CHAPTER II

THE OPERATIONAL CALCULUS FOR BASIC IMPULSE CIRCUITS

General

In impulse work the main concern is the sudden application of a direct voltage wave to an equivalent impulse circuit made up of the usual circuit parameters R, L and C. These parameters are invariant with time, and the mathematical equations[9,10] describling the behavior of these linear time invariant systems are linear integro-differential equations with constant coefficients given as

$$A \frac{d^2u}{d\,t^2} + B \frac{du}{dt} + C = 0 \qquad \ldots\ 2\text{-}1$$

In solving this linear equation, the Heaviside operational method is much prefer as a simplicity of use.

Heaviside Operational Calculus[11,12]

In handling circuit problems described above, the Heaviside operational calculus is an extremely useful method. Since it is a shorthand method that is well suited to engineers in dealing with impulse phenomena.

Heaviside's Expansion Theorem

The Heaviside expansion theorem is very important, and it is reality easy to use; given as

$$\frac{Y(p)}{Z(p)} = \frac{Y(o)}{Z(o)} + \sum_{p=p_1,p_2} \frac{Y(p)\,\epsilon^{pt}}{pZ'(p)} \qquad \ldots\ 2\text{-}2$$

Where, $Y(p)$ and $Z(p)$ are two polynomials in p.

$$Z'(p) = \frac{dZ(p)}{dp}$$

The symbol p replaces the Heaviside operator $\frac{d}{dt}$, and it is also the roots of the characteristic equation

$$Z\ (p) = 0 \qquad \ldots\ 2\text{-}3$$

and that, the operational form of the circuit parameters are given below :

inductance L is pL

capacitance C is $\frac{1}{pC}$ ... 2-4

resistance R is R

Basic Impulse Circuit

(R - C Circuit)

The basic impulse circuit is shown in figure 2-1. It is the fundamental circuit from which all impulse generator circuits are derived. In this circuit a capacitor C is charged from a direct voltage U_o, and then discharge through a resistance R across which the impulse voltage u is developed. Discharging is performed by closing the switch S.

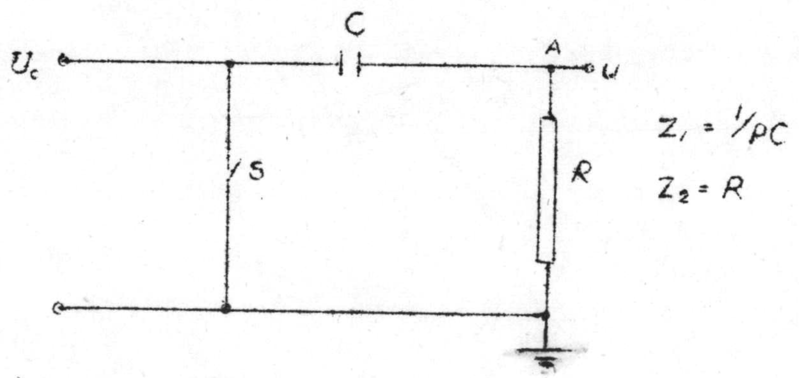

Fig. 2-1[13] Basic Impulse Circuit

Treating the two elements of the basic circuit as a potential divider, the impulse voltage u across the resistance is given by

$$u = -\frac{Z_2}{Z_1+Z_2} U_o$$

$$= -\frac{pCR}{pCR + 1} U_o \qquad \ldots \quad 2\text{-}5$$

$$= -\frac{Y(p)}{Z(p)} U_o$$

$$= -\left[\frac{Y(0)}{Z(0)} + \sum \frac{Y(p)\,\epsilon^{pt}}{pZ'(p)}\right] U_o$$

Here,

$$Y(p) = pCR \; ; \; \text{so that} \; Y(o) = 0$$

$$Z(p) = pCR + 1, \text{ and } Z(o) = 1 \qquad \ldots \quad 2\text{-}6$$

$$Z'(p) = CR$$

The roots of $Z(p) = 0$

$$p = p_1 = -\frac{1}{CR} \qquad \ldots \quad 2\text{-}7$$

Therefore,

$$u = -\left[0 + \frac{p_1 CR\,\epsilon^{p_1 t}}{p_1 CR}\right] U_o$$

$$= -U_o \epsilon^{-t/CR} \qquad \ldots \quad 2\text{-}8$$

Note that, if positive potential U_o is applied, the capacitor is positively charged, one plate being at $+U_o$ potential, the other being at zero potential due to ground connection the closing of the switch is equivalent to applying instantaneously a unit voltage of value $-U_o$; that is, the point A, which previously was at zero potential, is now suddenly raised to a potential $-U_o$. The voltage wave across the resistance then decays exponentially to zero at a rate dependent on the time constant, CR of the circuit. And the wave is given in figure 2-2.

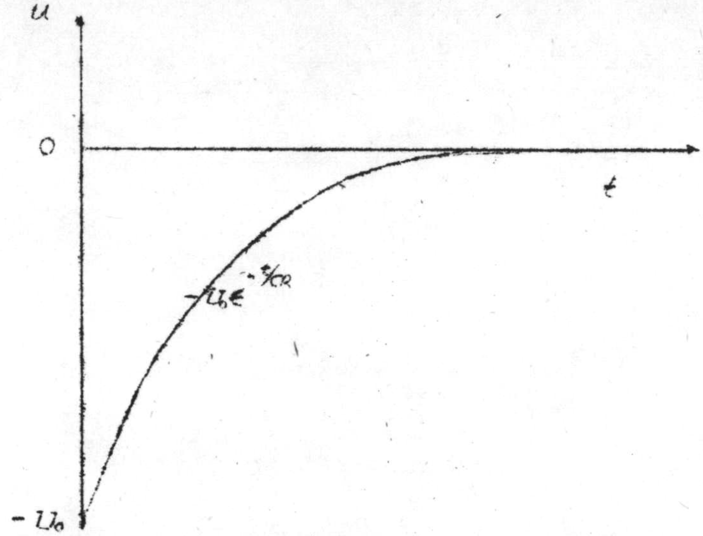

Fig. 2-2 Basic Impulse Wave form

$$u = -U_0 \epsilon^{-t/CR}$$

Again, neglecting the negative sign, the time taken for the wave to fall to half value $U_0/2$ is

$$\frac{u}{U_0} = 0.5 = \epsilon^{-t/CR} \quad \ldots \quad 2\text{-}9$$

or $\epsilon^{t/CR} = 2$

so that $t/CR = \ln 2 = 0.693$

and $t = 0.693\ CR\ \mu S \quad \ldots \quad 2\text{-}10$

if R is in ohms and C is in μF

Basic Impulse Circuit with Included

Inductance (RLC circuit)

The series RLC circuit is shown in figure 2-3. It is the same of the RC circuit, but with inductance L included. Neglecting the negative sign, which only means that if charging is positive, the resulting impulse is negative, and vice versa.

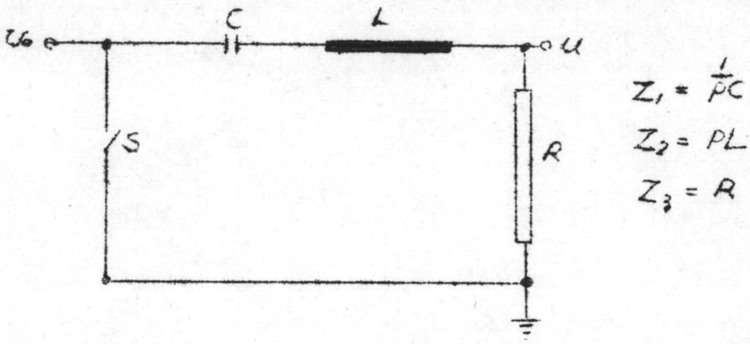

Fig. 2-3[13] RLC circuit

Again, by potential divider action the voltage u across the resistance R is given by

$$u = \frac{Z_3}{Z_1 + Z_2 + Z_3} U_o$$

$$= \frac{R}{R + pL + \frac{1}{pC}} U_o$$

$$= \frac{R}{L} \frac{p}{p^2 + \frac{pR}{L} + \frac{1}{LC}} U_o \qquad \text{... 2-11}$$

Here $Y(p) = \left(\frac{R}{L}\right)p, \quad Y(0) = 0$

$$Z(p) = p^2 + p\left(\frac{R}{L}\right) + \frac{1}{LC} \qquad \text{... 2-12}$$

$$Z'(p) = 2p + \frac{R}{L}$$

For $Z(p) = p^2 + p\left(\frac{R}{L}\right) + \frac{1}{LC} = 0$

the roots of p are

$$p = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

so that $p_1 = -(a-b)$

$$p_2 = -(a + b) \qquad \text{... 2-13}$$

Where $$a = +\left(\frac{R}{2L}\right)$$

$$b = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Therefore, from the expansion theorem

$$\frac{Y(p)}{Z(p)} = 0 + \frac{R}{L}\left[\frac{p_1 \epsilon^{p_1 t}}{p_1(2p_1 + \frac{R}{L})} + \frac{p_2 \epsilon^{p_2 t}}{p_2(2p_2 + \frac{R}{L})}\right]$$

$$= \frac{R}{L}\left\{\frac{\epsilon^{-(a-b)t}}{-2(a-b) + \frac{R}{L}} \mp \frac{\epsilon^{-(a+b)t}}{-2(a+b) + \frac{R}{L}}\right]$$

$$= \frac{R}{L}\left[\frac{1}{2b}\left(\epsilon^{-(a-b)t} - \epsilon^{-(a+b)t}\right)\right]$$

from which

$$u = U_o \frac{a}{b}\left(\epsilon^{-(a-b)t} - \epsilon^{-(a+b)t}\right) \qquad \ldots \quad 2\text{-}14$$

This is a typical double exponential wave of $U_o\left(\epsilon^{-\alpha t} - \epsilon^{-\beta t}\right)$ form, and if values for R, L and C are inserted the shape of wave can be found by plotting see figure 2-4.

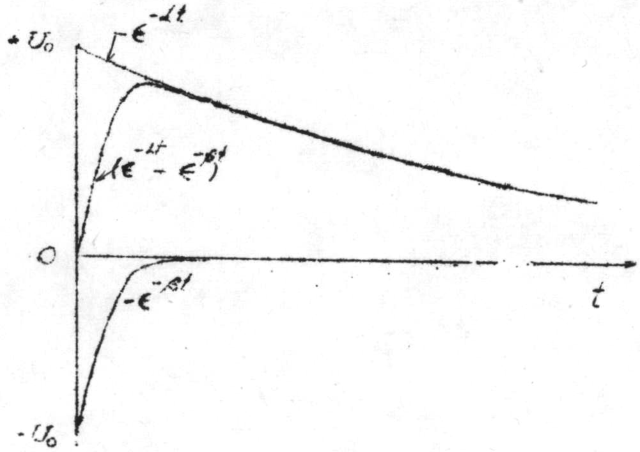

Fig. 2-4[14] Double Expontial Wave Shape

$$u = U_o\left(\epsilon^{-\alpha t} - \epsilon^{-\beta t}\right)$$

The time to peak value can also be obtained by differentiating the wave equation and equating to zero ; i.e.,

$$\frac{du}{dt} = 0 = \beta\epsilon^{-\beta t} - \alpha\epsilon^{-\alpha t} \qquad \dots 2\text{-}15$$

or $$\beta\epsilon^{-\beta t} = \alpha\epsilon^{-\alpha t}$$

from which $$\frac{\epsilon^{-\alpha t}}{\epsilon^{-\beta t}} = \frac{\beta}{\alpha}$$

$$\epsilon^{(\beta-\alpha)t} = \frac{\beta}{\alpha}$$

$$(\beta-\alpha)t = \ln.\frac{\beta}{\alpha}$$

$$t = \frac{\ln.\frac{\beta}{\alpha}}{(\beta-\alpha)}\ \mu S \qquad \dots 2\text{-}16$$

But for the sake of convenient, the time to peak value as well as the time to half-value of the wave-tail are advisely obtained by the wave shape discussed in the previous chapter.

Procedure in the case of Double Exponential Wave-forms

When desiring to know the transient response of a circuit to an impulse, the use of the single exponential wave provides adequate information. If it is imperative that the double exponential wave be used, then the two parts can be treated as separate single exponential waves, and the difference subsequently taken.

For the international 1/50 μsec wave, the accompanying table 1* provides the impulse wave constants for this wave shape

$$u = U_o\left(\epsilon^{-\alpha t} - \epsilon^{-\beta t}\right) \qquad \dots 2\text{-}17$$

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* A comprehensive list of waveforms is given in Appendix.1.

The value of U_o in the table is such that the amplitude of the wave is unity.

Wave shape t_1 / t_2	U_o	$\alpha \times 10^6$	$\beta \times 10^6$	Time to peak value μsec
1 / 50	1.036	0.0146	2.56	2.03

It will be of assistance to show a convenient way of plotting the wave $u = 1.036\ (\epsilon^{-0.0146t} - \epsilon^{-2.56t})$.[15] First draw up a table in the following manner, obtaining the values of the exponential terms from tables or from the graphs given in Appendix 2.

2. Table of Values for plotting equation

$$u = 1.036\ (\epsilon^{-0.0146t} - \epsilon^{-2.56t})$$

t	0.0146t	2.56t	$\epsilon^{-0.0146t}$	$\epsilon^{-2.56t}$	$(\epsilon^{-0.0146t} - \epsilon^{-2.56t})$	u
0	0	0	1	1	0	0
0.1	0.00146	0.256	0.9985	0.7741	0.2244	0.233
0.2	0.00292	0.512	0.9971	0.5993	0.3778	0.412
0.3	0.00438	0.768	0.9956	0.4639	0.5317	0.550
0.4	0.00584	1.024	0.9942	0.3592	0.6350	0.657
0.5	0.00730	1.280	0.9927	0.2780	0.7147	0.740
0.6	0.00876	1.536	0.9912	0.2152	0.7760	0.803
0.7	0.01022	1.792	0.9898	0.1667	0.8231	0.853
0.8	0.01168	2.048	0.9884	0.1290	0.8594	0.890
0.9	0.01314	2.304	0.9870	0.0999	0.8871	0.919
1.0	0.0146	2.560	0.9855	0.0773	0.9082	0.941
1.2	0.01752	3.072	0.9826	0.0455	0.9371	0.971
1.4	0.02044	3.584	0.9797	0.0279	0.9518	0.987
1.5	0.02190	3.840	0.9783	0.0215	0.9568	0.992
2.0	0.02920	5.120	0.9712	0.0060	0.9652	1.000
2.5	0.03650	6.400	0.9642	0.0017	0.9625	0.998
3.0	0.0438	7.680	0.9571	0.0005	0.9566	0.991

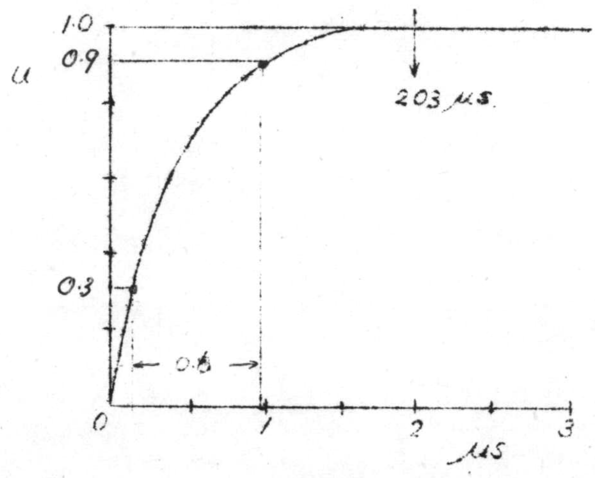

Fig. 2-5 Plot of the equation $u = 1.036\ (\epsilon^{-0.0146t} - \epsilon^{-2.56t})$

representing a standard unity-amplitude 1/50 μsec wave

Simplified circuit of Impulse generator and Load

Impulse generators are always operated in conjunction with other appearatus or loads, which results in a modification of the impulse wave shape. In case of capacitance load, its capacitive value mainly affects the wave shape by lengthening the front of the impulse.

Usually, a series resistance is introduced into the circuit, between the actual out put terminal of the generator and the test piece, for the purpose of removing unwanted ascillations from the peak of the generator wave.

The resistance potential divider is necessary to put in parallel across the whole generator resistors with ones having lower values for the sake of shorten of wave tail.

The equivalent circuit of the complete impulse set up assumes the form shown in figure 2-6.

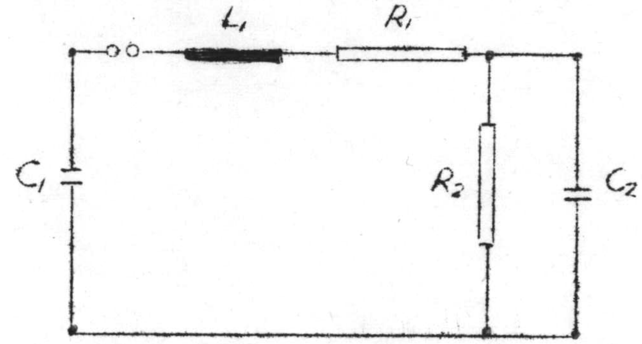

Fig. 2-6 Simplified circuit of impulse generator and load for calculation purpose

The voltage across the load C_2 is that across the parallel R_2C_2 branch, which combined give an impedance

$$Z_2 = \frac{R_2}{(1 + pC_2R_2)} \quad \ldots \quad 2\text{-}18$$

The total circuit impedance is then.

$$Z = \frac{1}{pC_1} + pL_1 + R_1 + \frac{R_2}{1 + pC_2R_2}$$

$$= \frac{p^3C_1C_2R_2L_1 + p^2\left[C_1L_1 + C_1C_2R_1R_2\right] + p\left[C_1R_1 + C_1R_2+C_2R_2\right] + 1}{pC_1\left\{1 + pC_2R_2\right\}} \quad \ldots \quad 2\text{-}19$$

so that $$u = \frac{Z_2}{Z} U_o$$

$$= \frac{pC_1R_2U_o}{p^3C_1C_2R_2L_1 + p^2\left(C_1L_1+C_1C_2R_1R_2\right) + p\left[C_1R_1+C_1R_2+C_2R_2\right] + 1} \quad \ldots \quad 2\text{-}20$$

For evaluating the critical resistance R_1 necessary to suppress any oscillation in in the circuit, R_2 can be neglected; i.e. $R_2 \rightarrow \infty$, then

$$u = \frac{pC_1U_o}{p^3C_1C_2L_1 + p^2C_1C_2R_1 + p(C_1 + C_2)}$$

$$u = \frac{C_1U_o}{p^2C_1C_2L_1 + p_1C_1C_2R_1 + (C_1 + C_2)} \qquad \ldots \quad 2\text{-}21$$

Oscillation will just not occur if

$$\sqrt{(C_1C_2R_1)^2 - 4C_1C_2L_1(C_1 + C_2)} = 0$$

i.e. if

$$R_1 = 2\sqrt{\frac{L_1(C_1 + C_2)}{C_1C_2}} \qquad \ldots \quad 2\text{-}22$$

Non-inductive Circuit

If L_1 is made zero, the circuit is said to be non-inductive, see figure 2-7

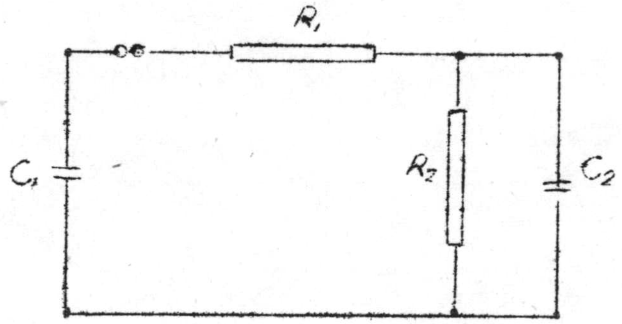

Fig. 2-7 Non-inductive Circuit

Then

$$u = \frac{pC_1R_2U_o}{p^2C_1C_2R_1R_2 + p\left[C_1R_1 + C_1R_2 + C_2R_2\right] + 1} \qquad \ldots \quad 2\text{-}23$$

Here, $\qquad Y(p) = pC_1R_2, \quad Y(0) = 0$

$$Z(p) = p^2 C_1 C_2 R_1 R_2 + p\left[C_1R_1 + C_1R_2 + C_2R_2\right] + 1$$

$$Z'(p) = 2pC_1C_2R_1R_2 + \left[C_1R_1 + C_1R_2 + C_2R_2\right]$$

Roots of $Z(p) = 0$ are

$$p_{1,2} = \frac{-\left[C_1R_1 + C_1R_2 + C_2R_2\right] \pm \sqrt{\left[C_1R_1+C_1R_2+C_2R_2\right]^2 - 4C_1C_2R_1R_2}}{2C_1C_2R_1R_2} \quad \ldots \text{ 2-24}$$

and the solution is of the double exponential form

$$u = \frac{C_1R_2}{\sqrt{\left[C_1R_1+C_1R_2+C_2R_2\right]^2 - 4C_1C_2R_1R_2}} U_o \left(\epsilon^{-p_1t} - \epsilon^{-p_2t}\right)$$

$$= \frac{U_o}{C_2R_1(p_2-p_1)} \left(\epsilon^{-p_1t} - \epsilon^{-p_2t}\right) \quad \ldots \text{ 2-25}$$

Simplified Non-oscillatory Inductive Circuit

If the tail of the wave is long compared with its front, as 1/50 wave, then little error results from ignoring the wave tail resistance when calculating the wave front duration.

The circuit then simplified to that shown in figure 2-8.

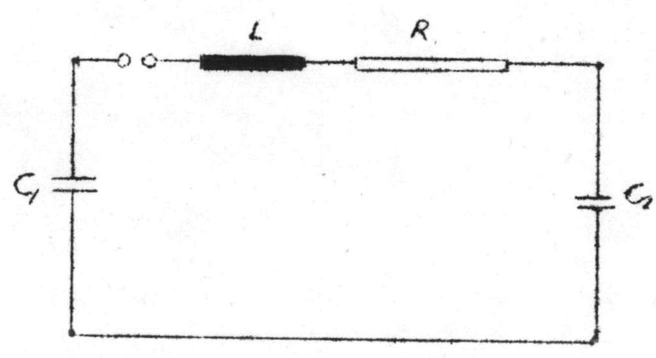

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Fig. 2-8 Simplified circuit for calculation of wave front

By potential divider action

$$u = \frac{1}{pC_2} \frac{pC_1C_2}{p^2C_1C_2L + pC_1C_2R + (C_1+C_2)} U_o \qquad \ldots \ 2\text{-}26$$

$$= \frac{1}{C_2L} \frac{1}{p^2 + p\frac{R}{L} + \frac{(C_1+C_2)}{C_1C_2t}} U_o$$

$$= \frac{1}{C_2L} \frac{1}{p^2 + p\frac{R}{L} + \frac{1}{LC}} U_o \qquad \ldots \ 2\text{-}27$$

where

$$C = \frac{C_1C_2}{C_1+C_2}$$

put

$$2\alpha = \frac{R}{L} \qquad \ldots \ 2\text{-}28$$

$$W_o^2 = \frac{1}{LC}$$

so that

$$u = \frac{1}{C_2L} \frac{1}{p^2 + 2\alpha p + W_o^2} U_o \qquad \ldots \ 2\text{-}29$$

then the solution,* is

$$u = \frac{U_o}{C_2L} \left\{ LC \left[1 - \epsilon^{-\alpha t}(1 + \alpha t) \right] \right\}$$

$$= \frac{CU_o}{C_2} \quad 1 - \epsilon^{-\frac{R}{2L}t} \left(1 + \frac{R}{2L} t \right) \qquad \ldots \ 2\text{-}30$$

If the circuit is critically damped, then $R = \sqrt{4L/C}$, so that u under this condition becomes

$$u = \frac{CU_o}{C_2} \left\{ 1 - \epsilon^{-\frac{2}{RC}t} \left(1 + \frac{2}{RC} t \right) \right\} \qquad \ldots \ 2\text{-}31$$

If α is reduced to zero, then

* W.G.Hawley, Impulse Voltage Testing, Chapman & Hall, Ltd., London, 1959, P.45.

$$u = \frac{C_1}{pC_1C_2R + (C_1 + C_2)} U_o$$

$$= \frac{U_o}{C_2R} \frac{1}{\left[p + \frac{C_1+C_2}{C_1C_2R}\right]}$$

$$= \frac{U_o}{C_2R} \frac{1}{p + \frac{1}{RC}} \qquad \text{... 2-32}$$

and the solution is

$$u = \frac{CU_o}{C_2} (1 - \epsilon^{-t/RC}) \qquad \text{... 2-33}$$

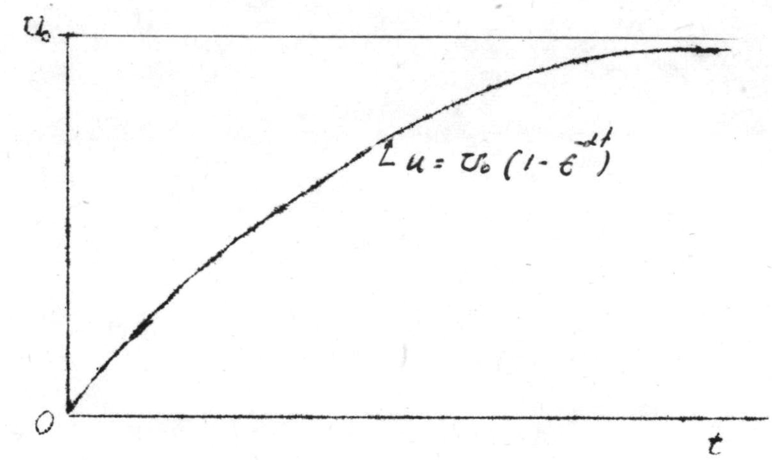


Fig. 2-9 Form of equation $u = U_o (1 - \epsilon^{-\alpha t})$

Effect of Inductance on Wave-front

The circuit inductance will lengthen the wave front of the impulse voltage wave, such that the inductive value must be kept as low as possible.

Figure 2-10 shows that when the load C_2 is 0.005 μF, the effect of the inductance on the wave front is a function of the generator capacitance C_1, and its value must not exceed 10 μH if a 1 μS wave front is desired.

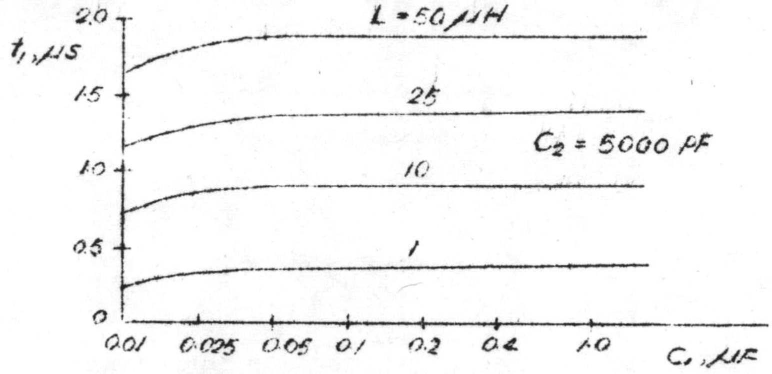


Fig. 2-10[15] Effect of inductance on wave-front, for a load of 5000 pF, as a function of generator capacitances.

Graphical Method[17]

Impulse Generator Circuits with Two Energy Stores

When the impulse generators are used in conjunction with the test objects, they form the simplified impulse generator circuits with two energy stores. The various circuits are given in figure 2-11.

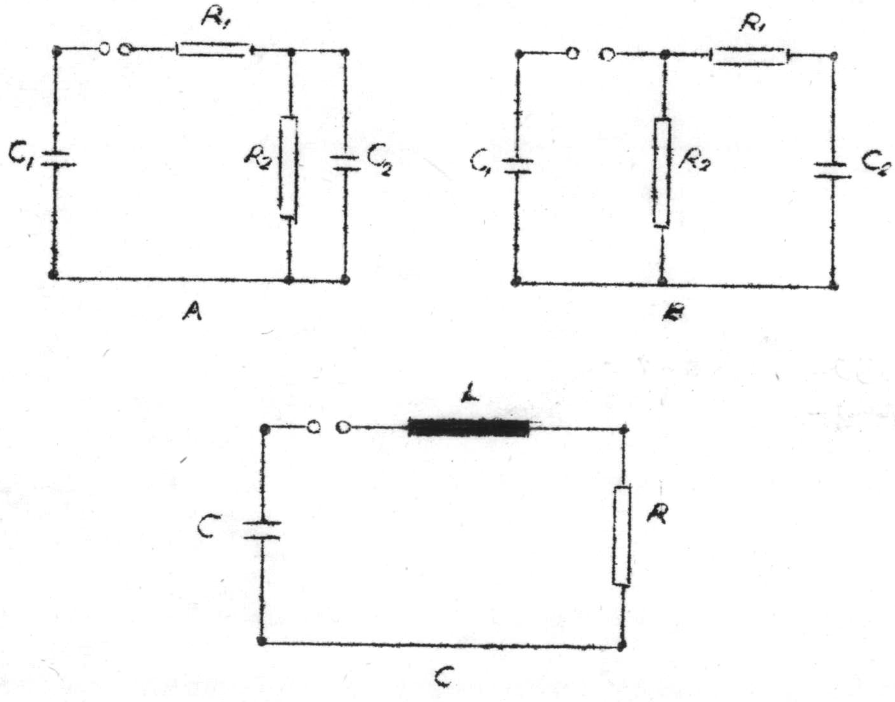


Fig. 2-11 Impulse generator circuits having two energy stores

Output Impulse Voltage

As seen in the previous calculations that, the two roots p_1 and p_2 of the characteristic equations depend upon the constant circuit parameters R, L and C. Thus all impulse generator circuits which have only two energy stores are characterized by the characteristic equation.

$$p^2 + ap + b = 0 \qquad \ldots \; 2\text{-}34$$

Where a and b are constant coefficients. If the roots p_1 and p_2 are real negative, that is

$$p_1 = -p_1$$

and

$$p_2 = -p_2 \qquad \ldots \; 2\text{-}35$$

Then the impulse voltage has the expected shape, as in general form.

$$u = \frac{U_o}{K(p_2 - p_1)} \left(\epsilon^{-p_1 t} - \epsilon^{-p_2 t} \right) \qquad \ldots \; 2\text{-}36$$

The constant coefficient K for any circuits A, B or C is given in table 3 below

Circuits	A	B	C
K	R_1C_2	R_1C_2	L / R

And the impulse voltage for each circuit is obtained by substituting and K into equation 2-36, and the wave shapes are plotted to scale.

Let the time function of the impulse voltage of equation 2-36 be written as

$$f(t, p_1, p_2) = \epsilon^{-p_1 t} - \epsilon^{-p_2 t} \qquad \ldots \; 2\text{-}37$$

But the impulse voltage wave shape are standardized by the times t_1, t_2 and the roots $-p_1$, $-p_2$. And since the relation between t and p is irrational, then it is most satisfactory to find the relationship in a graphic way.

Supposition $\quad |p_2| > |p_1| \quad (p_2 \neq p_1)$

$$p_1 t = \nu$$
$$p_2 t = \alpha \nu$$
$$\alpha = \frac{p_2}{p_1} \qquad \ldots \; 2\text{-}38$$

Then

$$f(t, p_1, p_2) = f(\nu, \alpha) \qquad (1 < \alpha < \infty)$$
$$= \left[\varepsilon^{-\nu} - \varepsilon^{-\alpha\nu} \right] \qquad \ldots \; 2\text{-}39$$

In practice, the value of α in y-axis is approximately equal to 10^4. The equation 2-39 represents a family of curves with the shape of impulse voltage of front time θ_1 and the time to half value θ_2. Thus it is possible to measure θ_1 and θ_2 by means of graphic construction, and the true values of θ_1 and θ_2 are given as

$$p_1 t_1 = \theta_1$$
$$p_1 t_2 = \theta_2 \qquad \ldots \; 2\text{-}40$$

The ratio

$$\frac{t_1}{t_2} = \frac{\theta_1}{\theta_2} \qquad \ldots \; 2\text{-}41$$

This ratio θ_1 / θ_2 is plotted against α ($\alpha = 1 \ldots 10^4$) in figure 2-12 and against θ_1 and θ_2 in figure 2-13. If t_1 and t_2 are given, then u may be found by the following process.

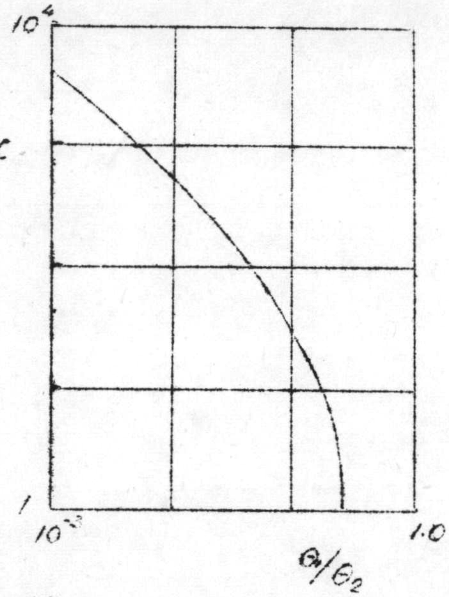


Fig.2-12 $\alpha = f(\theta_1 / \theta_2)$

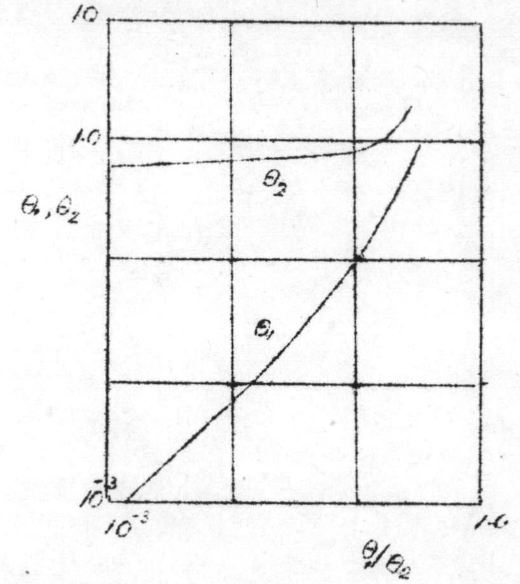


Fig. 2-13 $\theta_1, \theta_2 = f(\theta_1 / \theta_2)$

$$\frac{t_1}{t_2} = \frac{\theta_1}{\theta_2} \begin{cases} \nearrow \text{Fig. 2-12} \longrightarrow \alpha \\ \searrow \text{Fig. 2-13} \longrightarrow \theta_1 \longrightarrow p_1 = \dfrac{\theta_1}{t_1} \longrightarrow p_2 = \alpha \cdot p_1 \end{cases} \qquad \ldots \text{ 2-42}$$

$$K \longrightarrow u \longleftarrow (p_1, p_2)$$

But from equations 2-38, 2-40; p_1 and p_2 are easier calculated from

$$p_1 = \frac{\theta_2}{t_2}$$

and

$$p_2 = \frac{\theta_1}{t_2} \qquad \ldots \text{ 2-43}$$

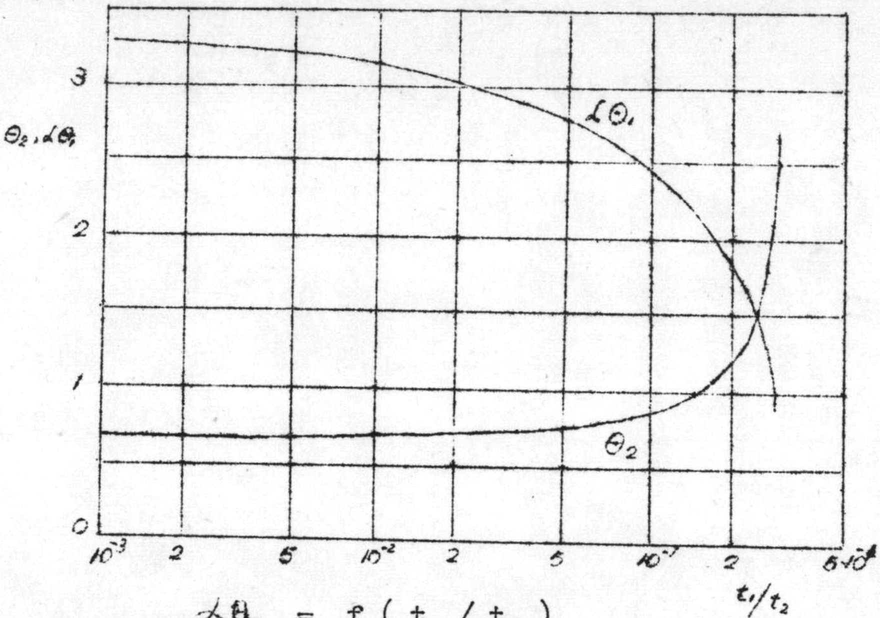
where it is sufficient to draw the graphs of

$$\alpha\theta_1 = f(t_1 / t_2)$$

and

$$\theta_2 = f(t_1 / t_2) \qquad \ldots \text{ 2-44}$$

given in figure 2-14



$$\measuredangle\theta_1 = f (t_1 / t_2)$$

Fig 2-14

$$\theta_2 = f (t_1 / t_2)$$

So that u can directly be found from t_1 / t_2 with the uses of figure 2-14 and equation 2-43 and the process is now

$$\frac{t_1}{t_2} \longrightarrow \text{Fig. 2-14} \begin{cases} \theta_2 \longrightarrow p_1 = \dfrac{\theta_2}{t_2} \\ \measuredangle\theta_1 \longrightarrow p_2 = \dfrac{\measuredangle\theta_1}{t_2} \end{cases} \qquad K \longrightarrow u \quad \ldots \quad 2\text{-}45$$

Voltage Efficiency (η)

The efficiency is given by the ratio of

$$\eta = \frac{u_{peak}}{U_o} \quad \ldots \quad 2\text{-}46$$

This ratio should be as high as possible. There exists a maximum for every circuit and every kind of impulse wave shape

If t_n is the time for impulse voltage to be peak, that is

$$u_{peak} = \frac{U_o}{K(p_2 - p_1)} \left[\epsilon^{-p_1 t_n} - \epsilon^{-p_2 t_n} \right] \quad \ldots \quad 2\text{-}47$$

Then

$$\eta = \frac{u_{peak}}{U_0} = \frac{1}{K(p_2 - p_1)} \left[\epsilon^{-p_1 t_n} - \epsilon^{-p_2 t_n} \right] \quad \ldots \quad 2\text{-}48$$

or

$$= \frac{1}{Kp_1(\alpha - 1)} \left[\epsilon^{-\vartheta_n} - \epsilon^{-\alpha \vartheta_n} \right] \quad \ldots \quad 2\text{-}49$$

For maximum efficiency

$$\frac{d\eta}{d\vartheta_n} = 0 \quad \ldots \quad 2\text{-}50$$

gets

$$\vartheta_n = \frac{\ln \alpha}{\alpha - 1} \quad \ldots \quad 2\text{-}51$$

Where ϑ_n is the value of standard time for the ratio peak value of impulse voltage to total charging voltage of the impulse generator becomes a maximum

and then

$$\eta = \frac{1}{Kp_1(\alpha - 1)} f(\alpha)$$

here

$$f(\alpha) = \left[\epsilon^{-\ln \alpha/(\alpha-1)} - \epsilon^{-\alpha(\ln \alpha/(\alpha - 1))} \right] \quad \ldots \quad 2\text{-}52$$

Since α and p_1 depend upon the voltage shape, and η depends only upon K for a given voltage shape, Then the impulse voltage will have a maximum efficiency whenever K is minimum.

The values of η for all three circuits of figure 2-10 are given in figure 2-15.

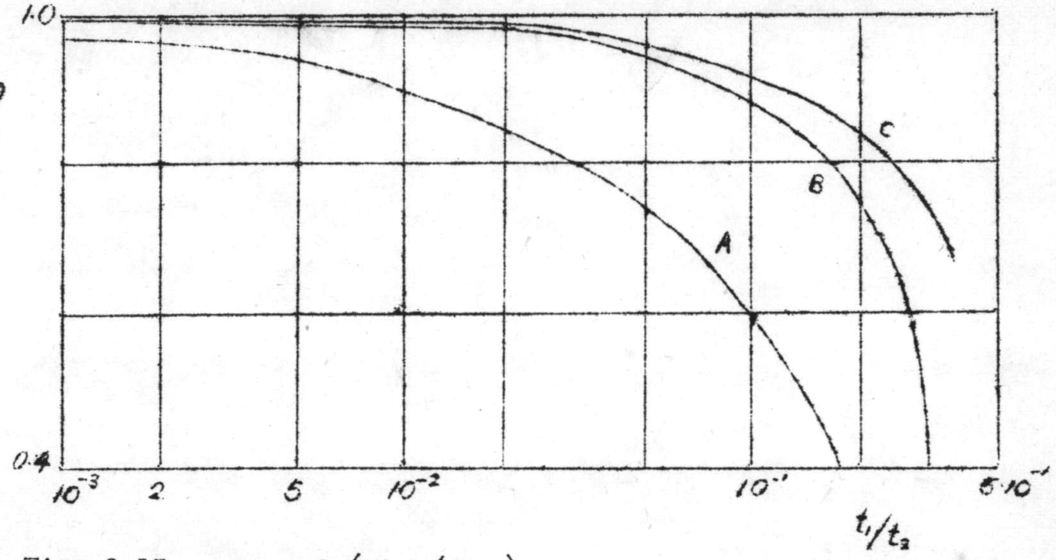


Fig. 2-15 $\eta = f\ (t_1\ /\ t_2\ )$

The Ratio C_1/C_2 for Maximum Efficiency

The efficiency will vary with changing ratios of generator C_1 to load capacitance C_2, and there is a wide divergence for large C_1/C_2 ratios for the two circuits locations of resistor R_1.

It can be shown for circuit A that is a maximum for a ratio C_1/C_2 which is itself a function of t_1/t_2, that means of impulse voltage wave shape. The function $C_1/C_2 = f\ (t_1/t_2)$ for maximum efficiency is shown in figure 2-16.

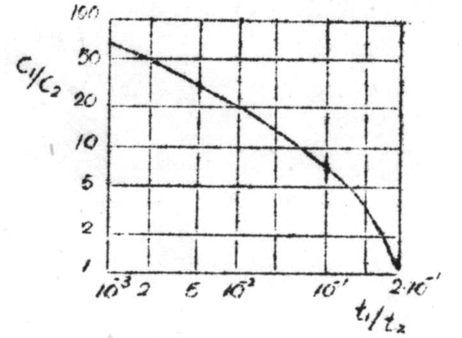


Fig. 2-16 $C_1/C_2 = f\ (t_1/t_2)_{\eta max.}$

valid for circuit A

In circuit B the ratio C_1/C_2 must be as high as possible. η reaches its maximum if $C_1/C_2 > 10^2$.

Procedure in Finding the Circuit Parameters

Circuit A

The two roots are

$$p_1, p_2 = \frac{-(C_1R_1+C_1R_2+C_2R_2) \pm \sqrt{(C_1R_1+C_1R_2+C_2R_2)^2 - 4C_1C_2R_1R_2}}{2C_1C_2R_1R_2} \quad \ldots \text{2-53}$$

so that,

$$\frac{p_1+p_2}{p_1p_2} = C_1R_1 + C_1R_2 + C_2R_2$$

$$\frac{1}{p_1p_2} = C_1C_2R_1R_2 \quad \ldots \text{2-54}$$

The condition for η is maximum is

$$C_1R_1 = C_2R_2 = \frac{1}{\sqrt{p_1p_2}} \quad \ldots \text{2-55}$$

But substitution, equation 2-54 is written as

$$\frac{2}{\sqrt{p_1p_2}} + \frac{1}{p_1p_2C_2R_1} = \frac{p_1+p_2}{p_1p_2} \quad \ldots \text{2-56}$$

then $$K_{min.} = (C_2R_1)_{min.} = \frac{1}{p_1+p_2-2\sqrt{p_1p_2}}$$

or $$C_2\,(R_1)_{min.} = \frac{p}{p_1+p_2-2\sqrt{p_1p_2}} \quad \ldots \text{2-57}$$

with $p_2 = \alpha p_1$,

$$C_2\,(R_1)_{min.}\,p_1 = \frac{1}{1+\alpha-2\sqrt{\alpha}} \quad \ldots \text{2-58}$$

Therefore the maximum efficiency is

$$\eta_{max.} = \frac{(\sqrt{\alpha}-1)^2}{\alpha-1}\; f(\alpha) \quad \ldots \text{2-59}$$

The value of α is obtained from figure 2-12, and the parameters can be calculated. But remember that the capacitive value of the test object must be included in C_2.

It is sufficient to write the ratio of C_1/C_2 in the function of t_1/t_2 for maximum efficiency condition; again write,

$$C_1R_2 = \frac{p_1 + p_2}{p_1p_2} - 2\frac{1}{\sqrt{p_1p_2}}$$

and

$$C_2R_2 = \frac{1}{\sqrt{p_1p_2}} \qquad \ldots \; 2\text{-}60$$

then

$$\frac{C_1}{C_2} = \frac{(\sqrt{\alpha} - 1)^2}{\sqrt{\alpha}} \qquad \ldots \; 2\text{-}61$$

For any t_1/t_2 wave, α is obtained from figure 2-11 and C_1/C_2 from figure 2-16 The values of R_1 and R_2 are given by

$$R_1 = \frac{1}{C_1\sqrt{p_1p_2}}$$

and

$$R_2 = \frac{1}{C_2\sqrt{p_1p_2}} \qquad \ldots \; 2\text{-}62$$

$$= \frac{C_1}{C_2} R_1$$

Circuit B

The two roots are given as

$$\frac{p_1 + p_2}{p_1p_2} = C_1R_1 + C_2R_1 + C_2R_2 \qquad \ldots \; 2\text{-}63$$

and

$$\frac{1}{p_1p_2} = C_1C_2R_1R_2$$

The efficiency of the circuit is

$$\eta = \frac{1}{C_2R_2}\,\frac{1}{p_1}\,\frac{1}{\alpha-1}\,f(\alpha) \qquad \ldots \; 2\text{-}64$$

Therefore

$$R_2 = \frac{p_1+p_2}{2p_1p_2C_2} \pm \sqrt{\left(\frac{p_1 + p_2}{2p_1p_2C_2}\right)^2 - \frac{1}{p_1p_2C_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)} \quad \ldots \quad 2\text{-}65$$

Condition for maximum efficiency, C_2R_2 is minimum. i.e.

$$(C_2R_2)_{min.} = \frac{1}{2p_1p_2}\left\{p_1 + p_2 \pm \sqrt{(p_1+p_2)^2 - 4p_1p_2}\right\} \quad \ldots \quad 2\text{-}66$$

Since C_2 is constant for any load capacitance,

$$(R_2)_{min.}C_2 = \frac{1}{p_1}\frac{1}{2\alpha}\left[1 + \alpha \pm (\alpha - 1)\right]$$

$$= \frac{1}{p_1}\frac{1}{\alpha} \quad \ldots \text{ only} \quad \ldots \quad 2\text{-}67$$

Then

$$\eta_{max.} = \frac{\alpha}{\alpha - 1} f(\alpha) \quad \ldots \quad 2\text{-}68$$

The value of η is given in figure 2-15. And the ratio C_1/C_2 must be greater than 10^2, 10^3... for maximum efficiency.

The resistances R_1 and R_2 are given by

$$R_2 = \frac{1}{p_1C_2}\frac{1}{\alpha} = \frac{1}{p_2C_2}$$

and

$$R_1 = \frac{1}{p_1p_2R_2C_1C_2} = \frac{1}{p_1C_1} \quad \ldots \quad 2\text{-}69$$

Where the values of p_1, p_2, C_1 and C_2 are obtained from figure 2-14 and equation 2-43.

Circuit C

The efficiency of the circuit is

$$\eta = \frac{R}{L} \cdot \frac{1}{p_1} \cdot \frac{1}{\alpha - 1} \cdot f(\alpha) \qquad \ldots 2\text{-}70$$

The characteristic equation of the circuit is

$$p^2 + \frac{R}{C} p + \frac{1}{LC} = 0 \qquad \ldots 2\text{-}71$$

Then the two roots are given as

$$p_1 + p_2 = \frac{R}{C}$$

$$p_1 p_2 = \frac{1}{LC} \qquad \ldots 2\text{-}72$$

For maximum efficiency,

$$R < \frac{1}{2} \sqrt{L / C} \qquad \ldots 2\text{-}73$$

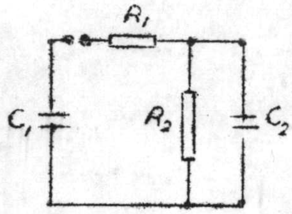
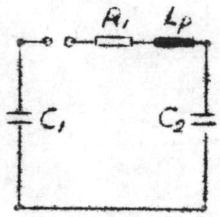
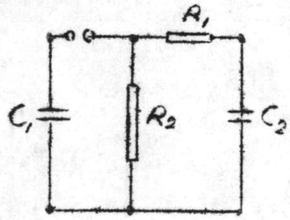
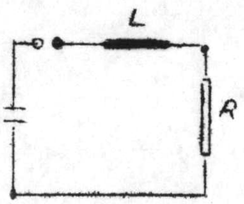
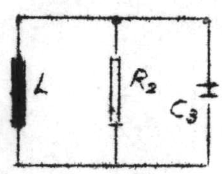
or

$$R < \frac{1}{2} \cdot \frac{1}{C} \cdot \frac{1}{R}$$

$$< \frac{1}{2} \cdot \frac{1}{C_{max.}} \cdot \frac{1}{p_1 + p_2} \qquad \ldots 2\text{-}74$$

$$\eta_{max.} = \frac{\alpha + 1}{\alpha - 1} f(\alpha) \qquad \ldots 2\text{-}75$$

Table 4. Circuits and parameters for maximum efficiency

Circuit	R_1	R_2	Critical damping
	$\frac{C_1}{C_2} R_2$	$\frac{1}{C_1\sqrt{p_1 p_2}}$	 $R_1 > 2\sqrt{L_p \frac{C_1 + C_2}{C_1 C_2}}$ or $L_p < \frac{R_1^2 C_1 C_2}{4(C_1 + C_2)}$
	$\frac{1}{p_1 C_1}$	$\frac{1}{p_2 C_2}$	
	$\frac{R}{L} = p_1 + p_2$ $\frac{1}{LC} = p_1 p_2$		 $R_2 < \frac{1}{2}\sqrt{\frac{L}{C_3}}$ or $C_3 < \frac{L}{4R_2^2}$

Samples of Application

Given $C_1 = 0.1\ \mu F$

$C_2 = 200 \ldots 300\ pF$

standard wave with $t_1 = 1.2\ \mu S$. and $t_2 = 50\ \mu S$

Find R_1 and R_2 for maximum efficiency.

solution

$$\frac{t_1}{t_2} = 0.024$$

Figure 2-14 $\quad \theta_1 = 2.95$

$$\theta_2 = 0.73$$

Equation 2-43 $\quad p_1 = 0.0146\ \mu S^{-1}$

$$p_2 = 2.46\ \mu S^{-1}$$

Circuit A

$$\eta = 85\ \%$$

$$\frac{C_1}{C_2} = 11.1$$

$$C_2 = 9\ nF$$

$$R_1 = 587\ \Omega$$

$$R_2 = 53\ \Omega$$

Circuit B

$$\eta = 97\ \%$$

$$\frac{C_1}{C_2} = 10^2$$

$$C_2 = 10\ \text{nF}$$

$$R_1 = 70\ \Omega$$

$$R_2 = 41\ \Omega$$

Circuit C

$$\eta = 97\ \%$$

No oscillation

$$R_2 < 674\ \Omega$$

$$L < 272\ \mu H$$

$$C > 0.1025\ \mu F$$

For $C = 0.15\ \mu F$

$$L = 186\ \mu H$$

$$R_2 = 460\ \Omega$$