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PART III

APPENDICES

APPENDIX A

DERIVATION OF EXPRESSIONS FOR RESULTANTS IN S-BEAM BETWEEN CENTRE-SPAN AND SUPPORT IN TERMS OF IMPOSED LOAD AND REDUNDANTS

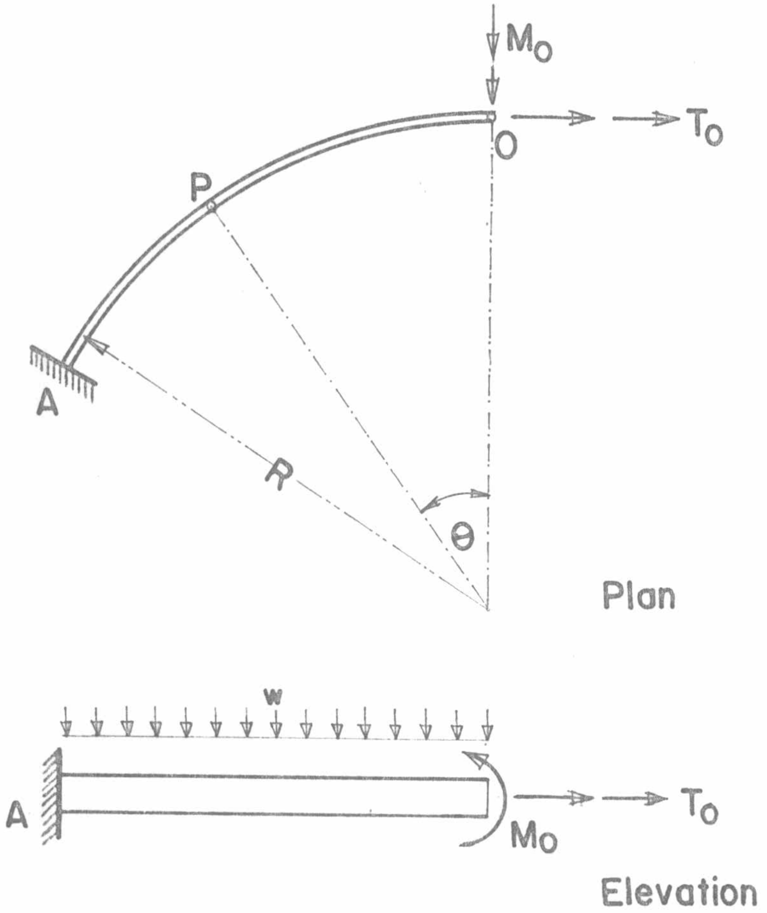

FIGURE A-1 Half-beam under Uniform Load and Redandants

As mentioned in Chapter 2 the derivation of the expressions for M and T at any point P in terms of w, M_o, and T_o for the S-beam's symmetric half is, for the sake of brevity of the arterial formulation via application of the strain energy principle, contained in this appendix. Figure A-1 once move diagrammatises the half-beam submitted

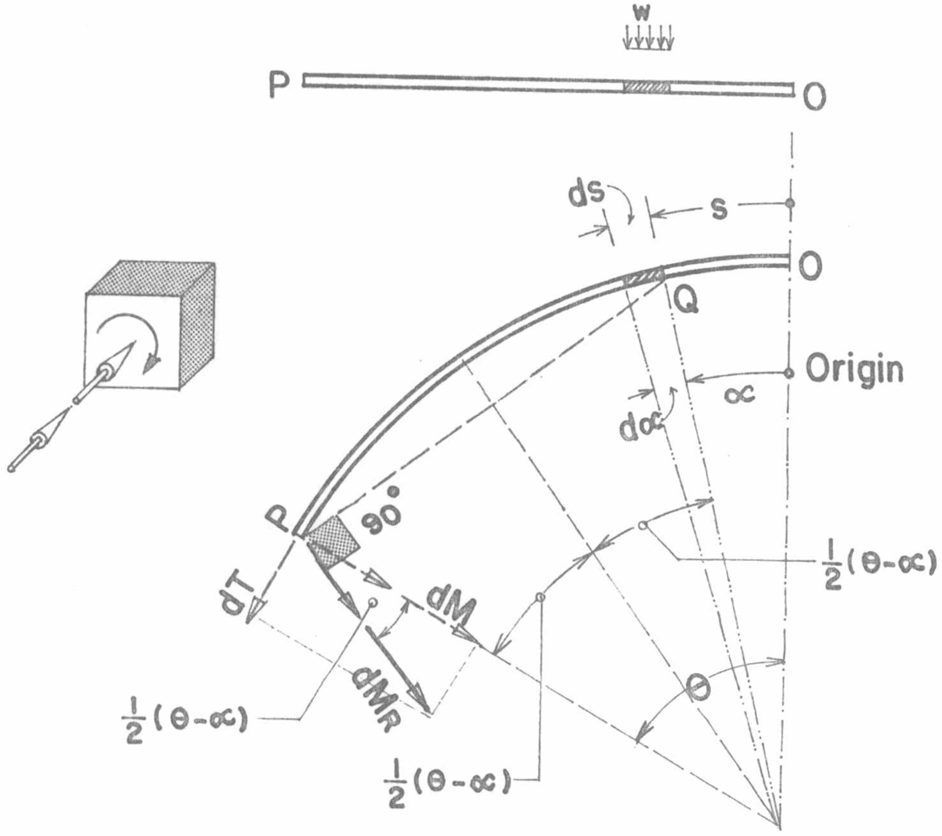

FIGURE A-2 Diagram Aiding Perception of Effect of w

to the combined action of w, M_o, and T_o. The influence of the uniform load w alone is first perused with reference to Figure A-2. On resorting to instrumentality of calculus an infinitesimal arc ds carrying the uniform load is focussed, the position of the arc identifying with a variable angle α measured from the origin O. The total load on this elemental arc is $dW = w\ ds = wRd\alpha$. Equilibrium of the part OP sustaining the elemental load dW is restored by a reactive resultant dM_R at P. Essentially the vectorial sense of dM_R bears a direction perpendicular to the chord connecting P with the location Q of the elemental strip. It simply follows that

$$dM_R = dW \times \text{arm } PQ$$

$$= (wRd\alpha)(2R \sin \frac{\theta-\alpha}{2} d\alpha)$$

$$= 2\ wR^2 \sin \frac{\theta-\alpha}{2}\ d\alpha$$

The elemental resultant bending and torsional moments, dM and dT, can be expressed via resolution of dM_R into components respectively orthogonal to and coincident with the tangent to the beam at P:

$$dM = dM_R \cos \frac{\theta-\alpha}{2}$$

$$dT = dM_R \sin \frac{\theta-\alpha}{2}$$

Mathematical integration of these elemental resultants between O and P leads to the formulation of M and T, the total resultants corresponding to the effect of the uniform load W extended between O and P. Note that in the following integral manipulation α is treated as a variable whereas θ, a constant.

$$M = \int_{\alpha=0}^{\alpha=\theta} dM_R \cos \frac{\theta-\alpha}{2}\ d\alpha$$

$$= \int_{\alpha=0}^{\alpha=\theta} 2wR^2 \sin \frac{\theta-\alpha}{2} \cos \frac{\theta-\alpha}{2}\ d\alpha$$

$$= wR^2 \int_0^\theta \sin(\theta-\alpha)\, d\alpha$$

$$= wR^2 (1-\cos\theta) \qquad \text{(A-11)}$$

$$T = \int_{\alpha=0}^{\alpha=\theta} dM_R \sin\frac{\theta-\alpha}{2}\, d\alpha$$

$$= \int_0^\theta 2\, wR^2 \sin^2\frac{\theta-\alpha}{2}\, d\alpha$$

$$= wR^2 \int_0^\theta \left[1-\cos(\theta-\alpha)\right] d\alpha$$

$$= wR^2 (\theta-\sin\theta) \qquad \text{(A-12)}$$

Next the effect of the resultant bending moment M_o alone (Figure A-3) comes forward. Again a resultant M_R with a vectorial sense parallel to that of M_o is needed at P to effectuated equilibrium, Resolution of M_R into components orthogonal to and coincident with the tangent at P results respectively in the expressions for M and T in terms of M_o.

$$M = M_o \cos\theta \qquad \text{(A-21)}$$

$$T = M_o \sin\theta \qquad \text{(A-22)}$$

Lastly the effect of T_o alone (Figure A-4) is assessed:

$$M = T_o \sin\theta \qquad \text{(A-31)}$$

$$T = -\, T_o \cos\theta \qquad \text{(A-32)}$$

Mark that the minus sign entangling with relation (A-32) emerges succeeding a review of the senses of the departmental torsional resultants registered in Figures A-2 and A-3.

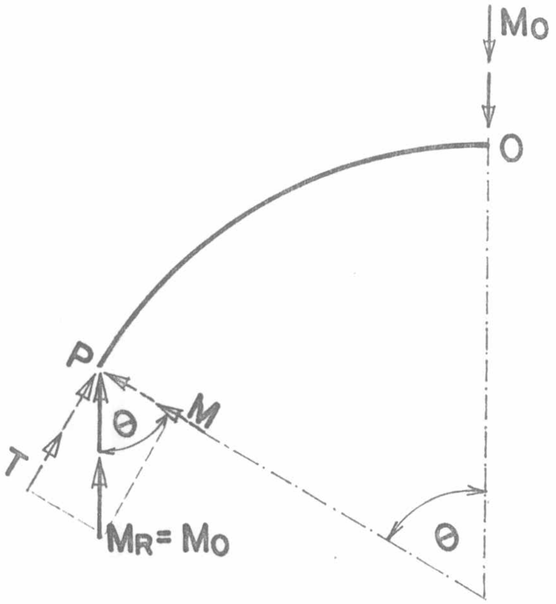

FIGURE A-3 Diagram Aiding Expression of M and T Due to Effect of Mo

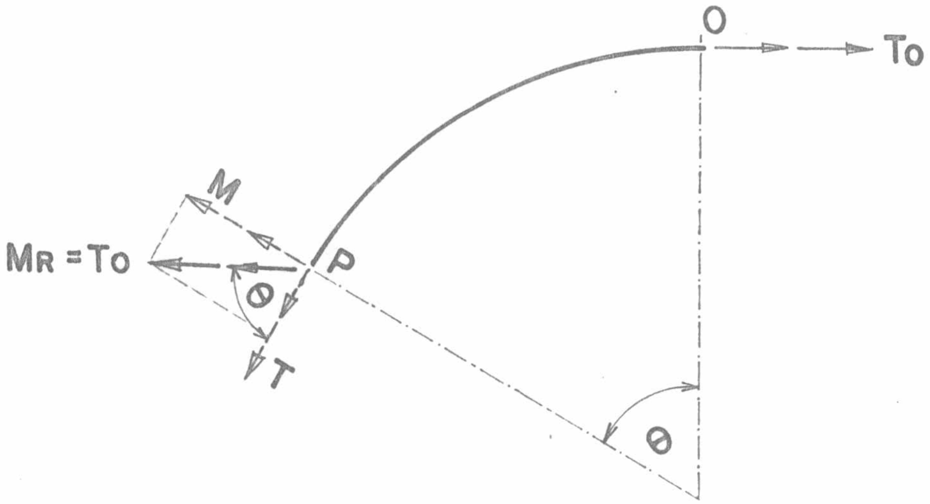

FIGURE A-4 Diagram Aiding Expression of M and T Due to Effect of To

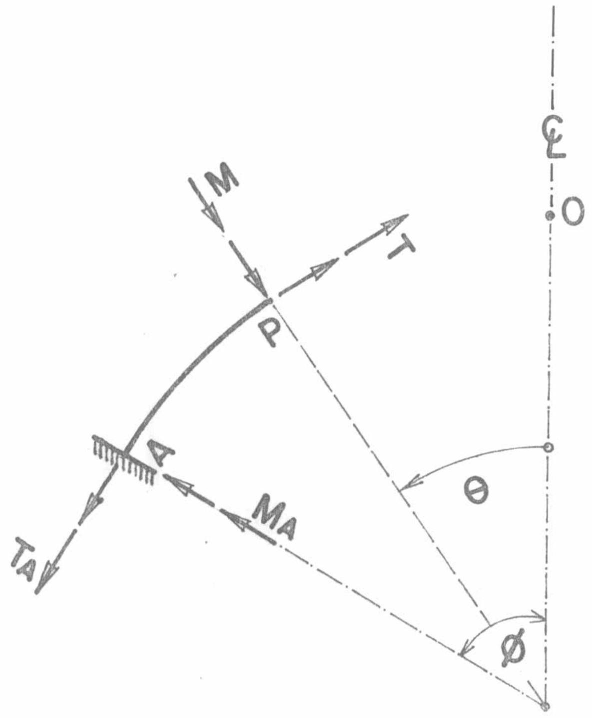

FIGURE A-5 Free-body Diagram of Portion of Beam between Any Point P and Support A

Apparently the accumulative resultants M and T at any point P on the half-beam due to the combined influence of w, M_o, and T_o are expressible via incorporation of the departmental effects instituted in the foregoing.

$$M = M_o \cos\theta + T_o \sin\theta - wR^2(1-\cos\theta)$$

$$T = M_o \sin\theta - T_o \cos\theta - wR^2(\theta-\sin\theta)$$

which personify relations (2.1) and (2.2) appearing in Chapter 2 prior to the application of the strain energy method. The representation of M and T associating with the portion of the beam between any point P and the fixed support A is shown in Figure A-5. The substitution of ϕ for θ yields expressions (4.1) and (4.2) respectively for the reactive bending and torsional moments at the support:

$$M_A = M_o \cos \phi + T_o \sin \phi - wR^2 (1-\cos \phi)$$

$$T_A = M_o \sin \phi - T_o \cos \phi - wR^2 (\phi-\sin \phi)$$

whose vectorial senses are also indicated in Figure A-5.

APPENDIX B

TABULATION OF VALUES OF REDUNDANTS RECONSTRUCTED FROM COMPUTER PRINTOUTS

$\frac{t}{b}$	ϕ deg	M_o coeff	T_o coeff
1.00	15	0.011329	0.001289
	30	0.044204	0.010226
	45	0.095340	0.033916
	60	0.159461	0.078443
	75	0.229578	0.148399
	90	0.297358	0.246481
	105	0.353582	0.373163
	120	0.388691	0.526418
1.10	15	0.011335	0.001239
	30	0.044296	0.009824
	45	0.095802	0.032628
	60	0.160885	0.075605
	75	0.232928	0.143394
	90	0.303950	0.238939
	105	0.364971	0.363172
	120	0.406446	0.514687

Table B.1 - Coefficients of M_o and T_o for S-beams

Note: M_o or T_o = appropriate coeff x wR^2

$\frac{t}{b}$	ϕ deg	M_o coeff	T_o coeff
1.20	15	0.011340	0.001191
	30	0.044377	0.009465
	45	0.096207	0.031465
	60	0.162148	0.073027
	75	0.235933	0.138808
	90	0.309935	0.231972
	105	0.375436	0.353864
	120	0.422951	0.503664
1.30	15	0.011344	0.001154
	30	0.044446	0.009144
	45	0.096561	0.030422
	60	0.163258	0.070693
	75	0.238607	0.134627
	90	0.315330	0.225570
	105	0.384982	0.345245
	120	0.438178	0.493381

Table B.1 - (Cont'd)

$\frac{t}{b}$	ϕ deg	M_o coeff	T_o coeff
1.40	15	0.011348	0.001115
	30	0.044506	0.008858
	45	0.096864	0.029483
	60	0.164227	0.068584
	75	0.240973	0.130823
	90	0.320166	0.219707
	105	0.393644	0.337300
	120	0.452147	0.483839
1.50	15	0.011351	0.001083
	30	0.044555	0.008600
	45	0.097122	0.028639
	60	0.165070	0.066675
	75	0.243065	0.127359
	90	0.324498	0.214336
	105	0.401495	0.329978
	120	0.464942	0.474996

Table B.1 - (Cont'd)

$\frac{t}{b}$	ϕ deg	M_o coeff	T_o coeff
1.60	15	0.011353	0.001054
	30	0.044598	0.008371
	45	0.097343	0.027881
	60	0.165798	0.064950
	75	0.244903	0.124212
	90	0.328362	0.209428
	105	0.408583	0.323255
	120	0.476612	0.466836
1.70	15	0.011355	0.001028
	30	0.044630	0.008164
	45	0.097527	0.027201
	60	0.166422	0.063394
	75	0.246509	0.121360
	90	0.331789	0.204962
	105	0.414947	0.317113
	120	0.487193	0.459350

Table B.1 - (Cont'd)

$\frac{t}{b}$	ϕ deg	M_o coeff	T_o coeff
2.00	15	0.011357	0.000964
	30	0.044694	0.007670
	45	0.097904	0.025541
	60	0.167787	0.059563
	75	0.250193	0.114277
	90	0.339920	0.193800
	105	0.430407	0.301667
	120	0.513361	0.440416
2.50	15	0.011360	0.000899
	30	0.044710	0.007110
	45	0.098132	0.023624
	60	0.168923	0.055065
	75	0.253789	0.105883
	90	0.348567	0.180489
	105	0.447695	0.283159
	120	0.543601	0.417596

Table B.1 - (Cont'd)

$\frac{t}{b}$	ϕ deg	M_o coeff	T_o coeff
3.00	15	0.011354	0.000856
	30	0.044658	0.006748
	45	0.098063	0.022350
	60	0.169243	0.052036
	75	0.255544	0.100216
	90	0.353543	0.171526
	105	0.458371	0.270723
	120	0.562987	0.402256
4.00	15	0.011329	0.000806
	30	0.044447	0.006310
	45	0.097542	0.020760
	60	0.168877	0.048266
	75	0.256673	0.093271
	90	0.358373	0.160709
	105	0.469897	0.255888
	120	0.584838	0.384056

Table B.1 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.00	0.0	0.041667	0.0
	0.10	0.059520	0.003412
	0.20	0.071073	0.012855
	0.30	0.079161	0.027505
	0.40	0.085139	0.046843
	0.50	0.089738	0.070524
	0.60	0.093385	0.098313
	0.70	0.096439	0.130040
	0.80	0.098804	0.165584
	0.90	0.100872	0.204852
	1.00	0.102638	0.247776
	1.10	0.104162	0.294300
	1.20	0.105492	0.344382
	1.30	0.106663	0.397989
	1.40	0.107700	0.455092
	1.50	0.108627	0.515669
	1.60	0.109460	0.579701
	1.70	0.110212	0.647174
	1.80	0.110894	0.718074
	1.90	0.111516	0.792391
	2.00	0.112086	0.870114

Table B.2 - Coefficients of M_o and T_o for Z-beams

Note: M_o or T_o = appropirate coeff x wL_2^2

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.10	0.0	0.041667	0.0
	0.10	0.060956	0.003126
	0.20	0.072994	0.011893
	0.30	0.081223	0.025642
	0.40	0.087203	0.043944
	0.50	0.091746	0.066508
	0.60	0.095314	0.093128
	0.70	0.098190	0.123654
	0.80	0.100559	0.157976
	0.90	0.102542	0.196010
	1.00	0.104228	0.237691
	1.10	0.105679	0.282968
	1.20	0.106940	0.331801
	1.30	0.108047	0.384157
	1.40	0.109025	0.440010
	1.50	0.109897	0.499338
	1.60	0.110679	0.562123
	1.70	0.111384	0.628349
	1.80	0.112022	0.698005
	1.90	0.112604	0.771080
	2.00	0.113135	0.847563

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.20	0.0	0.041667	0.0
	0.10	0.062450	0.002865
	0.20	0.074935	0.010996
	0.30	0.083266	0.023880
	0.40	0.089220	0.041171
	0.50	0.093687	0.062626
	0.60	0.097162	0.088072
	0.70	0.099944	0.117379
	0.80	0.102219	0.150451
	0.90	0.104116	0.187213
	1.00	0.105721	0.227606
	1.10	0.107098	0.271583
	1.20	0.108290	0.319109
	1.30	0.109334	0.370152
	1.40	0.110255	0.424688
	1.50	0.111074	0.482696
	1.60	0.111806	0.544159
	1.70	0.112466	0.609063
	1.80	0.113062	0.677395
	1.90	0.113605	0.749146
	2.00	0.114100	0.824305

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.30	0.0	0.041667	0.0
	0.10	0.063988	0.002628
	0.20	0.076878	0.010168
	0.30	0.085271	0.022229
	0.40	0.091172	0.038540
	0.50	0.095546	0.058908
	0.60	0.098919	0.083188
	0.70	0.101598	0.111274
	0.80	0.103778	0.143082
	0.90	0.105587	0.178548
	1.00	0.107112	0.217622
	1.10	0.108414	0.260262
	1.20	0.109540	0.306435
	1.30	0.110523	0.356115
	1.40	0.111388	0.409278
	1.50	0.112155	0.465907
	1.60	0.112841	0.525985
	1.70	0.113457	0.589499
	1.80	0.114014	0.656438
	1.90	0.114519	0.726792
	2.00	0.114980	0.800553

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.40	0.0	0.041667	0.0
	0.10	0.065559	0.002413
	0.20	0.078804	0.009406
	0.30	0.087222	0.020691
	0.40	0.093045	0.036064
	0.50	0.097313	0.055375
	0.60	0.100575	0.078509
	0.70	0.103149	0.105382
	0.80	0.105232	0.135926
	0.90	0.106953	0.170087
	1.00	0.108398	0.207824
	1.10	0.109629	0.249101
	1.20	0.110690	0.293890
	1.30	0.111614	0.342168
	1.40	0.112426	0.393917
	1.50	0.113145	0.449118
	1.60	0.113786	0.507759
	1.70	0.114362	0.569827
	1.80	0.114881	0.635313
	1.90	0.115352	0.704208
	2.00	0.115781	0.776505

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.50	0.0	0.041667	0.0
	0.10	0.067157	0.002219
	0.20	0.080706	0.008707
	0.30	0.089112	0.019262
	0.40	0.094836	0.033740
	0.50	0.098986	0.052028
	0.60	0.102132	0.074044
	0.70	0.104599	0.099721
	0.80	0.106585	0.129008
	0.90	0.108219	0.161863
	1.00	0.109587	0.198253
	1.10	0.110749	0.238150
	1.20	0.111747	0.281531
	1.30	0.112615	0.328379
	1.40	0.113376	0.378676
	1.50	0.114050	0.432410
	1.60	0.114649	0.489568
	1.70	0.115186	0.550141
	1.80	0.115670	0.614121
	1.90	0.116109	0.681501
	2.00	0.116508	0.752274

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.60	0.0	0.041667	0.0
	0.10	0.068768	0.002044
	0.20	0.082567	0.008067
	0.30	0.090928	0.017942
	0.40	0.096537	0.031571
	0.50	0.100560	0.048880
	0.60	0.103587	0.069811
	0.70	0.105946	0.094320
	0.80	0.107837	0.122369
	0.90	0.109387	0.153929
	1.00	0.110680	0.188976
	1.10	0.111775	0.227489
	1.20	0.112715	0.269453
	1.30	0.113530	0.314853
	1.40	0.114243	0.363677
	1.50	0.114873	0.415916
	1.60	0.115434	0.471559
	1.70	0.115935	0.530601
	1.80	0.116387	0.593035
	1.90	0.116795	0.658855
	2.00	0.117167	0.728055

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
1.70	0.0	0.041667	0.0
	0.10	0.070376	0.001886
	0.20	0.084373	0.007486
	0.30	0.092659	0.016729
	0.40	0.098138	0.029561
	0.50	0.102030	0.045940
	0.60	0.104937	0.065832
	0.70	0.107190	0.089211
	0.80	0.108989	0.116054
	0.90	0.110457	0.146344
	1.00	0.111679	0.180067
	1.10	0.112711	0.217209
	1.20	0.113595	0.257760
	1.30	0.114361	0.301713
	1.40	0.115030	0.349059
	1.50	0.115619	0.399792
	1.60	0.116143	0.453906
	1.70	0.116612	0.511396
	1.80	0.117033	0.572259
	1.90	0.117415	0.636491
	2.00	0.117761	0.704089

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
2.00	0.0	0.041667	0.0
	0.10	0.075113	0.001501
	0.20	0.089401	0.006041
	0.30	0.097327	0.013663
	0.40	0.102366	0.024404
	0.50	0.105853	0.038294
	0.60	0.108409	0.055357
	0.70	0.110363	0.075613
	0.80	0.111905	0.099079
	0.90	0.113153	0.125770
	1.00	0.114184	0.155697
	1.10	0.115050	0.188871
	1.20	0.115787	0.225302
	1.30	0.116423	0.264996
	1.40	0.116977	0.307961
	1.50	0.117463	0.354203
	1.60	0.117894	0.403726
	1.70	0.118278	0.456535
	1.80	0.118623	0.512635
	1.90	0.118934	0.572029
	2.00	0.119217	0.634721

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
2.50	0.0	0.041667	0.0
	0.10	0.082513	0.001065
	0.20	0.096489	0.004353
	0.30	0.103546	0.009987
	0.40	0.107802	0.018073
	0.50	0.110650	0.028701
	0.60	0.112688	0.041951
	0.70	0.114219	0.057891
	0.80	0.115412	0.076582
	0.90	0.116367	0.098077
	1.00	0.117149	0.122422
	1.10	0.117801	0.149659
	1.20	0.118353	0.179824
	1.30	0.118826	0.212950
	1.40	0.119237	0.249067
	1.50	0.119596	0.288200
	1.60	0.119913	0.330375
	1.70	0.120195	0.375611
	1.80	0.120448	0.423928
	1.90	0.120675	0.475345
	2.00	0.120880	0.529876

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
3.00	0.0	0.041667	0.0
	0.10	0.088953	0.000789
	0.20	0.102002	0.003257
	0.30	0.108115	0.007543
	0.40	0.111660	0.013770
	0.50	0.113975	0.022049
	0.60	0.115606	0.032479
	0.70	0.116816	0.045148
	0.80	0.117750	0.060139
	0.90	0.118493	0.077522
	1.00	0.119097	0.097364
	1.10	0.119599	0.119725
	1.20	0.120022	0.144660
	1.30	0.120384	0.172218
	1.40	0.120697	0.202444
	1.50	0.120970	0.235381
	1.60	0.121210	0.271067
	1.70	0.121423	0.309537
	1.80	0.121614	0.350823
	.90	0.121785	0.394955
	2.00	0.121940	0.441962

Table B.2 - (Cont'd)

$\frac{t}{b}$	k	M_o coeff	T_o coeff
4.00	0.0	0.041667	0.0
	0.10	0.098817	0.000480
	0.20	0.109468	0.002004
	0.30	0.113959	0.004690
	0.40	0.116436	0.008651
	0.50	0.118005	0.013990
	0.60	0.119088	0.020803
	0.70	0.119881	0.029182
	0.80	0.120486	0.039211
	0.90	0.120963	0.050970
	1.00	0.121349	0.064533
	1.10	0.121668	0.079969
	1.20	0.121935	0.097345
	1.30	0.122163	0.116724
	1.40	0.122359	0.138161
	1.50	0.122530	0.161714
	1.60	0.122680	0.187434
	1.70	0.122813	0.215369
	1.80	0.122932	0.245565
	1.90	0.123038	0.278067
	2.00	0.123134	0.312916

Table B.2 — (Cont'd)

APPENDIX C

DESIGN CRITERIA

This supplement compiles necessary formulae for proportioning of reinforced concrete members under combined action of bending, torsion, and shear, as quoted from 1977 ACI Code. The notation and the numeration of equations and clauses as adopted by the Code are preserved.

Note that metric conversion of equation (11-22) appearing in the Code is misleading and adjustment has been made to register the correct form in this appendix.

C.1. Notation

a = depth of equivalent rectangular stress block

A_ℓ = total area of longitudinal reinforcement to resist torsion

A_s = area of nonprestressed tension reinforcement

A'_s = area of compression reinforcement

A_t = area of one leg of a closed stirrup resisting torsion within a distance S

A_v = area of shear reinforcement within a distance S

b = width of compression face of member

b_s = shorter center-to-center dimension of closed rectangular stirrup

C_t = factor relating shear and torsional stress properties

$$= \frac{d}{bt}$$

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

d' = distance from extreme compression fiber to centroid of compression reinforcement

D = dead loads

f'_c = specified compressive strength of concrete

f_y = specified yield strength of nonprestressed reinforcement

L = live loads

M_u = factored moment at section

p = ratio of nonprestressed tension reinforcement

p_b = reinforcement ratio producing balanced strain conditions

S = spacing of web reinforcement in direction parallel to longitudinal reinforcement

t = overall thickness of member

t_s = longer center-to-center dimension of closed rectangular stirrup

T_c = nominal torsional moment strength provided by concrete

T_n = nominal torsional moment strength

T_s = nominal torsional moment strength provided by torsion reinforcement

T_u = factored torsional moment at section

V_c = nominal shear strength provided by concrete

V_n = nominal shear strength

V_s = nominal shear strength provided by shear reinforcement

V_u = factored shear force at section

U = required strength to resist factored load

α_t = coefficient as a function of $\frac{t_s}{b_s}$

β_1 = parameter

ϕ = strength reduction factor

C.2. Rules

$$U = 1.4D + 1.7L \quad \cdots\cdots\cdots\cdots \quad \text{(9-1.....Art. 9.2.1)}$$

$$p_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{6117}{(6117+f_y)} \quad \cdots\cdots\cdots \quad \text{(8-1.....Art. 8.4.3)}$$

$$p_{max} = 0.75\ p_b \quad \cdots\cdots\cdots\cdots \quad \text{(Art. 10.3.3)}$$

$$p_{min} = \frac{14}{f_y} \quad \cdots\cdots\cdots\cdots \quad \text{(10-3.....Art. 10.5.1)}$$

$$a = \frac{(A_s - A'_s) f_y}{0.85\ bf'_c} \quad \cdots\cdots\cdots\cdots \quad \text{(Art. 10.2.7)}$$

$$M_{u_1} = 0.90(A_s - A'_s)\ f_y(d - \frac{a}{2}) \quad \cdots\cdots\cdots \quad \text{(Art. 10.2.1)}$$

$$M_{u_2} = M_u - M_{u_1}$$

$$= 0.90\ A'_s f_y\ (d-d') \quad \cdots\cdots\cdots\cdots \quad \text{(Art. 10.2.1)}$$

$$T_c = \frac{0.21\sqrt{f'_c}\, b^2 t}{\sqrt{1+(\frac{0.4V_u}{C_t\ T_u})^2}} \quad \cdots\cdots\cdots\cdots \quad \text{(11-22....Art. 11.6.6.1)}$$

$$T_n = T_c + T_s \quad \cdots\cdots\cdots\cdots \quad \text{(11-21....Art. 11.6.5)}$$

$$T_s \leqslant 4T_c \qquad \text{(Art. 11.6.9.4)}$$

$$T_s = \frac{A_t \alpha_t b_s t_s f_y}{S} \quad \cdots\cdots\cdots\cdots \quad \text{(11-23....Art. 11.6.9.1)}$$

$$\alpha_t = 0.66 + 0.33 \frac{t_s}{b_s} \leqslant 1.50 \quad \cdots\cdots\cdots \quad \text{(Art. 11.6.9.1)}$$

$$A_\ell = \frac{2A_t(b_s+t_s)}{S} \qquad \text{(11-24...Art. 11.6.9.3)}$$

$$A_\ell = \left[\frac{28\ b.S}{f_y}\left(\frac{T_u}{T_u + \frac{V_u}{3C_t}}\right) - 2A_t\right] \frac{(b_s+t_s)}{S} \quad \cdots \quad \text{(11-25....Art. 11.6.9.3)}$$

$$V_c = \frac{0.53\sqrt{f'_c}\ bd}{\sqrt{1 + (2.5\ C_t \frac{T_u}{V_u})^2}} \quad \cdots\cdots\cdots\cdots \quad \text{(11-5.....Art. 11.3.1.4)}$$

$$V_n = V_c + V_s \quad \cdots\cdots\cdots\cdots \quad \text{(11-2.....Art. 11.1.1)}$$

$V_s = \leqslant 2.12\sqrt{f'_c}\, bd$ .... (Art. 11.5.6.8)

$V_s = \frac{A_v f_y d}{S}$ .... (11-17.....Art. 11.5.6.2)

$A_v + 2A_t \geqslant \frac{3.5\ bS}{f_y}$ .... (11-16.....Art. 11.5.5.5)

APPENDIX D

PROPERTIES OF MODEL-FORMING MATERIALS

Early investigation of necessary physical properties of the designated model-forming materials as conducted in January 1980 in the King Mongkut's Institute of Technology's laboratory conforms to the standards enforced by the well-known American Society for Testing of Materials.

D.1 Concrete Materials

The designated river sand and crushed limestone each represented by five samples exhibit the following properties.

(a) Sand:

bulk specific gravity = 2.62

absorption = 1.92 %

fineness modulus = 3.13

(b) Crushed limestone:

maximum size = 1.0 cm

bulk specific gravity = 2.73

dry unit rodded weight = 1,481.40 kg/m^3

absorption = 0.50 %

fineness modulus = 3.39

Tables D.1, D.2, and D.3 furnish information on gradation. Figure D.1 gives a graphical representation of the gradation characteristics.

Sieve Size	% retained	% Accumulative	% passing	ASTM % passing
½"				
3/8"	0	0	100	100
$4^{\#}$	3.3	3.3	96.7	95-100
$8^{\#}$	11.6	14.9	85.1	80-100
$16^{\#}$	24.6	39.5	60.5	50-85
$30^{\#}$	29.5	69.0	31.0	25-60
$50^{\#}$	20.3	89.3	10.7	10-30
$100^{\#}$	7.6	96.9	3.1	2-10
Pan	3.0	99.9	0	-
Fineness modulus		3.13		

Table D.1 - Gradation of the River Sand

Sieve Size	% Retained	% Accumulative	% passing	ASTM % passing
½"	0	0	100	90-100
3/8"	42	42	58	40-70
4#	56	98	2	0-15
8#	1.3	99.3	0.7	0-5
16#	0.7	100	0	-
30#				
50#				
100#				
Pan				
Fineness modulus		3.39		

Table D.2 - Gradation of the Crushed Limestone

Sieve Size	% retained		% retained -combined aggregates	% Accumulative
	Sand 58.50 %	Stone 41.50 %		
½"	0	0	0	0
3/8"	0	17.43	17.43	17.43
$4^{\#}$	1.93	23.24	25.17	42.60
$8^{\#}$	6.79	0.54	7.33	49.93
$16^{\#}$	14.39	0.29	14.68	64.61
$30^{\#}$	17.26		17.26	81.87
$50^{\#}$	11.88		11.88	93.75
$100^{\#}$	4.45		4.45	98.20
Pan	1.76		1.76	99.96

Table D.3 - Gradation of the Combined Aggregates

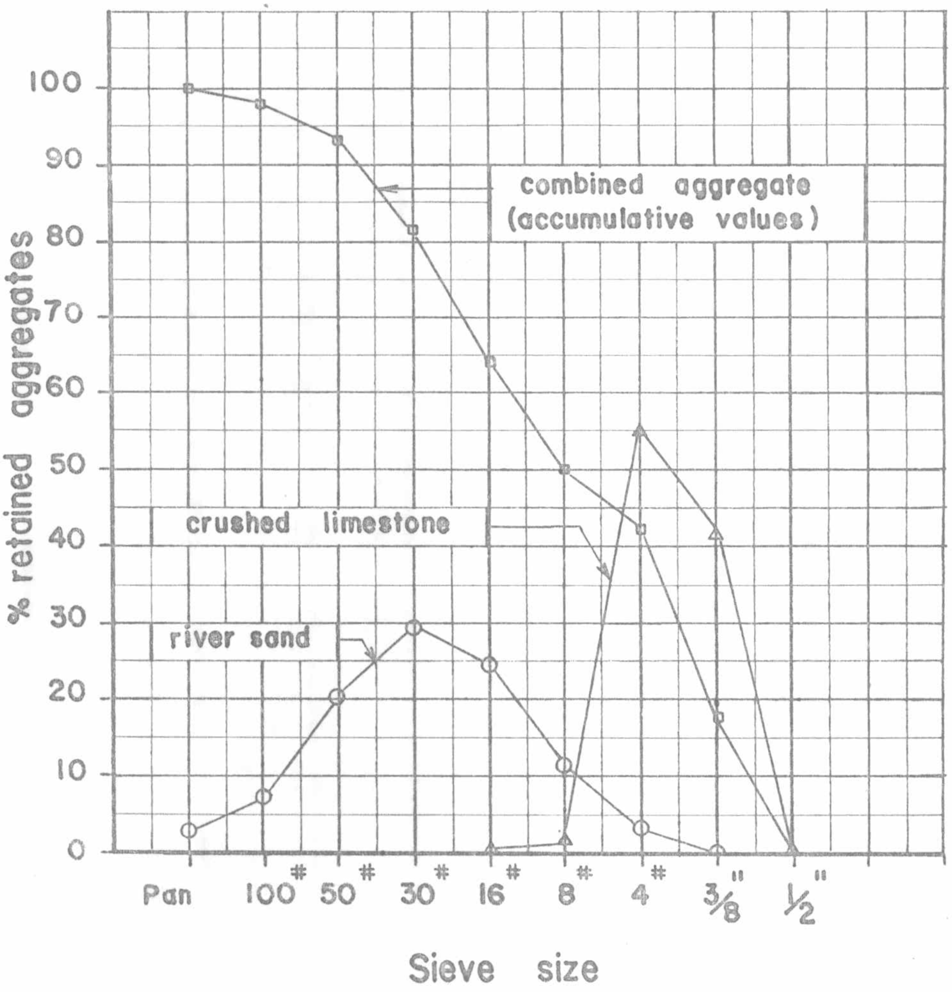

FIGURE D-1 Graphical Representation of Gradation Characteristics

Specimen No.	Cylinder strength kg/cm^2
1	214
2	214
3	203
4	209
5	210
6	205
7	215
8	212
9	208
10	214
Average	210.40

Table D.4 - Compressive Strength of Representative Cylinders

D.2 Reinforcing Steel

Table D.5 enwraps particulars of the representative structural-grade round bars employed for model reinforcement.

Norminal Size Ø 4 mm.					
Specimen No.	Actual size Ø mm.	Cross-sectional area cm^2.	Perimeter Σ_o cm.	Tensile Stresses kg/cm^2	
				Yield Strength	Ultimate Strength
1	4	0.126	1.257	4,500	5,500
2	3.75	0.110	1.178	4,020	5,420
3	3.75	0.110	1.178	4,110	5,480
4	4.10	0.132	1.288	4,700	5,620
5	4.01	0.126	1.260	4,570	5,450
Average		0.121	1.232	4,380	5,494
Norminal size Ø 6 mm.					
1	6.12	0.294	1.923	3,601	5,266
2	6.02	0.285	1.891	3,689	5,516
3	5.75	0.260	1.806	3,314	5,060
4	5.75	0.260	1.806	3,402	5,154
5	6.00	0.283	1.885	3,470	5,179
Average		0.276	1.862	3,495	5,235

Table D.5 - Results of Structural-grade Round Bars

Norminal Size Ø 9 mm.					
Specimen No.	Actual size Ø mm.	Cross-sectional area cm.^{2}	Perimeter Σ_o cm.	Tensile Stresses kg/cm^2	
				Yield Strength	Ultimate Strength
1	8.75	0.601	2.749	3,360	5,150
2	8.75	0.601	2.749	3,390	4,985
3	8.60	0.581	2.702	3,420	4,990
4	8.80	0.608	2.765	3,400	5,000
5	9.00	0.636	2.827	3,510	5,014
Average		0.605	2.758	3,416	5,027
Norminal Size Ø 12 mm.					
1	11.73	1.082	3.685	3,467	4,956
2	11.71	1.077	3.679	3,380	4,995
3	11.75	1.084	3.691	3,439	5,293
4	11.75	1.084	3.691	3,395	5,144
5	11.74	1.082	3.688	3,340	4,985
Average		1.082	3.687	3,404	5,074

Table D.5 - (Cont'd)

CIRCULAR VITA

NAME Khoollapath Vathanakhool

YEAR OF BIRTH : 1949

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EDUCATION : B. Eng., Hons (Civil)
King Mongkut's Institute of Technology 1973,
Dip. in Ed.
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REGISTRATION : Registered professional engineer of Thailand
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EXPERIENCE :

1973 - PRESENT : Instructor, King Mongkut's Institute of Technology at North Bangkok.