CHAPTER II

THEORY

2.1 Radioactive Neutron Sources (13) (8)

In this category we will condider sources of neutrons made up of a target material mixed or alloyed with a naturally decaying radioactive component which supplies the bombarding radiation for the release of neutrons. Radioactive sources are usually of relatively small volume. Thus they are readily portable and adaptable to particular experimental arrangements. These sources also can be calibrated quite accurately, and the neutron output is practically constant. In general it may be said that neutron sources which depend upon a radioactive preparation for the bombarding radiation are limited in the rate of neutron emission which can be conveniently achieved. The radioactive sources which have been used in the past have an order of neutron emission not greatly exceeding 107 neutron per sec.

There is a variety of reactions which lead to neutron production. In such reactions compound nucleus excited with the sum of the binding energy and the kinetic energy (in the center of mass system) of the projectiles first are formed by bombardment of target nuclei with x- particles, protons, deuterons, or gamma-rays. If the excitation energy is larger than the binding energy of the "last neutron in the compound neucleus, then a neutron is very likely to be emitted."

The discussion will be held on radioactive $(\alpha_1 n)$ sources only because they have been found to be most useful. The fact that the alpha particles from radioactive substances do not have energies extending much above 5 Mev. Those nuclides which have thresholds for the $(\alpha_1 n)$ reaction within this region of energy at first show a slow rise in the neutron yield as the alpha-particle energy increases beyond the threshold. This excitation curve rises more rapidly as the alpha-particle energy approaches the maximum energy of the alpha particles from the radioactive preparation. Because of the high yield of neutrons from beryllium, this element has been used almost exclusively as the target material in $(\alpha_1 n)$ radioactive neither source.

Americium-Beryllium (~,n) sources had been used in the experimental part of this thesis. Americium, is alloyed with Beryllium to form Am-Req3. Americium 241 itself has a half-life of about 470 years. Although this isotope decay by emitting alphaparticles of about 5.4 Mev. these alpha particles are followed by gamma-rays in the 40 to 60 kev region in the majority of the disintegrations. The (~,n) reaction by which alpha particles release neutrons from beryllium can be represented by

Hence the reaction is excergic. The distribution of neutrons energy would be expected to range from about 6.7 New to 10.9 Nev.



depending on whether the neutron is emitted the same direction as that of the incident alpha particle or in the opposite direction. The details of energetic consideration (E_n) and the strength of neutron source were given by Bockurts. The neutrons from Am (\checkmark,n) Be sources are not monoenergetic.

2.2 Specifications of Neutrons (6) (10)

Neutron energy distribution in water is composed of two companents, say, a Maxwellian Thermal and a 1/E epithermal flux.

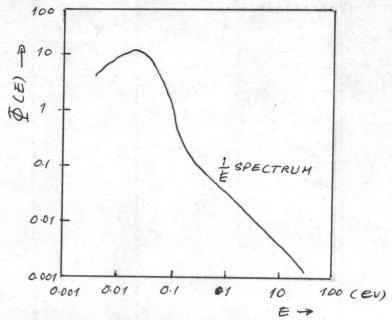


Fig 2.2.1 Neutron spectra in pure water.

2.2.1 Thermal Neutrons (14)

When fast neutrons have been slowed down by collisions until neutrons reach an energy that is comparable to that of the atoms of the moderator, the neutrons are called thermal neutrons. The energy and corresponding velocities of the neutrons.

then depend upon the temperature of the medium. The distribution of the velocities approaches the Maxwell distribution,

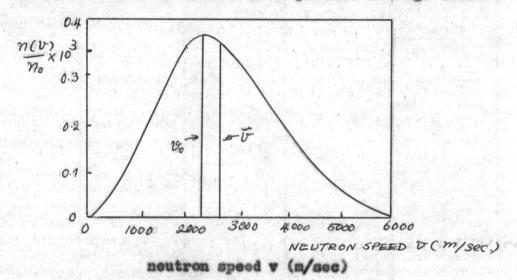


Fig. 2.2.2 Maxwellian distribution of neutrons in medium at 20°C. in medium at 20°C.

By the integration of eqn. (2.2.1) with some specified limit of velocity, the following velocities \mathbf{v}_0 , $\vec{\mathbf{v}}$, $\mathbf{v}_{\mathbf{g}}$ are obtained where \mathbf{v}_0 is the standard velocity of 2,200 m/sec, $\vec{\mathbf{v}}$ is the Maxwellian average velocity and $\mathbf{v}_{\mathbf{g}}$ is the most probable velocity. For a Maxwellian spectrum the relation among them are $\mathbf{v}_{\mathbf{g}}$ / $\vec{\mathbf{v}}$ = $\sqrt{\pi}/2$ and \mathbf{v}_0 / $\vec{\mathbf{v}}$ = $(\sqrt{\pi}/2)^{1/2}$

Cross section: A common and simple situation is the combination of Maxwellian distribution and absorber with 1/v cross section, $\delta = K/v$. The constant K may be any product of speed and known cross section. Thus for 1/v cross section absorber

Fluxes Neutron flux may be defined as a differential flux:

An extension to the case where the neutrons selected have a variety of speeds can be made by writing the flux as an integral:

Where n(v) the neutron velocity distribution function; is the number of neutrons in a unit velocity range evaluated at v. By the definition of average velocity.

$$\vec{\nabla} = \int_0^\infty \frac{n(v)v \, dv}{\int_0^\infty n(v) \, dv}$$
 the total flux is

Where n is the total density of neutron and $\overline{\phi}$ = $n\overline{v}$ called the "true" thermal flux.

Usually the flux nv is computed for the thermal component only of the neutrons. However, the flux is often measured by a method which leads to another convention. The reaction rate in the detector is devided by the 2,200 m/sec cross section (o) to obtain the "flux ".

Reaction rates : Since for a 1/v law detector, $G = G_V/v$ and the reaction rate $R = \int G(v) \cdot v n(v) dv$ $= \int_0^\infty G_0 \cdot n(v) dv = n v \cdot G_0 = \Phi_0 \cdot f_0 \cdot$

and its corresponding reaction rate is $\overline{\sigma}\overline{\Phi}$

2.2.2 Resonance Neutrons. (13) (17)

The neutrons that are in the process of being slowed down in the moderator, frequently called resonance neutrons and the energy distribution is known as 1/2 spectrum. The 1/2 distribution is ploted in Fig. 2.2.3.

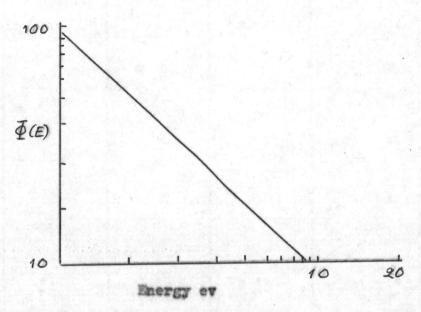


Fig. 2.2.3 Spectrum of 1/E Region.

Cross Section: The resonance cross section was determined by the useful Breit-Wigner formula for the absorption cross section with the eleminated perturbation caused by the Droppler effect. Beckurts (13) gave the derivation of the resonance cross section, finally $O_{\tau}(E)$ is:

$$\mathcal{O}_{\Gamma}(E) = \frac{\mathcal{O}_{o}}{1 + \left(\frac{E - E_{o}}{\Gamma/2}\right)^{2}} \\
\mathcal{O}_{\Gamma}(E) \propto \frac{1}{1 + \left(\frac{E - E_{o}}{\Gamma/2}\right)^{2}} \\$$

Where E_0 is the energy of the resonance peak and Γ is the total width of the resonance peak.

Fluxes. Since the energy-dependent flux distribution is defined as the number of neutrons in a range dE at energy E

(velocity v) is n(E) dE, and the corresponding flux is

 $\Phi(E)$ dS = n(E) dE v(E). Thus $\Phi(E)$ is the flux per unit energy at energy E. In the region that $\Phi(E)$ is inversly proportional to the energy, $\Phi(E)$ can be obtained by introduced a dimensionless variable, the lethargy, as a substitute for energy. Thus

$$\theta = \overline{\Phi}(u) = E \overline{\Phi}_{p}(E)$$
 -----(2.2.8)

Where u is the lethargy.

Reaction rates. The epithermal or resonance reaction rate Re is given in terms of a resonance integral $\mathcal{T}_{\mathbf{p}}$ as

(6.2.2.9)
$$R_0 = 0 \int_{R_0}^{\infty} \frac{\sigma(p)}{p} dp$$

where
$$\int_{E_c}^{\infty} \frac{G(E)}{E} dE = G_r$$
 (2.2.10)

B is the energy out-off of a neutron filter.

Summarizes of neutrons specification was constructed in table 2.2.1

Table 2.2.1

Type of neutron	Reaction rate	Flux	Cross section
Shermal Spithermal	ovn(v) dv. o ∫ o(z) dz Ecc z	ā	telion organis

2.2.3 The Gadmium Ratio. (1) (8)

Cadmium is commonly used in neutron activation experi-

ments as an absorber for thermal neutrons. One means of determining the ratio of the intensity of the thermal neutrons to that of the resonance neutrons is by measurement of the cadmium ratio. The reason for the use of cadmium for excluding the effects of resonance neutrons becomes apparent from the curve in Fig. 2.2.4 The curve shows the total absorption cross section of cadmium as a function of the neutron energy in the range of energies from 0.01 to 10 ev. The curve reveals that cadmium is practically opaque to thermal neutrons, but transparent to neutrons having energies greater than about 0.5 ev. The highest thermal absorption cross section is at 0.178 ev. The cadmium ratio is defined as the ratio between the observed neutron intensity as revealed by a porticular detector (bare foil reaction rates) to the intensity observed with the same detector covered by a layer of cadmium (Cadmium-covered reaction rate). In the first measurement the detector responds to the sum of the thermal plus the resonance neutrons. In the second measurement the detector responds only to the resonance neutrons. The cadmium ratio is then

Where R_b and R_e are bare foil and cadmium-covered foil reaction rates respectively.

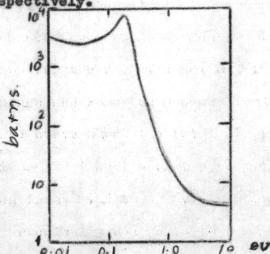


Fig. 2.2.4 Total cross section of cadmium as a function of neutron energy.

Thus the eqn. (2.2.12) may be taken as a measure of how well the neutrons are thermalized. The greater the ratio the greater the degree of thermalization. There is a method to determine the ratio of epithermal flux per unit lethargy to the true thermal flux by measuring the cadmium ratio of the detecting foils. For bare foil the reaction rate is $\overline{\Phi}\overline{\sigma} + \theta \overline{G}$, and the reaction rate for cadmium covered foil is approximately $\theta \overline{G}$ (true reaction rate = $\theta \overline{G} < \theta \overline{G}$). By definition the cadmium ratio is

$$GdR = \frac{\overline{\phi}\overline{\sigma} + \theta \overline{\sigma}_{\underline{r}}}{\theta \overline{\sigma}_{\underline{r}}}$$

$$GdR - 1 = \frac{\overline{\phi}\overline{\sigma}}{\theta \overline{\sigma}_{\underline{r}}}$$

$$GdR - 1 = \frac{\overline{\phi}\overline{\sigma}}{\theta \overline{\sigma}_{\underline{r}}}$$

$$(2.2.14)$$
and
$$B = \frac{\theta}{\overline{\phi}} = \frac{\overline{\sigma}}{\sigma_{\underline{r}}} \frac{1}{(GdR - 1)}$$

$$(2.2.15)$$

2.3 Westcott's Formalism (7) (10) (12)

The convention proposed has the feature that for any reaction whose cross section varies inversely as the neutron velocity, the effective value for any neutron spectrum is just the 2,200 m/sec cross section. Then the reaction rate is proportional to the neutron density and is independent of the neutron spectrum. Hence the effective cross section (ô) is defined by equating the reaction rate per atom present, R, to the product of ô with nvew where

epithermal neutrons.

and v_o = 2,200 m/sec.

The reaction rate is the following:

R = mv_6(2.3.1)

when the neutron spectrum is the sum of two components, an epithermal dE/E flux distribution cut off are a suitable lower limit of energy and a Maxwellian distribution corresponding to T^OK. These two components overlap in energy since no cut-off is imposed on the Maxwellian component. An epithermal index r represents the relative strength of the epithermal component. The resulting relation for the effective cross section is

6 = 6 (g + rs)(2.3.2)

where δ_0 is the cross section for 2,200 m/sec neutron and g and s are functions of the temperature T depending on the departure of the cross section law from the 1/v form (for a 1/v law, g = 1 and s = 0).

2.3.1 Betermination of \hat{O} Westcott (10) gave the derivation of \hat{O} from the reaction rate of eqn (2.3.1) that the reaction rate per atom with a cross section O(v) in the neutron spectrum composed of Maxwellian and epithermal density distribution functions is given by

$$R = nv_0 \hat{\sigma} = \int_0^\infty n(v) \, \sigma(v) \cdot v dv$$

$$= n(1 - f) \int_0^\infty \frac{1}{\sqrt{\pi}} \frac{v^3}{\sqrt{2}} e^{-(v/v_0)^2} \, \sigma(v) dv$$

$$+ nf_0 v_0 v_0^{2/2} \stackrel{\triangle}{\sim} \sigma(v) dv \qquad (2.3.3)$$

where $v_T = v_0 (T/T_0)^{1/2}$ and a unit step-function is used for $\triangle (\triangle = 1 \text{ if } E) \text{ pkT}, \triangle = 0 \text{ if } E < \text{pkT} \text{ and } \mu = 5).$

f = fraction of the total density in the epithermal

In the case, the spectrum being pure Maxwellian f=0. The effective cross section for this case is \hat{O}_m and the g factor is given by $g=\hat{O}_m/\hat{O}_0$.

When f is not zero, we may proceed as follows:

$$R = nv_0 \hat{\sigma}_m + nf \left[v_T \mu^2 \right] \stackrel{\triangle}{=} \sigma(v) dv - v_0 \hat{\sigma}_m$$
and
$$\hat{\sigma} = g \hat{\sigma}_0 + \left[\frac{4T}{T} \right]^2 r \int_{\mu kT}^{\infty} \left[\sigma(v) - \hat{\sigma}_{mv} \right] \stackrel{\triangle}{=} dE \qquad (2.3.4)$$

Where r = f(Tm) 1/4

Note that Westcott called the integral in the eqn (2.3.4) Z .

It is a "resonance integral " of the usual type with an effective lower energy limit of uET and with a 1/v term subtracted.

From eqn. (2.3.4) Westcott defined a quantity S by the equation S = 1 (42) (2.3.5)

and then obtain the same equation for \widehat{G} given as eqn.(2.3.2) above. 2.3.2 <u>Gadmium Ratios and The Determination of r(T/T₀)^{1/2} (10) (16)</u>

The cadmium ratios can be measured and from these r is easily deduced, if the neutron temperature is uncertain the quantity obtained is $r(T/T_0)^{\frac{1}{2}}$ rather than r. The bare foil reaction rate may be written as

Westcott assumed that the cadmium transmits all neutrons of energy greater than E_{cd} (E_{cd} = cadmium cut-off energy) and absorbs all neutrons of energy less than E_{cd} . Then the epi-cadmium activation can be written as.

When the detector has a resonance near thermal energies or

near the cadmium cut-off, the integral term in eqn (2.3.7) may

$$\int_{\sqrt{2E_{cd}}}^{\infty} \int_{\sqrt{2\mu kT}}^{\infty} - \int_{\sqrt{2\mu kT}}^{\sqrt{2E_{cd}}}$$

Considering the last integral

$$\sqrt{2} \text{ E}_{cd}$$
= 1/v component + excess ever 1/v component
$$\sqrt{2}\mu kT$$
= $\frac{8v_0 G_0}{(2\pi c_0)^{1/2}} - \frac{8v_0 G_0}{(2\mu kT)^{1/2}} + \frac{6\Sigma}{2} \dots (2.3.8)$

Where δZ is the contribution to the excess over 1/v resonance integral between ukT and S_{cd} (\triangle = 1 for $E > \mu kT$). Write $W = (\delta Z/Z)(S_o/g)$ where $S_o = S(T/T_o)^{\frac{1}{2}}$

Then
$$\int_{2\mu kT}^{\sqrt{2E}cd} \frac{dv}{v} = \frac{80_0 v_0}{(2B_{cd})^{1/2}} - \frac{80_0 v_0}{(2\mu kT)^{1/2}} + \frac{1180_0 \sqrt{1}}{4} \dots (2.3.9)$$

Thus the epi-cadsius reaction rate is

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$$\frac{1}{\sqrt{17}} = \frac{4nrv_0 \sigma_0}{\sqrt{17}} = \frac{1}{4} \left(\frac{T^2}{2}\right)^2 + \frac{8v_0}{(28)} - \frac{1}{4} \left(\frac{1}{2}\right)^2$$

and

$$ddR = \frac{R_b}{R_o} = \frac{g + rs}{\sqrt{\frac{2}{T_o} \left[\sqrt{\frac{2}{T_o} + 4g\sqrt{\frac{2}{T_o}}}{\sqrt{112}gd}\right]}} \dots (2.3.10)$$

For a1/v absorber S = W = 0 and g = 1.

Westcott proposed a factor K by considering the actual variation of the cadmium cross section with energy to define an effective sharp cut-off of cadmium. Thus

Hence we may write for non 1/v cross section between pkT and

The cadmin ratio need a correction factor F due to some attenuation of the resonance neutrons by cadmium. In addition the factor G_p and G_{th} are introduced also. Then $r(T/T_0)^{\frac{1}{2}}$ in eqn.

$$r(2/2_0)^{1/2} = \frac{G_{th}}{(r \text{ CdR} - 1) \frac{S_0G_r}{s} +(2.3.14)}$$

Equation 2.3.14 is well known as Westcott epithermal index.

Epithermal index for fail where G, and Gth neglected may be expressed as

$$2(2/2)^{\frac{1}{2}} = \frac{1}{(FGAR - 1) \frac{S_0}{c} + GAR(1/R - B)}$$
(2.3.15)

2.4 Flux Perturbation by Detecting Foils. (11), (17)

A standard technique employed in the measurement of low energy neutron fluxes in various media is based upon the measurement of the activity induced in thin metal foil. But the presence of the detecting foil within the flux to be measured creates a number of perturbations which cause the activation per unit mass of the foil to be not exactly proportional to the undisturbed neutron flux.

The perturbations involving neutron effects can be separated into two categories: self-shielding and flux depression. Self-shielding arises from the attenuation of the neutron flux as it penetrates the foil, so that the interior of the foil has a lower saturated activity than the surface layers. Flux depression describes the decrease of the flux in the medium near the foil due to absorptions in the foil.

2.4.1 Corrections for Flux Depression Effect. (17)

Within the assumptions of (4) one-velocity diffusion theory in the medium, (b) no scattering in an infinite-radius foil, and (c) the usual P₄ expansion of the neutron flux, the total foil perturbation G_{th} is given by

$$G_{th} = \frac{2x}{1 + \frac{\alpha_1(x)[3]}{2[2^{\lambda_{th}} + 1]}} = \frac{1}{2[2^{\lambda_{th}} + 1]}$$

The factor G_{th} is readily recognized as the thermal self-shielding factor, and therefore the flux depression factor F_{f} (x) is given by

$$F_g(x) = \left[1 + \frac{\infty_1(x)}{2} \left(\frac{3L}{2\lambda_{tr}} \frac{R}{R+L} - 1\right)\right]^{-1} \cdot \cdot \cdot (2.4.3)$$

This factor was suggested by Bothe. He assumed that a circular foil of radius R would be equivalent in effect on the neutron density to a spherical shell of a radius 2R/3.

2.4.2 Corrections for Self-Shielding Effect. (/),(15)

Thermal Self-Shielding Factor (Gth). (11)

Self-shielding correction factor may be expressed as the ratio of the average flux over the foil to the unperturbed flux and it may be estimated from the recent paper of Baumann (2) For monoenergotic neutrons and a purely absorbing foil, the selfshielding correction factor is given by

$$\frac{\overline{\Phi}}{\overline{\Phi}} = \left[\frac{1}{2} - E_3(x) \right] (1/x) \dots (2.4.4)$$

$$\frac{\overline{Q}}{\overline{Q}} = G_{th} = 1 - \frac{x}{2} \left[\frac{3}{2} - \delta - \ln x \right] - \frac{x^2}{1.3!} + \frac{x^3}{2.4!} - \dots (2.4.5)$$
where X is the product of the foil thickness t gm/cm.² and the effective capture cross-section of the foil in cm.²/gm.

$$X = \frac{N_0}{\Lambda} \sigma_{eff} \cdot t$$
 $\Lambda = \text{atomic weight}$ $= \sum_{off} \cdot t$ $N_o = \text{Avogadro's number} \cdot t$

$$B_3(x) = \int_1^\infty u^{-3} \cdot e^{-Xu} du$$

$$0.5772157$$

The expression above is called the thermal self-chielding factor and can. 2.4.5 is used for thin foil. Baumann choose $X = \sum_{\text{eff}}$ instead of \sum_{a} in eqn. (2.4.2) because it has been point out that a thin 1/v detector reduces a normally incident Maxwellian flux by the factor $(1 - (\sqrt{m}/2) \sum_{\text{o}} t)$ where \sum_{o} is the cross section appropriate to v_{o} (2200 meters/sec at 20.4°G), but that the response of a thin 1/v detector to this flux is reduced by the factor $(1 - (2/\sqrt{m}) \sum_{\text{o}} t$. It might be expected that for an isotropic Maxwellian distribution cross section value of $(\sqrt{m}/2) \sum_{\text{o}}$ should be used to calculate the flux degreession, and one of $(2/\sqrt{m}) \sum_{\text{o}}$ for the activation degreession.

For a 1/v absorber in an isotropically incident

Maxwellian flux, the necessary integrals have been evaluated as convergent series

$$\frac{1}{\overline{Q}} = 1 - \frac{1}{10} \sum_{0}^{1} \left[\frac{5}{2} - \frac{31}{2} - \log_{2} \Sigma_{0}^{1} \right] - \frac{(\Sigma_{0}^{1})^{2}}{2} + 0.036926 (\Sigma_{0}^{1})^{3} \dots (2.4.6)$$

Resonance Self-Shielding Factor. (15)

The other self-shielding correction factor is the resonance self-shielding factor (G,) which accounts for the attenuation due to self-shielding in a foil of thickness t of the resonance neutrons. G, may be obtained from the M.Sc. Thesis of Sudhiravuth (9) or may be derived from cadmium ratio measurements. The later method is considered more accurate.

To calculate G_p from cadmium ratio measurements, CdR is measured for any thickness and then converted to CdR₀ by the method mentioned in annex III. The equation (2.3.15) is solved for $r(T/T_0)^{1/2}$, $S_{o,j}$ Kand W are tabulated by Westcott and F by Martin (4). G_p may be evaluated from eqn. 2.3.14 for foil of thickness to