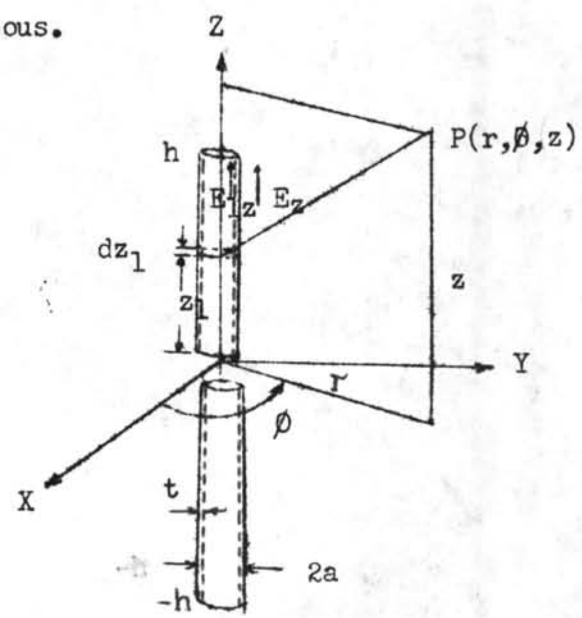
CHAPTER II

CURRENT DISTRIBUTION AND IMPEDANCE

A. CURRENT DISTRIBUTION.

The method of calculating current distribution of a center-fed cylindrical antenna will be discussed. This method bases on the boundary-value problem, and follows nearly the same way as Hallen's (4) but with some considerations on antenna conductivity which causes the Hallen's Integral Equation becomes nonhomogeneous.



Fig, 2.1 The center-driven cylindrical antenna and its cylindrical coordinates.

A cylindrical dipole antenna is shown in Fig.2.1 The tangential component of the electric field must be continuous at the boundary, that is

$$E_{1z} = E_z \tag{3}$$

Where E_{1z} = Electric field just inside the conductor

E_z = Electric field just outside the conductor

To simplify the problem it is assumed that $I(z)$ is equal to zero at $z = \pm h$ and the end effect will be neglected.

Consider the electric field inside the conductor

$$E_z = ZI_z \tag{4}$$

Where Z = The conductor impedance in ohms per meter length of conductor under consideration of skin effect.

I_z = Total current in the z-component

The electric field outside the cylinder can be determined from the vector potential $\overline{A}$

A Maxwell's equation can be written as

$$\nabla \times \overline{E} + \frac{\partial \overline{B}}{\partial t} = 0 \tag{5}$$

Since $$\overline{B} = \nabla \times \overline{A} \tag{6}$$

Hence, $$\nabla \times \left(\overline{E} + \frac{\partial \overline{A}}{\partial t}\right) = 0 \tag{7}$$

By vector identity: $\nabla x(-\nabla\phi) = 0$ (8)

Where ϕ is a scalar potential.

Comparison of eq.(7) and eq.(8) we may write

$$\overline{E} + \frac{\partial \overline{A}}{\partial t} = -\nabla\phi$$

Or

$$\overline{E} = -\nabla\phi \ jw\overline{A} \tag{9}$$

By vector condition

$$\nabla.\overline{A} = -j \ w\phi \tag{10}$$

Substitute eq. (10) into eq.(9) gives

$$\overline{E} = \frac{1}{jw\mu\epsilon} \nabla(\nabla.\overline{A}) - jw\overline{A}$$

$$= -jw\left[\frac{c^2}{w^2} \nabla(\nabla.\overline{A}) + \overline{A}\right]$$

$$= -j\frac{w}{k^2}\left[\nabla(\nabla.\overline{A}) + k^2\overline{A}\right] \tag{11}$$

Since the current has only z component, hence the vector potential $\overline{A}$ has only A_z, then eq. (11) becomes

$$E_z = -j\frac{w}{k^2}\left[\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z\right] \tag{12}$$

From eq. (3), eq.(4) and eq.(12), we obtain

$$-j\frac{w}{k^2}\left(\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z\right) = ZI_z$$

$$\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z = j\frac{k^2 ZI_z}{w} \tag{13}$$

Eq.(13) may be written as

$$\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z = F(z) \tag{14}$$

Eq.(14) is a second order first degree wave equation which is homogeneous when Z is zero. But Z is not equal to zero, therefore the solution can be represented by the sum of a complimentary function (A_c) and a particular integral (A_p)

That is $$A_z = A_c + A_p \tag{15}$$

Consideration of eq.(14), we can write that

$$A_c = B_1 \cos kz + B_2 \sin kz \tag{16}$$

Where B_1 and B_2 are constants

A_p can be found by using "The Method of Variation of Parameters"[1] as follows,

Let $$A_p = U_1(z)Y_1(z) + U_2(z)Y_2(z) \tag{17}$$

Where $Y_1(z)$, $Y_2(z)$ are the solutions of eq.(13) for which Z = 0

Hence $$Y_1(z) = \cos kz \tag{18}$$

$$Y_2(z) = \sin kz \tag{19}$$

$U_1(z)$, $U_2(z)$ are two unknown functions of z and can be obtained as follows,

$$U_1'(z) = -\frac{Y_2(z)}{Y_1(z)\,Y_2'(z) - Y_2(z)\,Y_1'(z)} \cdot F(z) \tag{20}$$

Substitute eq.(18) and eq.(19) into eq.(20), we have

$$U_1'(z) = -\frac{\sin kz \; F(z)}{k(\cos^2 kz + \sin^2 kz)}$$

$$= -\frac{\sin kz \, F(z)}{k}$$

$$U_1(z) = -\int_0^z \frac{\sin ks}{k} \, F(s) \, ds \tag{21}$$

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[1]C.R. Wylie, Jr. "Advance Engineering Mathematics", 3rd Edition, pp 49-51, 1966.

Similarly $$U_2(z) = \int_0^z \frac{\cos ks}{k} F(s)\, ds \qquad (22)$$

Substitute eq.(18),eq.(19), eq.(21),eq.(22) into eq.(17) we have

$$\begin{aligned} A_p &= -\frac{\cos kz}{k} \int_0^z \sin ks\ F(s)\, ds + \frac{\sin kz}{k} \int_0^z \cos ks\, F(s)\, ds \\ &= \frac{1}{k} \int_0^z (\cos ks \sin kz - \sin ks \cos kz)\, F(s)\, ds \\ &= \frac{1}{k} \int_0^z \sin k(z-s)\, F(s)\, ds \qquad (23) \end{aligned}$$

From eq.(13) and eq.(14), it can be shown that

$$F(s) = j\frac{k^2 Z I(s)}{w}$$

Hence $$A_p = j\frac{kZ}{w} \int_0^z \sin k(z-s) I(s) ds \qquad (24)$$

A_z can be obtained from eq.(15), eq.(16) and eq.(24) as follows:

$$A_z(z) = B_1 \cos kz + B_2 \sin kz + j\frac{kZ}{w} \int_0^z \sin k(z-s) I(s) ds$$

Or $$A_z(z) = -j/c \left[C_1 \cos kz + C_2 \sin kz - Z \int_0^z \sin k(z-s) I(s)\, ds \right] \qquad (25)$$

Where $$c = k/w$$

$$C_1, C_2 = \text{constants}$$

For a center - driven antenna; the current and vector potential are symmetrical with respect to the origin.

Hence $$I(z) = I(-z)$$

$$\bar{A}(z) = \bar{A}(-z)$$

Then eq.(25) becomes:

$$A_z = -j/c \left[C_1 \cos kz + C_2 \sin k|z| - Z \int_0^z \sin k(z-s) I(s) ds \right] \qquad (26)$$

The applied voltage, V is defined as

$$V = \lim_{z \to 0} \left[\phi(z) - \phi(-z) \right] \qquad (27)$$

From eq.(10) when only z component is considered, we have

$$\frac{\partial A_z}{\partial z} = -jw\mu\epsilon\phi \tag{28}$$

Substitute eq.(28) into eq.(27), we get

$$V = \lim_{z\to 0} j\frac{1}{w\mu\epsilon}\left[\frac{\partial A_z(z)}{\partial z} - \frac{\partial A_z(-z)}{\partial z}\right]$$

$$= j\frac{c^2}{w}\left[\lim_{z\to 0}\frac{\partial A_z(z)}{\partial z} - \lim_{z\to 0}\frac{\partial A_z(-z)}{\partial z}\right] \tag{29}$$

From eq.(25), we can show that

$$\lim_{z\to 0}\frac{\partial A_z(z)}{\partial z} = -j/c\left[kC_2\right] \tag{30}$$

$$\lim_{z\ 0}\frac{\partial A_z(-z)}{\partial z} = -j/c\left[-kC_2\right] \tag{31}$$

Substitute eq.(30) and eq.(31) into eq.(29), we obtain

$$V = 2C_2 \tag{32}$$

The vector potential $A_z(z)$ can also be expressed in term of the surface current $I(z_1)$ as follows:

$$A_z(z) = \frac{\mu}{4\pi}\int_{-h}^{h}\frac{I(z_1)e^{-jkR}}{R}\,dz_1 \tag{33}$$

$$R = \left[(z-z_1)^2 + r^2\right]^{1/2} \tag{34}$$

The combination and the rearrangement of eq.(26), eq.(32) and eq.(33), result in

$$j\frac{c\mu}{4\pi}\int_{-h}^{h}\frac{I(z_1)e^{-jkR}}{R}\,dz_1 = C_1\cos kz + \frac{V}{2}\sin kz - Z\int_{0}^{h} I(z)\sin k(z-s)ds \tag{35}$$

The integration on the left-hand side of eq.(33) can be written as

$$\int_{-h}^{h}\frac{I(z_1)e^{-jkR}}{R}\,dz_1 = \int_{-h}^{h}\frac{I(z)}{R}\,dz_1 + \int_{-h}^{h}\frac{I(z_1)e^{-jkR} - I(z)}{R}\,dz_1 \tag{36}$$

Integrating the right hand side of eq.(36) and putting r = a, we thus obtain.

$$I(z)\int_{-h}^{h}\frac{dz_1}{R} = I(z)\int_{-h}^{h}\frac{dz_1}{\sqrt{(z-z_1)^2 + a^2}}$$

$$= I(z)\ln\left[\frac{\{(h-z)^2 + a^2\}^{1/2} + (h-z)}{\{(h-z)^2 + a^2\}^{1/2} - (h+z)}\right]$$

$$= I(z)\left[\Omega + \ln(1-(\tfrac{z}{h})^2) + L(z)\right] \qquad (37)$$

Where

$$\Omega = 2\ln\left[\frac{2h}{a}\right] \qquad (38)$$

$$= \ln\left[\frac{1}{4}\left\{\left[1+(\frac{a}{h-z})^2\right]^{1/2}+1\right\}\left\{\left[1+(\frac{a}{h+z})^2\right]^{1/2}+1\right\}\right] \qquad (39)$$

From eq.(35),eq.(36) and eq.(37), it follows that

$$\frac{jc\mu}{4\pi}\left[I(z)\left\{\Omega + \ln(1-(\tfrac{z}{h})^2)+\delta\right\} + \int_{-h}^{h}\frac{I(z_1)e^{-jkR} - I(z)}{R}\,dz_1\right]$$

$$= C_1\cos kz + \frac{V}{2}\sin k|z| - Z\int_0^z I(s)\sin k(z-s)\,ds$$

$$I(z) = -\frac{j4\pi}{\mu c\Omega}\left[C_1\cos kz + \frac{V}{2}\sin k|z| - Z\int_0^z I(s)\sin k(z-s)\,ds\right]$$

$$-\frac{1}{\Omega}\left[I(z)\ln(1-(\tfrac{z}{h})^2) + I(z)\delta + \int_{-h}^{h}\frac{I(z_1)e^{jkR} - I(z)}{R}\,dz_1\right] \qquad (40)$$

At $z = h$, $I(z) = I(h) = 0$, Therefore eq.(40) becomes

$$0 = -\frac{j4\pi}{Z_0\Omega}\left[(C_1\cos kh + \frac{V}{2}\sin kh)\right] - \frac{1}{\Omega}\left[-\frac{j4\pi Z}{Z_0}\cdot\right.$$

$$\left.\int_0^h I(s)\sin(h-s)\,ds + \int_{-h}^{h}\frac{I(z_1)e^{-jkR_1}}{R_1}\,dz_1\right] \qquad (41)$$

Where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ = intrinsic impedance of free space

$$R_1 = \left[(h-z_1)^2 + a^2\right]^{1/2}$$

Subtracting eq.(41) from eq.(40), the result is as follows:

$$I(z) = \frac{-j4\pi}{Z_o\Omega}\left[C_1(\cos kz-\cos kh) + \frac{V}{2}(\sin k\,z-\sin kh)\right]$$
$$-\frac{1}{\Omega}\left[I(s)\ \ln(1-(\frac{z}{h})^2) + I(z)\delta + \int_{-h}^{h}\frac{\left[I(z_1)e^{-jkR}-I(z)\right]dz_1}{R}\right.$$
$$\left.-\frac{j4\pi Z}{Z_o}\int_0^z I(s)\sin k(z-s)ds\right] + -\frac{1}{\Omega}\left[\int_{-h}^{h}\frac{I(z)e^{-jkR_1}}{R_1}\,dz_1\right.$$
$$\left.-\frac{j4\pi Z}{Z_o}\int_0^h I(s)\sin k(h-s)\ ds\right] \tag{42}$$

Eq.(42) is the integral equation for the total current distribution on the center-fed cylindrical dipole. The zero order approximation can be obtained by neglecting the terms in the last two brackets in eq.(42)

$$I_o(z) = -\frac{j}{30\Omega}\left[C_1F_o(z) + \frac{V}{2}G_o(z)\right] \tag{43}$$

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 120\pi$$

$$F_o(z) = \cos kz-\cos kh = f_o(z)-f_o(h) \tag{43a}$$

$$G_o(z) = \sin k|z| - \sin kh = g_o(z) - g_o(h) \tag{43b}$$

Substituting $I_o(z)$ for $I(z)$ on the right hand side of eq.(42) gives an approximation for the remaining term

$$I_{o1}(z) = \frac{-j}{30\Omega^2}\left[C_1F_1(z) + \frac{V}{2}G_1(z)\right] \tag{44}$$

Where

$$F_1(z) = f_1(z) - f_1(h)$$

$$f_1(z) = -F_o(z)\ln(1-\frac{z^2}{h^2}) - F_o(z)\delta - \int_{-h}^{h}\frac{F_o(z_1)e^{-jkR}-F_o(z)}{R}\,dz_1$$
$$+\frac{j4\pi Z}{Z_o}\int_0^Z F_o(s)\sin k(z-s)\ ds \tag{45}$$

$$f_1(h) = -\int_{-h}^{h} \frac{F_o(z_1)\, e^{-jkR_1}}{R_1}\, dz_1 + \frac{4\pi Z}{Z_o}\int_{0}^{h} F_o(s)\sin k(h-s)ds \tag{46}$$

$G_1(z)$ is exactly like $F_1(z)$ with G written for F.

The first order approximation is then,

$$I_1(z) = I_o(z) + I_{o1}(z)$$

Or

$$I_1(z) = \frac{j}{30\Omega}\left[C_1\left\{F_o(z) + \frac{F_1(z)}{\Omega}\right\} + \frac{V}{2}\left\{G_o(z) + \frac{G_1(z)}{\Omega}\right\}\right] \tag{47}$$

Repeating these processes indefinitely, the final result will be obtained as follows:

$$I(z) = \frac{-j}{30\Omega}\left[C_1\left\{F_o(z) + \frac{F_1(z)}{\Omega}, \frac{F_2(z)}{\Omega^2} + \ldots\ldots\right\} + \frac{V}{2}\left\{G_o(z) + \frac{G_1(z)}{\Omega} + \frac{G_2(z)}{\Omega^2} + \ldots\ldots\right\}\right] \tag{48}$$

The expression for C_1 can be obtained by substituting eq.(48) into eq.(41), therefore

$$C_1 = -\frac{1}{2} V\left[\frac{g_o(h) + g_1(h)/\Omega + g_2(h)/\Omega^2 + \ldots\ldots}{f_o(h) + f_1(h)/\Omega + f_2(h)/\Omega^2 + \ldots\ldots}\right] \tag{49}$$

Substituting C_1 into eq.(48) and rearranging the terms, we get

$$I(z) = j\frac{V}{60\Omega}\left[\frac{\sin k(h-z) + K_1/\Omega + K_2/\Omega^2 + \ldots\ldots}{\cos kh + A_1/\Omega + A_2/\Omega^2 + \ldots\ldots\ldots}\right] \tag{50}$$

Neglecting the higher order terms of eq.(50), the first order solution for the antenna current distribution becomes

$$I(z) = j\frac{V}{30\Omega}\left[\frac{\sin k(h-z) + K_1/\Omega}{\cos kh + A_1/\Omega}\right] \tag{51}$$

Where

$$K_1 = f_1(z)\sin kh - f_1(h)\sin k|z| + g_1(h)\cos kz - g_1(z)\cos kh \quad (52)$$

$$A_1 = g_1(h) \quad (53)$$

When the cylindrical antennais infinitesimally thin ($\Omega \rightarrow \infty$)

eq.(51) becomes

$$I(z) = N\sin k(h-z) \quad (51a)$$

Where

$$N = j\frac{V}{60\Omega \cos kh}$$

Hence the sinusoidal current distribution is obtained.

B. IMPEDANCE the input impedance of the center-fed cylindrical antenna can be obtained from eq.(51) by putting z = o

$$Z_{in} = \frac{V}{I(o)}$$

$$= j60\Omega\frac{(\cos kh + A_1/\Omega)}{(\sin kh + K_1/\Omega)} \quad (54)$$

The input impedance of the cylindrical stub antenna over a perfectly ground plane is one-half of eq.(54), hence

$$Z_{in} = -j30\Omega\frac{(\cos kh + A_1/\Omega)}{(\sin kh + K_1/\Omega)} \quad (55)$$

Due to the complication of the expression of the input impedance, it is advisable to use a computer programmed for this solution.

C. DATA PRECALCULATED FOR NUMERICAL ANALYSIS

The purpose of this analysis is to compute the impedance of the cylindrical stub antenna over a perfect ground plane. In this thesis, the values of h = 50 cm and a = 3/2″ (therefore h/a = 13.1) are considered and the value of h/a = 75 is computed so that it can be compaired with the results obtained from the others.

From eq.(55), it is shown that

$$Z_{in} = -j30\Omega\left[\frac{\cos kh + A_1/\Omega}{\sin kh + K_1/\Omega}\right] \quad (56)$$

From eq.(43a), eq.(43b), eq.(45), eq.(46), eq(52) and eq.(53), we obtain

$$K_1(0) = f_1(0)\sin kh + g_1(h) - g_1(z)\cos kh \quad (57)$$

$$A_1(0) = f_1(h) \quad (58)$$

$$f_1(0) = \int_{-h}^{h} \frac{F_o(z)e^{-jkR_2} - Fo(0)}{R_2}\, dz_1, \text{for } \delta(o) \ll 1 \quad (59)$$

$$R_2 = \sqrt{z_1^2 + a^2} \quad (60)$$

$$f_1(h) = \int_{-h}^{h} \frac{F_o(z_1)e^{-jkR_1}}{R_1}\, dz_1 + j\frac{2\pi Z}{Z_o}\int_o^h F_o(s)\sin k(h-s)\, ds \quad (61)$$

$g_1(h)$ and $g_1(0)$ are exactly the same as $f_1(h)$, $f_1(0)$ with g written for h

Considering the last term of eq.(61), we have

$$j\frac{2\pi Z}{Z_o}\int_o^h F_o(s)\sin k(h-s)\, ds = j\frac{Z}{30} M \quad (62)$$

Where

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$$Z_o = 120\pi$$

$$M = \int_o^h F_o(s)\sin k\,(h-s)\, ds \quad (63)$$

Integrating eq.(63), we obtain

$$M = \frac{h}{z}\sin kh - \frac{\cos kh}{k}(1-\cos kh) \qquad (64)$$

For $h = 0$ to 1.0, $k = 2\pi$, it is clear from eq.(64) that

$$M < 1 \qquad (65)$$

For aluminium. $f = 300$ MHz

δ =depth of penetration

$$\approx 4.76 \times 10^{-4} \text{ cm} \qquad (66)$$

From eq.(66) therefore the antenna thickness, t is much greater than δ

$$\text{Hence } Z = \frac{(1+j)}{2\pi a}\sqrt{\frac{\pi f \mu}{\sigma}}^{2} \quad \text{ohms/m}$$

$$|Z| = \sqrt{\frac{f\mu}{2\pi\sigma a^2}} \quad \text{ohms/m}$$

$$0.033 \quad \text{ohm, for } h = 1.0,\ a = 0.0381\text{m} \qquad (67)$$

From eq.(65) and eq.(67), it can be seen that

$$\frac{|Z|}{30}\cdot M < 0.0011 \text{ ohms.} \qquad (68)$$

Therefore, from eq.(62) and eq.(68) we can neglect the last term of eq.(61)

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[2] Simon Ramo, "Field and waves in Communication Electronic" John Willey & Sons, Inc., P 294, 1965

From eq.(56), let

$$A_1 = AR + jAI$$

$$K_1 = KR + jKI$$

From eq.(58) and eq.(61), let

$$AR = -\int_{-h}^{h} \frac{\cos\theta_1(\cos kz - \cos kh)}{R_1}\,dz$$

$$AI = \int_{-h}^{h} \frac{\sin\theta_1(\cos kz - \cos kh)}{R_1}\,dz$$

$$Z_{in} = ZR + jZI$$

$$ZR = -30\Omega\,\frac{(KI\cos kh + AR.KI - AI.\sin kh - AI.KR)}{\{(\sin kh + KR)^2 + KI^2\}}$$

$$ZI = -30\Omega\,\frac{(\cos kh\sin kh + KR\cos kh + AR\sin kh + AR.KR + AI.KI)}{\{(\sin kh + KR)^2 + KI^2\}}$$

From eq.(57), let

$$K_1 = B\sin kh + C - D\cos kh$$

$$KR = BR\sin kh + CR - DR\cos kh$$

$$KI = BI\sin kh + CI - DI\cos kh$$

From eq. (59), let

$$BR = -\int_{-h}^{h} \frac{\cos\theta_2(\cos kz - \cos kh)dz - (1-\cos kh)}{R_2}dz - (1-\cos kh)\int_{-h}^{h}\frac{1}{R_2}\,dz$$

$$= -\int_{-h}^{h} \frac{\cos\theta_2(\cos kz - \cos kh)}{R_2}\,dz - (1-\cos kh)\ln\left(\frac{h}{a} + \sqrt{\frac{h^2}{a^2}+1}\right)$$

$$BI = \int_{-h}^{h} \frac{\sin\theta_2(\cos kz - \cos kh)}{R_2}\,dz$$

Similary

$$CR = -\int_{-h}^{h} \frac{\cos\theta_1(\sin k|z| - \sin kh)}{R_1}\,dz$$

$$CI = \int_{-h}^{h} \frac{\sin\theta_1(\sin k|z| - \sin kh)}{R_1}\,dz$$

$$DR = -\int_{-h}^{h} \frac{\cos\theta_2(\sin k|z| - \sin kh)\,d_z}{R_2} - \int_{-h}^{h} \frac{\sin kh}{R_2}\,dz$$

$$DI = \int_{-h}^{h} \frac{\sin\theta_2(\sin k|z| - \sin kh)}{R_2}\,dz$$

$$R_1 = \sqrt{(h-z)^2 + a^2}$$

$$R_2 = \sqrt{z^2 + a^2}$$

$$\theta_1 = 2\pi R_1$$

$$\theta_2 = 2\pi R_2$$

D. COMPUTER PROGRAM FOR THEORETICAL ANALYSIS

This theoretical analysis is run by the computer NEAC-SERIES 2200 which is installed at The Computer Science Center, Chulalongkorn University.

```
FORTRAN    200    SOURCE LISTING AND DIAGNOSTICS          PROGRAM: 0

C     PROGRAM SOLVING IMPEDANCE OF STUB CYLINDRICAL RADIATOR
C     ZR=RESISTANCE OF STUB CYLINDRICAL ANTENNA
C     ZI=REACTANCE OF STUB CYLINDRICAL ANTENNA
C     XL=ANTENNA LENGTH
      EXTERNAL R1,R2,FUN1,FUN2,SIN,COS
      PI=3.14159
      DO 40 I=1,2
      READ(2,5)AAA
    5 FORMAT(F1 .3)
      DT = 0.016
      XL=0.2
      WRITE(3,10)
   10 FORMAT(1H1,8X,38HIMPEDANCE OF STUB CYLINDRICAL RADIATOR,///)
      WRITE(3,15)
   15 FORMAT(1 X,14HANTENNA LENGTH,10X,10HRESISTANCE,10X,9HREACTANCE,/
   20 CALL INTG(R1,COS,FUN1,XL,DT,PI,AAA,AR)
      CALL INTG(R1,SIN,FUN1,XL,DT,PI,AAA,AI)
      AA =2.*ALOG(AAA)
      EL =-30.*AA
      AAAA=AAA/2.
      RL=2.*ALOG(AAAA+SQRT((AAAA)**2+1.))
      AR=-AR/AA
      AI=AI/AA
      S=SIN(2.*PI*XL)
      C=COS(2.*PI*XL)
      CALL INTG(R2,COS,FUN1,XL,DT,PI,AAA,BR)
      CALL INTG(R2,SIN,FUN1,XL,DT,PI,AAA,BI)
      BR=(1.-C)*RL-BR
      CALL INTG(R1,COS,FUN2,XL,DT,PI,AAA,CR)
      CALL INTG(R1,SIN,FUN2,XL,DT,PI,AAA,CI)
      CALL INTG(R2,COS,FUN2,XL,DT,PI,AAA,DR)
      CALL INTG(R2,SIN,FUN2,XL,DT,PI,AAA,DI)
      DR=-RL*S-DR
      XKR=(BR*S-CR-DR*C)/AA
      XKI=(BI*S+CI-DI*C)/AA
      DV=S**2+2.*XKR*S+XKR**2+XKI**2
      ZR=EL*(XKI*C+XKI*AR-AI*S-AI*XKR)/DV
      ZI=EL*(S*C+AR*S+XKR*C+AR*XKR+AI*XKI)/DV
      WRITE(3,25)XL,ZR,ZI
   25 FORMAT(1 X,F5.2,20X,F9.3,11X,F9.3,/)
      XL=XL+0. 5
      IF(XL.LE. .7)GO TO 20
   40 CONTINUE
      STOP
      END
```

FORTRAN 200 SOURCE LISTING AND DIAGNOSTICS PROGRA

```
      FUNCTION R1(X,XL,AAA)
      R1=SQRT((XL-X)**2+(2.*XL/AAA)**2)
      RETURN
      END
```

FORTRAN 200 SOURCE LISTING AND DIAGNOSTICS PROGRA

```
      FUNCTION R2(X,XL,AAA)
      R2=SQRT(X**2+(2.*XL/AAA)**2)
      RETURN
      END
```

FORTRAN 200 SOURCE LISTING AND DIAGNOSTICS PROGRA

```
      FUNCTION FUN1(X,XL,PI)
      PII=2.*PI
      FUN1=COS(PII*X)-COS(PII*XL)
      RETURN
      END
```

FORTRAN 200 SOURCE LISTING AND DIAGNOSTICS PROGRA

```
      FUNCTION FUN2(X,XL,PI)
      PII=2.*PI
      FUN2=SIN(PII*ABS(X))-SIN(PII*XL)
      RETURN
      END
```

```
FORTRAN     200        SOURCE LISTING AND DIAGNOSTICS

      SUBROUTINE INTG(F1,F2,F3,XL,DT,PI,AAA,SUM)
      X=-XL
    1 R=F1(X,XL,AAA)
      A=F2(2.*PI*R)
      B=F3(X,XL,PI)
      CL=A*B/R
      IF(X+XL)3,2,3
    2 SUM=CL/2.
      GO TO 4
    3 SUM=SUM+CL
    4 X=X+DT
      IF(X-XL)1,1,50
   50 SUM=SUM-CL/2.
      SUM=SUM*DT
      RETURN
      END
```

IMPEDANCE OF STUB CYLINDRICAL RADIATOR

$h/a = 13.1$

ANTENNA LENGTH	RESISTANCE	REACTANCE
.20	16.113	-24.319
.25	27.751	5.929
.30	45.156	30.956
.35	72.196	51.510
.40	117.162	61.459
.45	181.515	29.593
.50	187.358	-72.527
.55	101.970	-111.181
.60	49.725	-82.757
.65	31.967	-50.097
.70	29.194	-23.867

IMPEDANCE OF STUB CYLINDRICAL RADIATOR

h/a = 75

ANTENNA LENGTH	RESISTANCE	REACTANCE
.20	17.347	-49.884
.25	31.274	12.575
.30	56.243	74.569
.35	102.979	142.786
.40	220.683	220.348
.45	511.262	155.350
.50	443.014	-296.365
.55	150.832	-276.116
.60	62.591	-174.047
.65	37.242	-98.957
.70	33.532	-40.307

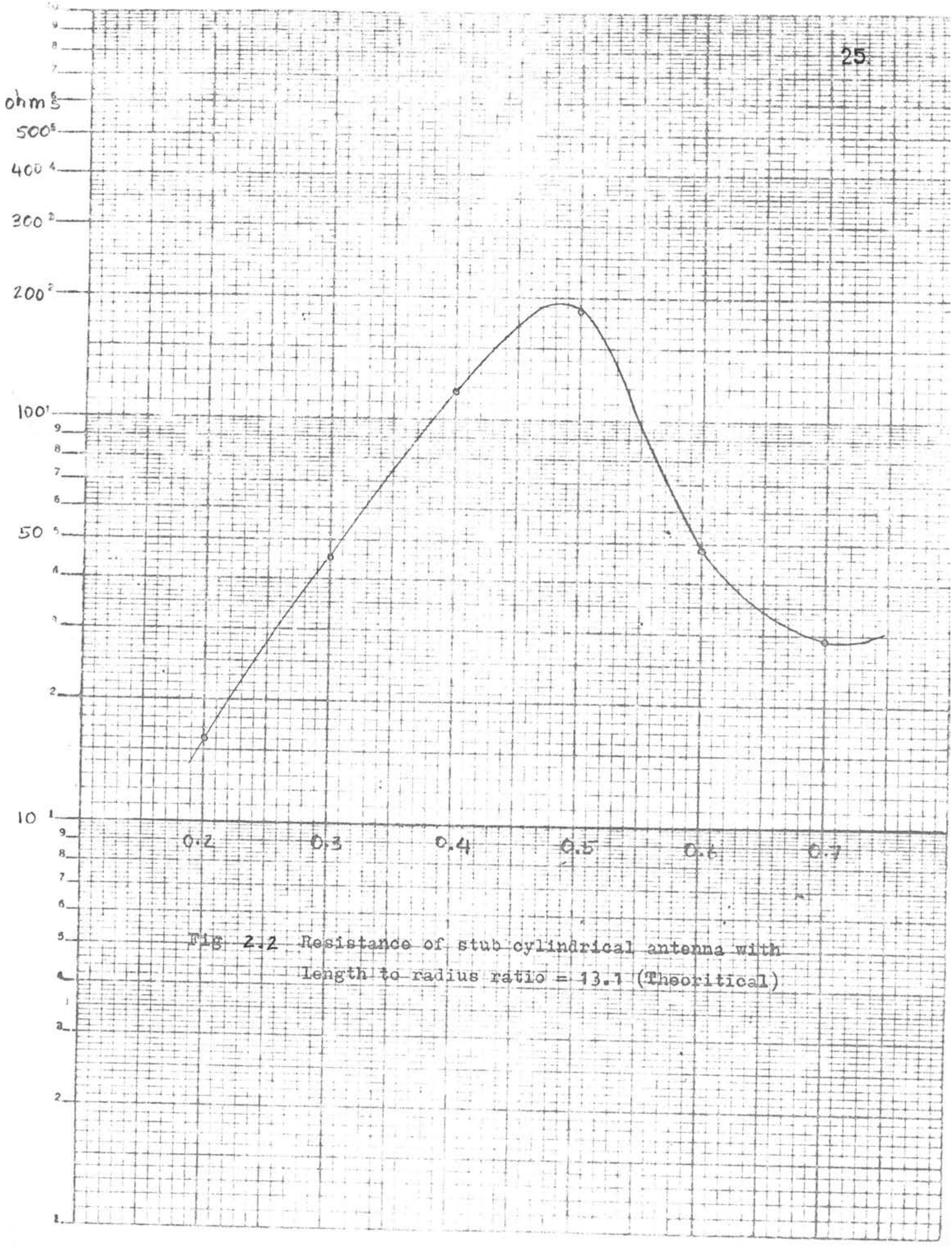

Fig 2.2 Resistance of stub cylindrical antenna with length to radius ratio = 13.1 (Theoritical)

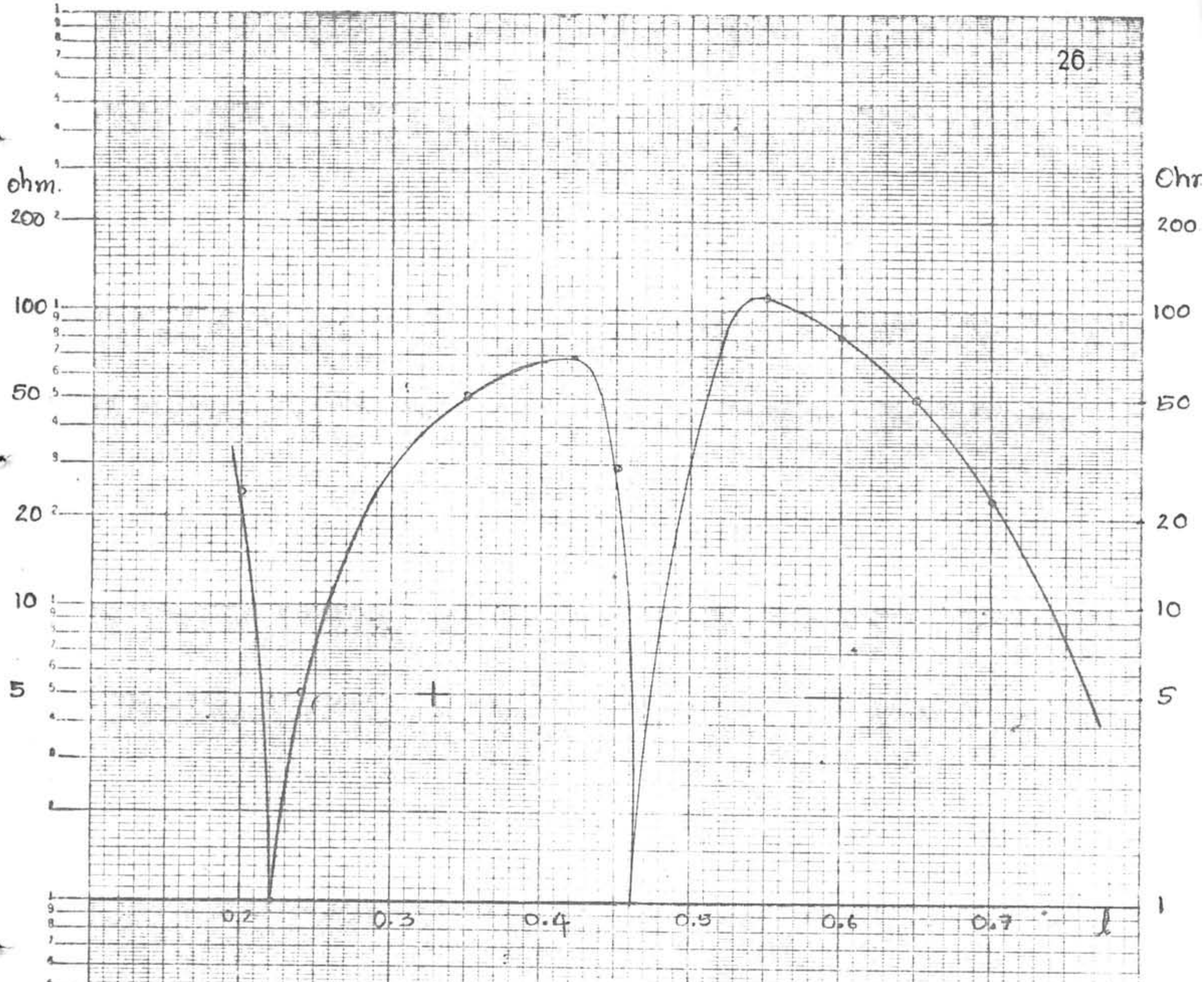

Fig2.3 Reactance of stub cylindrical antenna with length to radius ratio = 13.1 (Theoritical)

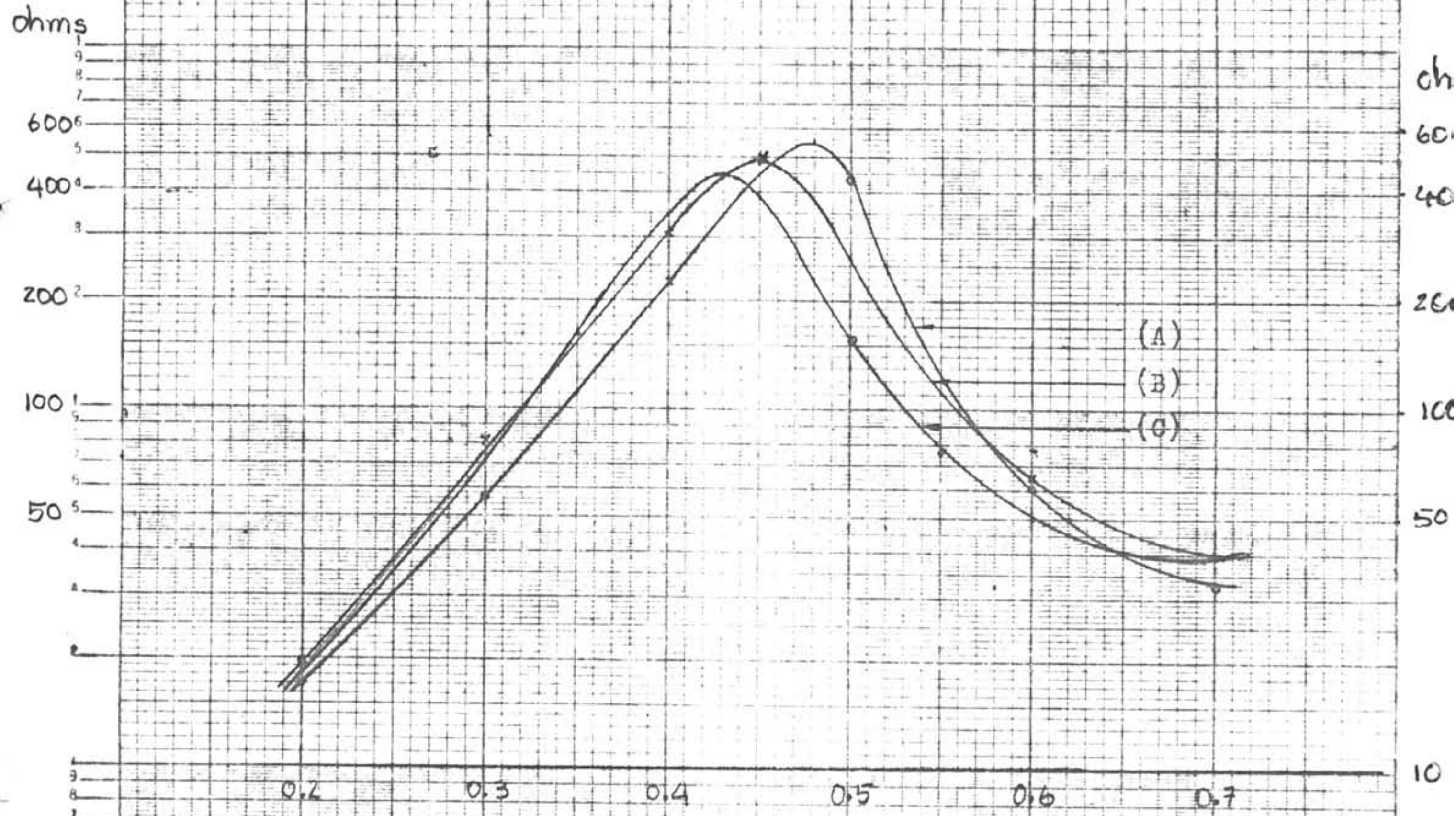

Fig2.4 Resistance of stub cylindrical antenna with length to radius ratio = 75 (OR Ω = 10) comparision between (A) first order app. (equation (56)) (B) Hallen, complete solution (C) R. King & Middleton

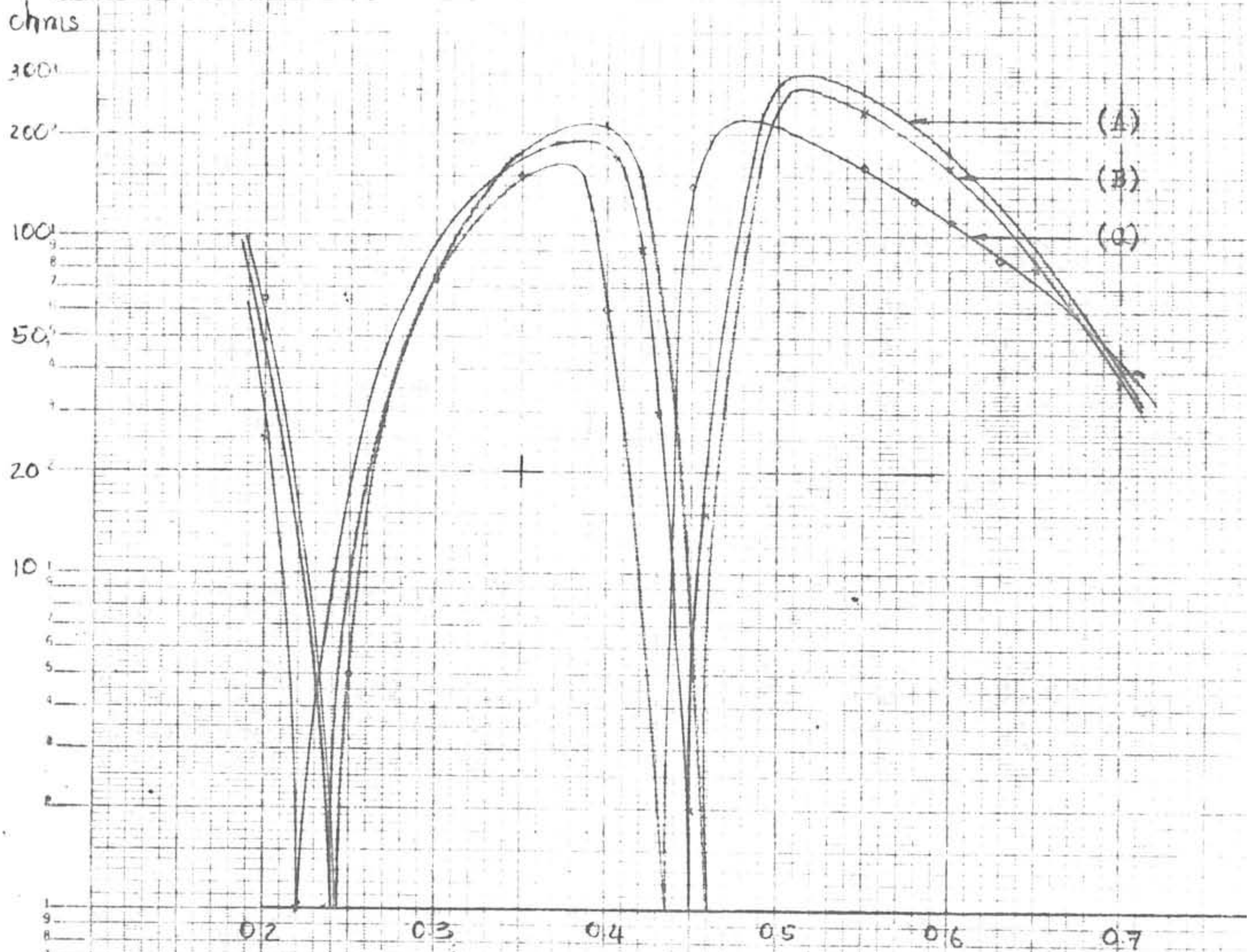

Fig2.5 Reactance of stub cylindrical antenna with length to radius ratio = 75 (OR Ω = 10) comparision between (A) first order app.(equation (56)) (B) Hallen, complete solusion (C) R.King & Middleton