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APPENDIX

DENSITY TAPERING METHOD OF DESIGN NONUNIFORMLY SPACED ARRAYS

It was shown that nonuniformly spaced array and the continuous line source will nearly have the same radiation pattern if the density of the equally excited, unequally spaced radiating elements as a function of the location within the array is decided to be of the same form as the continuous current density function of the line source. This introduces the approximate way of design nonuniformly spaced array and is known as the "density tapering" or "space tapering" method.

It is of interest to inquire, therefore, what relation the pattern of the density tapered array has to the pattern of the amplitude-tapered line source.

The antenna pattern produced by the continuous line source extending from $-a$ to $+a$ with amplitude taper $i_o(z)$ is

$$f_o(u) = \int_{-a}^{a} {}_{-o}(z) \cos 2\pi zu \, dz \qquad (A-1)$$

The nonuniformly spaced array which the element location is symmetrical on the array axis has a directional pattern given by

$$f(u) = 1 + 2\sum_{n=1}^{N} \cos 2\pi z_n u \qquad (A-2)$$

The array illumination function can be expressed as a summation of delta function, or

$$i(z) = \delta(z) + \sum_{n=1}^{N} \delta(z \pm z_n) \qquad (A-3)$$

The array pattern may be written similar to the pattern of (A-1) as

$$f(u) = \int_{-a}^{a} i(z) \cos 2\pi zu \, du \qquad (A-4)$$

The difference in the two radiation patterns is related to the difference in the aperture current densities by a Fourier transform

$$f_o(u) - f(u) = \int_{-a}^{a} \left[i_o(z) - i(z) \right] \cos 2\pi zu \, dz \qquad \text{(A-5)}$$

By integrating by parts, introducing the definition of the cumulative current distribution $I_o(z) = \int_{-a}^{a} i_o(z)\, dz$, and assuming that the cumulative distribution of the model aperture illumination and the array illumination are equal at the end points $\left[\text{i.e } I_o(-a)=I(-a) \text{ and } I_o(a)=I(a) \right]$, it can be shown that

$$\frac{f_o(u) - f(u)}{2\pi u} = \int_{-a}^{a} \left[I_o(z) - I(z) \right] \sin 2\pi zu \, dz \qquad \text{(A-6)}$$

This establishes a Fourier sine transform relationship between the pattern difference and the distribution difference. By applying Parseval's theorem eq.(A-6) may be written as

$$\frac{\left[f_o(u) - f(u) \right]^2}{u^2} du = 4\pi \int_{-a}^{a} \left[I_o(z) - I(z) \right]^2 dz \qquad \text{(A-7)}$$

The difference between the model continuous source pattern and the nonuniformly spaced array pattern may be expressed in a number of ways If the least-mean-square difference between the two is selected as the criterion for expressing how well the density-tapered pattern matches that of the model , the difference between the two or the error is expressed in the general form as

$$\epsilon = \int_{-a}^{a} \left[f_o(u) - f(u) \right]^2 W(u) \, du \qquad \text{(A-8)}$$

Comparing Eq.(A7) with Eq.(A8), $W(u)$ is taken to be $\frac{1}{u^2}$ which is a weighting function that expresses the relative importance of agreement as a function of the angular variable u , so the difference between the two pattern becomes progressively greater with increasing distance from the main beam.

Minimizing the mean-square difference of the patterns with $\frac{1}{u^2}$ weighting left side of Eq.(A7) is thus equivalent to minimizing the mean square difference between the current distributions right side of Eq(A7) if the total integrated current across the apertures in the two case are equal; that is , $I_0(z) = I(z)$.

The function has the form of a sum of steps of equal height, as shown in Fig.A1

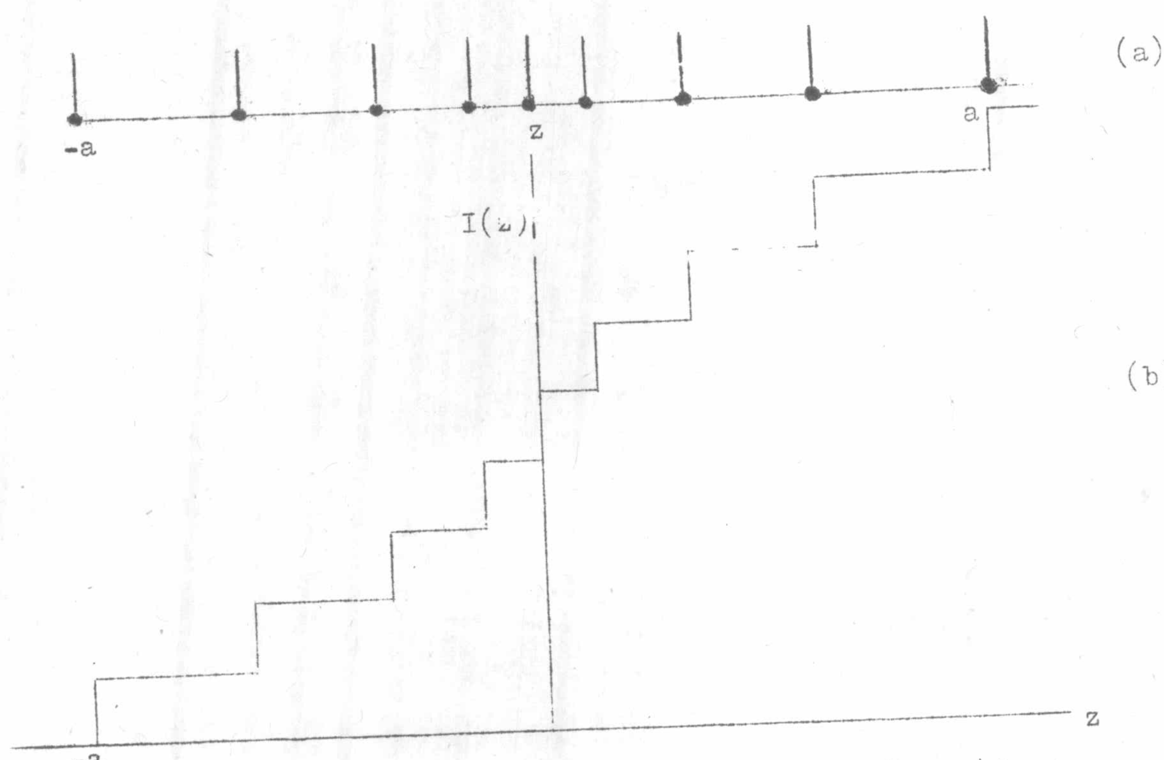

Fig.A1 (a) location of density-tapered elements

(b) its cumulative current distributions

To minimize Eq.(A-7), $I_o(z)$ should pass through each step of $I(z)$. If the k^{th} equal interval of $I_o(z)$ lying between (k-1) and k defines the region on the z axis, $\alpha < z < \beta$ (Fig.A2) the problem is to determine z_k within the interval (α, β) so that the mean square difference between the model distribution and the array distribution is a minimum.

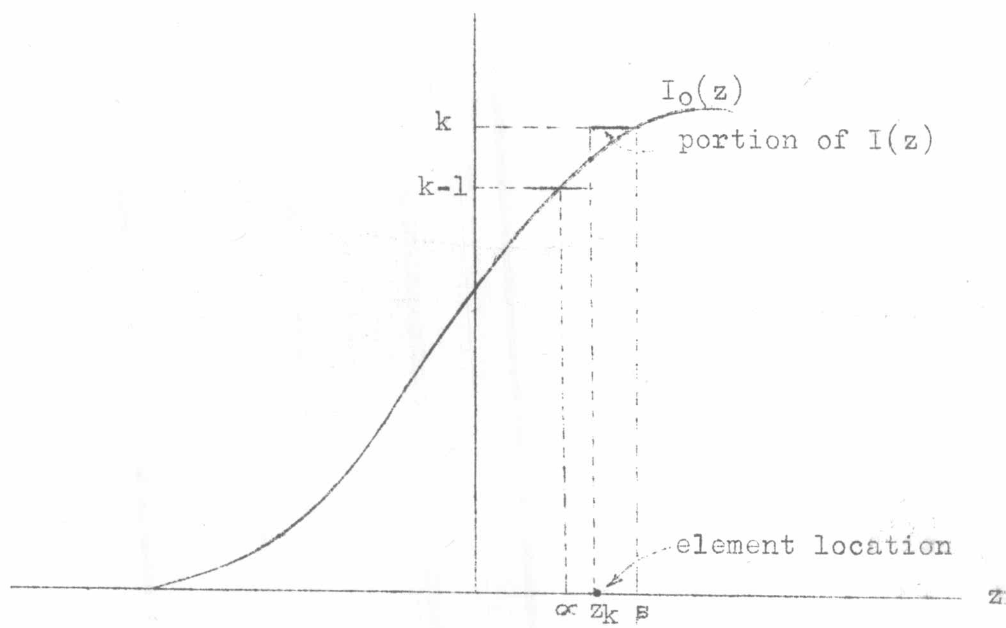

Fig.A2 Fitting of I(z) to $I_o(z)$ so as to minimize the mean-square difference Eq.(A-7)

Thus it is required to minimize

$$\int_{\alpha}^{\beta} \left[I_o(z) - I(z) \right]^2 dz = \int_{\alpha}^{z_k} \left[I_o(z) - (k-1) \right]^2 dz + \int_{z_k}^{\beta} \left[k - I_o(z) \right]^2 dz \qquad \text{(A-9)}$$

Differentiating with respect to z_k and solving gives

$$I_o(z_k) = k - \tfrac{1}{2} \qquad \text{(A-10)}$$

which states that each step of I(z) should be chosen that $I_o(z)$ crosses the middle of the step.

Then the element locations of a nonuniformly spaced array may be determined with the equal area approximation applied to the cumulative current distribution $I_o(z)$ of the model line source illumunation. The ordinate of $I_o(z)$ is divided into the number of equal increments which is equal to the total number of elements contained in the array, then projecting then projecting these points onto the z axis . The elements are then located within the center of each interval.

For example, the 8element nonuniformly spaced array having the pattern approximated to the line source which current distribution shown in Fig.A3 is found as illustrated in F g.A4 .

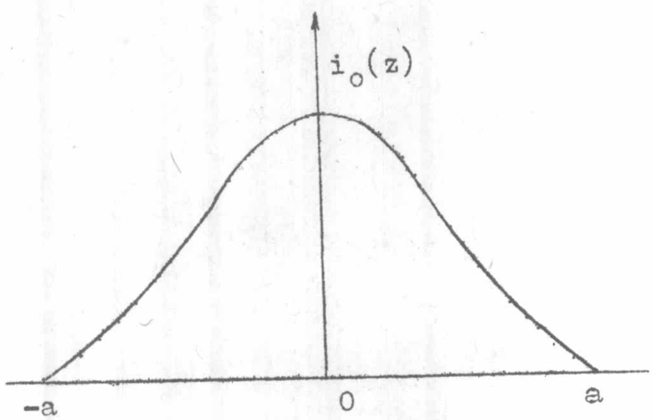

Fig. A-3

The model current distribution of the continuous line source.

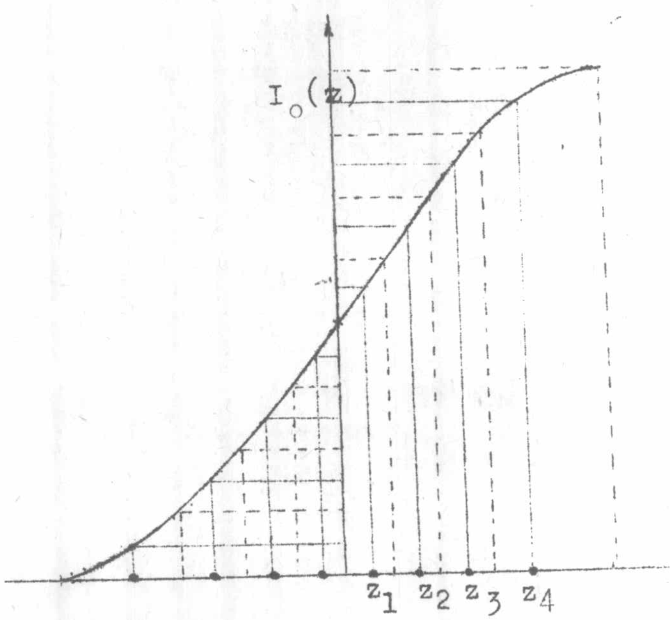

Fig. A-4

The cumulative current distribution and the location of 8 element nonuniformly spaced array.

Continuous Line Source with Taylor Distribution

A sidelobe level control technique similar to that for arrays has been developed for continuous current distribution by Taylor. Of considerable interest is the relationship between the half-power beamwidth vs. the sidelobe ratio of the ideal array factor (i.e, the equal sidelobe type of array factor) since this relationship represents the limits of the minimum beamwidth that can be obtained for a given sidelobe level.

Consider a radiation pattern of the form

$$f_o(\psi) = \cos\left(\sqrt{\psi^2 - c^2}\right) \qquad (A-11)$$

For the angular range $c^2 \leq \psi^2 < \infty$ this function oscillates between 1 and -1 ; however, when $= 0$

$$f_o(\psi = 0) = \cosh c$$

and the sidelobe level of a pattern with a main beam at $\psi = o$ is given by

$$R = \cosh c \qquad (A-12)$$

Using Eq.(A-12) the value of c can be determined for a given sidelobe level.

To determine the current distribution that produces the pattern of Eq.(A11) let us use the Woodward technique. Sampling the pattern function $f_o(\psi)$ at the intervals $\psi = n\pi$ yields the Fourier expansion of the current as

$$i_o(z) = \frac{1}{2a} \sum_{n=-\infty}^{\infty} \cos\sqrt{(n\pi)^2 - c^2}\; e^{-j\pi nz/a} \qquad (A-13)$$

If the pattern of Eq.(A13) is allowed to continue for large values of ψ, the coefficients of the current expansion for large n will tend to cos $n\pi$. These are the terms that provide impulsive behavior at the edges of the line source. Large values of current or aperture field at the edges of the antenna are difficult to produce and represent an unrealistic result from two standpoints. First, for many antennas it is desirable to have the edge current values low, not high; second, the reason that we are led to these peaks is to sustain the equal-lobe pattern for large values of ψ. In fact, this is rarely a feature. It would generally be preferred if the side lobes decreased in value for large ψ. Calculation of the pattern directivity shows that power expended in the far out lobes is at the expense of the main beam gain.

A simple method of eliminating the higher terms of the current distribution that contribute to the impulsive behavior is to terminate the series of Eq.(A13) after the N^{th} term. In effect, this requires that the pattern function of the terminated series must have zeros occuring at the sample points $\psi_n = n\pi$, when $n > N$. The pattern is thus forced to revert to $\sin\psi/\psi$ behavior for $\psi > \psi_N$. The choice of N is dependent upon the desired sidelobe level,

Some important characteristics of this continuous line source shown in Fig.A5is taken from Taylor's paper. When Taylor current distributions are used as a model in density tapering method of designing nonuniformly spaced array, their integrated current distributions must be determined. Fig.A6 gives the function $I_o(z)$ for various sidelobe levels, together with levels for various numbers of elements, from which their corresponding element locations are determined .

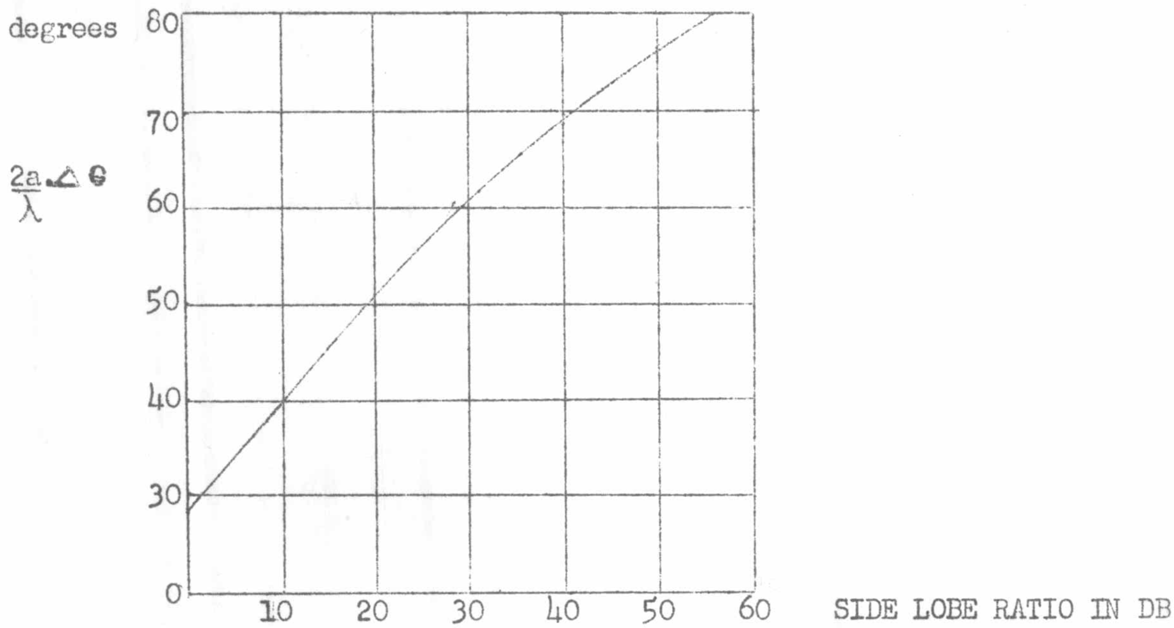

Fig. 5 Half power beamwidth-total length product vs. side lobe ratio for an ideal space factor defined by $f(\psi) = \cos(\psi^2 - c^2)^{\frac{1}{2}}$

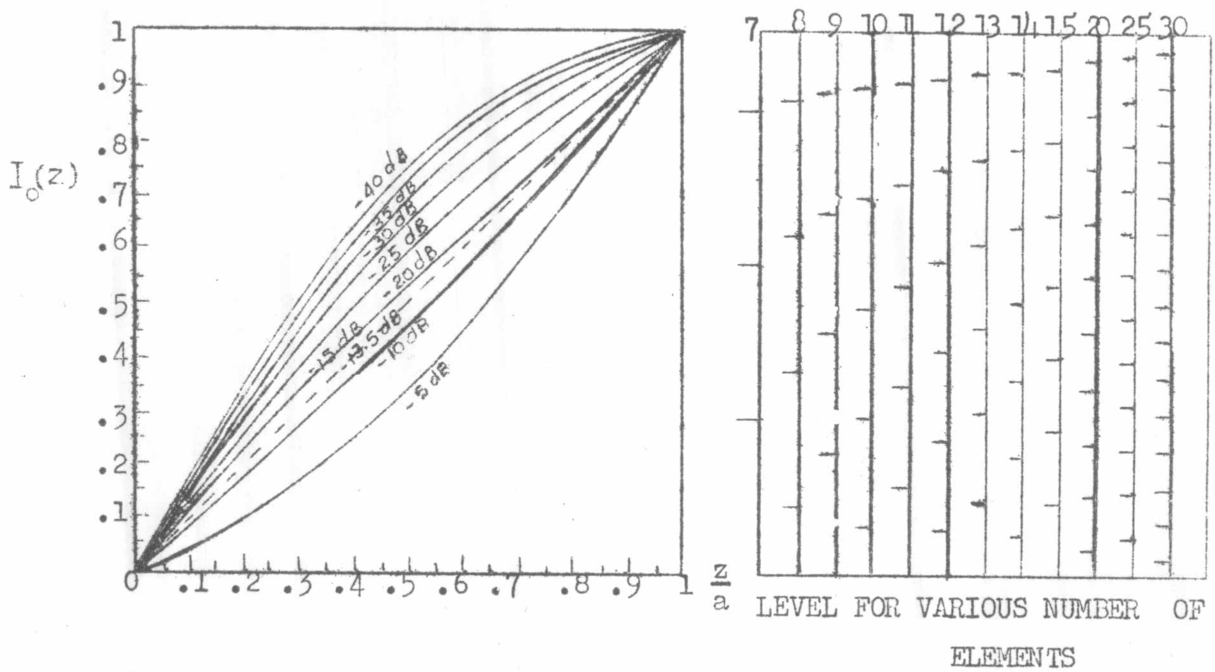

Fig. 6 Taylor cumulative current distributions $I_o(z)$

One Parameter Taylor Continuous Line Source

Another method of partial-pattern synthesis that is intended for side lobe control was developed by Taylor . This method, which is not based - on an error criterion, uses a modified $(\sin\psi)/\psi$ pattern as its basis and benefits from the high efficiency that is achieved by using patterns- resembling $(\sin\psi)/\psi$. A more suitable pattern recieved by this method would be one in which the side lobes decrease as the angle from the main beam. or the parameter increases. Basically, the approach is to represent the directivity pattern by a function of the form

$$f(\psi) = \frac{\sin\sqrt{\psi^2 - c^2}}{\sqrt{\psi - c^2}} \qquad \text{(A-14)}$$

where ψ = ka $\sin\theta$

c = constant

For large values of ψ, this pattern behaves as $(\sin\psi)/\psi$. For $\psi=0$ the pattern height is

$$f(0) = \frac{\sin jc}{jc} = \frac{\sinh c}{c} \qquad \text{(A-15)}$$

From Eq.(A-14)the first sidelobe occurs when

$$\sqrt{\psi^2 - c^2} = 4.603 \qquad \text{(A-16)}$$

and has a height of

$$f_1(\psi) = \frac{\sin 4.603}{4.603}$$

$$= \frac{\sin 26\,5}{4.603}$$

$$= \frac{1}{4.603} \qquad \text{(A-17)}$$

Thus the ratio of the main beam to the first side lobe is given by

$$R = 4.603 \frac{\sinh c}{c}$$

If R , the height of the main beam over the first side lobe,is specified the constant c can be determined. When c is equal to zero, the directivity pattern reduces to the familiar $\sin\psi/\psi$ for the case of a uniform - distribution and the first side lobe ratio is 4.603 or 13.2 db . As c is increased in value, the side lobe ratio increases, and by the appropriate choice of c a theoretical side lobe level as low as desired can be chosen

It will be noted that because of the expression for the directivity , at large value of ψ, the side lobe decrease as $1/\psi$. Hence we have a directivity pattern whose side lobes decrease in a similar to that of the uniform distribution but has the property that the maximum side lobe level can be arbitrarily chosen.

Fortunately, the aperture distribution function for this type of patt ern has a relatively simple form and is given by

$$i(z) = \frac{1}{2a} J_o\left[jc\sqrt{1 - (\tfrac{z}{a})^2}\right] \qquad |z| \leq a$$

or

$$i(z) = \frac{1}{2a} I_o(c\sqrt{1 - z^2}) \qquad |z| \leq 1$$

where J_o is the Bessel function of the first kind with an imaginary- argument.

I_o is the modified Bessel function of zero order.Tables of this function are readily available .

The value of this method is that it is a simple way of obtaining an aperture distribution that produces a main beam with relatively high efficiency and side lobes that do not exceed a specified value.

Some important characteristics of this type of current distribution are plotted in Fig. A8 and Fig. A7

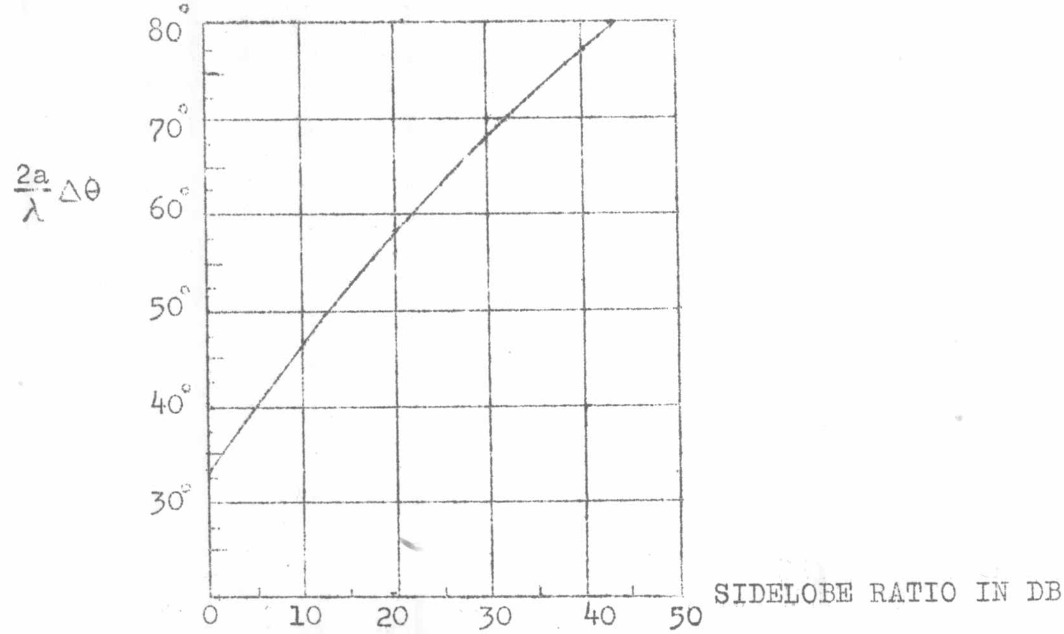

Fig. A7 Half-power beamwidth vs sidelobe ratio for space factor defined by $f(\psi) = \dfrac{\sin\sqrt{\psi^2 - c^2}}{\sqrt{\psi^2 - c^2}}$

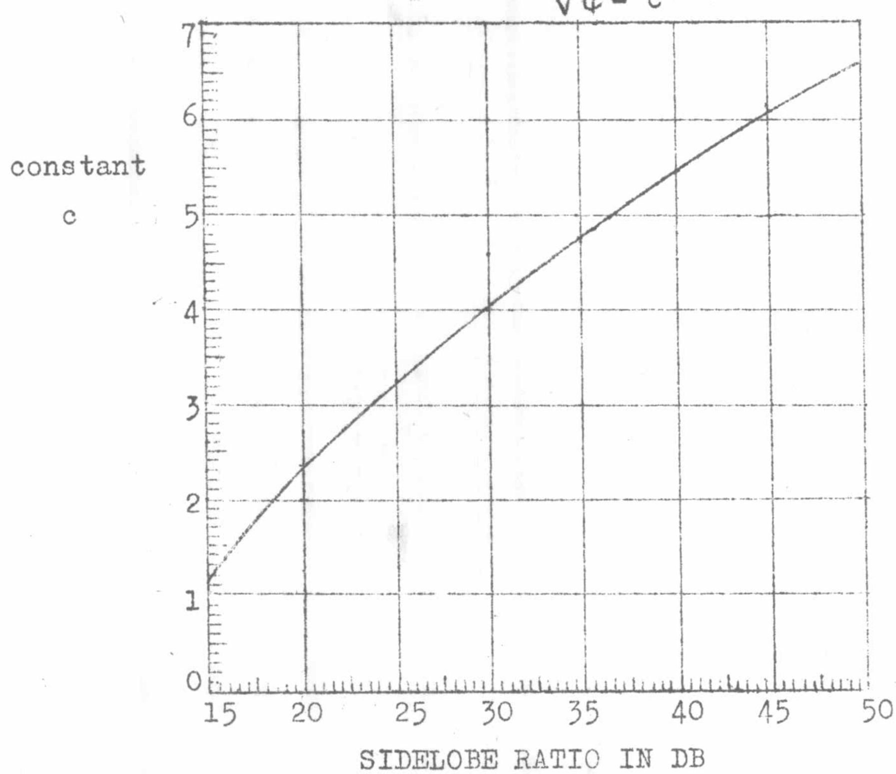

Fig. A8 The value of constant c VS side lobe ratio

Following the density tapering method, various nonuniformly spaced arrays were obtained from this line source current distribution .

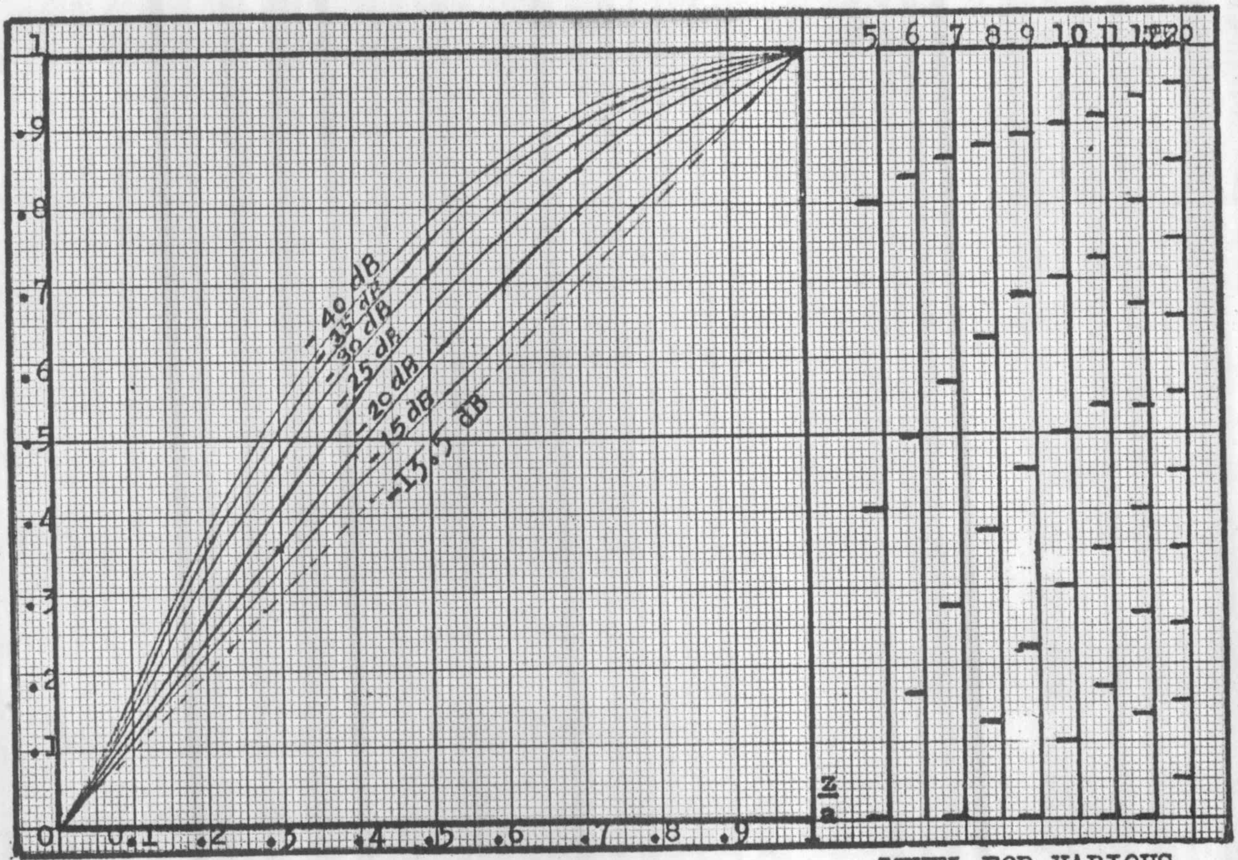

Fig. A9 The cumulative current distributions for spaced factor defined by $\dfrac{\sin\sqrt{\psi^2 - c^2}}{\sqrt{\psi^2 - c^2}}$

FigA9' gives the function $I_0(z)$ for various sidelobe levels, together with levels for various numbers of elements, from which their corresponding-element locations are determined.

The dotted line in the figure is for uniform arrays, and, thus, it corresponds -13.5 dB. Note that all curves for low sidelobe levels are above this line.

VITA

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