

## CHAPTER 3

### PRESENTATION OF ANALYTICAL AIDS

#### 3.1 The Rigidity Ratio

Expressed as the ratio of the flexural rigidity to the torsional rigidity the parameter  $m$  plies with the expressions for redundants as derived in Chapter 2. As this research identifies solely with rectangular beam sections of reinforced concrete the rigidity ratio  $m$  can be written in terms of the cross-sectional dimensions  $t$  and  $b$ , the thickness  $t$  being always greater than, at least equal to, the width  $b$ . The imposition of a Poisson's ratio of 0.15 on the known relation

$$G = \frac{E}{2(1+\mu)}$$

results in  $\frac{G}{E} = \frac{1}{2.3}$ . The moment of inertia of the beam section with respect to the horizontal axis assumes the familiar expression

$$I = \frac{1}{12} bt^3$$

With reference to Timoshenko and Goodier<sup>1</sup> the expression for the polar moment of inertia of the beam section takes the form

$$J = Cb^3t$$

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<sup>1</sup> S. Timoshenko and J.N. Goodier, Theory of Elasticity

(New York; McGraw-Hill Book Co., 1951), pp. 275-278.

wherein

$$C = \frac{1}{3} \left( 1 - \frac{192}{\pi^5} \cdot \frac{b}{t} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{1}{n^5} \tanh \frac{n\pi t}{2b} \right)$$

It follows that

$$m = \frac{2.3}{12C} \cdot \frac{t^2}{b^2} \quad (3.1)$$



### 3.2 The S-beam

Evidently the S-beam solution as directed by relations (2.9) to (2.16) exhibits a complex nature as they assimilate compound trigonometric terms. Such trait furnishes justification for development of a graphical presentation enabling a rapid handling of the problem in the design office. The graphical aids represented by Charts 3.1 and 3.2 bring forward values of the redundants  $M_0$  and  $T_0$  for practical ranges of values of  $\phi$  and  $\frac{t}{b}$ . Computerised numerical values of  $M_0$  and  $T_0$  forming the basis for the institution of the graphical aids, as tabulated in Appendix B, draw on the following input values of  $\phi$  and  $\frac{t}{b}$ :

$\phi = 15, 30, 45, \dots, \text{and } 120 \text{ degrees; and}$

$\frac{t}{b} = 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 2.0, 2.5, 3.0, \text{ and } 4.0$

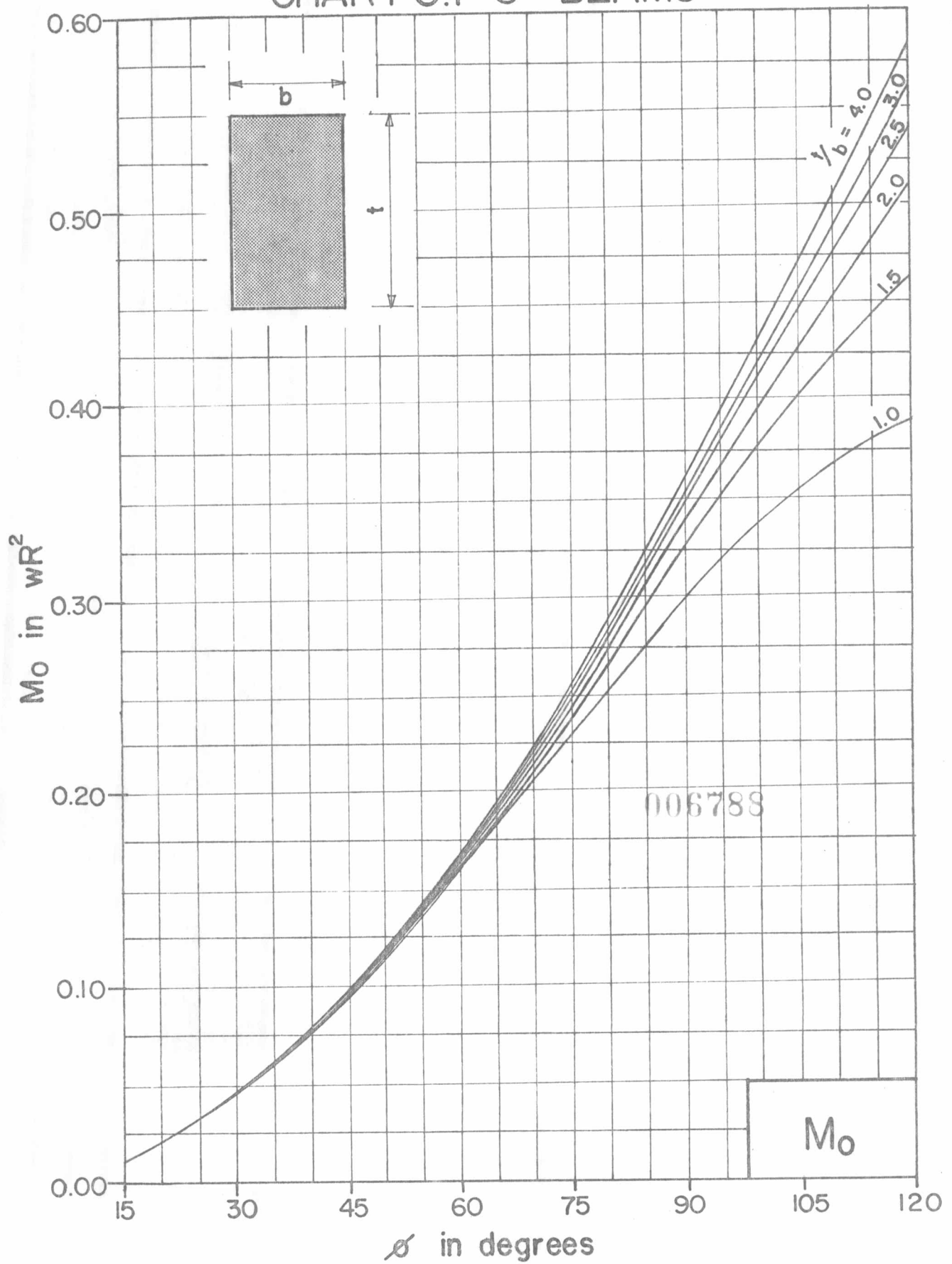
### 3.3 The Z-beam

Although expressions (2.23) and (2.24) for the redundant  $M_0$  and  $T_0$  appear relatively simple their graphical presentation, Charts

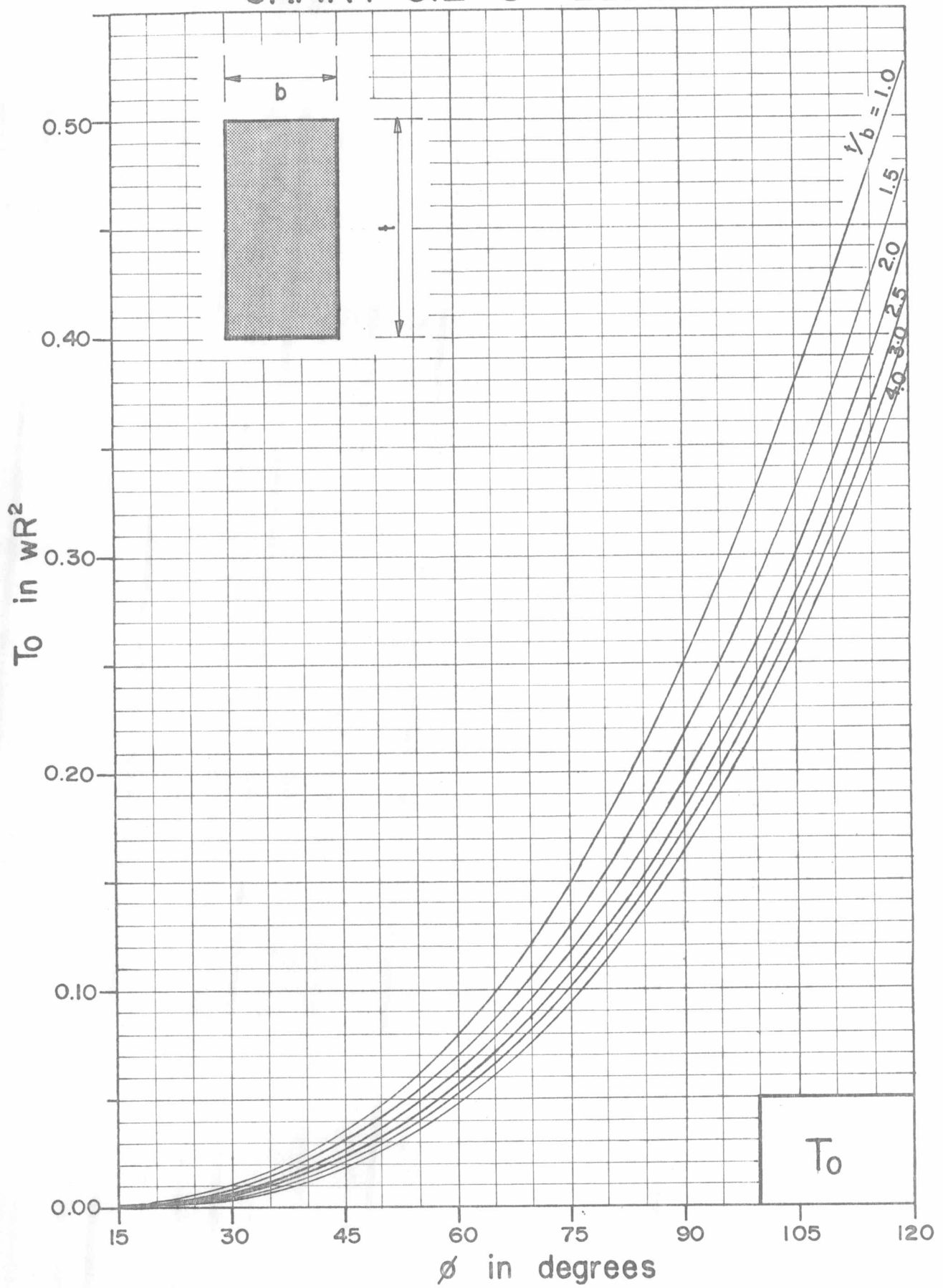
3.3, and 3.4 has been registered. Computerised numerical values of these redundants, as listed in Appendix B, correspond to identical input values of  $\frac{t}{b}$  set forth in Article 3.2, and to the following input values of k:

k = 0, 0.1, 0.2, 0.3, ....., and 2.00

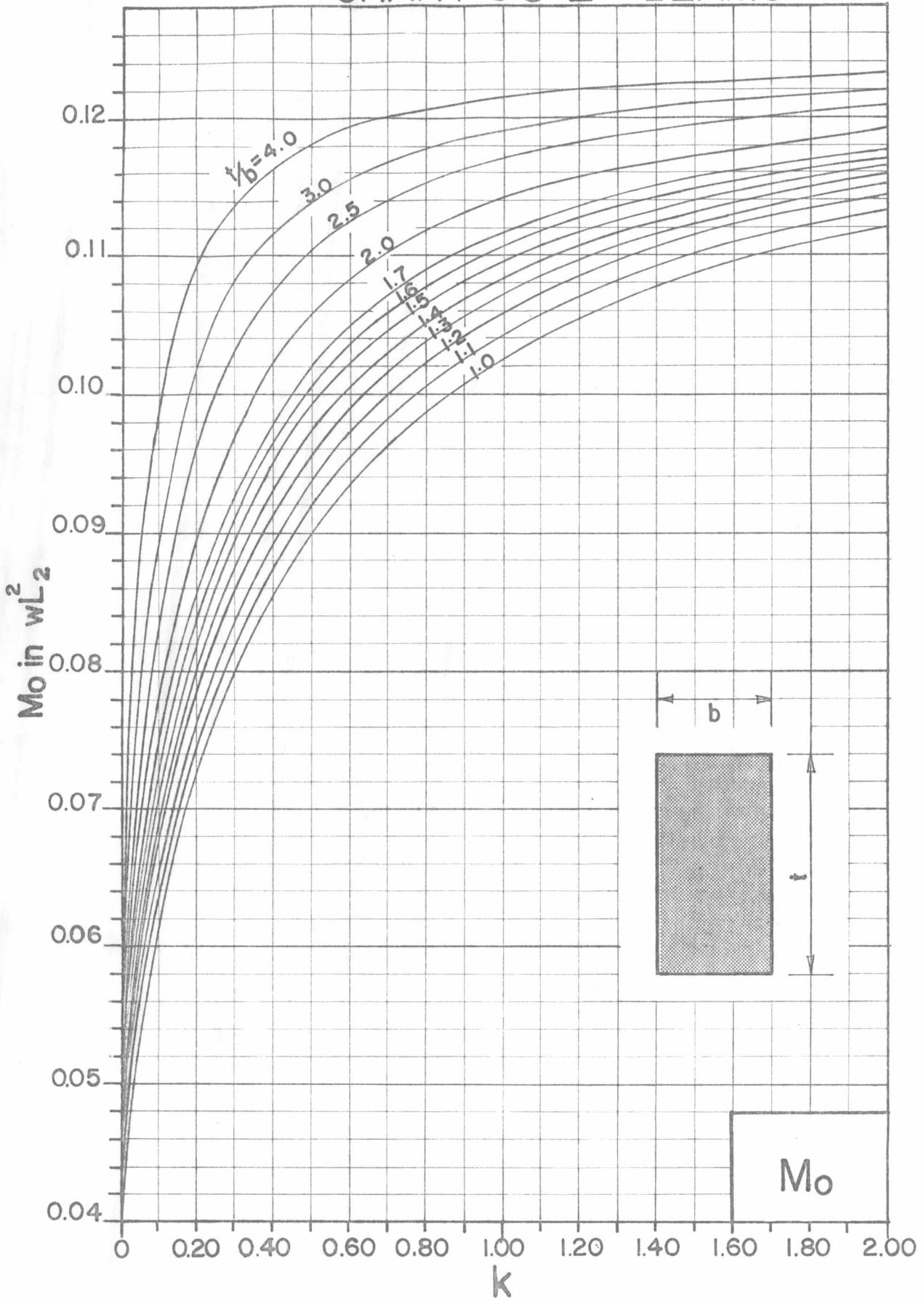
## CHART 3.1 S-BEAMS



## CHART 3.2 S-BEAMS



# CHART 3.3 Z - BEAMS



## CHART 3.4 Z - BEAMS

