CHAPTER 2

INPUT - OUTPUT ANALYSIS

2.1 The Structural Nature of an Input - Output table

2.1.1 What is input - output ?

An input - output table can be regarded as a collection of data describing the particular structural characteristics of an economic system, and/or as an analytic technique for explaining and influencing the behavior of the system at a certain point in time or over the course of time. The basic notion of input-output analysis rests on the belief what economy of any country can be divided into a distinct number of sectors, called industries (or sometimes 'activities') each consisting of one or more firms producing a similar but not necessarily homogeneous product. Each industry requires certain inputs from other sectors in order to produce its own output. Similarly, each industry sells some of its gross output to other industries so that they too can satisfy their intermediate material needs. The input - output table provides a convenient framework for measuring and tracing their interindustry flows of current inputs and outputs among the various sectors of the economy.

2.1.2 The make - up of a typical table

In table2.1 we have a representation of the structural make - up of a typical input Output table. Most of the important information contained in the table is located within the three main quadrants; [1/] the 'Interindustry Transactions' quadrant 1, the Value Added' quadrant 2, and the 'Final Use' quadrant 3.

The producing sectors of the economy are listed as rows 1,2, ...n, in the transactions quadrant while these same industries are also listed by column an using sectors. The transactions quadrant is thus always a 'square' matrix with the same number of rows as columns, one for each sector of the economy. Reading from left to right along any row, say 2, which represents sector or industry 2, we see that part of the total output of sector 2 is sold to the other sectors for intermediate use in the production of their own output (e.g. cotton growers sell their output to the textile industry who use this raw material input to produce cotton fabrics). Reading down any column in the transactions quadrant, we note that each particular sector (sector 2) also uses the outputs of other industries as material inputs for its own production (e.g. cotton growers purchase fertilizer and chemicals to increase and protect their per - acre cotton yield). In short, rows designate outputs,

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1/ Micheal P. Todaro, 'Development Planning : Models and Methods', Oxford University Press, 1971.

Table 2-1 The Make - up of a Typical Input - output Table

Producing Sectors (Output) → / Using Sectors (Inputs) ↓	Intermediate Use: Sector (1)	Sector (2)	Sector (N)	Total Intermediate Production	Final Use (Demand)	Total Final Demand	Total Output (Sales)
(1) Sector 1	Interindustry Transactions Quadrantl.				Final Use Quandrant 3.		
(2) Sector 2							
.							
.							
.							
(N) Sector N							
Total Intermediate Use							
	Value Added Quadrant 2 (Primary Inputs).						
Value Added							
Total Inputs (Purchases)							

columns represent inputs, and each sector is both a user of inputs and a producer of outputs.

Turning to the final use quadrant, we see that the outputs of each sector are also normally demanded for ultimate use in the form in which they are produced, e.g. as final demand products which are consumed for their own sake and not for use in further production. They may be purchased as consumption goods by individual consumers or by the central government, as investment goods by the state or by private investors; or they may be sold to foreign demanders in the form of exports. The horizontal summation of the outputs of sector 2 sold as intermediate goods to the other sectors plus the output sold to private, public and foreign final demanders yields the total output of sector 2.

Reading down columns 1 to n in the value added quadrant we obtain information about the amount of primary inputs (i.e. employees incomes, indirect tax, depreciation etc.) used by each sector in the production of its output. For any particular sector the elements in each column of this quadrant when added to the elements in the corresponding column of the transaction quadrant immediately above yield a value for total inputs purchased or used up by this industry during the accounting period, i.e. the total costs of operation. For each sector, the value of total output must equal the value of total input.

2.1.3 A hypothetical example.

Let us turn now to an input - output table using hypothetical data and review the above procedures with the aid of numerical **figures table** a2.2 provides such a **numerical illustration** of the basic input - output accounting system. For simplicity, we have divided our table into only 5 sectors (agriculture, sector 1 ; mining,sector 2 ; manufacturing, sector 3 ; transportation, sector 4 ; and service, sector 5).

Note that each sector appears in the accounting system twice, as a producer of outputs (rows 1 to 5) and as a user of inputs (columns 1 to 5). The elements in each row show how a particular sector disposed of its total output during the given accounting period. For example, of the total available output of agricultural products (125 units), 15 units are used by agricultural **itself**, 20 units by manufacturing, and 10 by service. The total intermediate use of agricultural products, i.e. use for further production, is 45 units. To this figure must be added the quantity of agricultural goods demanded by final users which in our table consists of the private consumption expenditure of 35 units, the government consumption expenditure of 5 units, the gross capital formation of 10 units and exports to foreign countries of 30 units. The sum of total intermediate and total final demand yields a gross output for agricultural production of 125 units. Similarly, the mining sector produce a total output of 40 units of which none is sold on an interindustry basis to other sectors while 10 units are purchased for gross capital formation and 30 units are exported. The total disposition of the total outputs of the other three sectors can be read off the table in the same manner.

The role of the agricultural sector as a purchaser of inputs is shown by column one. Reading down this column we see that in order to produce its total output of 125 units, agriculture had to use 15 units of its own output (e.g. using a portion of cereal seed for replanting), 10 units of manufacturing output (e.g. fertilizers, insecticides, etc.), 5 units of transportation (e.g. for transporting perishable goods to local markets or to the coast for export), and 5 units for service. Thus the total domestic interindustry purchases of intermediate material goods and services by agriculture was 35 units. The remaining 90 units of total inputs purchased consisted of the importation of 15 units of foreign goods and the creation of 75 units of value added in the form of payments of 40 units to households as wages, 20 units to the government as taxes, 5 units for the use of capital (depreciation),

Table 2-2 Hypothetical Input-output Table

Producing Sectors (Outputs) → / Using Sectors (Inputs) ↓	Intermediate Use: Agriculture	Mining	Manufacture	Transportation	Service	Total Intermediate Demand	Final Use (Demand): Private Consumption	Government Consumption	Gross Capital Formation	Exports	Total Final Demand	Total Outputs
Agriculture	15(x_{11})	0(x_{12})	20(x_{13})	0(x_{14})	10(x_{15})	45 $\sum_{j=1}^{5} x_{1j}$	35	5	10	30	80(F_1)	125(X_1)
Mining	0(x_{21})	0(x_{22})	0(x_{23})	0(x_{24})	0(x_{25})	0 $\sum_{j=1}^{5} x_{2j}$	0	0	10	30	40(F_2)	40(X_2)
Manufacture	10(x_{31})	0(x_{32})	25(x_{33})	15(x_{34})	5(x_{35})	55 $\sum_{j=1}^{5} x_{3j}$	15	5	20	5	45(F_3)	100(X_3)
Transportation	5(x_{41})	15(x_{42})	15(x_{43})	0(x_{44})	15(x_{45})	50 $\sum_{j=1}^{5} x_{4j}$	5	10	10	0	25(F_4)	75(X_4)
Service	5(x_{51})	10(x_{52})	15(x_{53})	0(x_{54})	5(x_{55})	35 $\sum_{j=1}^{5} x_{5j}$	5	2	8	0	15(F_5)	50(X_5)
Total Purchases	35	25	75	15	35	185	60	52	22	65	205	390
Imports	15	0	10	30	5	60						
Employees Incomes	40(L_1)	5(L_2)	6(L_3)	5(L_4)	2(L_5)	58(L_T)						
Indirect Taxes	20	5	3	7	2	37						
Depreciation	5	3	5	12	4	29						
Profits	10	2	1	6	2	21						
Value Added	75	15	15	30	10	145						
Total Inputs	125	40	100	75	50	390						

and 10 units for the profit of the entrepreneur. It is immediately evident that the value of the total output of agriculture is equal to the total value of all input purchased, i.e. 125 units. This same procedure can be followed in analysing the input - output structure of each and every sector of the economy.

So far, we have been concerned primarily with a descriptive explication of the input - output table. Even if input - output were used only as an accounting mechanism, its highly interdependent, disaggregrated contents would still represent a descriptive tool of considerable analytic value. However, the major theoretical and practical value of input - output tables is that they can easily be transformed into a consistent mathematical model and utilized either as a forecasting tool to predict the effects of autonomous changes in final demand on total output and employment in all sectors of the economy. In order to tranform the accounting table into a workable mathematical model, however, certain necessary assumptions about the production process must be made.

2.2 The Economic Model

An economic model is based on three sets of relation:2/

2/ Schaffer W.A., ' On the Use of Input - Output Models for Regional Planning', Martinus Nijhoff Social Science Division, Leiden 1976.

1. definitions or identities. 2. technical or behavioral conditions, and 3. equilibrium conditions.

2.2.1 Identities: the transactions table

The transactions table provides set of identities. It can be defined in terms of the following equations.

$$
\begin{array}{ccccccc}
x_{11} & + x_{12} & + x_{13} & + x_{14} & + x_{15} & + F_1 & = X_1 \\
x_{21} & + x_{22} & + x_{23} & + x_{24} & + x_{25} & + F_2 & = X_2 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots \\
x_{51} & + x_{52} & + x_{53} & + x_{54} & + x_{55} & + F_5 & = X_5
\end{array}
$$

2.2.2 Technical conditions : the direct requirements table

The crucial assumption of input - output analysis is the assumption that a single process production function exists in every industry. The first is the assumption that each commodity produced by a single industry ; it follows from this that there is one method of production in each sector with only one primary output or no joint production. The second is the assumption of constant returns to scale. Technically we may say that the production process is assumed to be characterized by the existence of constant 'technical coefficients of production', i.e. each additional unit of new output is produced by an unchanging proportional combination of material inputs from the other sectors.

Stated mathematically, $a_{ij} = x_{ij} / X_j \quad i = 1, 2, \ldots . n. \quad (2)$
$j = 1, 2, \ldots . n.$

The matrix of technical coefficients of production with 5 sectors would consist of 25 technical coefficients which can be arranged as follows:

Table 2 - 3 Matrix of Technical coefficients

INPUTS \ OUTPUT		INTERMEDIATE DEMAND				
		1. Agriculture	2. Mining	3. Manufacture	4. Transport.	5. Service
INTERMEDIATE INPUTS	1. Agriculture	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
	2. Mining	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
	3. Manufacture	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
	4. Transportation	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
	5. Service	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Substitute $x_{ij} = a_{ij} X_j$ into equation (1)

$$\begin{array}{l} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + a_{15}X_5 + F_1 = X_1 \\ a_{21}X_2 + a_{22}X_2 + a_{23}X_3 + a_{24}X_4 + a_{25}X_5 + F_2 = X_2 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots + .. = .. \\ a_{51}X_1 + a_{52}X_2 + a_{53}X_3 + a_{54}X_4 + a_{55}X_5 + F_5 = X_5 \end{array} \quad (3)$$

This set of equations can be expressed in the form of matrices and vectors:

$$\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_5 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots\cdots & a_{15} \\ a_{21} & a_{22} & \cdots\cdots & a_{25} \\ \cdots & \cdots & & \cdots \\ a_{51} & a_{52} & \cdots\cdots & a_{55} \end{pmatrix} X \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ \lambda_5 \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_5 \end{pmatrix}$$

The block of terms in 'a' are the **entries in the** matrix A and the above can be simplified to yield the basic input - output accounting system.

$$X = AX + F \qquad (5)$$

Using equation (2) to calculate the a_{ij}'s for hypothetical 5 sector input - output table, we can arrive at the following 'A' matrix of technical coefficients of production.

Table 2-4 The 'A' Matrix of Technical Coefficients for Hypothetical 5 Sectors Economy.

(Direct Input per unit of Output)

OUTPUT → / ↓ INPUT	INTERMEDIATE DEMAND				
	1. Agriculture	2. Mining	3. Manufacture	4. Transportation	5. Service
INTERMEDIATE INPUT 1. Agriculture	15/125=.12	0/40=0	20/100=.20	0/75=0	2 10/50=.20
2. Mining	0/125=0	0/40 = 0	0/100 = 0	0/75 = 0	0/50=0
3. Manufacture	10/125=.08	0/40=0	25/100=.25	15/75=.2	5/50=.30
4. Transportation	5/125 =.04	15/40=.375	15/100=.15	0/75=0	15/50=.30
5. Service	5/125=.04	10/40=.25	15/100=.15	0/75=0	5/50 =.10

Table 2-5 The 'A' Matrix of Technical Coefficients for Hypothetical 5 Sectors Economy.

(Direct Input per 1000 units of Output)

INPUT \ OUTPUT		INTERMEDIATE DEMAND				
		Agricuture	Mining	Manufacture	Transportation	Service
INTERMEDIATE INPUT	1.Agriculture	120	0	200	0	200
	2.Mining	0	0	0	0	0
	3.Manufacture	80	0	250	200	100
	4.Transportation	40	375	150	0	300
	5.Service	40	250	150	0	100

2.2.3 Equilibrium condition: supply equals demand

The analytical and mathematical content of the input-output model rests on a dual foundation. The first consists of a set of accounting equations, one for each producing sector of the economy. The first of these equations states that the total output of sector 1 is equal to the sum of the seperate amounts sold by sector 1 to the other industries plus the amounts produced to satisfy final demands. The second equation say the same thing for sector 2 - and so on for all n industries. In terms of our input-output table these equations state that for any sector total output is equal to the sum of all the entries in that sector's row in the table. Thus, an implicit assumption of input - output analysis common to all general equilibrium models is that in all sectors the entire product produced is consumed either by other industries as intermediate inputs or by final demanders. In short, supply always equals demand.

2.3 The Inverse Matrix[3/]

The input structure discussed in the previous section show what each sector requires to produce its output but it tells nothing about the further effect. The effect of the production of a motor - vehicle does not end with the steel, components and tyres purchased. It begins a long chain of production since each of the products purchased will, in their turn, require various inputs; production will be required in some industries not directly supplying the motor industry and the direct suppliers may be called on also to supply other industries.

It is thus possible to define two types of inputs. Direct inputs are those purchased by the industry under consideration; indirect inputs are those purchased by all industries in which

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3/ United Nations, '<u>Input-Output Tables andAnalysis</u>', New York, 1973.

production is required in order to supply inputs to the first industry. It can be seen from Table 2-5 that 1000 units of final demand in agricultural products require 120 units of agriculture sector, 80 units of manufacturing sector, 40 units of transportation sector, and 40 units of service sector. But the production of 120 units agriculture sector require some units of manufacturing sector, transportation sector, and service sector, all of which in turn will generate other inputs from all sectors from Table 2-4 , 2-5 these direct and indirect requirements can be traced through the system, and it is possible to know the full impact of production in each of the sectors on the other sectors.

Table 2 - 6 Direct and Indirect Inputs

Producing Sector	Initial Output	Direct Inputs	Indirect Inputs					Total
			Round1	Round 2	Round 3	Round4	Round n	
1. Agriculture	1000	120	38.40	17.00	8.10	3.92		1190
2. Mining		0	0	0	0	0		0
3. Manufacture		80	41.60	19.90	10.13	4.97		160
4. Transportation		40	28.80	14.0	6.64	3.24		100
5. Service		40	20.80	9.90	4.66	2.31		80

The direct input requirements of 1000 units of agriculture sector are shown in the second column of Table 6. The indirect input requirements of the first round can be derived from technical coefficients of Table 4 and direct input components of Table 5.

Table 2 - 7 Indirect Inputs (Round 1).

Producing Sector / Direct Inputs	Agriculture		Mining		Manufacture		Transportation		Service	
1.	2	3=1x2	4	5=1x4	6	7=1x6	8	9=1x8	10	11=1x10
120	0.12	14.4	0	0	0.08	9.6	0.04	4.80	0.04	4.80
0	0	0	0	0	0	0	0.375	0	0.25	0
80	0.20	16.6	0	0	0.25	20	0.15	12.0	0.15	1.20
40	0	0	0	0	0.20	8.0	0	0	0	0
40	0.20	8.0	0	0	0.10	4.0	0.30	12.0	0.10	4.0
Total		38.4		0		14.6		28.8		20.8

The second round of indirect inputs is calculated in the same way, the successive rounds rapidly diminish and we can calculate the full impact of the demand for 1000 units of outputs of agriculture sector.

The most convenient way to obtain these results is to use the basic equation (5) and to solve this for X as follows:

$$X = AX + F$$

$$X - AX = F$$

$$(I - A)X = F$$

$$X = (I - A)^{-1}F \qquad (6)$$

The matrix, $(I - A)^{-1}$ is known as the Leontief inverse or the matrix multiplier (an analogy to the Keynesian multiplier).

The matrix (I - A) is formed and inverted, yielding the matrix in Tabl^ 2 - 8

Table 2 - 8 The Inverse Matrix, $(I - A)^{-1}$

	Agriculture	Mining	Manufacture	Transportation	Service
Agriculture	1.19	0.11	0.40	0.08	0.34
Mining	0	1.00	0	0	0
Manufacture	0.16	0.19	1.50	0.30	0.30
Transportation	0.10	0.50	0.32	1.06	0.41
Service	0.08	0.31	0.27	0.05	1.18

The columns of this matrix show the total input requirements, both direct ant indirect, of one unit of output of the five sectors. From Table 2-6 the 1000 units of final demand in agricultural products multiplied with the first column giving the same result obtained in Table 2-9 It will be noted that the entries in this matrix are considerably larger than those in Table 2-4 showing direct inputs; the difference between the two are the indirect inputs.

Table 2 - 9 Direct and Indirect Inputs.

Agriculture	1.19 x 1000 = 1190
Mining	0 x 1000 = 0
Manufacture	0.16 x 1000 = 160
Transportation	0.10 x 1000 = 100
Service	0.08 x 1000 = 80

This inverse matrix $(I-A)^{-1}$, is fundamental to input - output analysis as it shows the full impact of the demand for the output of each sector on all the other sector. With such a matrix it is possible to unravel the technological in interdependence of the productive system and to trace the generation of demand from final consumers back throughout the system. It is then possible to calculate what output levels would be required to meet various postulated levels of final demand and consequently how output levels would be required to change to meet postulated changes in final demand, such as change in private consumption expenditure.

Table 2 - 10 Primary Input Requirements

Item / Sector	Direct&Indirect Input for 1000 units Final Demand of Agri. Products (1)	Primary Input Coefficients (2)	Primary Input Requirements (3)=(1) x (2)
Agriculture	1190	90/125=0.72	856
Mining	0	15/40 =0.375	0
Manufacture	160	25/100=0.25	40
Transportation	100	60/75 =0.80	80
Service	80	15/50 = 0.30	24
			1000

The output of product required were calculated in Table2.9and are shown in the first column of Table 2.10Each of the producing sectors will require primary inputs to produce its required output. The primary input coefficients of each sector are calculated from Table2.2and shown in column 2 of Table2.10. The third column is obtained as the product of the first two columns and shows the primary inputs required by each of the five sectors. It will be noticed that the total primary input requirement is 1000 units exaxtly equal to the level of final demand assumed in the example.

2.4 Economic Change in Input-Output Models

An input - output model is designed to trace the effect of changes in an economy which has been represented in an input - output table. Such models show the consequences of change in terms of monies through an economy and in terms of incomes generated for primary resource owners. The models themselves do not show the causes of change; these causes are exogeneous to the system.

Economic change as traced through an input - output model can take two forms:4/ (1) structural change (2) change in final demand.

2.4.1 Structural change

Change in the economic structure of an country can be initiated in several ways. It can be through public investment in schools, highways, public investment in schools, highways, public facilities, etc., or it can be through private investment in new production facilities, or it can be through changes in the marketing structure of the economy.

Structural change in an input - output context can be interpreted to mean ' changes in production coefficients'. In turn, this can be interpreted as either changes in technology or changes in marketing patterns or both.

The addition of a completely new industry to the

4/ Schaffer W.A.., ' On the Use of Input - Output Models for Regional Planning', Martinus Nijhoff Social Science Division, Leiden, 1976.

system means adding another row and column to the interindustry table to represent the new industry.

2.4.2 Changes in final demand.

Changes in final demand are basically changes in private and government expenditure patterns, gross **capital** formation, and changes in the demands by other countries for the goods produced in the country.

Accounting for structural changes in an input - output model requires substantial skill and familiarity with the mechanics of the model on the part of the analyst. This is not the case when accounting for the effects of changes in final demand.

2.4.3 Solution to the system: the total - requirements table

Since we assume the production coefficients to be stable over time, we can rewrite the equation system to apply to a later period by substituting. F' for F, X_i' for X_i and $X_{ij}' = a_{ij}X_j'$ for X_{ij} in each of our equations:

$$
\begin{array}{ccccccc}
a_{11}X_1' + & a_{12}X_2' + & a_{13}X_3' + & a_{14}X'_4 + & a_{15}X'_5 + & F_1' & = X_1' \\
a_{21}X_1' + & a_{22}X_2' + & a_{23}X_3' + & a_{24}X_4' + & a_{25}X_5' + & F_2' & = X_2' \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdot\cdot & \cdot\cdot \\
a_{51}X_1' + & a_{52}X_2' + & a_{53}X_3' + & a_{54}X_4' + & a_{55}X_5' + & F_5' & = X_5'
\end{array}
\qquad (7)
$$

The set of equations can **be expressed in the form of** matrices and vectors.

$$\begin{bmatrix} X'_1 \\ X'_2 \\ \cdots \\ X'_5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} X \begin{bmatrix} X'_1 \\ X'_2 \\ \cdot\cdot \\ X'_5 \end{bmatrix} + \begin{bmatrix} F'_1 \\ F'_2 \\ \cdot\cdot \\ F'_5 \end{bmatrix} \quad (8)$$

$$X' = AX' + F'$$

$$(I-A)\ X' = F'$$

$$X' = (I-A)^{-1}\ F' \quad (9)$$

We can easily derive projections of the expected gross output (X') of industries in the later year.
