CHAPTER II

REVIEW OF LITERTURE

The analysis of the thinned array and the inaccurate positioning in equally spaced array lead into the subject of nonuniformly spaced arrays. A number of studies on the property of these arrays have been made, in which attempts were made to use the additional freedom of the element placement to improve the radiation pattern and to reduce the number of elements required.

Historically, the first significant work on nonuniformly spaced array was carried out by Unz. H[1], and was reported in his University of California (Berkeley, 1956) doctoral dessertation. Basically this paper dealt with the linear arrays with arbitrarily distributed element, and the prescribed array factor was developed into an infinite Fourier series which is then approximated by a finite set of the series but because of the need to invert the matrix equal to the number of elements contained in the array this method is difficult to use in practice.

Later, Harrington R.F[2], pointed out that the slight variations of the element positions from those of equally spaced arrays can reduce the sidelobe level. This is an important property that leads to a number of pattern synthesis by nonuniformly spaced array. Many techniques have been developed mostly for producing a radiation pattern with a specific main beamwidth and low sidelobe level. The orthogonal and eigen value-function method were also suggested by Unz. H,[3]. The broadband nonuniformly spaced arrays were studied by Bruce J.D[4], King D.D , R.F Pack and Y.S Chen[6].

A more general approach has been described by Ishimaru [7], who viewed the synthesis problem for nonuniformly spaced array as being the question of approximating to the continuous line source. An approximate directional pattern was obtained by relating a pattern function obtained from a continuous line source to a distribution function of the element spacings. This technique was also examined in the case of constant amplitude tapering by C.H Tang [8], and Doyle [9]. The latter has shown that the density-tapered pattern is equivalent to the least-mean-square approximation to the model continuous source (amplitude-tapered)pattern with weighting proportional to the inverse square of the normalized pattern argument.

Further development of the pattern synthesis based on the approximation theory has been described by C.H Tang [10], who selected the - Chebyshev pattern as a model pattern to be realized by obtaining first an unequally spaced array consisting of isotropic antenna elements of equal amplitudes and then getting the excitation condition for maximum gain by keeping the positions of the antenna elements unchanged.

When the number of elements are large the statistical method of designing nonuniformly spaced array was introduced by Skolnik, Sherman and Ogg [11], . In this approach, called statistical density taper, the model amplitude taper illumination function is employed to determined , on a probabilistic basis, whether or not an element should be located at a particular point within the array. The model illumination function serves as a role analogous to that of the probability density function of probability theory.

In nonuniformly spaced array there should be no restriction on the element spacings. In practice, however, array elements can not be located much closer. Closer spacing results in increased mutual coupling which changes the array illumination. This case of study has been considered by Bruce J.D and Unz H,[12] and give the limitation to the minimum spacing between adjacent element.

Recently the application of the digital computer to the nonuniformly spaced array pattern synthesis was suggested. The technique of dynamic programming was introduced by Skolnik , Nemhauser, and Sherman,[13]. This is a method for determining an optimum solution of many possible configurations of element locations. The critereon for selecting the optimum radiation pattern must be carefully formulated and programmed into the computer. One critereon that has been used with dynamic programming is that of minimizing the maximum sidelobe level over a specified region - of angle. Dynamic programming applied to design of linear arrays of unequally spaced elements, has achieved results equal or more superior to results with array designed by other methods and is widely used today.

A review of the development of the synthesis theory for nonuniformly spaced array has been given by Merrill I.Skomik[14] and Ishimaru[15].

Since a large number of contributions to this field have been made only an important selected few of many different approaches that relate to the basis of the thesis were stated here.

Analysis of Nonuniformly Spaced Arrays

For a nonuniformly spaced array of N identical elements locating along the Z axis as shown in Fig.1, the far field can be written as

$$E(\theta,\phi) = f(\theta,\phi)\sum_{n=0}^{N-1} i_n \cdot \frac{e^{-jkr_n}}{r_n} \qquad (1)$$

where $f(\theta,\phi)$ the radiation pattern of each element

i_n n^{th} element current

r_n distance from the n^{th} element to the far field point

Also let $r_n = r - z_n \sin\theta$ (2)

where z_n the distance of the n^{th} element from the center

r distance from the center to the far field point and is much greater than z_n ($r \gg z_n$).

Combining Eq.(1) and Eq.(2) gives

$$E(\theta,\phi) = f(\theta,\phi) \cdot \frac{e^{-jkr}}{r}\sum_{n=0}^{N-1} i_n \cdot e^{jkz_n \sin\theta} \qquad (3)$$

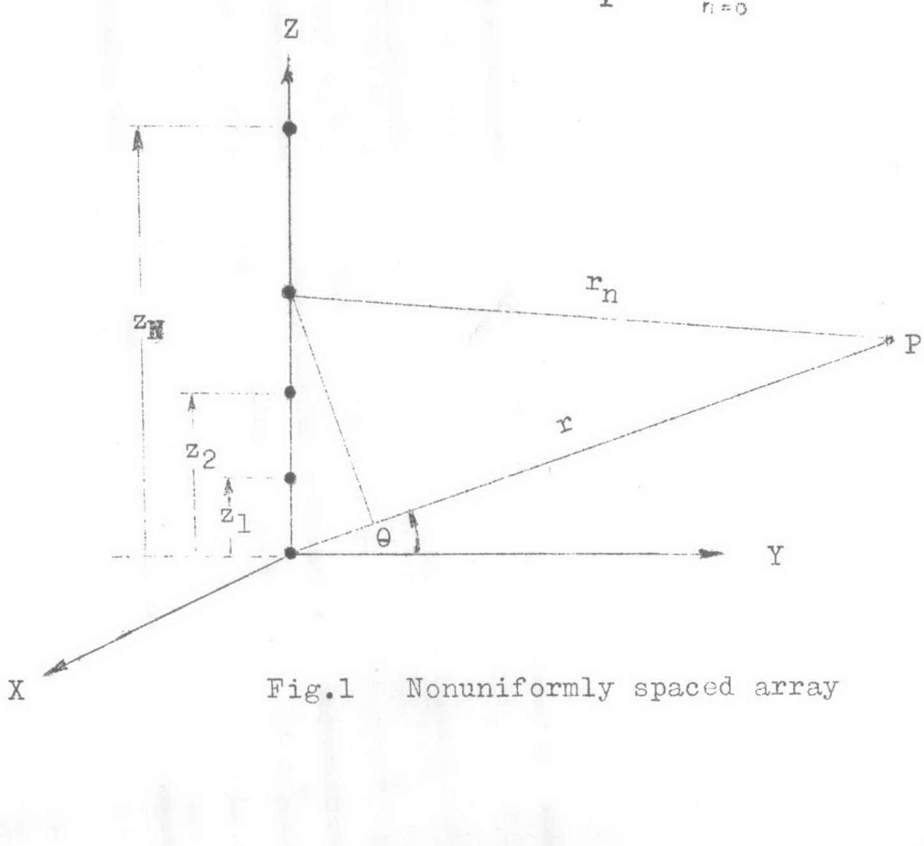

Fig.1 Nonuniformly spaced array

The associated polynomial in Eq.(3), sometimes called the array factor

$$f(\theta) = \sum_{n=o}^{N-1} i_n . e^{jkz_n \sin\theta} \qquad (4)$$

is the far field pattern of unequally spaced array of isotropic elements. Since the pattern has rotational symmetry about the z axis, the angle variable is chosen for convenience as $\sin\theta$ instead of $\sin\theta \cos\phi$. This is equivalent to looking at the pattern in the xz or $\phi = 0$ plane which is sufficient in view of the rotational symmetry.

The exponentials of Eq.(4) have the series expansions

$$e^{jkz_n \sin\theta} = \sum_{m=-\infty}^{\infty} e^{jm\theta} . J_m(kz_n) \qquad (5)$$

Combining Eq.(4) and Eq.(5) gives

$$f(\theta) = \sum_{n=o}^{N-1} i_n \sum_{m=-\infty}^{\infty} e^{jm\theta} . J_m(kz_n) \qquad (6)$$

Since $f(\theta)$ is periodic in θ, it can be expanded in the Fourier series

$$f(\theta) = \sum_{m=-\infty}^{\infty} g_m e^{jm\theta} \qquad (7)$$

where $$g_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta)\, e^{-jm\theta}\, d\theta \qquad (8)$$

Then from Eq.(6) and (7)

$$g_m = \sum_{n=o}^{N-1} i_n . J_m(kz_n) \qquad (9)$$

Eq.(9) gives the coefficients g_m of the Fourier series expansions of the far field pattern of Eq.(7) in terms of the element currents i_n and the distribution of the elements along the z axis. By using matrix notation Eq(9) is

$$\begin{bmatrix} g_o \\ g_1 \\ . \\ . \\ g_m \end{bmatrix} = \begin{bmatrix} J_o(kz_o) & J_o(kz_1) & \ldots & \ldots & J_o(kz_{n-1}) \\ J_1(kz_o) & J_1(kz_1) & \ldots & \ldots & J_1(kz_{n-1}) \\ & & & & \\ J_m(kz_o) & J_m(kz_1) & \ldots & \ldots & J_m(kz_{n-1}) \end{bmatrix} . \begin{bmatrix} i_o \\ i_1 \\ . \\ . \\ i_{N-1} \end{bmatrix}$$

The power pattern of the array is designated as $S(\theta)$, thus

$$S(\theta) = f(\theta).f^*(\theta) \tag{10}$$

where $f^*(\theta)$ is the complex conjugate function of $f(\theta)$.

Then from Eq.(4)

$$S(\theta) = \sum_{n=o}^{N-1} i_n . e^{jkz_n \sin\theta} \cdot \sum_{m=o}^{N-1} i_m^* . e^{-jkz_m \sin\theta} \qquad 1?$$

By letting z_{nm} equal to $z_n - z_m$, thus

$$S(\theta) = \sum_{n=o}^{N-1} \sum_{m=o}^{N-1} i_n i_m^* \cdot e^{jkz_{nm} \sin\theta} \tag{11}$$

The total radiated power is proportional to

$$W = \int_0^{2\pi}\int_0^{\pi} S(\alpha) \sin\alpha \, d\alpha . d\phi$$

where $\sin\alpha \, d\alpha . d\phi$ is the element of the solid angle.

Then

$$W = 2\pi \int_0^{\pi} S(\alpha) \sin\alpha \, d\alpha \tag{12}$$

From Eq.(11)

$$S(\alpha) = \sum_{n=o}^{N-1} \sum_{m=o}^{N-1} i_n i_m^* . e^{jkz_{nm} \cos\alpha} . \tag{13}$$

Combining Eq.(12) and Eq.(13) gives

$$W = 2\pi \sum_{n=o}^{N-1} \sum_{m=o}^{N-1} i_n . i_m^* \int_0^{\pi} e^{jkz_{nm} \cos\alpha} . \sin\alpha d\alpha \tag{14}$$

The integral can be written as

$$\int_0^{\pi} e^{jkz_{nm}\cos\alpha} \cdot \sin\alpha \, d\alpha = \int_1^{-1} -e^{jkz_{nm}\cos\alpha} \, d(\cos\alpha)$$

$$= \left[\frac{-e^{jkz_{nm}\cos\alpha}}{jkz_{nm}} \right]_1^{-1}$$

$$= \frac{e^{jkz_{nm}} - e^{-jkz_{nm}}}{jkz_{nm}}$$

$$= \frac{2\sin kz_{nm}}{kz_{nm}}$$

so that Eq.(14) is

$$W = 4\pi \sum_{n=o}^{N-1} \sum_{m=o}^{N-1} \frac{i_n i_m^* . \sin kz_{nm}}{kz_{nm}} \qquad (15)$$

The directivity of the array can be found from

$$D = \frac{S(\theta)_{max}}{S(\theta)_{average}}$$

$$S(\theta)_{max} = S(\theta)_{\theta=90^\circ} = \sum_{n=o}^{N-1} \sum_{m=o}^{N-1} i_n . i_m^*$$

$$S(\theta)_{average} = \frac{W}{4\pi} = \sum_{n=o} \sum_{m=o} i_n . i_m^* . \frac{\sin kz_{nm}}{kz_{nm}}$$

so that the directivity is

$$D = \frac{\sum_{n=o}^{N-1} \sum_{m=o}^{N-1} i_n . i_m^*}{\sum_{n=o}^{N-1} \sum_{m=o}^{N-1} i_n . i_m^* . \frac{\sin kz_{nm}}{kz_{nm}}} \qquad (16)$$

Now consider the special case where the current at each elements are equal. The polynomial of Eq.(4) may be written as

$$f(\theta) = \sum_{n=o}^{N-1} e^{jkz_n \sin\theta} \qquad (17)$$

By letting $u = \sin\theta$ and the spacing z_n is expressed in wavelengths , Eq.(17) becomes

$$f(u) = \sum_{n=o}^{N-1} e^{j2\pi z_n u} \qquad (18)$$

or

$$f(u) = \sum_{n=o}^{N-1} (\cos 2\pi z_n u + j\sin 2\pi z_n u)$$

Thus

$$|f(u)| = \left[\left(\sum_{n=o}^{N-1} \cos 2\pi z_n u \right)^2 + \left(\sum_{n=o}^{N-1} \sin 2\pi z_n u \right)^2 \right]^{\frac{1}{2}} \qquad (19)$$

In general, this expression , called the array factor is difficult to handle analytically. For convenience , it is usually assumed that elements are arranged symmetrically in pairs about the center and that the center - element is the reference from which the phase is measured (Fig.2) which these assumptions the array pattern of Eq.(18) becomes

$$f(u) = 1 + 2 \sum_{n=1}^{N} \cos 2\pi z_n u \qquad (20)$$

where the total number of elements is designated as $2N + 1$.

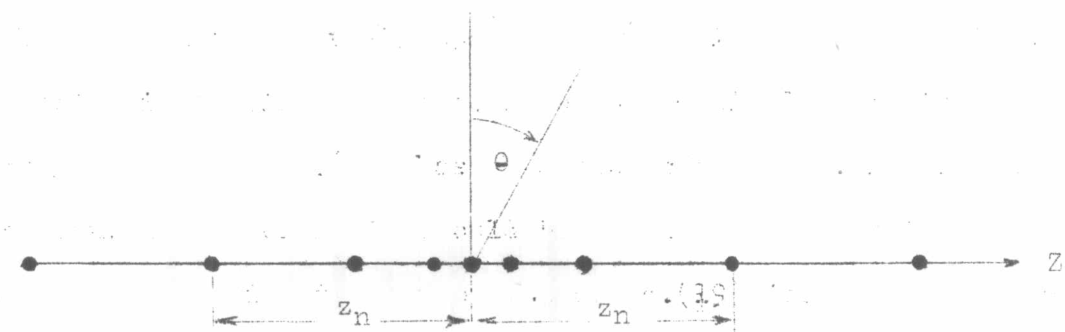

Fig.2 Symmetrical arrangement of nonuniformly spaced elements.

The problem in unequally spaced array design is to select the N values of element pair spacing z_n to achive some desired radiation patterns . The array pattern of Eq.(14) has N degree of freedom with which to specify the pattern. In principle it should be possible to approximate a desired radiation pattern with the expression of Eq.(19) just as with a conventional , equally spaced array of 2N + 1 elements occupying the same aperture . The pattern of equally spaced array is

$$f_{eq}(u) = 1 + 2\sum_{n=1}^{N} i_n.\cos 2\pi du \qquad (21)$$

where d is the spacing between adjacent elements. In the equally spaced array, it is the N values of the current i_n at the element pairs that are to be determined. With the unequally spaced array, it is the N values of z_n .