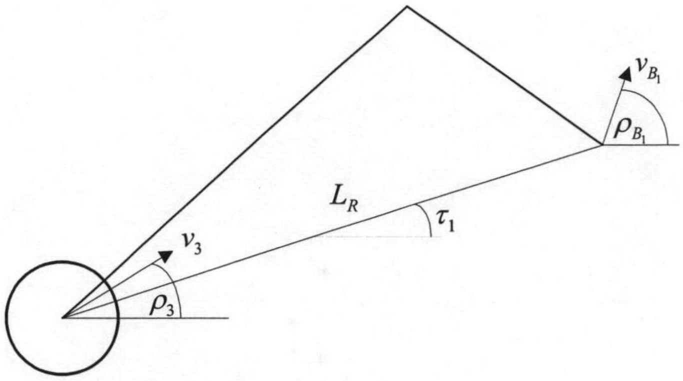


Figure A.3: Left Rocker on uneven terrain

$$v_{B_1}\cos(\rho_{B_1} - \tau_1) = v_3\cos(\rho_3 - \tau_1)$$

$$\cos(\rho_3 - \tau_1) = \frac{v_{B_1}}{v_3}\cos(\rho_{B_1} - \tau_1)$$

Estimated contact angle of left back wheel

$$\rho_3 = \arccos\left[\frac{v_{B_1}}{v_3}\cos(\rho_{B_1} - \tau_1)\right]$$

A.2 Right Side

A.2.1 Right Bogie

In the same way to left side, we can estimate contact angle of the wheels on the right side as follow:

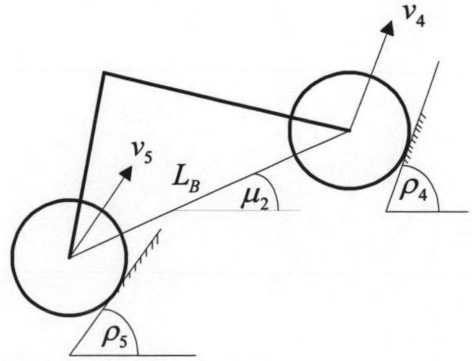


Figure A.4: Right Bogie on uneven terrain

$$a_2 = \frac{L_B\dot{\mu}_2}{v_4} \qquad b_2 = \frac{v_5}{v_4}$$

Estimated contact angle of the right front wheel

$$\rho_4 = \mu_2 + \arcsin\left(\frac{a_2^2 - b_2^2}{2a_2}\right)$$

Estimated contact angle of the right middle wheel

$$\rho_5 = \mu_2 + \arcsin\left(\frac{1 + a_2^2 - b_2^2}{2a_2}\right)$$

A.2.2 Right Bogie joint

$$r_4 = \frac{L_B \sin(90 + \rho_5 - \mu_2)}{\sin(\rho_4 - \rho_5)}$$

$$r_5 = \frac{L_B \sin(90 - \rho_4 + \mu_2)}{\sin(\rho_4 - \rho_5)}$$

From robot geometry

$$\lambda = \arctan(\frac{85}{127}) = 33.8$$

$$d = 152.82 \text{ mm}$$

$$r_{B_2} = \sqrt{r_5^2 + d^2 - 2r_5 d\cos(90 + \rho_5 - \mu_2 - \lambda)}$$

$$\eta_2 = \arcsin\left[\frac{d\sin(90 + \rho_5 - \mu_2 - \lambda)}{r_{B_2}}\right]$$

$$\rho_{B_2} = \rho_5 + \eta_2$$

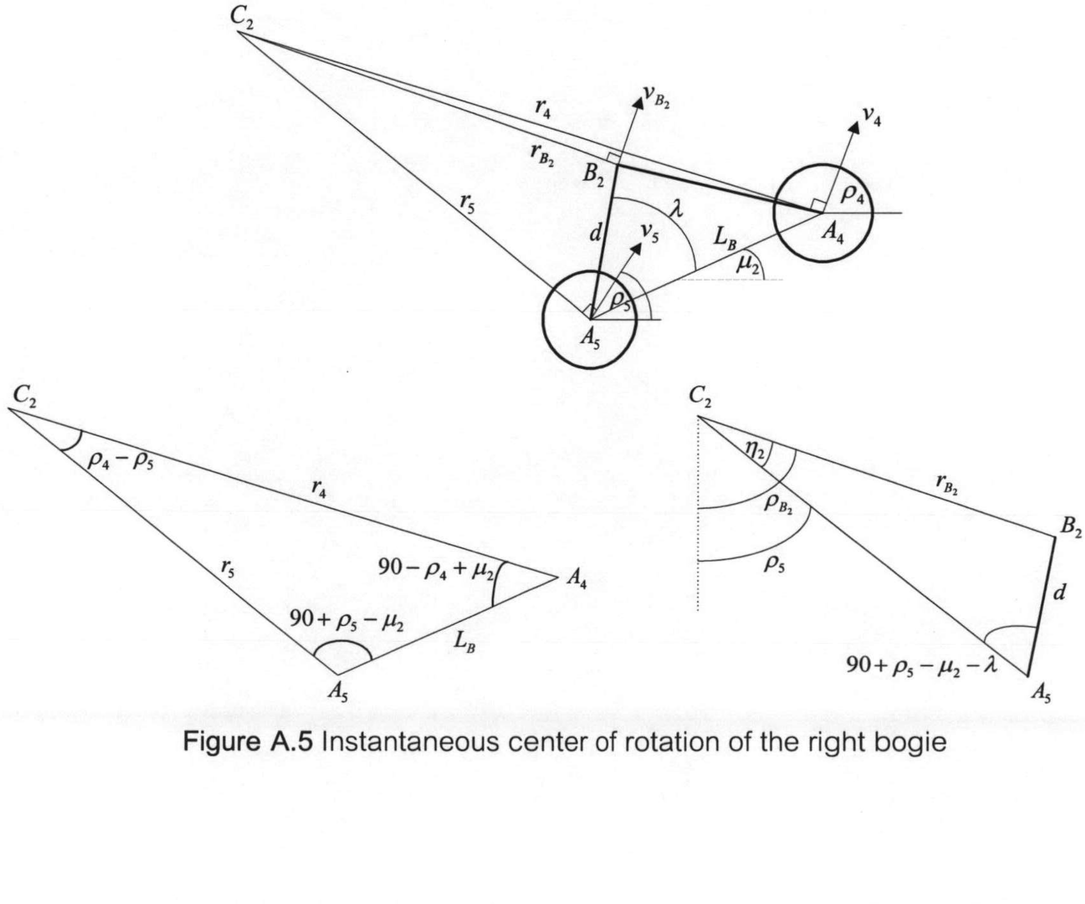


Figure A.5 Instantaneous center of rotation of the right bogie

Estimated bogie joint velocity

$$v_{B_2} = r_{B_2}\dot{\mu}_2$$

A.2.3 Right Rocker

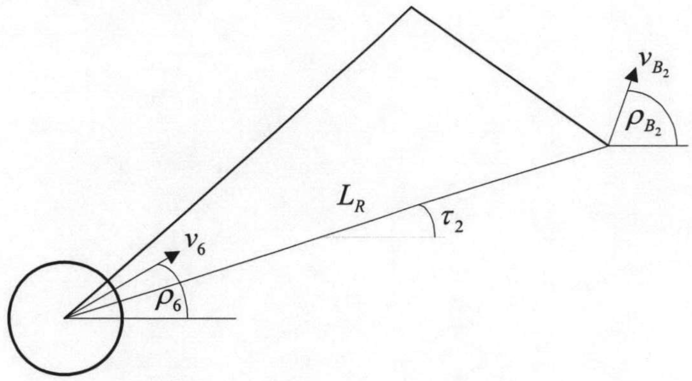


Figure A.6: Right Rocker on uneven terrain

Estimated contact angle of right back wheel

$$\rho_6 = \arccos\left[\frac{v_{B_2}}{v_6}\cos(\rho_{B_2} - \tau_2)\right]$$

APPENDIX B

FORWARD KINEMATICS

B.1 Coordinate Assignment

Define :

φ	:	pitch angle between robot body and horizontal
β	:	angle between left rocker with respect to body
$\therefore -\beta$	:	angle between right rocker with respect to body
γ_1	:	angle between left bogie with respect to left rocker
γ_2	:	angle between right bogie with respect to right rocker
τ_1	:	angle between left rocker with respect to horizontal
τ_2	:	angle between right rocker with respect to horizontal
μ_1	:	angle between left bogie with respect to horizontal
μ_2	:	angle between right bogie with respect to horizontal

From Robot's geometry :

l_1	:	168 mm	l_5	:	85 mm
l_2	:	109 mm	l_6	:	293.5 mm
l_3	:	115 mm	l_7	:	200 mm
l_4	:	127.5 mm	l_8	:	127 mm

Then

$$\tau_1 = \varphi + \beta \qquad \tau_2 = \varphi - \beta$$

$$\mu_1 = \varphi + \beta + \gamma_1 \qquad \mu_2 = \varphi - \beta + \gamma_2$$

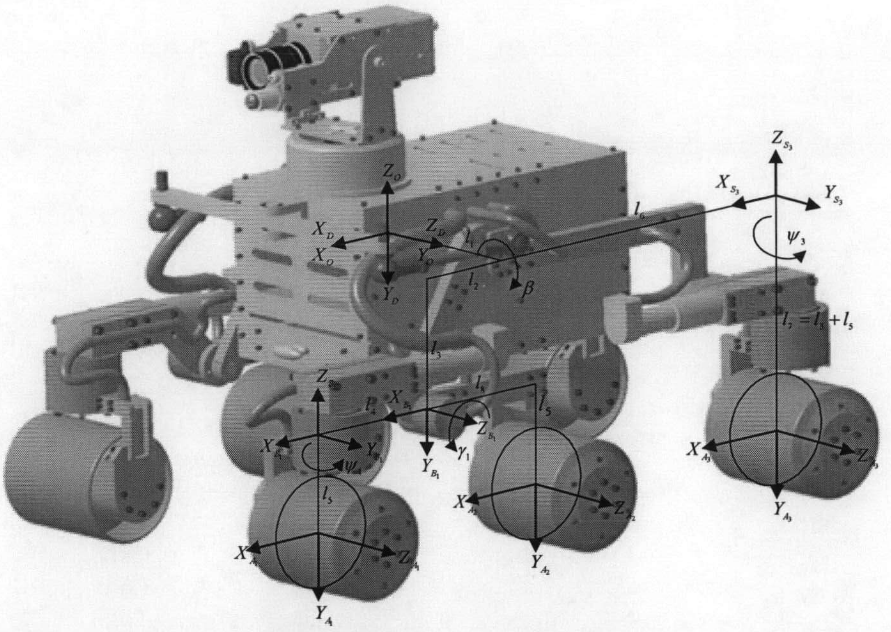


Figure B.1: Left coordinate frames

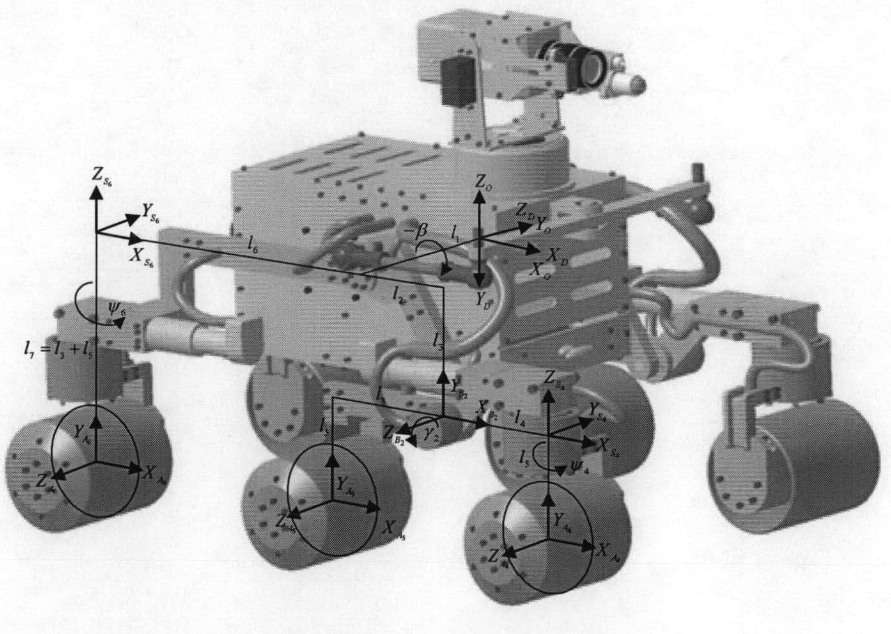


Figure B.2 Right coordinate frames

B.2 Forward Kinematics

Define :

a_i	:	the distance from $\hat{z}_{i-1}$ to $\hat{z}_i$ along $\hat{x}_i$
α_i	:	the angle between $\hat{z}_{i-1}$ to $\hat{z}_i$ about $\hat{x}_i$
d_i	:	the distance from $\hat{x}_{i-1}$ to $\hat{x}_i$ along $\hat{z}_{i-1}$

Θ_i : the angle between $\hat{x}_{i-1}$ to $\hat{x}_i$ along $\hat{z}_{i-1}$

j	i	a	α	d	Θ
O	D	0	-90	0	0
D	B_1	l_2	0	l_1	β
B_1	S_1	l_4	90	0	γ_1
S_1	A_1	0	-90	$-l_5$	ψ_1
B_1	A_2	$-l_8$	0	0	γ_1
D	S_3	$-l_6$	90	l_1	β
S_3	A_3	0	-90	$-l_7$	ψ_3
D	B_2	l_2	180	$-l_1$	$-\beta$
B_2	S_4	l_4	-90	0	γ_2
S_4	A_4	0	90	$-l_5$	ψ_4
B_2	A_5	$-l_8$	0	0	γ_2
D	S_6	$-l_6$	90	$-l_1$	$-\beta$
S_6	A_6	0	90	$-l_7$	ψ_6

Table B.1:Denavit-Hartenburg parameters

Transformation from a coordinate frame i to coordinate frame j can be written as

$$\mathbf{T}_{j,i} = \begin{bmatrix} C\Theta_j & -S\Theta_j C\alpha_j & S\Theta_j S\alpha_j & a_j C\Theta_j \\ S\Theta_j & C\Theta_j C\alpha_j & -C\Theta_j s\alpha_j & a_j S\Theta_j \\ 0 & S\alpha_j & C\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{O,D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{D,B_1} = \begin{bmatrix} C\beta & -S\beta & 0 & l_2 C\beta \\ S\beta & C\beta & 0 & l_2 S\beta \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{B_1,S_1} = \begin{bmatrix} C\gamma_1 & 0 & S\gamma_1 & l_4 C\gamma_1 \\ S\gamma_1 & 0 & -C\gamma_1 & l_4 S\gamma_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{S_1,A_1} = \begin{bmatrix} C\psi_1 & 0 & -S\psi_1 & 0 \\ S\psi_1 & 0 & C\psi_1 & 0 \\ 0 & -1 & 0 & -l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{B_1,A_2} = \begin{bmatrix} C\gamma_1 & -S\gamma_1 & 0 & -l_8C\gamma_1 \\ S\gamma_1 & C\gamma_1 & 0 & -l_8S\gamma_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{D,S_3} = \begin{bmatrix} C\beta & 0 & S\beta & -l_6C\beta \\ S\beta & 0 & -C\beta & -l_6S\beta \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{S_3,A_3} = \begin{bmatrix} C\psi_3 & 0 & -S\psi_3 & 0 \\ S\psi_3 & 0 & C\psi_3 & 0 \\ 0 & -1 & 0 & -l_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{D,B_2} = \begin{bmatrix} C\beta & -S\beta & 0 & l_2C\beta \\ -S\beta & -C\beta & 0 & -l_2S\beta \\ 0 & 0 & -1 & -l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{B_2,S_4} = \begin{bmatrix} C\gamma_2 & 0 & -S\gamma_2 & l_4C\gamma_2 \\ S\gamma_2 & 0 & C\gamma_2 & l_4S\gamma_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

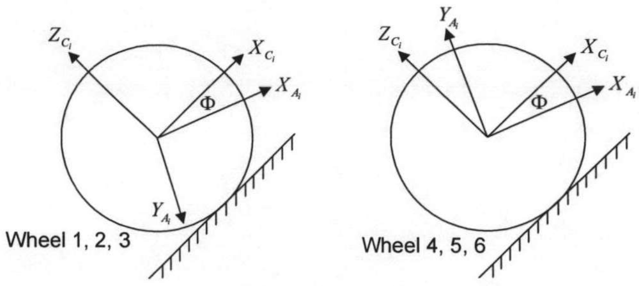
$$T_{S_4,A_4} = \begin{bmatrix} C\psi_4 & 0 & S\psi_4 & 0 \\ S\psi_4 & 0 & -C\psi_4 & 0 \\ 0 & 1 & 0 & -l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{B_2,A_5} = \begin{bmatrix} C\gamma_2 & -S\gamma_2 & 0 & -l_8C\gamma_2 \\ S\gamma_2 & C\gamma_2 & 0 & -l_8S\gamma_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{D,S_6} = \begin{bmatrix} C\beta & 0 & -S\beta & -l_6C\beta \\ -S\beta & 0 & -C\beta & l_6S\beta \\ 0 & 1 & 0 & -l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{S_6,A_6} = \begin{bmatrix} C\psi_6 & 0 & S\psi_6 & 0 \\ S\psi_6 & 0 & -C\psi_6 & 0 \\ 0 & 1 & 0 & -l_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B.2.1 Contact Coordinate Frame



Wheel 1 : $\Phi = \mu_1 - \rho_1$

Wheel 2 : $\Phi = \mu_1 - \rho_2$

Wheel 3 : $\Phi = \tau_1 - \rho_3$

Wheel 4 : $\Phi = \mu_2 - \rho_4$

Wheel 5 : $\Phi = \mu_2 - \rho_5$

Wheel 6 : $\Phi = \tau_2 - \rho_6$

Figure B.3: Contact Coordinate Frame

Transform using Z-X-Y Euler Angle

$$\mathbf{T}_{A_i,C_i} = \begin{bmatrix} Cp_iCr_i - Sp_iSq_iSr_i & Cr_iSp_i + Cp_iSq_iSr_i & -Cq_iSr_i & 0 \\ -Cq_iSp_i & Cp_iCq_i & Sq_i & 0 \\ Cr_iSp_iSq_i + Cp_iCr_i & -Cp_iCr_iSq_i + Sp_iSr_i & Cq_iCr_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

j	i	p	q	r
A_1	C_1	$-(\mu_1 - \rho_1)$	90	0
A_2	C_2	$-(\mu_1 - \rho_2)$	90	0
A_3	C_3	$-(\tau_1 - \rho_3)$	90	0
A_4	C_4	$(\mu_2 - \rho_4)$	–90	0
A_5	C_5	$(\mu_2 - \rho_5)$	–90	0
A_6	C_6	$(\mu_2 - \rho_6)$	–90	0

Table B.2: Parameters for Contact Coordinate Frame

Transformation Matrices

$$\mathbf{T}_{A_1,C_1} = \begin{bmatrix} C(\mu_1 - \rho_1) & -S(\mu_1 - \rho_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S(\mu_1 - \rho_1) + C(\mu_1 - \rho_1) & -C(\mu_1 - \rho_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_{A_2,C_2} = \begin{bmatrix} C(\mu_1 - \rho_2) & -S(\mu_1 - \rho_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S(\mu_1 - \rho_2) + C(\mu_1 - \rho_2) & -C(\mu_1 - \rho_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_{A_3,C_3} = \begin{bmatrix} C(\tau_1-\rho_3) & -S(\tau_1-\rho_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S(\tau_1-\rho_3)+C(\tau_1-\rho_3) & -C(\tau_1-\rho_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_{A_4,C_4} = \begin{bmatrix} C(\mu_2-\rho_4) & S(\mu_2-\rho_4) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S(\mu_2-\rho_4)+C(\mu_2-\rho_4) & C(\mu_2-\rho_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_{A_5,C_5} = \begin{bmatrix} C(\mu_2-\rho_5) & S(\mu_2-\rho_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S(\mu_2-\rho_5)+C(\mu_2-\rho_5) & C(\mu_2-\rho_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_{A_6,C_6} = \begin{bmatrix} C(\tau_2-\rho_6) & S(\tau_2-\rho_6) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S(\tau_2-\rho_6)+C(\tau_2-\rho_6) & C(\tau_2-\rho_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B.2.2 Wheel Motion Frame

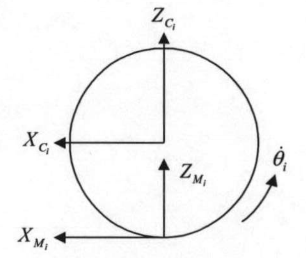


Figure B.4: Wheel Motion Frame

	a	α	d	Θ
$C_1 \to M_1$	$-R\theta_1$	0	$-R$	0
$C_2 \to M_2$	$-R\theta_2$	0	$-R$	0
$C_3 \to M_3$	$-R\theta_3$	0	$-R$	0
$C_4 \to M_4$	$-R\theta_4$	0	$-R$	0
$C_5 \to M_5$	$-R\theta_5$	0	$-R$	0
$C_6 \to M_6$	$-R\theta_6$	0	$-R$	0

Table B.3: Parameters for Wheel Motion Frame

Transformation matrices for all wheels can be written as

$$\mathbf{T}_{O,M_1} = \mathbf{T}_{O,D}\mathbf{T}_{D,B_1}\mathbf{T}_{B_1,S_1}\mathbf{T}_{S_1,A_1}\mathbf{T}_{A_1,C_1}\mathbf{T}_{C_1,M_1}$$

$$\mathbf{T}_{O,M_2} = \mathbf{T}_{O,D}\mathbf{T}_{D,B_1}\mathbf{T}_{B_1,A_2}\mathbf{T}_{A_2,C_2}\mathbf{T}_{C_2,M_2}$$

$$\mathbf{T}_{O,M_3} = \mathbf{T}_{O,D}\mathbf{T}_{D,S_3}\mathbf{T}_{S_3,A_3}\mathbf{T}_{A_3,C_3}\mathbf{T}_{C_3,M_3}$$

$$\mathbf{T}_{O,M_4} = \mathbf{T}_{O,D}\mathbf{T}_{D,B_2}\mathbf{T}_{B_2,S_4}\mathbf{T}_{S_4,A_4}\mathbf{T}_{A_4,C_4}\mathbf{T}_{C_4,M_4}$$

$$\mathbf{T}_{O,M_5} = \mathbf{T}_{O,D}\mathbf{T}_{D,B_2}\mathbf{T}_{B_2,A_5}\mathbf{T}_{A_5,C_5}\mathbf{T}_{C_5,M_5}$$

$$\mathbf{T}_{O,M_6} = \mathbf{T}_{O,D}\mathbf{T}_{D,S_6}\mathbf{T}_{S_6,A_6}\mathbf{T}_{A_6,C_6}\mathbf{T}_{C_6,M_6}$$

In order to obtain the wheel Jacobian matrix, we must express the motion of the robot to the wheel motion, by applying the instantaneous transformation $\dot{\mathbf{T}}_{\hat{O},\hat{M}_i}$ as follows

$$\dot{\mathbf{T}}_{\hat{O},O} = \mathbf{T}_{\hat{O},\hat{M}_i}\dot{\mathbf{T}}_{M_i,O}$$

$\dot{\mathbf{T}}_{\hat{O},O}$ is found to have the following form

$$\dot{\mathbf{T}}_{\hat{O},O} = \begin{bmatrix} 0 & -\dot{\phi} & \dot{p} & \dot{x} \\ \dot{\phi} & 0 & -\dot{r} & \dot{y} \\ -\dot{p} & \dot{r} & 0 & \dot{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

ϕ : yaw angle of the robot

p : pitch angle of the robot

r : roll angle of the robot

Once the instantaneous transformations of each wheel are obtained, we can extract a set of equations relating the robot's motion in vector form $[\dot{x} \quad \dot{y} \quad \dot{z} \quad \dot{\phi} \quad \dot{p} \quad \dot{r}]^T$ to the joint angular rates.

The results for wheel 1 (Left front wheel) and 4 (Right front wheel) are found to be

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} A_i & 0 & B_i & C_i \\ D_i & 0 & E_i & F_i \\ G_i & 0 & H_i & I_i \\ 0 & 0 & 0 & J_i \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & K_i \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \dot{\beta} \\ \dot{\gamma}_i \\ \dot{\psi}_i \end{bmatrix} \qquad i = 1,4$$

The results for wheel 2 (Left middle wheel) and 5 (Right middle wheel) are found to be

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} A_i & 0 & B_i \\ C_i & 0 & 0 \\ D_i & 0 & E_i \\ 0 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \dot{\beta} \\ \dot{\gamma}_i \end{bmatrix} \qquad i = 2,5$$

The results for wheel 2 (Left back wheel) and 5 (Right back wheel) are found to be

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} A_i & 0 & B_i \\ C_i & 0 & D_i \\ E_i & 0 & F_i \\ 0 & 0 & G_i \\ 0 & -1 & 0 \\ 0 & 0 & H_i \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \dot{\beta} \\ \dot{\psi}_i \end{bmatrix} \qquad i = 3,6$$

A_i to H_i are in terms of wheel-ground contact angle ρ_1 to ρ_6 and joint angle, such as β, γ and ψ.

It is seen that these set of equation are in the general form

$$\dot{\mathbf{u}} = \mathbf{J}_i \dot{\mathbf{q}}_i \qquad i = 1-6$$

where $\mathbf{J}_i$ is the Jacobian matrix of wheel i, and $\dot{\mathbf{q}}_i$ is the joint angular rate vector.

BIOGRAPHY

Mongkol Thianwiboon was born on November 25, 1976 in Lampang, Thailand and went to Chulalongkorn University, where he studied and obtained his Bachelor's Degree in Mechanical Engineering in 1997. He continued to attend in the Master of Engineering program with "Control of an Omni-Directional Wheeled Mobile Robot" as his research topic. Afterthat, he worked as a system administrator at Engineering Computer Center, Chulalongkorn University while attending the Doctor of Philosophy Program in Mechanical Engineering in 2000.