CHAPTER 3

SIMULATION STUDIES OF THE DIFFUSION MODEL

FOR THE LINEAR CASE

3.1 Study of the interrelationship between R_x and P_x, P_y, r_x, r_y

The experimental determination of concentration profiles from experiment is complicated. But some points in the concentration profile are known such as the feed concentrations and the outlet concentrations. The inlet-end interior concentration at the entrances of both phases (where there is a jump) can be found experimentally. The approach of this section is to use these two interior points and the other exterior points to find R_x instead of relying on the entire concentration profile. The parameters P_x and P_y are known to be ill-defined using the concentration profile approach and in this section will be assumed known for the R_x identification.

In this section the equation will be set in such a way as to show the interrelationship between R_x and P_x, P_y, r_x and r_y

For both phases, the jump ratio (r) is the difference between the feed concentration and the inlet-end interior concentration, divided by the difference between feed and outlet concentrations [8] . See figure 3-1, in this case :

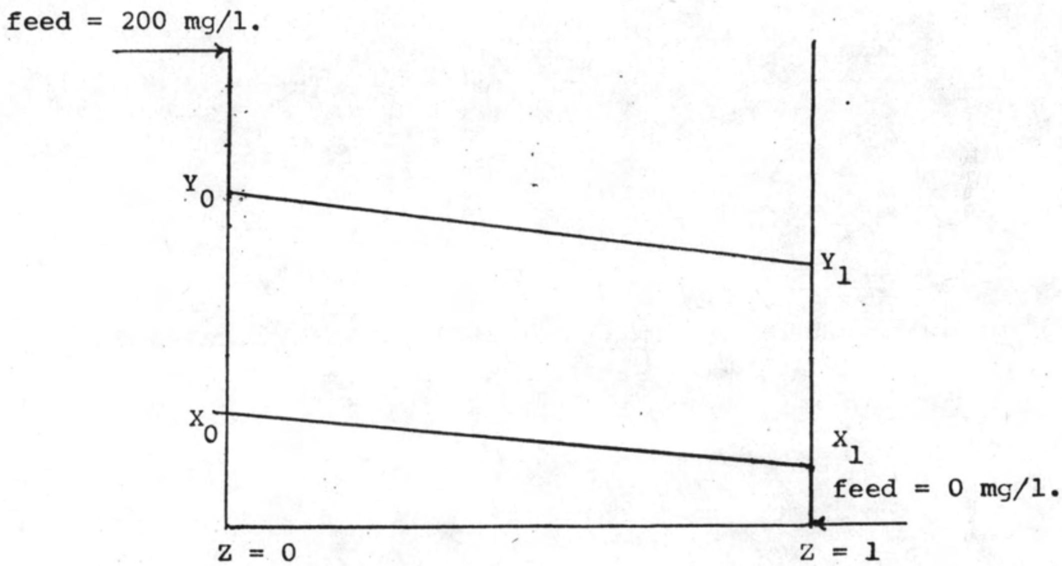


Figure 3-1

Concentration profiles for jump ratio

	in X phase (mg/l)	in Y phase (mg/l)
The feed concentration	0	200
The inlet-end interior concentration	X_1	Y_0
The outlet concentration	X_0	Y_1

So, in X phase $\quad r_x = X_1/X_0$

in Y phase $\quad r_y = (200-Y_0)/(200-Y_1)$

Now, we set the equation :

$$R_x = A^a P_x^b P_y^c \qquad (3\text{-}1)$$

taking the logarithm to this equation, we obtain.

$$\log R_x = a \log A + b \log P_x + c \log P_y \qquad (3\text{-}2)$$

From the output data at $P_x = P_y = 5$

when $A = r_x$ see table 3-1 and figure 3-2

$A = r_y$ see table 3-2 and figure 3-3

$A = r_x r_y$ see table 3-3 and figure 3-4

$A = r_y/r_x$ see table 3-4 and figure 3-5

$A = r_y + r_x$ see table 3-5 and figure 3-6

$A = r_y - r_x$ see table 3-6 and figure 3-7

$A = r_y^2 - r_x^2$ see table 3-7 and figure 3-8

$A = (r_y + r_x)/(r_y - r_x)$ see table 3-8 and figure 3-9

From equation (3-2), when the following plot is made $\log R_x$ V.S. $\log A$, for the original assumption to hold the plot must be a straight line with slope a and intercept $\log R_x$ equal to $b \log P_x + c \log P_y$.

TABLE 3-1

R_x	log R_x	X_0	X_1	$r_x = X_1/X_0$	log r_x
1	0	194.51660	38.56710	0.1982715	-0.702740
2	0.30103	381.10400	75.26428	0.1974901	-0.704455
3	0.47712	559.17800	110.10950	0.1969131	-0.705725
4	0.60206	729.54740	143.16570	0.1962390	-0.707215
5	0.69897	892.59910	174.63970	0.1956530	-0.708513
8	0.90309	1342.42969	260.29370	0.1938974	-0.712428
20	1.30103	2700.39746	505.97827	0.1873717	-0.727296
40	1.60206	4062.53687	722.59229	0.1778672	-0.749904

TABLE 3.2

R_x	$\log R_x$	Y_0	$200-Y_0$	Y_1	$200-Y_1$	$r_y = \frac{200-Y_0}{200-Y_1}$	$\log r_y$
1	0	199.02590	0.97410	195.12130	4.87870	0.1996638	-0.699701
2	0.30103	198.08850	1.91150	190.45800	9.54200	0.2003248	-0.698265
3	0.47712	197.18540	2.81460	186.00640	13.99360	0.2011348	-0.696513
4	0.60206	196.31450	3.68550	181.74660	18.25340	0.2019075	-0.694847
5	0.69897	195.47390	4.52610	177.67020	22.32980	0.2026932	-0.693161
8	0.90309	193.11647	6.88353	166.42804	33.57196	0.2050381	-0.688165
20	1.30103	185.54515	14.45485	132.48058	67.51942	0.2140843	-0.669415
40	1.60206	176.85167	23.14833	98.42735	101.57265	0.2278992	-0.642257

TABLE 3-3

R_x	log R_x	r^*_x	r^{**}_y	$r_x r_y$	$\log(r_x r_y)$
1	0	0.1982715	0.1996638	0.0395876	-1.40244
2	0.30103	0.1974901	0.2003248	0.0395621	-1.40272
3	0.47712	0.1969131	0.2011348	0.0396060	-1.40224
4	0.60206	0.1962390	0.2019075	0.0396221	-1.40206
5	0.69897	0.1956530	0.2026932	0.0396575	-1.40167
8	0.90309	0.1938974	0.2050381	0.0397563	-1.40059
20	1.30103	0.1873717	0.2140843	0.0401133	-1.39671
40	1.60206	0.1778672	0.2278992	0.0405357	-1.39216

* from table 3-1

** from table 3-2

TABLE 3-4

R_x	log R_x	r^*_y	r^{**}_y	r_y/r_x	log r_y/r_x
1	0	0.1982715	0.1996638	1.0070221	3.03904×10^{-3}
2	0.30103	0.1974901	0.2003248	1.0143536	6.18939×10^{-3}
3	0.47712	0.1969131	0.2011348	1.0214394	9.21261×10^{-3}
4	0.60206	0.1962390	0.2019075	1.0288856	0.0123671
5	0.69897	0.1956530	0.2026932	1.0359830	0.0153527
8	0.90309	0.1938974	0.2050381	1.0574566	0.0242626
20	1.30103	0.1873717	0.2140843	1.1425647	0.0578808
40	1.60206	0.1778672	0.2278992	1.2812885	0.1076470

* from table 3-1

** from table 3-2

TABLE 3-5

R_x	$\log R_x$	r^*_x	r^{**}_y	r_y+r_x	$\log(r_y+r_x)$
1	0	0.1982715	0.1996638	0.3979353	-0.400188
2	0.30103	0.1974901	0.2003248	0.3978149	-0.400319
3	0.47712	0.1969131	0.2011348	0.3980479	-0.400065
4	0.60206	0.1962390	0.2019075	0.3981465	-0.399957
5	0.69897	0.1956530	0.2026932	0.3983462	-0.399739
8	0.90309	0.1938974	0.2050381	0.3989355	-0.399097
20	1.30103	0.1873717	0.2140843	0.4014560	-0.396362
40	1.60206	0.1778672	0.2278992	0.4057664	-0.391724

* from table 3-1

** from table 3-2

TABLE 3-6

R_x	$\log R_x$	r_x^*	r_y^{**}	$r_y - r_x$	$\log(r_y - r_x)$
1	0	0.1982715	0.1996638	1.3923×10^{-3}	-2.85627
2	0.30103	0.1974901	0.2003248	2.8347×10^{-3}	-2.54748
3	0.47712	0.1969131	0.2011348	4.2217×10^{-3}	-2.37451
4	0.60203	0.1962390	0.2019075	5.6685×10^{-3}	-2.24653
5	0.69897	0.1956530	0.2026932	7.0402×10^{-3}	-2.15242
8	0.90309	0.1938974	0.2050381	0.0111407	-1.95309
20	1.30103	0.1873717	0.2140843	0.0267126	-1.57328
40	1.60206	0.1778672	0.2278992	0.050032	-1.30075

* from table 3-1

** from table 3-2

TABLE 3-7

R_x	$\log R_x$	$(r_y+r_x)^*$	$(r_y-r_x)^{**}$	$(r_y^2-r_x^2)$	$\log(r_y^2-r_x^2)$
1	0	0.3979353	1.3923×10^{-3}	5.54045×10^{-4}	-3.25645
2	0.30103	0.3978149	2.8347×10^{-3}	1.12768×10^{-3}	-2.94781
3	0.47712	0.3980479	4.2217×10^{-3}	1.68043×10^{-3}	-2.77458
4	0.60206	0.3981465	5.6685×10^{-3}	2.25689×10^{-3}	-2.64649
5	0.69897	0.3983462	7.0402×10^{-3}	2.80443×10^{-3}	-2.55215
8	0.90309	0.3989355	0.0111407	4.44442×10^{-3}	-2.35218
20	1.30103	0.4014560	0.0267126	0.0107239	-1.96965
40	1.60206	0.4057664	0.050032	0.0203013	-1.69248

* from table 3-5

** from table 3-6

TABLE 3-8

R_x	$\log R_x$	(r_y+r_x)*	(r_y-r_x)**	$\frac{(r_y+r_x)}{(r_y-r_x)}$	$\log \frac{(r_y+r_x)}{(r_y-r_x)}$
1	0	0.3979353	1.3923×10^{-3}	285.81146	2.45608
2	0.30103	0.3978149	2.8347×10^{-3}	140.33756	2.14717
3	0.47712	0.3980479	4.2217×10^{-3}	94.286164	1.97445
4	0.60206	0.3981465	5.6685×10^{-3}	70.238422	1.84657
5	0.69897	0.3983462	7.0402×10^{-3}	56.581659	1.75268
8	0.90309	0.3989355	0.0111407	35.808836	1.55399
20	1.30103	0.4014560	0.0267126	15.028713	1.17692
40	1.60206	0.4057664	0.050032	8.110137	0.909028

* from table 3-5

** from table 3-6

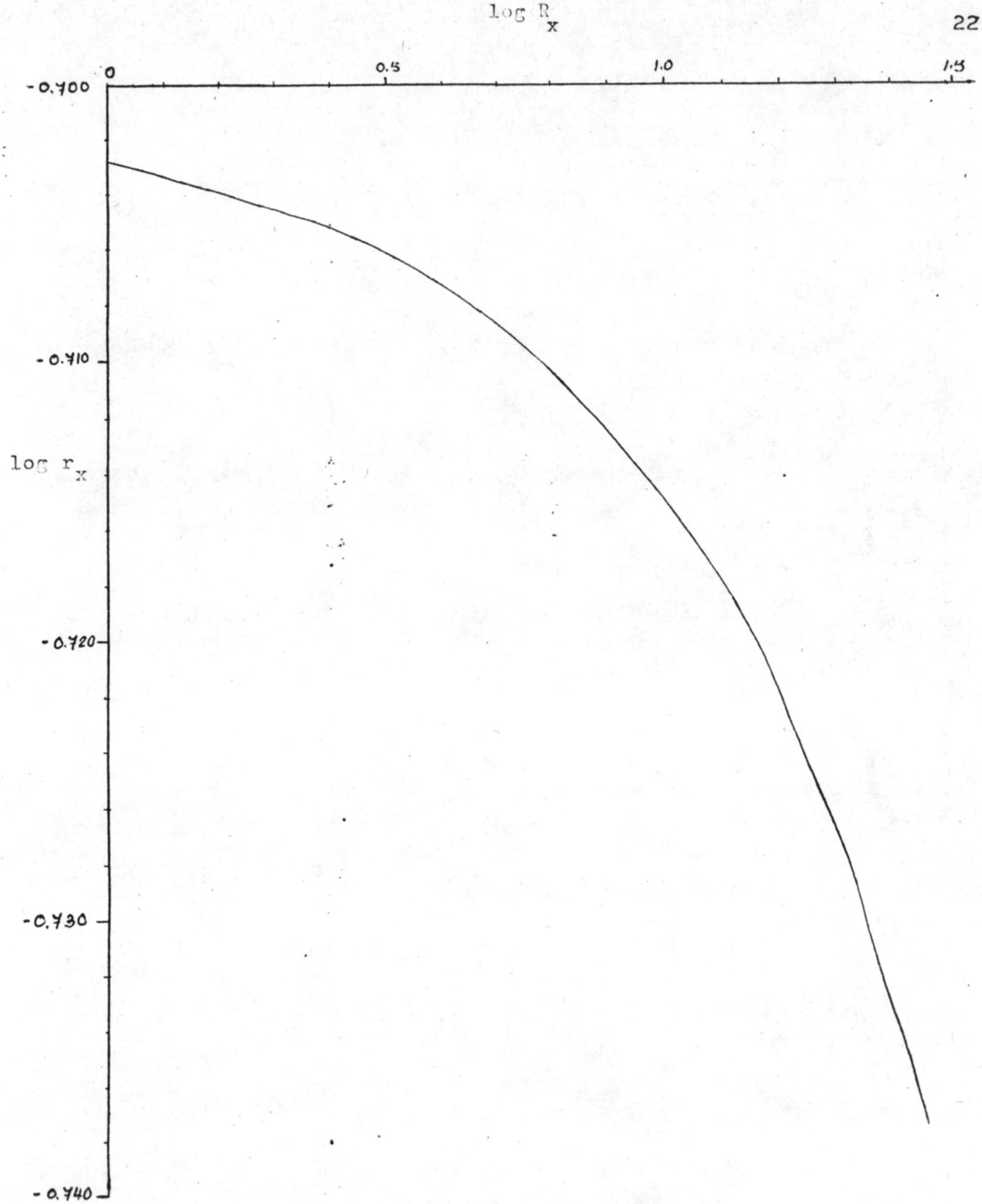


Figure 3-2 The relation of log R_x with log r_x when $P_x = P_y = 5$

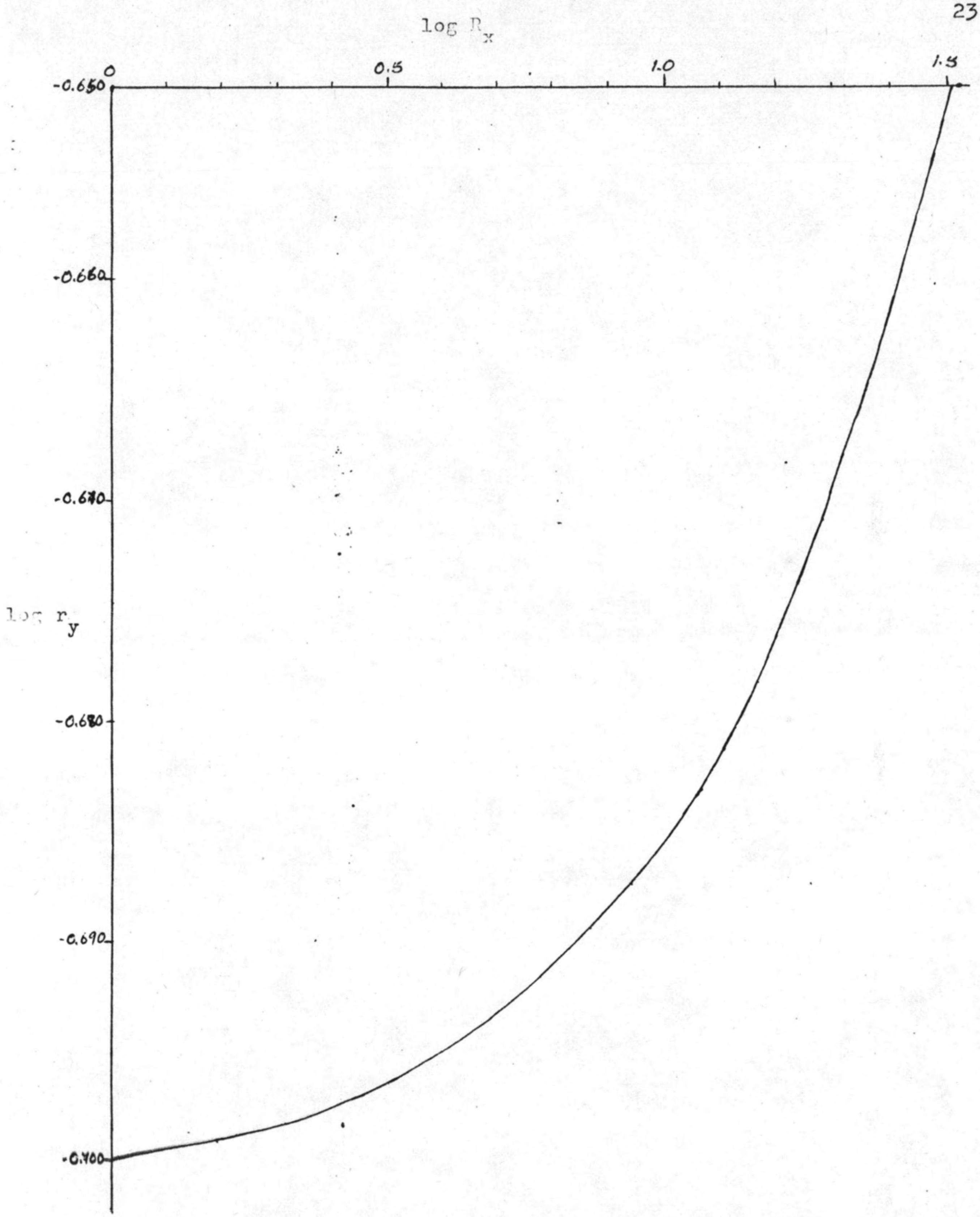


Figure 3-3 The relation of log R_x with log r_y when $P_x = P_y = 5$

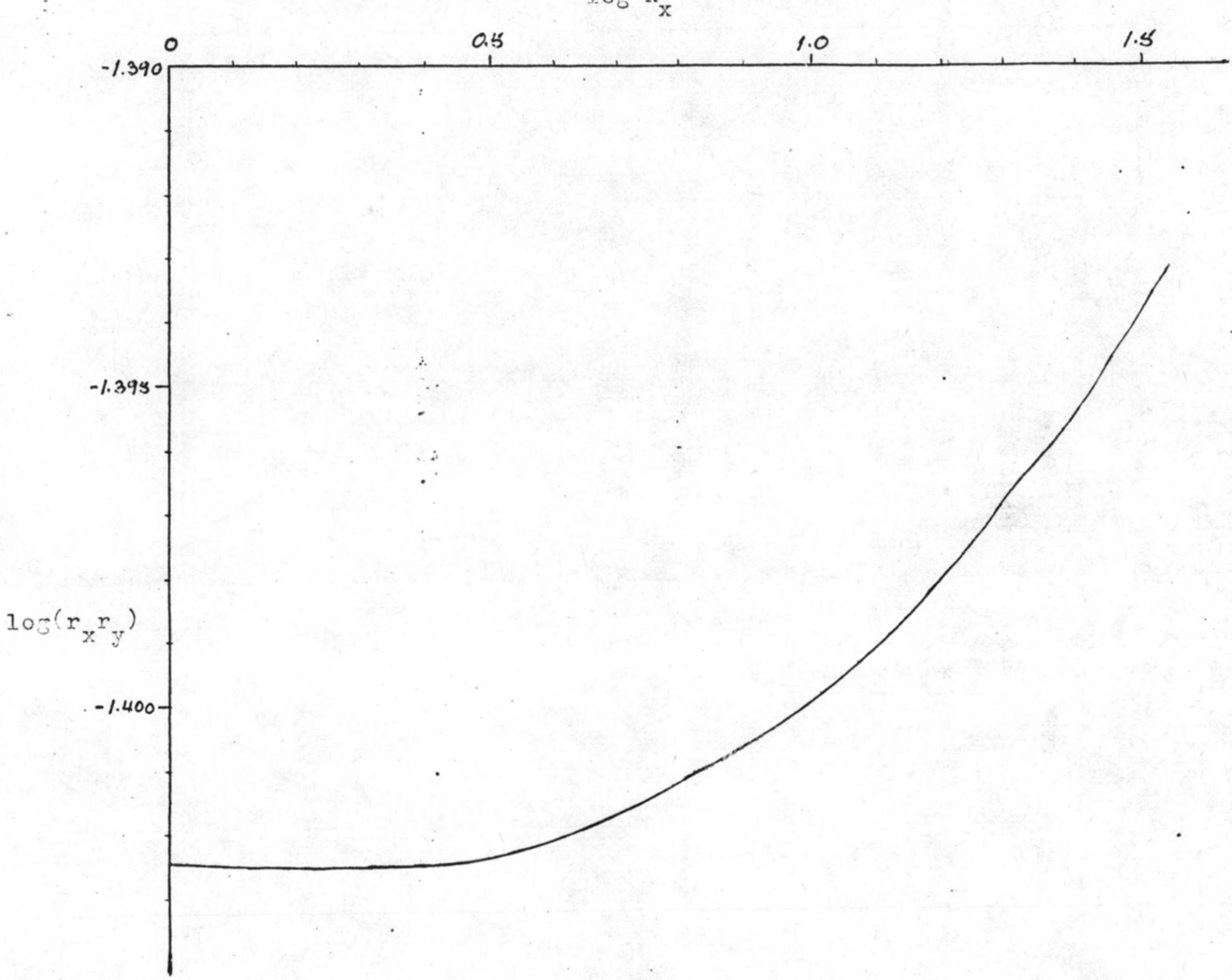


Figure 3-4 The relation of log R_x with log $(r_x r_y)$ when $P_x = P_y = .5$

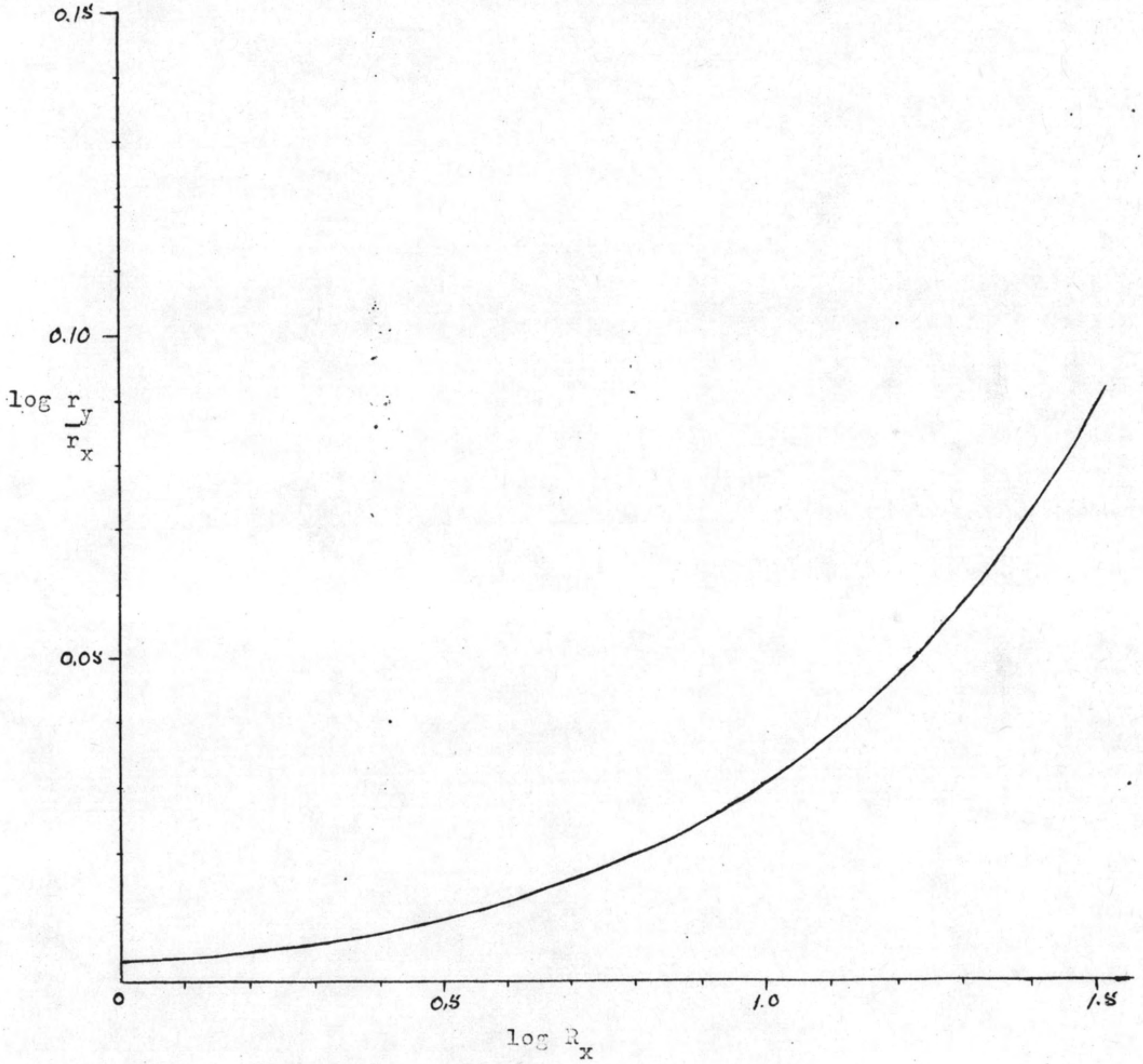


Figure 3-5 The relation of log R_x with log r_y/r_x

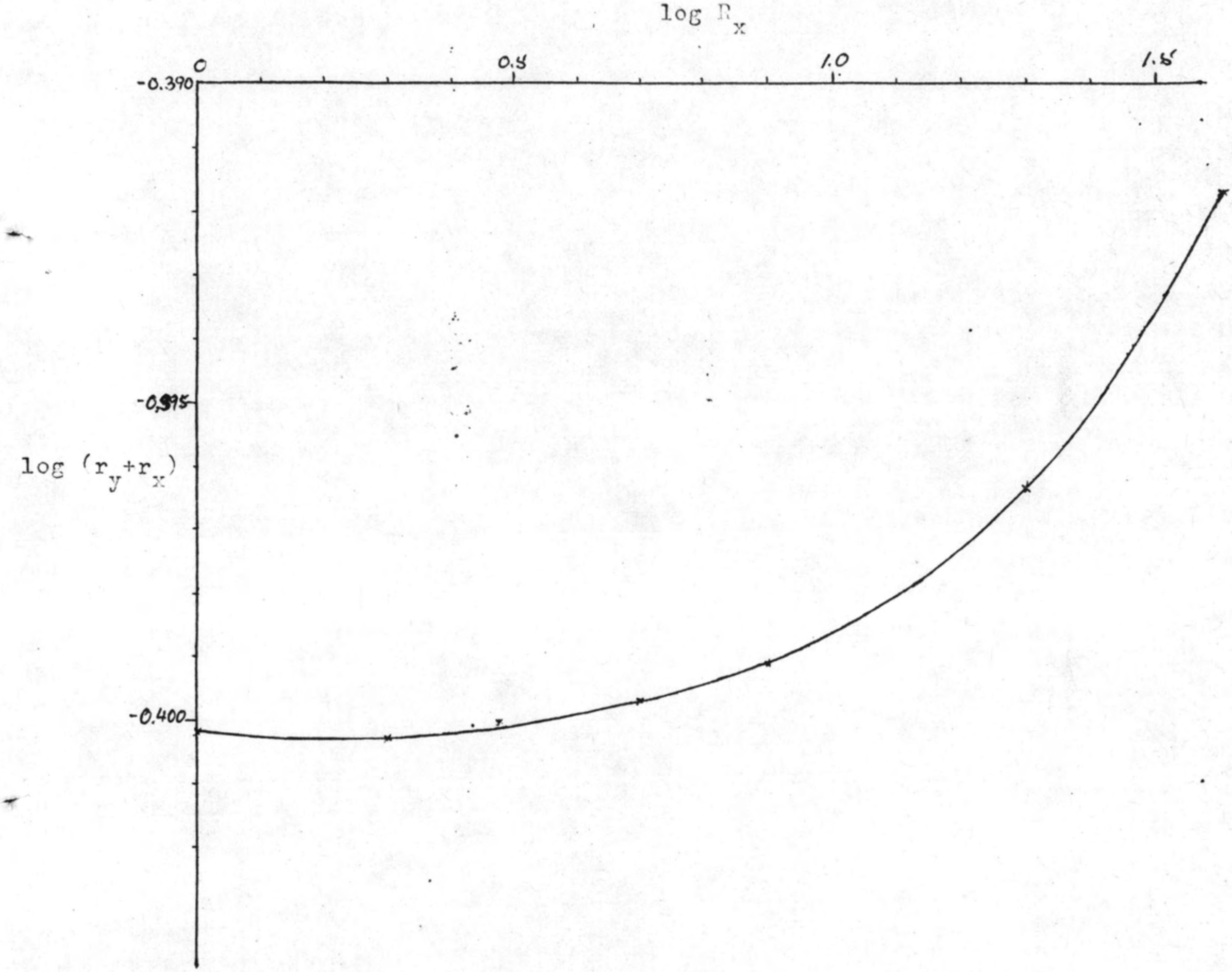


Figure 3-6 The relation of log R_x with log (r_y+r_x) when $P_x=P_y=5$

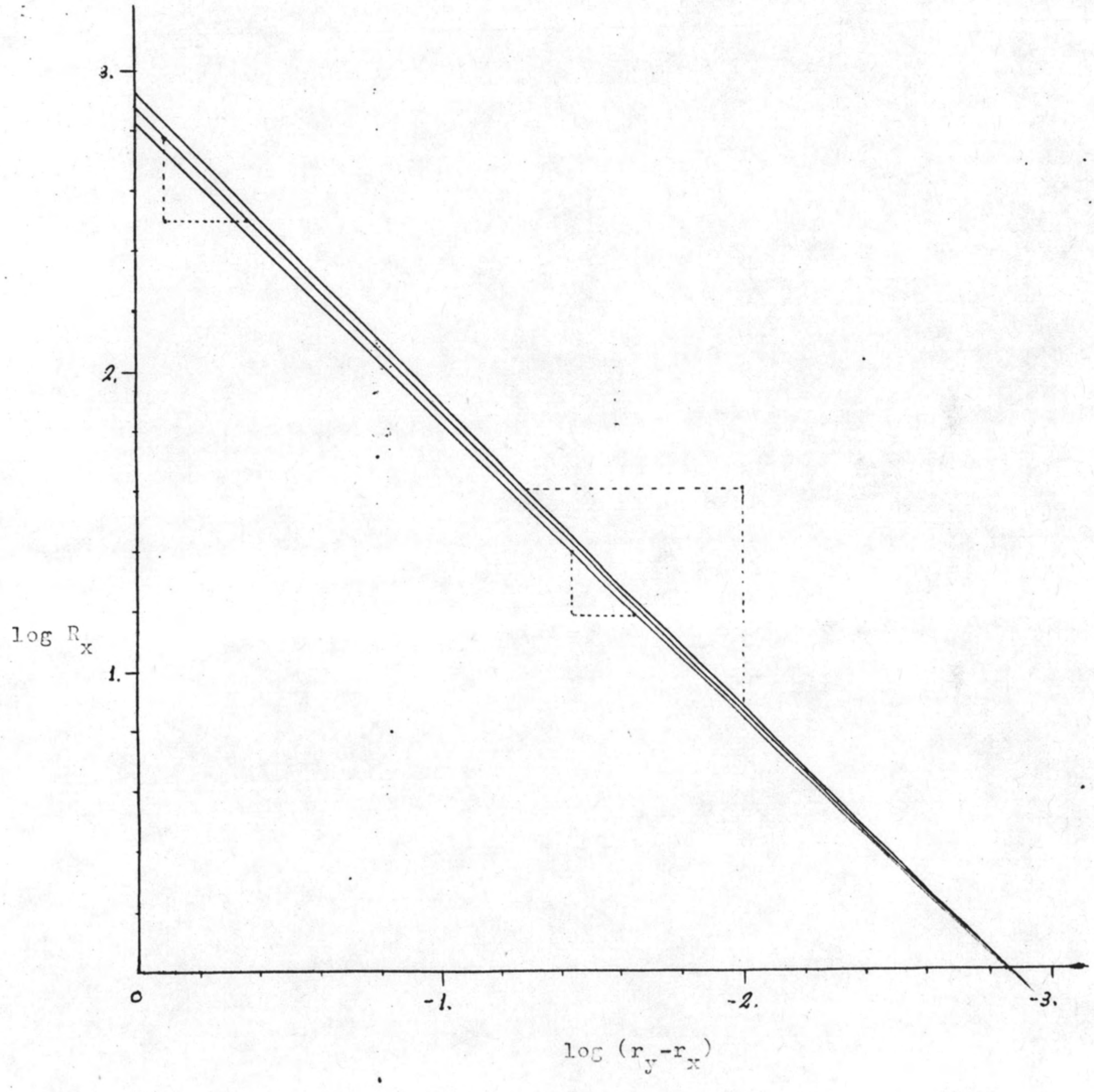


Figure 3-7 The relation of log R_x with log (r_y-r_x) when $P_x=P_y=5$

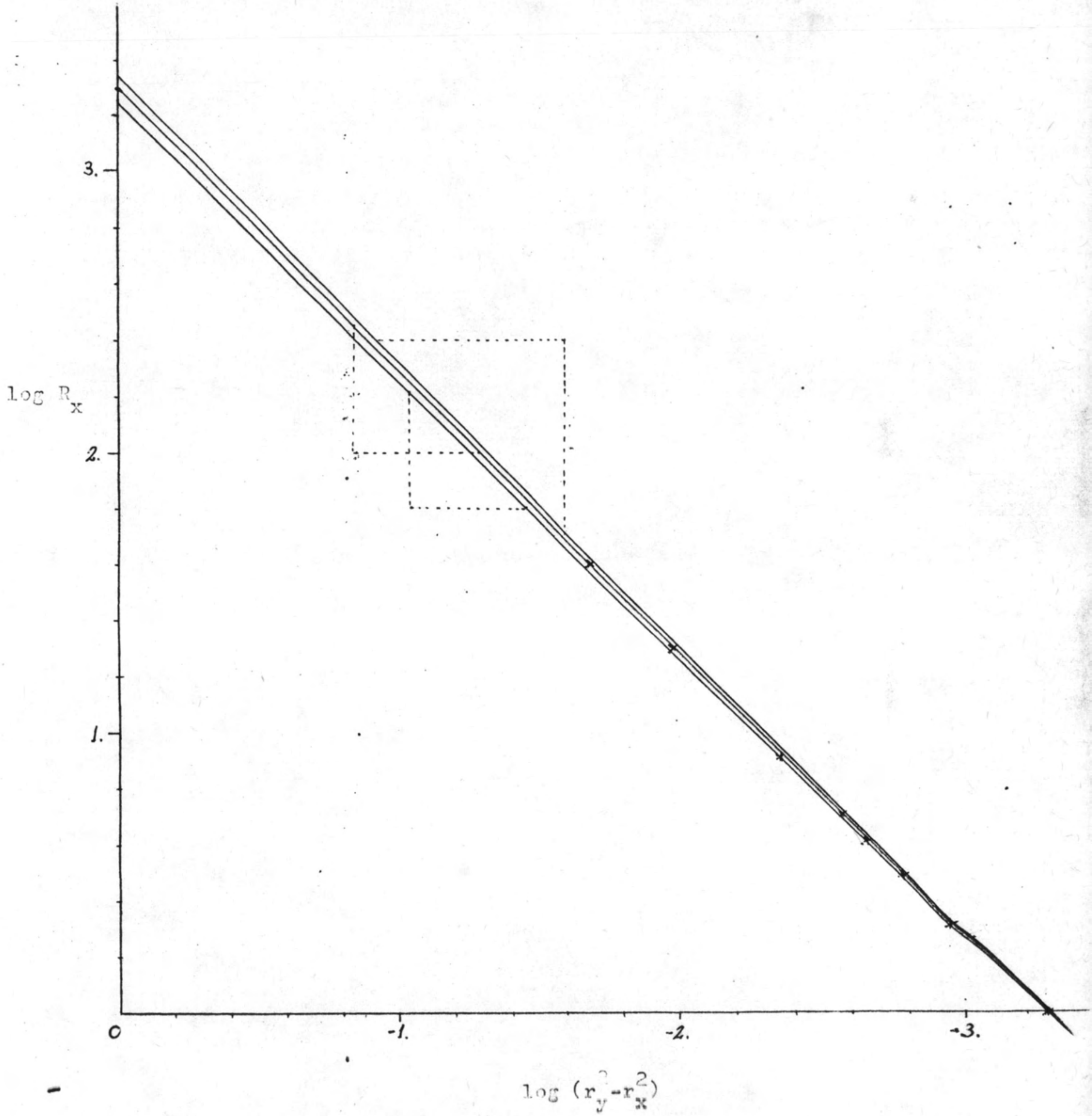


Figure 3-8 The relation of log R_x with log $(r_y^2-r_x^2)$ when $P_x = P_y = 5$

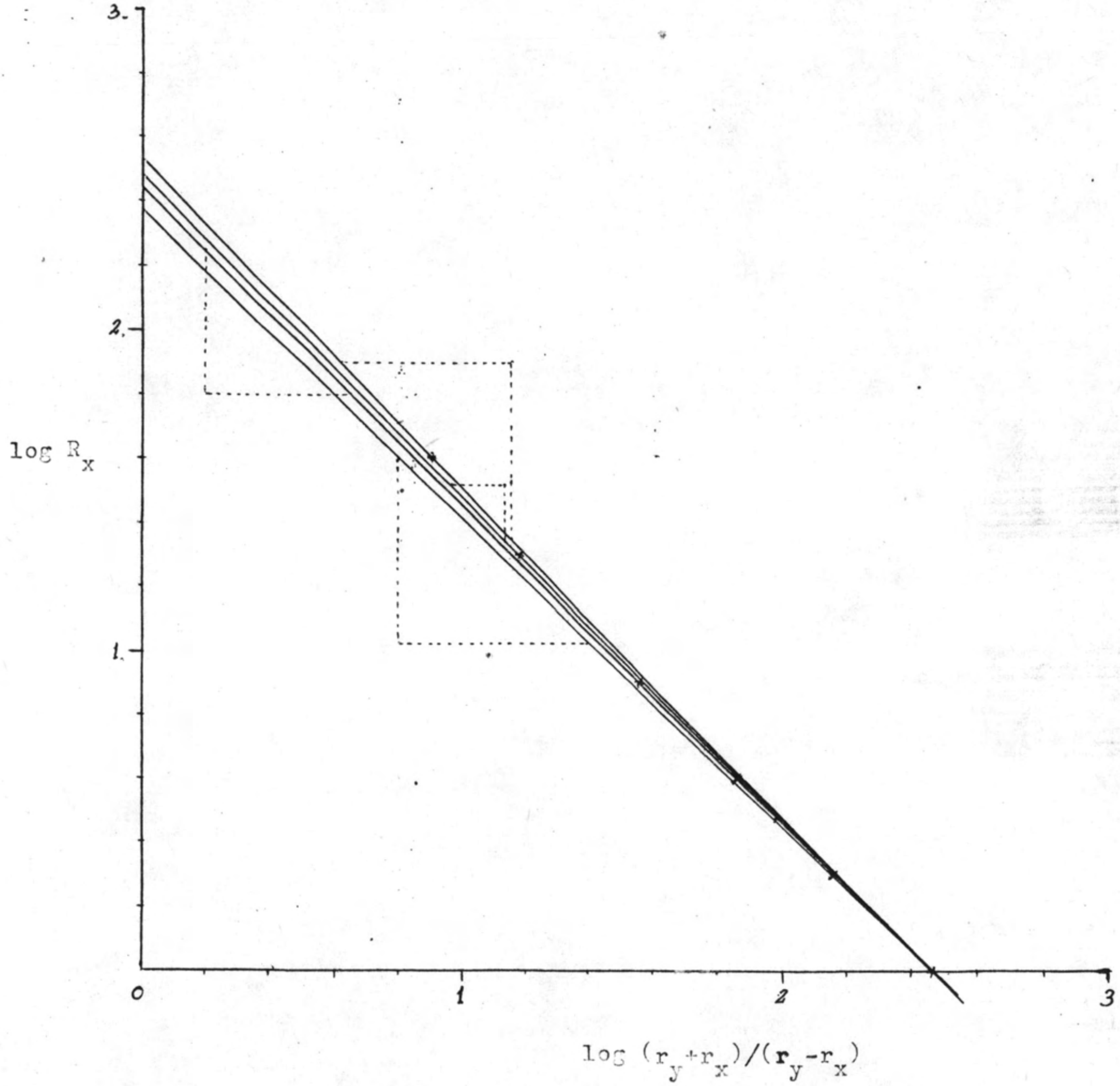


Figure 3-9 The relation of log R_x with log $(r_y+r_x)/(r_y-r_x)$ when $P_x = P_y = 5$

However figures 3-2 to 3-6 are all curves, but figures 3-7 to 3-9 seem to be straight lines. To check with others data.

when $P_x = P_y = 10$	see table 3-9 and figure 3-10, 3-11, 3-12, 3-13.
$P_x = P_y = 1$	see table 3-10 and figure 3-14
$P_x = 2,\ P_y = 1$	see table 3-11 and figure 3-15
$P_x = 4,\ P_y = 1$	see table 3-12 and figure 3-16
$P_x = 4,\ P_y = 2$	see table 3-13 and figure 3-17
$P_x = 2,\ P_y = 4$	see table 3-14 and figure 3-18
$P_x = 3,\ P_y = 5$	see table 3-15 and figure 3-19

From these figures we can see that when P_x equal to P_y and A equals to $(r_y - r_x)$, $(r_y^2 - r_x^2)$ or $(r_y + r_x)/(r_y - r_x)$ straight lines like figures 3-7, 3-8 and 3-9 can be obtained

Then, we can set equation when P_x equal to P_y as

$$R_x = (r_y - r_x)^a\ P_x^{b+c} \qquad = (r_y - r_x)^a\ P_y^{b+c} \tag{3-3}$$

$$R_x = (r_y^2 - r_x^2)^a\ P_x^{b+c} \qquad = (r_y^2 - r_x^2)^a\ P_y^{b+c} \tag{3-4}$$

$$R_x = \left(\frac{r_y + r_x}{r_y - r_x}\right)^a P_x^{b+c} \qquad = \left(\frac{r_y + r_x}{r_y - r_x}\right)^a P_y^{b+c} \tag{3-5}$$

and we obtain

$$\log R_x = a \log (r_y - r_x) + (b + c) \log P_x \tag{3-6}$$

$$\log R_x = a \log (r_y^2 - r_x^2) + (b + c) \log P_x \tag{3-7}$$

$$\log R_x = a \log \left(\frac{r_y + r_x}{r_y - r_x}\right) + (b + c) \log P_x \tag{3-8}$$

TABLE 3-9

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	194.73164	19.38852	199.50761	195.10422	0.49239	4.89578
2	382.50562	37.88814	199.02992	190.40900	0.97008	9.59100
3	562.88184	55.49200	198.56609	185.90268	1.43391	14.09732
4	735.98193	72.25497	198.11531	181.57431	1.88469	18.42569
5	902.55811	88.22232	197.67696	177.41415	2.32304	22.58585
10	1645.82715	157.19740	195.64943	158.83035	4.35057	41.16965
20	2793.20581	255.17299	192.23213	130.15015	7.76787	69.84985

TABLE 3-9 (Cont)

R_x	$r_x = X_1/X_0$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	$r_y + r_x$	$r_y - r_x$	$r_y^2 - r_x^2$	$\frac{(r_y+r_x)}{(r_y-r_x)}$
1	0.0995653	0.1005743	0.0100137	1.010134	0.2001396	1.009×10^{-3}	2.0194×10^{-4}	198.354410
2	0.0990525	0.1011448	0.0100186	1.0211231	0.2001973	2.0923×10^{-3}	4.18872×10^{-4}	95.682884
3	0.0985855	0.1017150	0.0100276	1.031744	0.2003005	3.1295×10^{-3}	6.2684×10^{-4}	64.003994
4	0.0981749	0.1022859	0.0100419	1.0418742	0.2004608	4.111×10^{-3}	8.24094×10^{-4}	48.762053
5	0.0977469	0.1028537	0.0100536	1.0522451	0.2006006	5.1068×10^{-3}	1.02442×10^{-3}	39.281076
10	0.0955127	0.1056742	0.0100932	1.1063889	0.2011869	0.0101615	2.04436×10^{-3}	19.798937
20	0.0913548	0.1112081	0.0101593	1.2173208	0.2025629	0.0198533	4.02154×10^{-3}	10.202983

TABLE 3-9 (Cont)

$\log R_x$	$\log r_x$	$\log r_y$	$\log r_x r_y$	$\log r_y/r_x$	$\log(r_y+r_x)$	$\log(r_y-r_x)$	$\log(r_y^2-r_x^2)$	$\log\frac{(r_y+r_x)}{(r_y-r_x)}$
0	-1.00189	-0.997513	-1.99940	4.17901×10^{-3}	-0.698667	-2.99611	-3.69478	2.29744
0.30103	-1.00413	-0.995056	-1.99919	9.07812×10^{-3}	-0.698542	-2.67938	-3.37792	1.98083
0.47712	-1.00619	-0.992615	-1.99880	0.0135720	-0.698318	-2.50453	-3.20284	1.80621
0.60206	-1.00800	-0.990184	-1.99818	0.0178153	-0.697971	-2.38605	-3.08402	1.68808
0.69897	-1.00990	-0.987780	-1.99768	0.0221169	-0.697668	-2.29185	-2.98952	1.59418
1	-1.01994	-0.976031	-1.99597	0.0439078	-0.696400	-1.99304	-2.68944	1.29664
1.30103	-1.03927	-0.953864	-1.99313	0.0854050	-0.693440	-1.70217	-2.39561	1.00873

TABLE 3-10

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	193.7255	122.4438	196.9341	195.1533	3.0659	4.8467
2	376.1146	237.6686	194.0464	190.5942	5.9536	9.4058
3	548.2551	346.1509	191.3216	186.2903	8.6784	13.7097
4	710.6684	448.5111	188.7461	182.2295	11.2539	17.7705
5	864.1637	545.2255	186.3078	178.3908	13.6922	21.6092
8	1278.9260	805.8995	179.7138	168.0230	20.2862	31.9770
20	2457.3980	1541.6590	160.8212	138.5622	39.1788	61.4378
40	3545.4760	2208.8600	143.0252	111.3603	56.9748	88.6397

TABLE 3-10 (Cont)

R_x	$r_x = X_1/X_0$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	$r_y + r_x$	$r_y - r_x$	$r_y^2 - r_x^2$	$\frac{(r_y + r_x)}{(r_y - r_x)}$
1	0.6320479	0.6325747	0.3998175	1.0008334	1.2646226	5.268×10^{-4}	6.66203×10^{-4}	2400.57440
2	0.6319047	0.6329711	0.3999774	1.0016875	1.2648758	1.0664×10^{-3}	1.34886×10^{-3}	1186.11750
3	0.6313683	0.6330116	0.3996634	1.0026027	1.2643799	1.6433×10^{-3}	2.07775×10^{-3}	769.41514
4	0.6311116	0.6332911	0.3996773	1.0034534	1.2644027	2.1795×10^{-3}	2.75576×10^{-3}	580.13429
5	0.6309284	0.6336282	0.3997740	1.0042790	1.2646566	2.6998×10^{-3}	3.41404×10^{-3}	468.38899
8	0.6301377	0.6343997	0.3997591	1.0067636	1.2645374	4.262×10^{-3}	5.38945×10^{-3}	296.70046
20	0.6273542	0.6376986	0.4000628	1.0164889	1.2650528	0.0103444	0.0130862	122.29349
40	0.6230080	0.6427684	0.4004498	1.0317177	1.2657764	0.0197604	0.0250122	64.05621

TABLE 3-10 (Cont)

log R_x	log r_x	log r_y	$\log(r_x r_y)$	$\log r_y/r_x$	$\log(r_y+r_x)$	$\log(r_y-r_x)$	$\log(r_y^2-r_x^2)$	$\log\frac{(r_y+r_x)}{(r_y-r_x)}$
0	-0.199250	-0.198888	-0.398138	3.61825×10^{-4}	0.101961	-3,27835	-3.17639	3.38032
0.30103	-0.199348	-0.198616	-0.397965	7.32296×10^{-4}	0.102048	-2.97208	-2.87003	3.07413
0.47712	-0.199717	-0.198588	-0.398306	1.1289×10^{-3}	0.101878	-2.78428	-2.68241	2.88616
0.60206	-0.199894	-0.198897	-0.398290	1.49722×10^{-3}	0.101885	-2.66164	-2.55976	2.76353
0.69897	-0.200020	-0.198166	-0.398185	1.85442×10^{-3}	0.101938	-2.56867	-2.46673	2.67061
0.90309	-0.200565	-0.197637	-0.398202	2.92751×10^{-3}	0.101932	-2.37039	-2.26845	2.47232
1.30103	-0.202487	-0.195385	-0.397872	7.10265×10^{-3}	0.102109	-1.98529	-1.88319	2.08740
1.60206	-0.205506	-0.191945	-0.397452	0.0135609	0.102357	-1.70420	-1.60185	1.80656

TABLE 3-11

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	194.29180	83.96660	196.93200	195.15270	3.06800	4.84730
2	377.04060	162.92570	194.03820	190.57560	5.96180	9.42440
3	550.15200	237.56970	191.30430	186,25470	8.69570	13.74530
4	713.44640	308.02430	188.71720	182.17060	11.28280	17.82940
5	867.93050	374.66720	186.26500	178.30160	13.73500	21.69840
8	1286.82666	554.96118	179.62093	167.82800	20.37907	32.17200
10	1533.69092	660.97656	175.70207	161.66087	24.29793	38.33913
20	2487.190092	1068.79150	160.49576	137.82094	39.50424	62.17906
40	3609.55688	1542.72241	142.40524	109.76231	57.59476	90.23769

TABLE 3-11 (Cont)

R_x	$r_x = \frac{X_1}{X_0}$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	r_y+r_x	r_y-r_x	$r_y^2-r_x^2$	$\frac{(r_y+r_x)}{(r_y-r_x)}$
1	0.4321674	0.6329296	0.2735315	1.4645473	1.0650970	0.0007622	0.2138312	5.3052666
2	0.4321171	0.6325919	0.2733537	1.4639362	1.0647090	0.2004748	0.2134473	5.3109368
3	0.4318255	0.6326307	0.2731860	1.4650146	1.0644562	0.2008052	0.2137483	5.3009394
4	0.4317413	0.6328199	0.2732144	1.4657386	1.0645612	0.2010786	0.2140604	5.2942540
5	0.4316788	0.6329959	0.2732509	1.4663585	1.0646747	0.2013171	0.2143372	5.2885457
8	0.4312633	0.6334411	0.2731798	1.4688036	1.0647044	0.2021778	0.2152595	5.2661785
10	0.4309711	0.6337632	0.2731336	1.4705468	1.0647343	0.0207921	0.2159197	5.2503736
20	0.4297183	0.6353302	0.2730130	1.4784806	1.0650485	0.2056119	0.2189866	5.1798971
40	0.4273993	0.6382561	0.2727902	1.4933484	1.0656554	0.2108568	0.2247006	5.0539294

TABLE 3-11 (Cont)

$\log R_x$	$\log r_x$	$\log r_y$	$\log(r_x r_y)$	$\log(r_y/r_x)$	$\log(r_y+r_x)$	$\log(r_y-r_x)$	$\log(r_y^2-r_x^2)$	$\log\frac{(r_y+r_x)}{(r_y-r_x)}$
0	-0.364348	-0.198645	-0.198645	0.1562993	0.0273892	-0.697318	-0.669929	0.724707
0.30103	-0.364399	-0.198876	-0.563275	0.165522	0.0272309	-0.697940	-0.670709	0.725171
0.47712	-0.364692	-0.198850	-0.563541	0.165842	0.0271278	-0.697225	-0.670097	0.724353
0.60206	-0.364776	-0.198720	-0.563496	0.166057	0.0271706	-0.696634	-0.669464	0.723805
0.69897	-0.364839	-0.198599	-0.563438	0.166240	0.0272169	-0.696119	-0.668902	0.723336
0.90309	-0.365257	-0.198294	-0.563551	0.166964	0.0272290	-0.694267	-0.667037	0.721496
1	-0.36552	-0.198073	-0.563625	0.167479	0.0272412	-0.692949	-0.665708	0.720190
1.30103	-0.366816	-0.197000	-0.563817	0.169816	0.0273694	-0.686952	-0.659582	0.714321
1.60206	-0.369166	-0.195005	-0.564171	0.174161	0.0276168	-0.676012	-0.648396	0.703629

TABLE 3-12

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	196.14380	47.74589	196.92950	195.14650	3.07050	4.85350
2	383.60440	92.90266	194.02920	190.56000	5.97080	9.44000
3	559.85220	135.65910	191.28510	186.21990	8.71490	13.78010
4	721.46540	176.00620	188.68390	182.10460	11.31610	17.89540
5	871.52650	214.24060	186.21430	178.20350	13.78570	21.79650
8	1298.73071	318.73364	179.51228	167.60791	20.48772	32.39209
20	2520.40015	621.28052	160.10559	136.98180	39.89441	63.01820
40	3682.79810	911.41016	141.65683	107.97124	58.34317	92.02876

TABLE 3-12 (Cont)

R_x	$r_x = \frac{X_1}{X_0}$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	r_y+r_x	r_y-r_x	$r_y^2-r_x^2$	$\frac{r_y+r_x}{r_y-r_x}$
1	0.2434228	0.6326362	0.1539980	2.5989192	0.8760590	0.3892134	0.3409739	2.2508449
2	0.2422466	0.6325000	0.1532209	2.6109757	0.8747466	0.3902534	0.3413728	2.2414830
3	0.2423123	0.6324264	0.1532446	2.6099640	0.8747387	0.3901141	0.3412479	2.2422637
4	0.2439565	0.6323468	0.1542651	2.5920473	0.8763033	0.3883903	0.3403477	2.2562440
5	0.2458222	0.6324731	0.1554759	2.5728884	0.8782953	0.3866509	0.3395936	2.2715459
8	0.2454193	0.6324914	0.1552255	2.5771868	0.8779107	0.3870721	0.3398147	2.2680805
20	0.2465007	0.6330617	0.1560501	2.5681943	0.8795624	0.3865610	0.3400045	2.2753521
40	0.2474776	0.6339667	0.1568925	2.5617134	0.8814443	0.3864891	0.3406686	2.2806446

TABLE 3-12 (Cont)

$\log R_x$	$\log r_x$	$\log r_y$	$\log(r_x r_y)$	$\log(r_y/r_x)$	$\log(r_y+r_x)$	$\log(r_y-r_x)$	$\log(r_y^2-r_x^2)$	$\log\frac{(r_y+r_x)}{(r_y-r_x)}$
0	-0.613639	-0.198846	-0.812485	0.414793	-0.0574666	-0.409812	-0.467279	0.352346
0.30103	-0.615742	-0.198939	-0.814682	0.416803	-0.0581177	-0.408653	-0.466771	0.350535
0.47712	-0.615624	-0.198990	-0.814615	0.416635	-0.0581217	-0.408808	-0.466930	0.350687
0.60206	-0.612688	-0.199045	-0.811732	0.413643	-0.0573456	-0.410732	-0.468077	0.353386
0.69897	-0.609379	-0.198958	-0.808337	0.410421	-0.0563594	-0.412681	-0.469040	0.356322
0.90309	-0.610091	-0.198945	-0.809037	0.411146	-0.0565497	-0.412208	-0.468758	0.355658
1.30103	-0.608182	-0.198554	-0.806736	0.409628	-0.0557333	-0.412782	-0.468515	0.357049
1.60206	-0.606464	-0.197934	-0.804398	0.408531	-0.0548051	-0.412863	-0.467668	0.358058

TABLE 3-13

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	194.20190	47.73690	197.89350	195.13650	2.10650	4.86350
2	381.97610	92.99256	195.89180	190.51740	4.10820	9.48260
3	556.75070	135.74900	193.98660	186.13450	6.01340	18.86550
4	723.82080	176.32980	192.17120	181.96830	7.82880	18.03170
5	882.79100	214.88790	190.43910	178.00370	9.56090	21.99630
10	1572.82031	381.51514	182.84062	160.71558	17.15938	39.28442
20	2588.46436	621.78394	171.47131	135.30960	28.52869	64.69040

TABLE 3-13 (Cont)

R_x	$r_x = \frac{X_1}{X_0}$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	r_y+r_x	r_y-r_x	$r_y^2-r_x^2$	$\frac{(r_y+r_x)}{(r_y-r_x)}$
1	0.2458106	0.4331242	0.1064665	1.7620240	0.6789348	0.1873136	0.1271737	3.6245889
2	0.2434512	0.4332356	0.1054717	1.7795582	0.6766868	0.1897844	0.1284245	3.5655554
3	0.2438236	0.4336951	0.1057451	1.7787248	0.6775187	0.1898715	0.1286414	3.5683011
4	0.2436097	0.4341687	0.1057677	1.7822307	0.6777784	0.1905590	0.1291567	3.5567902
5	0.2434187	0.4346594	0.1058042	1.7856450	0.6780781	0.1912407	0.1296761	3.5456788
10	0.2425675	0.4367986	0.1059531	1.8007301	0.6793661	0.1942311	0.1319540	3.4977204
20	0.2402134	0.4410034	0.1059349	1.8358817	0.6812168	0.2007900	0.1367815	3.3926829

TABLE 3-13 (Cont)

$\log R_x$	$\log r_x$	$\log r_y$	$\log(r_x r_y)$	$\log r_y/r_x$	$\log(r_y+r_x)$	$\log(r_y-r_x)$	$\log(r_y^2-r_x^2)$	$\log\frac{(r_y+r_x)}{(r_y-r_x)}$
0	-0.6093990	-0.3633870	-0.9727870	0.2460120	-0.1681720	-0.7274310	-0.8956030	0.5592590
0.30103	-0.6135880	-0.2621760	-0.9768640	0.2503120	-0.1696120	-0.7217390	-0.8913520	0.5521270
0.47712	-0.6129240	-0.3628150	-0.9757400	0.2501090	-0.1690790	-0.7215400	-0.8906190	0.5524620
0.60206	-0.6133050	-0.3623410	-0.9756470	0.2509640	-0.1689120	-0.7199710	-0.8888830	0.5510580
0.69897	-0.6136460	-0.3618510	-0.9754970	0.2517950	-0.1687200	-0.7184200	-0.8871400	0.5496990
1	-0.6151673	-0.3597187	-0.9748863	0.2554486	-0.1678961	-0.7116812	-0.8795774	0.5437850
1.30103	-0.6194030	-0.3555580	-0.9749610	0.2638450	-0.1667150	-0.6972580	-0.8639730	0.5305430

TABLE 3-14

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	194.44470	83.97559	198.79920	195.13090	1.20080	4.86910
2	379.88140	163.58200	197.64830	190.49350	2.35170	9.50650
3	556.16630	239.06200	196.54390	186.08630	3.45610	13.91370
4	724.26130	310.71230	195.48310	181.88490	4.51690	18.11510
5	884.49010	378.77560	194.46280	177.87910	5.53720	22.12090
8	1323.82178	563.63892	191.62048	166.89661	8.37952	33.10339

TABEL 3-14 (Cont)

R_x	$r_x = \frac{X_1}{X_0}$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	r_x+r_y	r_x-r_y	$r_x^2-r_y^2$	$\frac{(r_x+r_y)}{(r_x-r_y)}$
1	0.4318738	0.2466164	0.1065071	0.5710381	0.6784902	0.1852574	0.1256953	3.6624188
2	0.4306133	0.2473781	0.1065242	0.5744785	0.6779914	0.1832352	0.1242318	3.7001154
3	0.4298390	0.2483954	0.1057700	0.5778800	0.6782344	0.1814436	0.1230612	3.7379902
4	0.4290058	0.2493444	0.1069701	0.5812145	0.6783502	0.1796614	0.1218733	3.7757147
5	0.4282417	0.2503153	0.1071954	0.5845187	0.6785570	0.1779264	0.1207332	3.8136948
8	0.4257664	0.2531317	0.1077749	0.5945318	0.6788981	0.1726347	0.1172013	2.9325703

TABLE 3-14 (Cont)

$\log R_x$	$\log r_x$	$\log r_y$	$\log(r_x r_y)$	$\log(r_y/r_x)$	$\log(r_x+r_y)$	$\log(r_x-r_y)$	$\log(r_x^2-r_y^2)$	$\log\frac{(r_x+r_y)}{(r_x-r_y)}$
0	-0.364643	-0.607978	-0.972621	-0.243335	-0.168456	-0.732224	-0.900381	0.563768
0.30103	-0.365913	-0.606639	-0.972551	-0.240726	-0.168776	-0.736991	-0.905767	0.568215
0.47712	-0.366694	-0.604856	-0.971551	-0.238162	-0.168620	-0.741258	-0.909879	0.572638
0.60206	-0.367537	-0.603200	-0.970737	-0.235664	-0.168546	-0.745545	-0.914091	0.576999
0.69897	-0.368811	-0.601513	-0.969824	-0.233202	-0.168414	-0.749760	-0.918173	0.581346
0.90309	-0.370829	-0.596653	-0.967482	-0.225825	-0.168195	-0.762872	-0.931067	0.594676

TABLE 3-15

R_x	X_0	X_1	Y_0	Y_1	$200-Y_0$	$200-Y_1$
1	194.75030	61.52756	199.02620	195.11960	0.97380	4.88040
2	380.79840	120.00750	198.08950	190.46910	1.91050	9.53090
3	558.14410	175.43080	197.18740	186.03610	2.81260	13.96390
4	727.75840	228.10320	196.31790	181.79530	3.68210	18.20470
5	889.60540	278,16850	195.47890	177.74930	4.52110	22.25070
10	1603.00073	494.83667	191.68147	159.91447	8.31853	40.08553

TABLE 3-15 (Cont)

R_x	$r_x = \frac{X_1}{X_0}$	$r_y = \frac{(200-Y_0)}{(200-Y_1)}$	$r_x r_y$	r_y/r_x	r_x+r_y	r_x-r_y	$r_x^2-r_y^2$	$\frac{(r_x+r_y)}{(r_x-r_y)}$
1	0.3159305	0.1995328	0.0630384	0.6315718	0.5154633	0.1163977	0.0599887	4.4284663
2	0.3151470	0.2004532	0.0631722	0.6360625	0.5156002	0.1146938	0.0591361	4.4954496
3	0.3143109	0.2014193	0.0633082	0.6408282	0.5157302	0.1128916	0.0582216	4.5683664
4	0.3134325	0.2022609	0.0633951	0.6453092	0.5156934	0.1111716	0.0573304	4.6387152
5	0.3126875	0.2031891	0.0635346	0.6498152	0.5158766	0.1094984	0.0564876	4.7112706
10	0.3086939	0.2075195	0.0640600	0.6722500	0.5162134	0.1011744	0.0522275	5.1022136

<u>TABLE</u> 3-15 (Cont)

$\log R_x$	$\log r_x$	$\log r_y$	$\log(r_x r_y)$	$\log(r_y/r_x)$	$\log(r_x+r_y)$	$\log(r_x-r_y)$	$\log(r_x^2-r_y^2)$	$\log\frac{(r_x+r_y)}{(r_x-r_y)}$
0	-0.500408	-0.699986	-1.20039	-0.199577	-0.287802	-0.934056	-1.22186	0.646253
0.30103	-0.501487	-0.697987	-1.19947	-0.196500	-0.287687	-0.940460	-1.22815	0.652773
0.47712	-0.502640	-0.695899	-1.19854	-0.193258	-0.287577	-0.947338	-1.23492	0.659761
0.60206	-0.503856	-0.694088	-1.19794	-0.190232	-0.287608	-0.954006	-1.24161	0.666398
0.69897	-0.504889	-0.692100	-1.19699	-0.187210	-0.287454	-0.960592	-1.24805	0.673138
1	-0.510472	-0.682941	-1.19341	-0.172469	-0.287171	-0.994929	-1.28210	0.707759

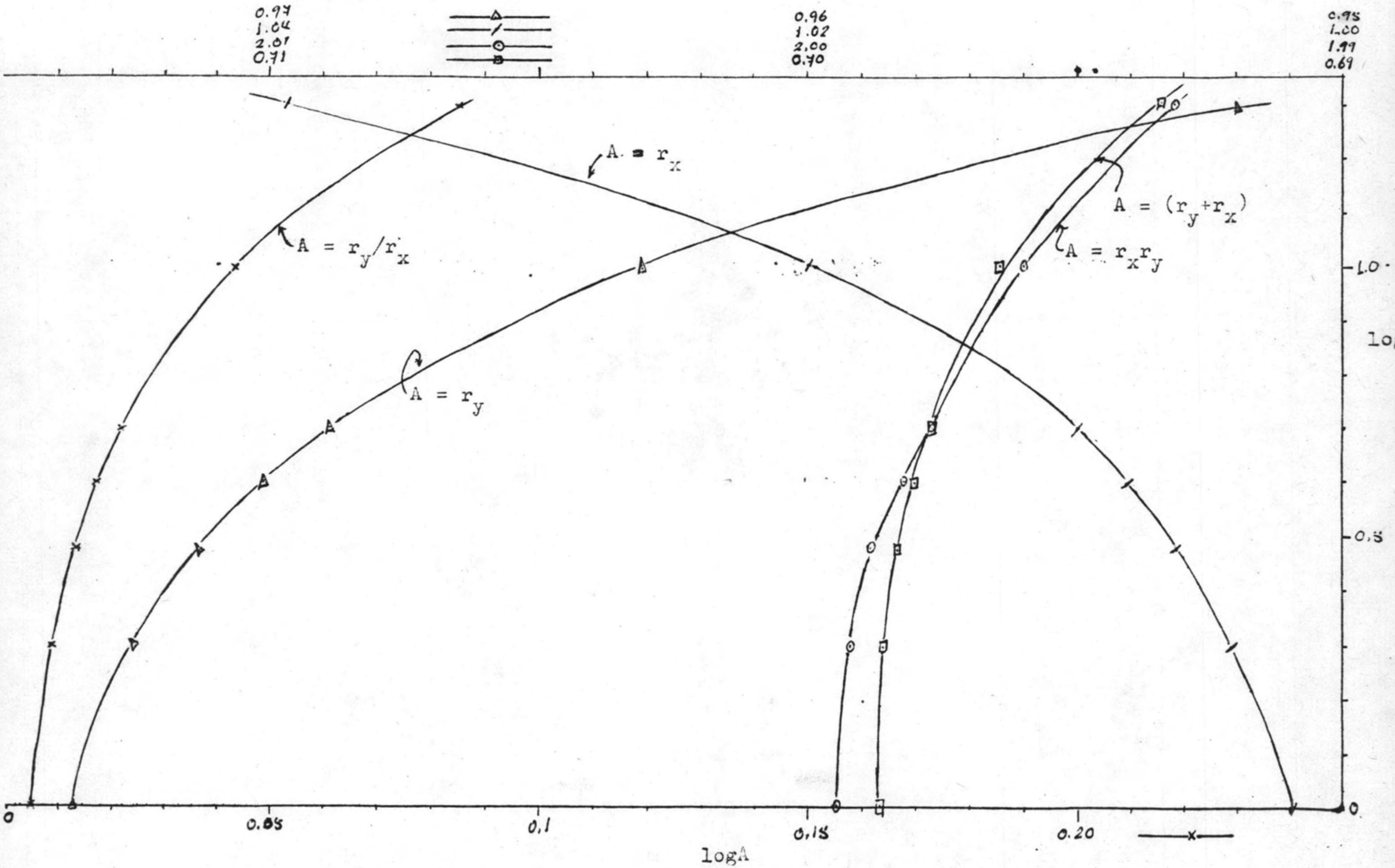


Figure 3-10 The relation of log R_x with logA when A = $r_x, r_y, r_x r_y, r_y/r_x, r_y+r_x$

at $P_x = P_y = 10$

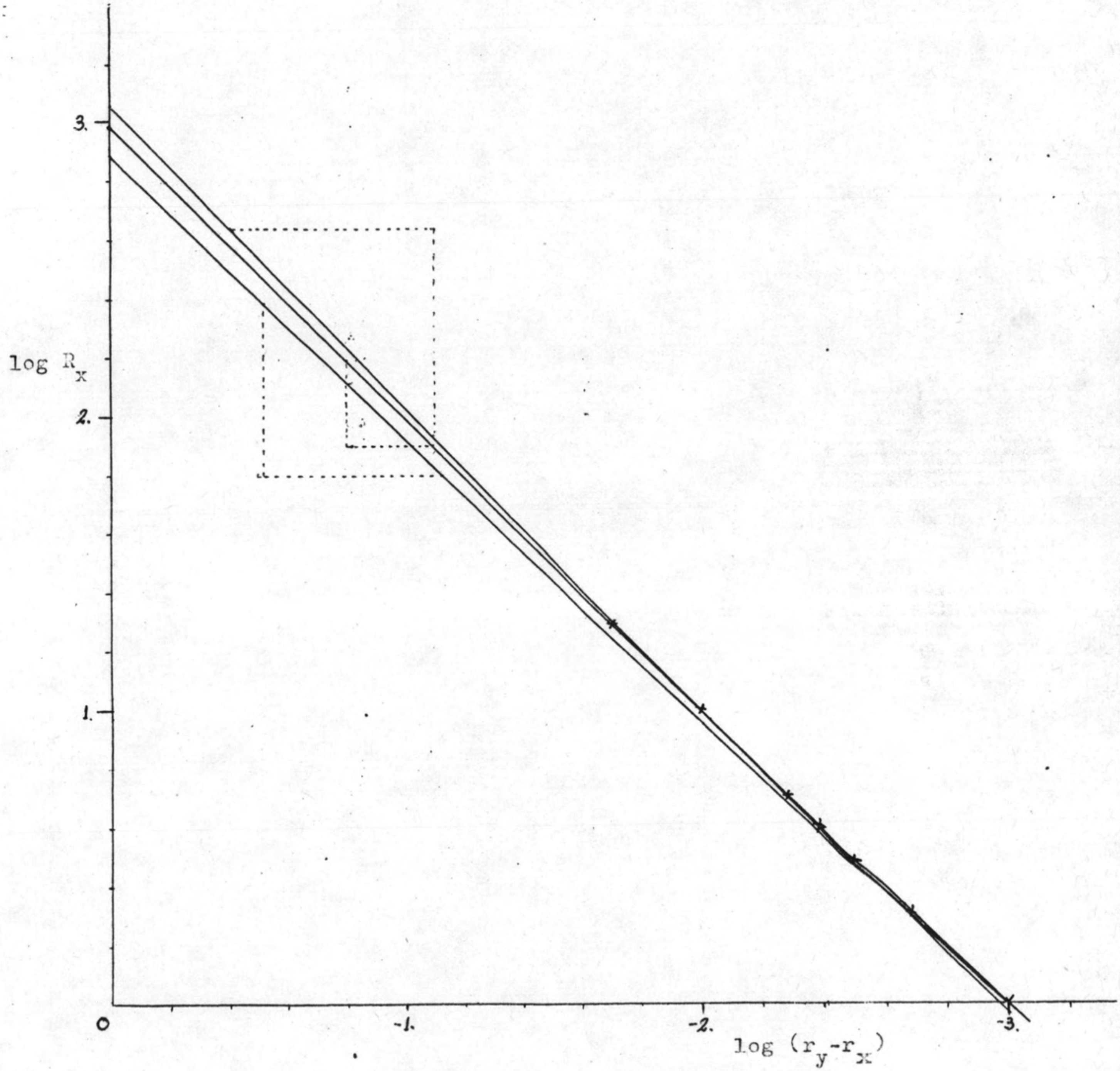


Figure 3-11 The relation of log R_x with log $(r_y - r_x)$ at $P_x = P_y = 10$

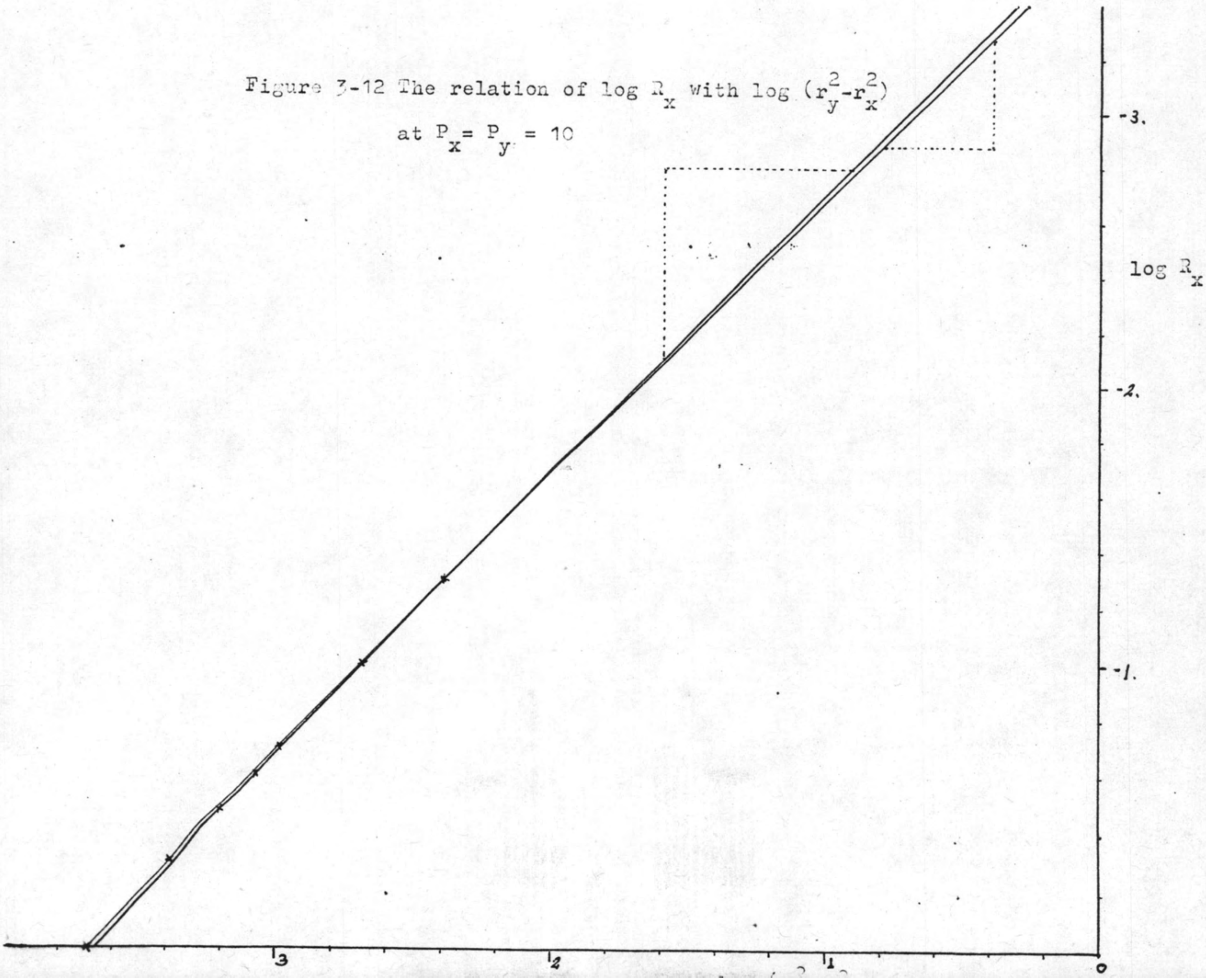


Figure 3-12 The relation of log R_x with log $(r_y^2-r_x^2)$ at $P_x = P_y = 10$

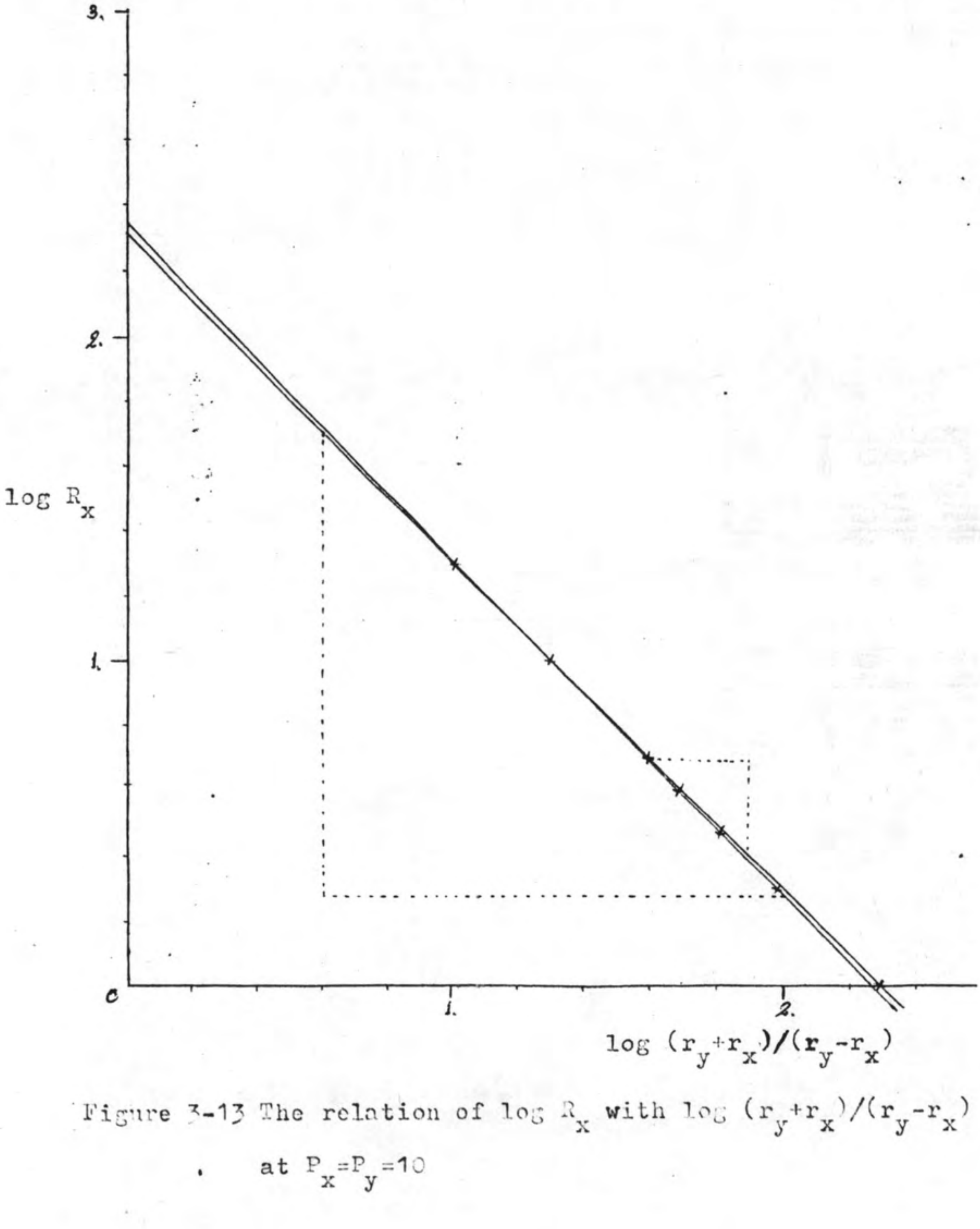


Figure 3-13 The relation of log R_x with log $(r_y+r_x)/(r_y-r_x)$ at $P_x=P_y=10$

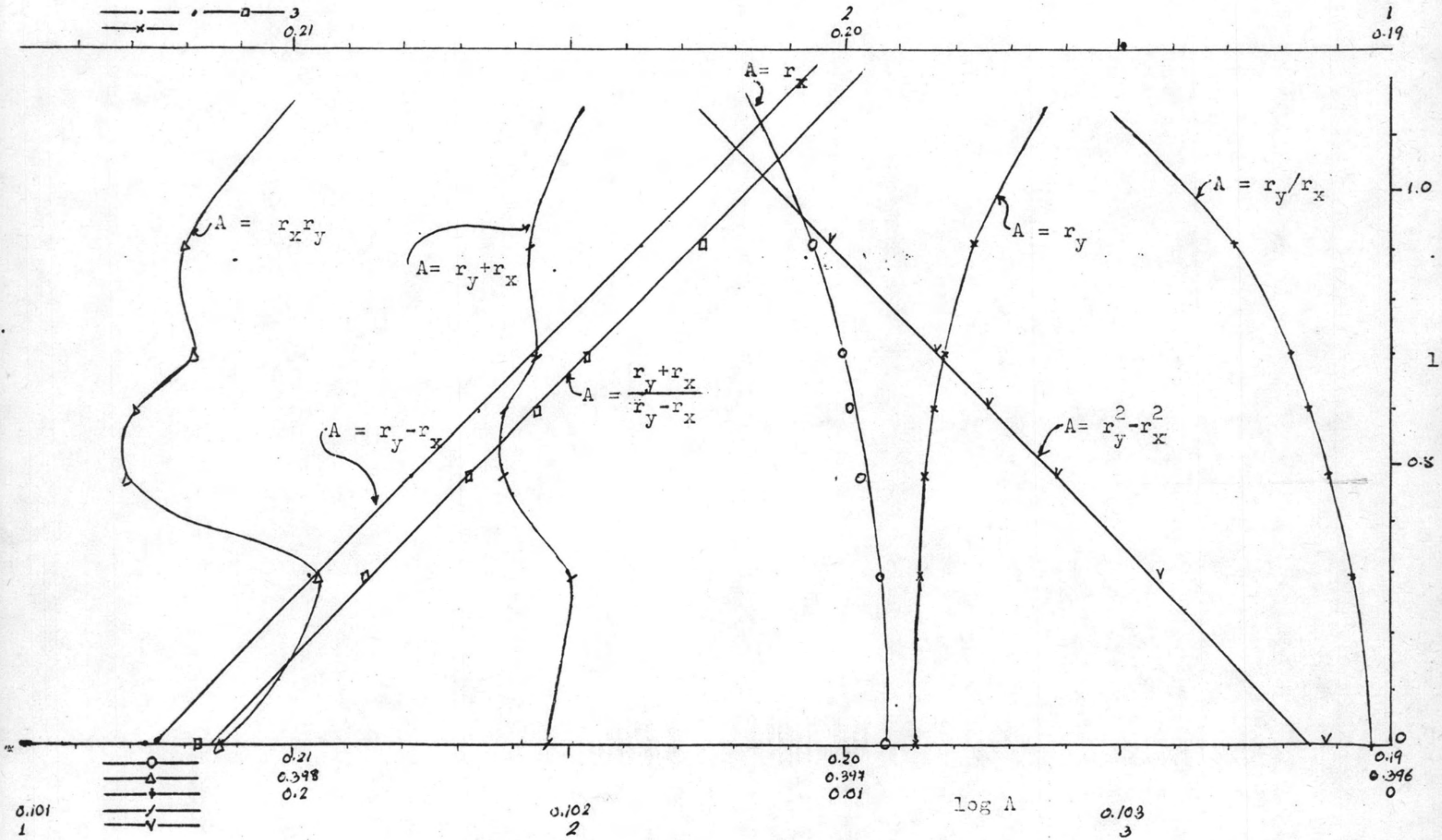


Figure 3-14 The relation of log R_x with log A at $P_x = P_y = 1$

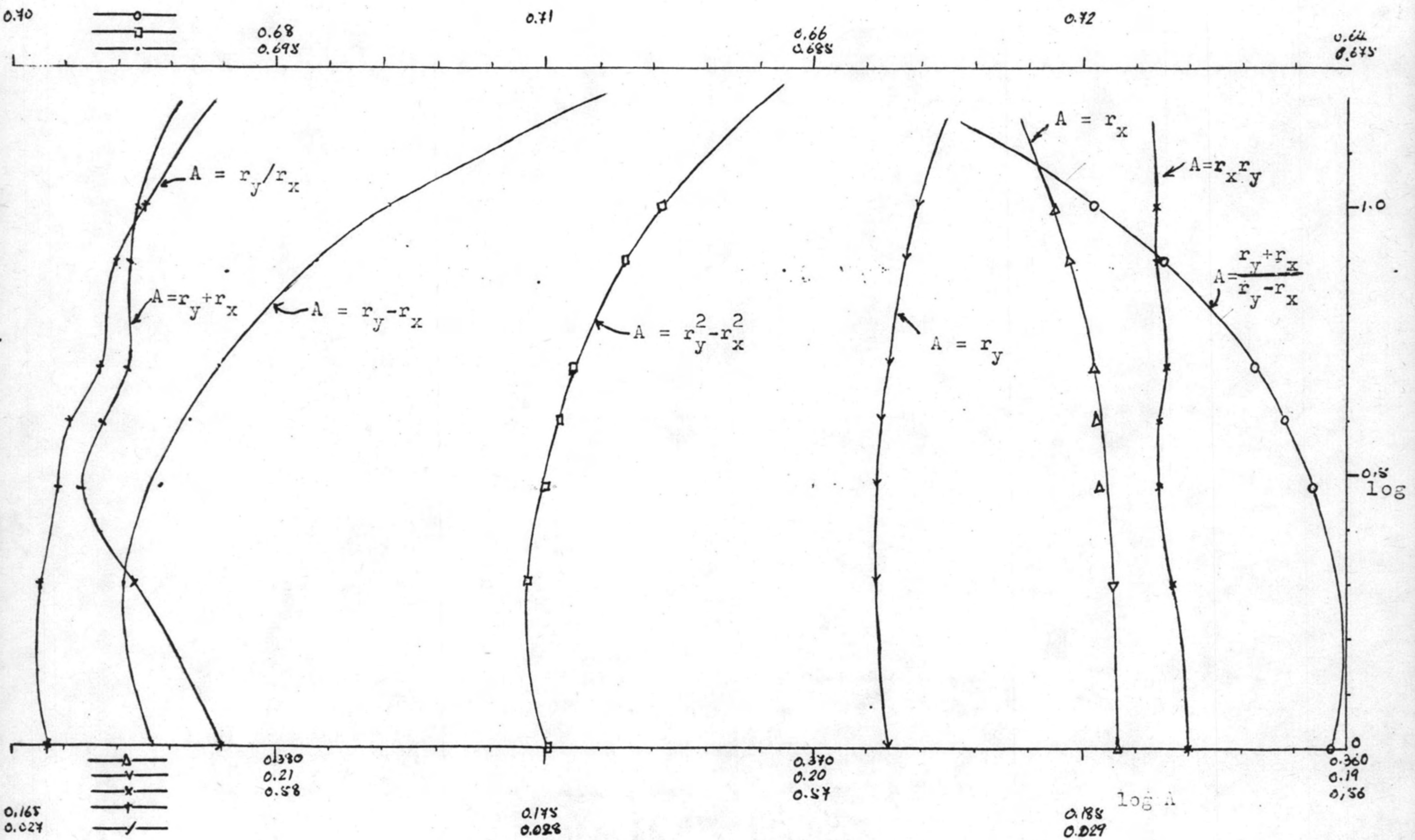


Figure 3-15 The relation of log R_x with log A at $P_x = 2$, $P_y = 1$

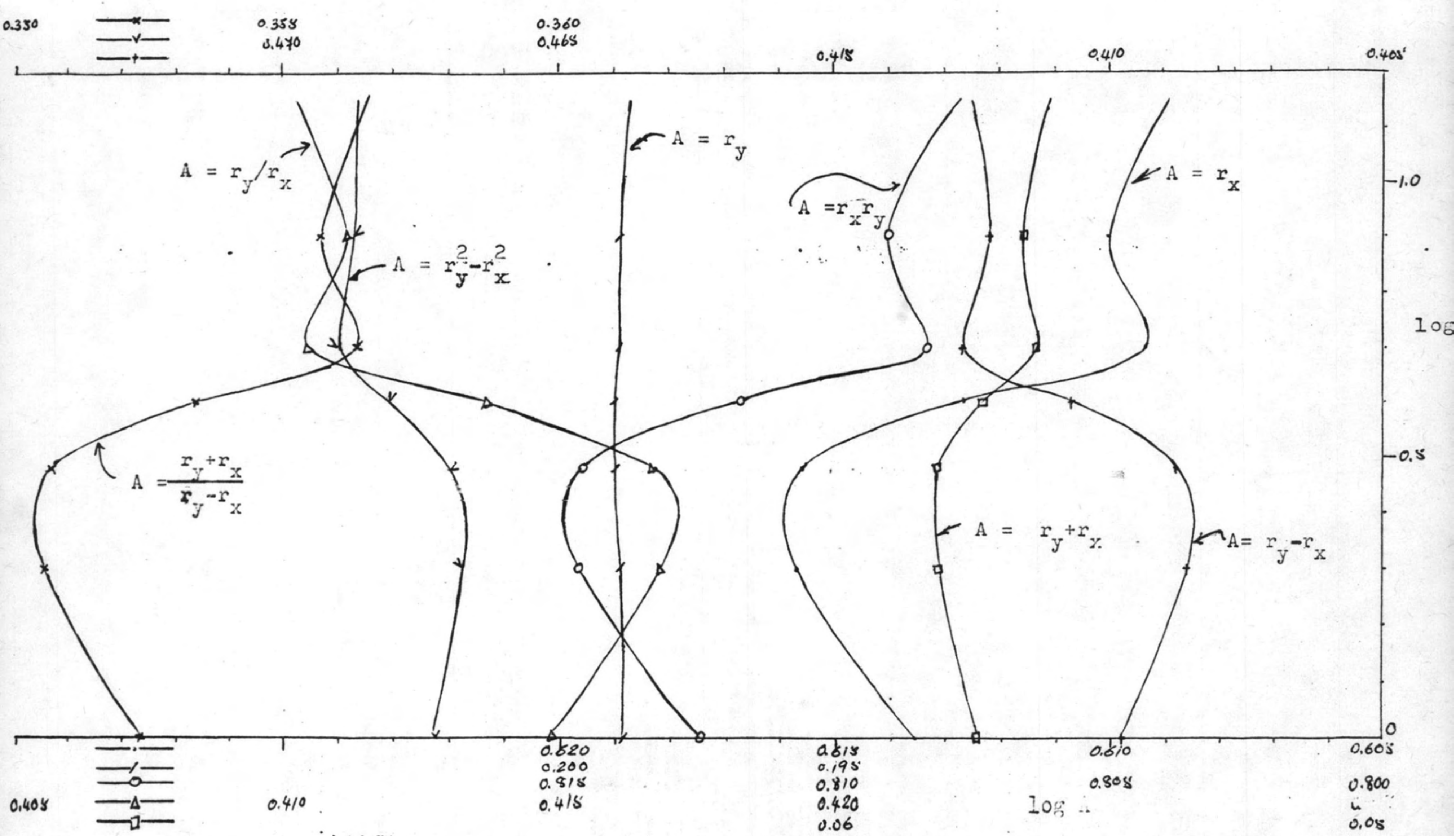


Figure 3-16 The relation of log R_x with log A at P_x=4, P_y=1

Figure 3-17 The relation of log R_x with log A at $P_x = 4$, $P_y = 2$

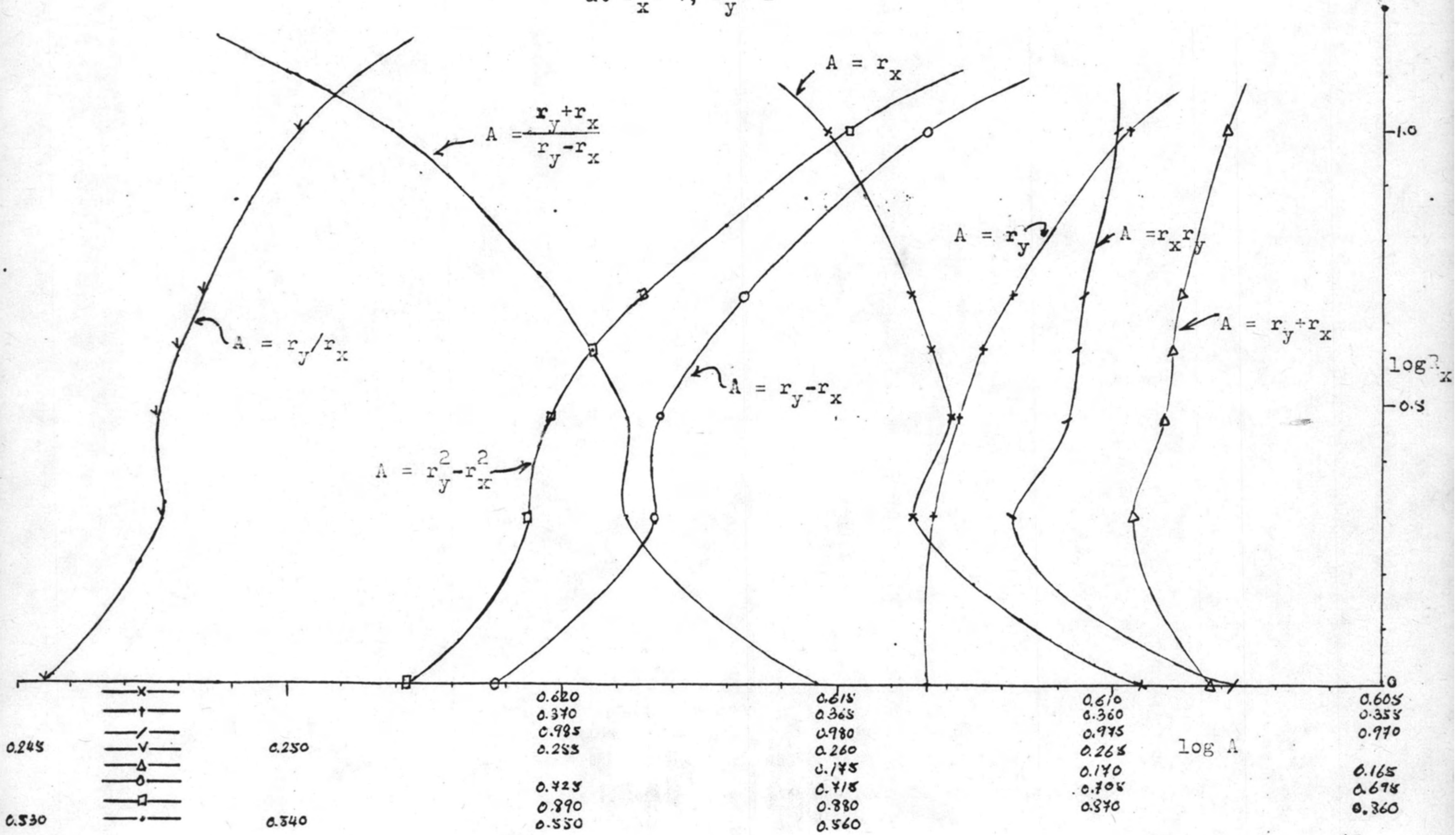


Figure 3-12 The relation of log R_x with log A

at $P_x = 2$, $P_y = 4$

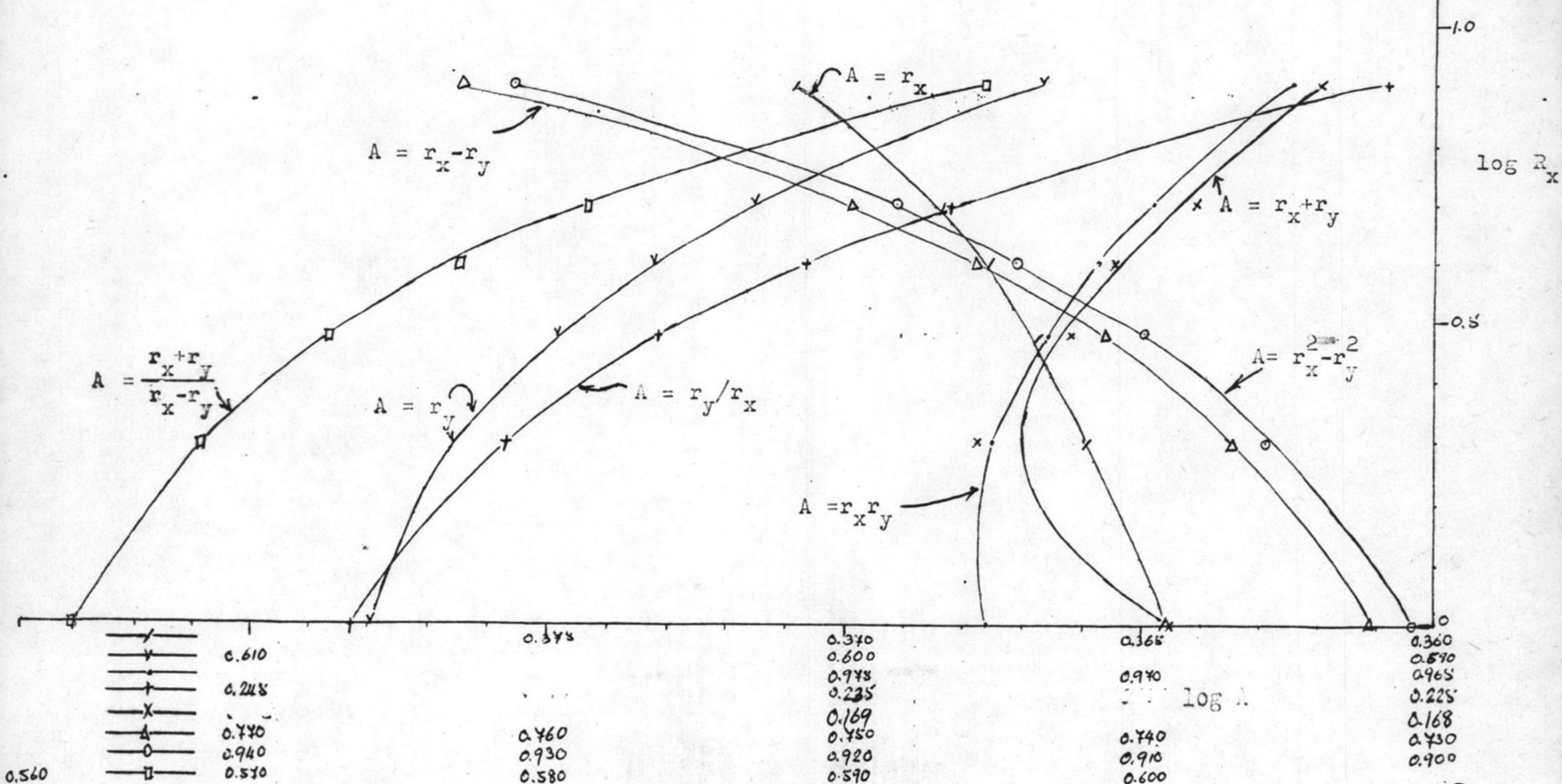
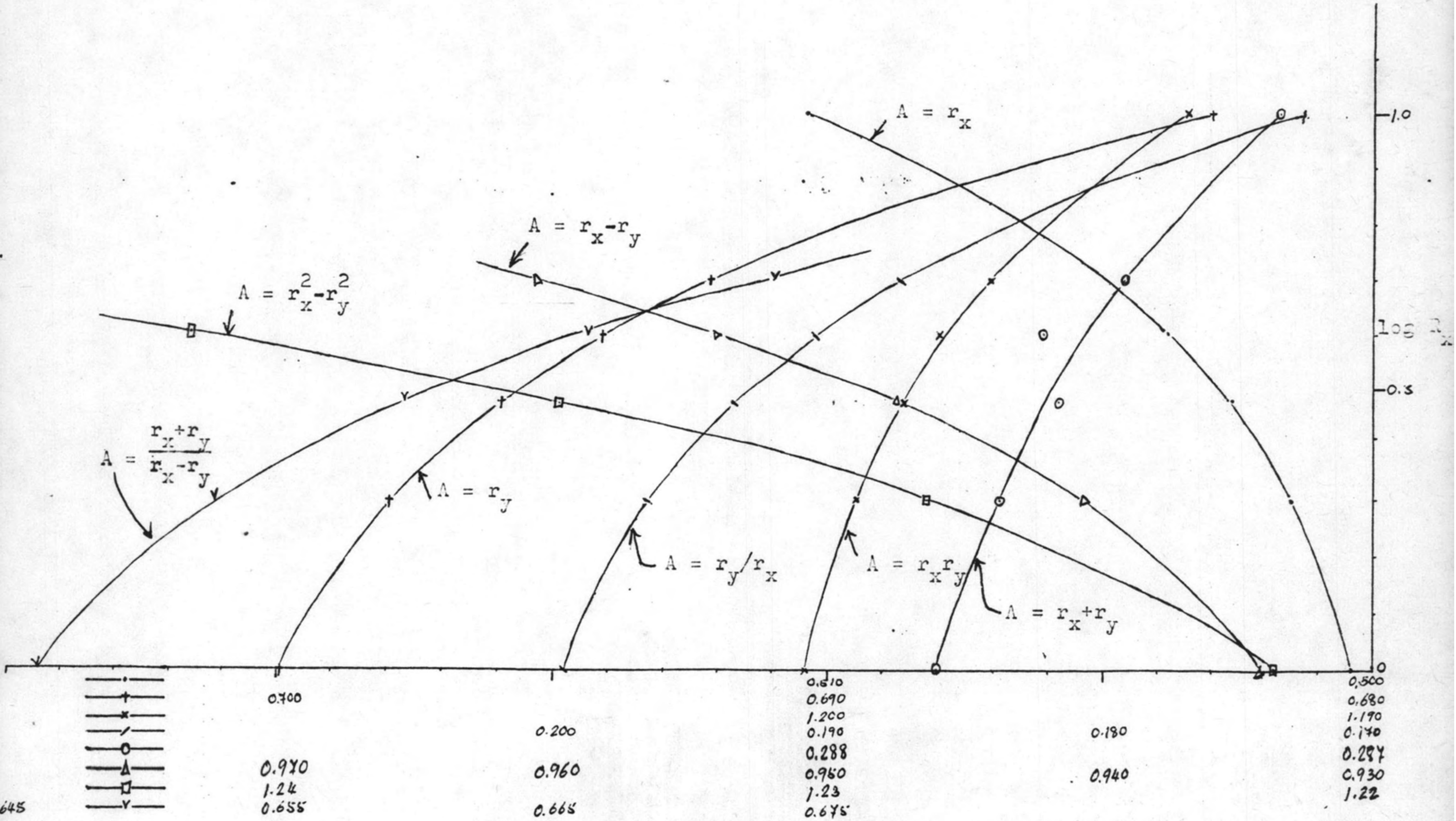


Figure 3-19 The relation of log R_x with log A

at $P_x = 3$, $P_y = 5$



To find the value of a and (b + c) from figures 3-7, 3-8, 3-9 see table 3-16 and from figures 3-11, 3-12, 3-13 see table 3-17. We can see that slope a must be minus a. So, the equations (3-3), (3-4) and (3-5) become.

$$R_x = \frac{P_x^{(b+c)}}{(r_y - r_x)^a} = \frac{P_y^{(b+c)}}{(r_y - r_x)^a} \tag{3-9}$$

$$\therefore R_x = \frac{P_x^{(b+c)}}{(r_y^2 - r_x^2)^a} = \frac{P_y^{(b+c)}}{(r_y^2 - r_x^2)^a} \tag{3-10}$$

$$R_x = \left(\frac{r_y - r_x}{r_y + r_x}\right)^a P_x^{(b+c)} = \left(\frac{r_y - r_x}{r_y + r_x}\right)^a P_y^{(b+c)} \tag{3-11}$$

See table 3-18 to find R_x from equation (3-9) at $P_x = P_y = 5$, $a = -1$ and table 3-21 to fine R_x from equation (3-9) at $P_x = P_y = 10$, $a = -1$ to compare the value of (b+c) in two tables, they are not equal, from table 3-18 $P_x = P_y = 5$, b+c = 4.153254 from table 3-21 $P_x = P_y = 10$, b+c = 3.004. Likewise a similar trend occurs, when we compare (b+c) values in table 3-19 with table 3-22 and table 3-20 with table 3-23.

Then we can not set only one equation for all data of P_x and P_y in order to find R_x, since if we set one equation like :

$$R_x = (r_y - r_x)^{-1} P_x^{4.153254} \tag{3-12}$$

TABLE 3-16 $P_x = 5$

figure	slope = a	(b+c) log P_x	(b+c)	$5^{(b+c)}$
3-7	0.22/-0.22=-1	2.82	4.034507	660.69250
	0.28/-0.28=-1	2.88	4.120348	758.57698
	0.72/-0.70=-1.028	2.92	4.177576	831.76303
3-8	0.40/-0.40=-1	3.25	4.649698	1778.2771
	0.44/-0.44=-1	3.29	4.706925	1949.8418
	0.66/-0.64=-1.031	3.34	4.778459	2187.7591
3-9	0.58/0.60=0.9667	2.37	3.390703	234.42271
	0.45/0.45=1	2.45	3.505157	281.83804
	0.18/0.18=1	2.50	3.576691	316.22756
	0.56/0.54=1.037	2.54	3.633918	346.73659

TABLE 3-17 $P_x = 10$

figure	slope= a	(b+c) log P_x	(b+c)	$10^{(b+c)}$
3-11	0.58/-0.60=-0.9667	2.88	2.88	758.57758
	0.29/-0.29=-1	2.99	2.99	977.23723
	0.72/-0.70=-1.0286	3.05	3.05	1122.01850
3-12	0.40/-40=-1	3.68	3.68	4786.3009
	0.10/-10=-1	3.72	3.72	5248.0746
3-13	0.30/0.30=1	2.31	2.31	204.17379
	1.46/1.42=1.0282	2.34	2.34	218.77616

TABLE 3-18 $P_x = P_y = 5$, slope $a = -1$

R_x	$(r_y - r_x)$	$(b+c) \log P_x$	$(b+c)$	$P_x^{(b+c)}$	$R_x = (r_y - r_x)^{-a} P_x^{(b+c)}$
1	1.3923×10^{-3}	2.88	4.120348	758.57698	1.0561667
		2.90	4.148962	794.32821	1.1059431
		2.903	4.153254	799.83419	1.1136091
2	2.8347×10^{-3}	2.88	4.120348	758.57698	2.1503381
		2.90	4.148962	794.32821	2.2516821
		2.903	4.153254	799.83419	2.2672899
3	4.2217×10^{-3}	2.88	4.120348	758.57698	3.2024844
		2.90	4.148962	794.32821	3.3534154
		2.903	4.153254	799.83419	3.3766599
4	5.6685×10^{-3}	2.88	4.120348	758.57698	4.2999936
		2.90	4.148962	794.32821	4.5026494
		2.903	4.153254	799.83419	4.5338601
5	7.0402×10^{-3}	2.88	4.120348	758.57698	5.3405336
		2.90	4.148962	794.32821	5.5922294
		2.903	4.153254	799.83419	5.6309926
8	0.0111407	2.88	4.120348	758.57698	8.4510785
		2.90	4.148962	794.32821	8.8493722
		2.903	4.153254	799.83419	8.9107127
20	0.0267126	2.88	4.120348	758.57698	20.263563
		2.90	4.148962	794.32821	20.752618
		2.903	4.153254	799.83419	21.365650
40	0.050032	2.88	4.120348	758.57698	37.953123
		2.90	4.148962	794.32821	39.741829
		2.903	4.153254	799.83419	40.017304

<u>TABLE</u> (3-19) $P_x = P_y = 5$, $a = -1$

R_x	$(r_y^2-r_x^2)$	$(b+c)\log R_x$	$(b+c)$	$P_x^{(b+c)}$	$R_x=(r_y^2-r_x^2)^{-a}P_x^{(b+c)}$
		3.290	4.706925	1949.8418	1.0803001
1	5.54045×10^{-3}	3.295	4.7140792	1972.4225	1.0928108
		3.300	4.7212326	1995.2622	1.1054650
		3.290	4.706925	1949.8418	2.1987976
2	1.12768×10^{-3}	3.295	4.7140792	1972.4225	2.2242614
		3.300	4.7212326	1995.2622	2.2500172
		3.290	4.706925	1949.8418	3.2765726
3	1.68043×10^{-3}	3.295	4.7140792	1972.4225	3.3145179
		3.300	4.7212326	1995.2622	3.3329458
		3.290	4.706925	1949.8418	4.4005784
4	2.25689×10^{-3}	3.295	4.7140792	1972.4225	4.4515406
		3.300	4.7212326	1995.2622	4.5030873
		3.290	4.706925	1949.8418	5.4681948
5	2.80443×10^{-3}	3.295	4.7140792	1972.4225	5.5315208
		3.300	4.7212326	1995.2622	5.5955731
		3.290	4.706925	1949.8418	8.6659158
8	4.44442×10^{-3}	3.295	4.7140792	1972.4225	8.7662740
		3.300	4.7212326	1995.2622	8.8677832
		3.290	4.706925	1949.8418	20.909908
20	0.0107239	3.295	4.7140792	1972.4225	21.152061
		3.300	4.7212326	1995.2622	21.396992
		3.290	4.706925	1949.8418	39.584323
40	0.0203013	3.295	4.7140792	1972.4225	40.042740
		3.300	4.7212326	1995.2622	40.506416

TABLE 3-20 $P_x = P_y = 5$, $a = 1$

R_x	$(r_y+r_x)/(r_y-r_x)$	$(b+c)\log P_x$	$(b+c)$	$P_x^{(b+c)}$	$R_x = \frac{r_y+r_x}{r_y-r_x} P_x^{-a(b+c)}$
		2.500	3.5766910	316.22756	1.1064201
1	285.81146	2.512	3.5938595	325.08700	1.1374176
		2.518	3.6024438	329.61000	1.1532427
		2.500	3.5766910	316.22756	2.2533351
2	140.33756	2.512	3.5938595	325.08700	2.3164646
		2.518	3.6024435	329.61000	2.3486941
		2.500	3.5766910	316.22756	3.3539126
3	94.286164	2.512	3.5938595	325.08700	3.4478759
		2.518	3.6024435	329.61000	3.4958469
		2.500	3.5766910	316.22756	4.5022019
4	70.238422	2.512	3.5938595	325.08700	4.6283357
		2.518	3.6024435	329.61000	4.6927307
		2.500	3.5766910	316.22756	5.5888704
5	56.581659	2.512	3.5988595	325.08700	5.7454483
		2.518	3.6024435	329.61000	5.8253859
		2.500	3.5766910	316.22756	8.8309924
8	35.808836	2.512	3.5938595	325.08700	9.0784017
		2.518	3.6024435	329.61000	9.2047113
		2.500	3.5766910	316.22756	21.041559
20	15.028713	2.512	3.5938595	325.08700	21.631060
		2.518	3.6024435	329.61000	21.932017
		2.500	3.576691	316.22756	38.991639
40	8.1101375	2.512	3.5938595	325.08700	40.08403
		2.518	3.6024435	329.61000	40.641727

TABLE 3-21 $P_x = P_y = 10$, $a = -1$

R_x	$(r_y - r_x)$	$(b+c)$	$P_x^{(b+c)}$	$R_x = (r_y - r_x)^{-a} P_x^{(b+c)}$
1	1.009×10^{-3}	2.990	977.23723	0.9860323
		3.004	1009.25000	1.0183332
		3.016	1037.53000	1.0468677
2	2.0923×10^{-3}	2.990	977.23723	2.0446734
		3.004	1009.25000	2.1116537
		3.016	1037.53000	2.170824
3	3.1295×10^{-3}	2.990	977.23723	3.0582639
		3.004	1009.25000	3.1584478
		3.016	1037.53000	3.2469501
4	4.111×10^{-3}	2.990	977.23723	4.0174222
		3.004	1009.25000	4.1490267
		3.016	1037.53000	4.2652858
5	5.1068×10^{-3}	2.990	977.23723	4.990555
		3.004	1009.25000	5.1540379
		3.016	1037.53000	5.2984582
10	0.0101615	2.990	977.23723	9.9301961
		3.004	1009.25000	10.255493
		3.016	1037.53000	10.542861
20	0.0198533	2.990	977.23723	19.401383
		3.004	1009.25000	20.036943
		3.016	1037.53000	20.598394

TABLE 3-22 $P_x = P_y = 10$, $a = -1$

R_x	$(r_y^2 - r_x^2)$	(b+c)	$P_x^{(b+c)}$	$R_x = (r_y^2 - r_x^2)^{-a} P_x^{(b+c)}$
1	2.0194×10^{-4}	3.680	4786.3009	0.9665456
		3.696	4965.9200	1.0028178
		3.698	4988.8400	1.0074463
2	4.18872×10^{-4}	3.680	4786.3009	2.0048474
		3.696	4965.9200	2.0800848
		3.698	4988.8400	2.0896853
3	6.2684×10^{-4}	3.680	4786.3009	3.0002448
		3.696	4965.9200	3.1128372
		3.698	4988.8400	3.1272044
4	8.24094×10^{-4}	3.680	4786.3009	3.9443618
		3.696	4965.9200	4.0923848
		3.698	4988.8400	4.1112731
5	1.0244×10^{-3}	3.680	4786.3009	4.9031823
		3.696	4965.9200	5.0871877
		3.698	4988.8400	5.1106674
10	2.04436×10^{-3}	3.680	4786.3009	9.7849221
		3.696	4965.9200	10.1521280
		3.698	4988.8400	10.1989840
20	4.02154×10^{-3}	3.680	4786.3009	19.248300
		3.696	1965.9200	19.970645
		3.698	1988.8400	20.062819

TABLE 3-23 $P_x = P_y = 10,\ a = 1$

R_x	$(r_y+r_x)/(r_y-r_x)$	$(b+c)$	$P_x^{(b+c)}$	$R_x \left(\frac{r_y+r_x}{r_y-r_x}\right)^{-a} P_x^{(b+c)}$
1	198.354410	2.308	203.23600	1.0246104
		2.309	203.70400	1.0269598
		2.310	204.17379	1.0293382
2	95.682884	2.308	203.23600	2.1240580
		2.309	203.70400	2.1289492
		2.310	204.27379	2.133859
3	64.003994	2.308	203.23600	3.1753643
		2.309	203.70400	3.1826763
		2.310	204.17379	3.1900163
4	48.762053	2.308	203.23600	4.1679131
		2.309	203.70400	4.1775107
		2.310	204.17379	4.1871450
5	39.281076	2.308	203.23600	5.1738908
		2.309	203.70400	5.1858049
		2.310	204.17379	5.1977646
10	19.798937	2.308	203.23600	10.264995
		2.309	203.70400	10.288633
		2.310	204.17379	10.312361
20	10.202983	2.308	203.23600	19.919272
		2.309	203.70400	19.965141
		2.310	204.17379	20.011185

Equation (3-12) can used only for $P_x = P_y = 5$

$$R_x = (r_y - r_x)^{-1} P_x^{3.004} \qquad (3\text{-}13)$$

Equation (3-13) can used only for $P_x = P_y = 10$

3.1.1 Discussion and Conclusions

The use of the Diffusion Model to modelize liquid-liquid extraction columns has been a topic of intensive academic and practical interest in the past several years. One problem concerning the scale up of columns from pilot scale concerns the measurement of the 3 basic paramenters of the model, P_x, P_y and R_x. It is known that the parameters P_x and P_y may best be identified using tracer analysis methods. However the determination of the parameter R_x is normally done using complete concentration profiles [7,12] however experimental measurement of concentration profiles along the length of the column is time consuming because if the column is divided into 10 equal sections the number of concentration measurements that need to be made is 26. The approach of this section is one of utilizing only 6 measurements, namely the inlet -outlet exterior and interior concentrations in both phases. The basic data is therefore a knowledge of P_x, P_y, r_x, r_y and the aim is to evaluate R_x. Several relationships between R_x and P_x, P_y, r_x r_y were attempted but a general relation allowing an approximation of R_x was not met with success. Only in the case of $P_x = P_y$ was a relation found to obtain R_x from P_x, P_y, r_x, r_y. Namely if A is set equal to $(r_y - r_x)$ or $(r_y^2 - r_x^2)$ or $(r_y + r_x)/(r_y - r_x)$.

3.2 Study of HETP extraction column of different height.

The Height Equivalent to a Theoretical Plate (HETP) is defined as the height of a section of column such that the concentration of solute in the phase leaving the top of the section is in equilibrium with the solute concentration of the other phase leaving the bottom of that section. It is found that the HETP varies very strongly with flow rates of each phases as well as the type and size of column. Here in this section a new approach to the study of the problem is presented, using the advantages of the diffusion model which is supposed to take into account the column operating factors that so much influenced the values of HETP. Instead of performing experiments to see whether HETP varies with the height of an extraction column for identical operating conditions we chose to use a theoretical simulation of the diffusion model to study the problem.

The procedures can be summarized as follow :-

1. The values of P_x, P_y, R_x and Λ were set (see appendix D), $c_{x,in}$ and $c_{y,in}$ are known

2. The concentration profiles were obtained from the analytical of the diffusion model.

3. From the concentration profile, the outgoing streams concentration $c_{x,out}$ and $c_{y,out}$ were read off.

4. Then evaluation of HETP was done through a graphical method. See figure 3-20

4.1 The operating line was obtained by connecting the two inlet outlet points.

4.2 The number of theoretical plates was found graphically.

4.3 HETP was computed by dividing the column height by the number of theoretical plates.

5. The height of the extraction column was then changed and the 4 parameters to suit the new column length. However the actual mixing properties of the column remained identical. Then, steps 1 to 4 were reused again and again.

The above described method was used to abtain the HETP from the diffusion model for identical columns, identical operating conditions, identical chemical systems (with straight equilibrium lines) but columns of different lengths.

In this theoretical simulation the following chemical system was used for the extraction of I_2 from an aqueous solution using CCl_4 as a solvent.

The data of this system are as follow:-

1. The Equilibrium constant or Distribution coefficient.

$$M= \frac{dc^*_x}{dc_y} = \frac{mg\ I_2\ /\ litre\ CCl_4}{mg\ I_2/\ litre\ H_2O} = 89.9$$

2. The Number of true-overall transfer units.

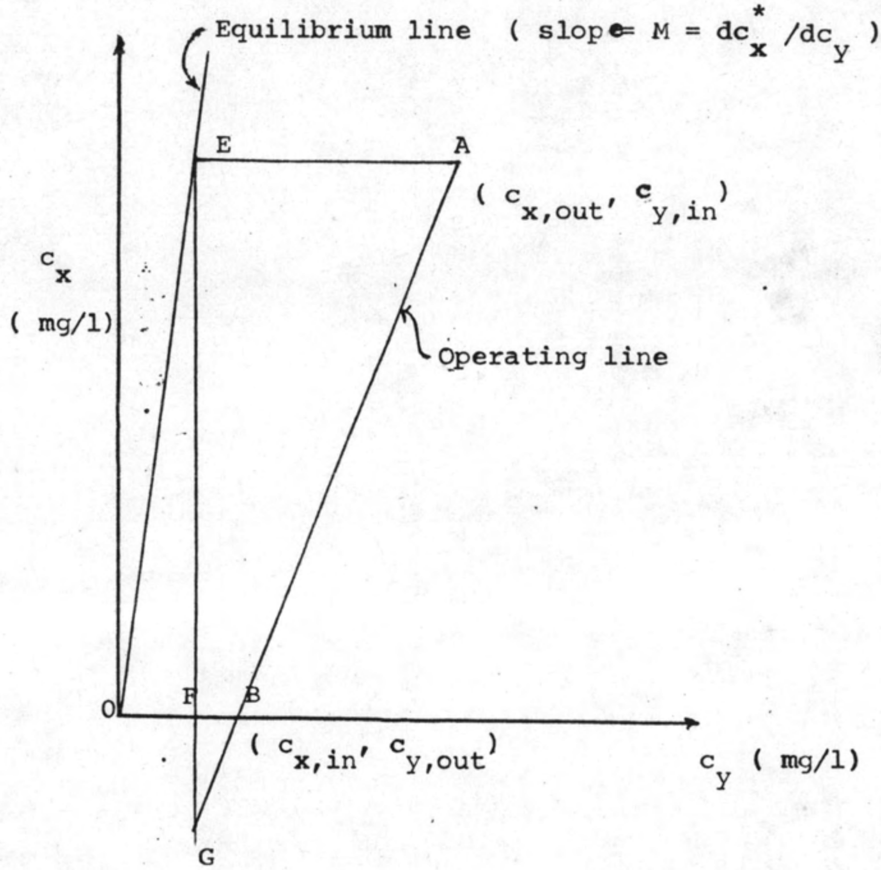


Figure 3-20

Theoretical plates evaluation

$$R_x = \frac{k_x a' L}{F_x} \quad ; \qquad R_y = \frac{R_x G_x}{G_y}$$

We assumed that k_{ox}, a, F_x, G_x and G_y were all constant. For example for an extraction column of 1 meter height (L) with $R_x = 1.0$, when the extraction column height was simulated at 5 meters the value of R_x was adjusted to 5.0, and so on.

3. The Peclet number

$$P_x = \frac{F_x L}{D_x} \qquad \text{and} \qquad P_y = \frac{F_y L}{D_y}$$

D_x, D_y, F_x and F_y are supposed to be constant along the height of the extraction column. For an extraction column of 1 meter height operating at $P_x = 1.0$ and $P_y = 1.0$ when the extraction column is 10 meters height the new parameter values are $P_x = 10.0$ and $P_y = 10.0$

4. The Extraction factor

For this system, let the ratio of F_x / F_y = 1/40

So, the extraction factor, $= m \dfrac{F_x}{F_y}$

$= 89.9/40 = 2.2475$

5. The Inlet concentrations.

Let the inlet concentration of phase X= 0.0 mg I_2/litre CCl_4

and the inlet concentration of phase Y= 200.0 mg I_2/litre H_2O.

We set the inlet concentrations constant throughout the experiment.

6. The Height of the extraction column

We shall vary the height of the column from 1 meter up to 10 meters (at an interval of 1 meter), there will be 10 values of HETP.

3.2.1 Evaluation of HETP

The various data were fed into the computer program (appendix D) and the output was programmed to give our relevent concentrations. For example, at $P_x = 10.0$, $R_x = 10.0$ the output data gives :

$$c_{x,out}\ (X_o) = 1645.82715 \text{ mg } I_2/\text{litre } CCl_4$$

$$c_{y,out}\ (Y_1) = 158.83035 \text{ mg } I_2/\text{litre } H_2O$$

Points A and B in figure 3-21 determine the operating line, from which the number of theoretical plates can be determined by drawing the line AB to cut line EF at G (Figure 3-20)

$$\text{Number of theoretical plates} = \frac{\text{length of EF}}{\text{length of EG}}$$

From Geometry, we can get $\frac{EF}{EG} = \frac{EG-FG}{EG}$

and $\frac{FG}{EG} = \frac{BF}{AE}$

Therefore, Number of theoretical plates

$$= 1- \frac{\text{length of BF}}{\text{length fo AE}}$$

$$= 1- \frac{140.83035}{182}$$

$$= 0.2262069$$

$$\text{HETP} = \frac{10.0}{0.2262069} = 44.207316 \text{ meters.}$$

See figure 3-21 and table 3-24

The others data are shown in figure 3-23, figure 3-25, table 3 -25 and table 3 -26

3.2.2 Discussion & Conclusion

In this section, we suggest a new approach to study the HETP by using the diffusion model instead of experimenting with real extraction columns. The results are shown in Table 3-24, 3-25 and 3-26 and also in Figure 3-27. It can clearly be seen that HETP is not constant when the height of the column is altered, it tends to increase rapidly to an asymptotic value. The reasons for this characteristic change of HETP with the height of the column may be attributed to 2 effects superimposed together when the height of the column is increased. The first one is the increase of the number of true overall transfer unit in X phase (R_x) when the column is higher. From our assumptions that R_x varies directly with the height of the column, so when R_x is increased the concentration which is the most important factor for mass transfer in the extraction column decresaes considerably as shown in Figure

Table 3-24

Column Height	P_x	P_y	R_x	X_o	X_1	Y_o	Y_1	HETP
1.0	1.0	1.0	1.0	193.72550	122.44380	196.93410	195.15330	40.852538
2.0	2.0	2.0	2.0	378.12830	163.18640	195.89680	190.54030	41.438945
3.0	3.0	3.0	3.0	555.62690	175.07120	195.5640	186.10060	41.872310
4.0	4.0	4.0	4.0	726.83250	176.60850	195.47650	181.81650	42.236092
5.0	5.0	5.0	5.0	892.59910	174.63970	195.47390	177,67020	42.544043
6.0	6.0	6.0	6.0	1053.05859	171.46513	195.50024	173.65858	42.822292
7.0	7.0	7.0	7.0	1208.30273	167.89717	195.53584	169.77524	43.077265
8.0	8.0	8.0	8.0	1358.68652	154.25102	195.57390	166.01289	43.54592
9.0	9.0	9.0	9.0	1504.47095	160.68581	195.61201	162.36629	43.763955
10.0	10.0	10.0	10.0	1645.82715	157.19740	195.64943	158.83035	44.085879

The unit of column height and HETP are meter.

HETP is calculated graphically (figure 3-21).

Table 3-25

Column Height	P_x	P_y	R_x	X_o	X_1	Y_o	Y_1	HETP
1.0	1.0	0.5	5	895.21045	542.92798	183.09421	178.52295	8.8466525
2.0	2.0	1.0	10	1533.69092	660.97656	175.70207	161.66087	9.4942164
3.0	3.0	1.5	15	2098.97461	658.17139	172.58713	147.53775	10.064379
4.0	4.0	2.0	20	2588.46436	621.78394	171.47131	135.30960	10.573439
5.0	5.0	2.5	25	3017.43848	578.29761	171.28996	124.52324	10.996762
6.0	6.0	3.0	30	3410.56592	535.33521	171.54280	114.91061	11.42328
7.0	7.0	3.5	35	3755.28345	496.41016	171.97086	106.27216	11.800122
8.0	8.0	4.0	40	4113.53125	456.49048	172.49077	98.47717	12.135201
9.0	9.0	4.5	45	4569.51563	447.12256	173.13623	91.63168	12.374465
10.0	10.0	5.0	50	4834.54688	425.90137	173.62961	85.24454	12.722706

The unit of column height and HETP are meter.

HETP is calculated grahically from(figure 3-23.)

Table 3-26

Column Height	P_x	P_y	R_x	X_o	X_1	Y_o	Y_1	HETP
1.0	1.0	2.0	4.0	715.96509	451.07886	192.20485	182.09683	10.724358
2.0	2.0	4.0	8.0	1323.82178	563.63892	191.62048	166.89661	11.177106
3.0	3.0	6.0	12.0	1858.59473	569.51685	191.79317	153.52463	11.580329
4.0	4.0	8.0	16.0	2334.82910	542.79370	192.02470	141.61861	11.921608
5.0	5.0	10.0	20.0	2761.68115	507.82593	192.24106	130.94511	12.236643
6.0	6.0	12.0	24.0	3146.25684	472.56616	192.43700	121.32863	12.583993
7.0	7.0	14.0	28.0	3494.18872	439.36377	192.31436	112.62833	12.938976
8.0	8.0	26.0	32.0	3810.08008	408.75854	192.77542	104.72823	13.225324
9.0	9.0	18.0	36.0	4097.89063	380.74292	192.92218	97.53186	13.596422
10.0	10.0	20.0	40.0	4360.67188	355.08252	193.05615	90.95630	13.884342

The unit of column height and HETP are meter.

HETP is calulated graphically from(figure 3-25)

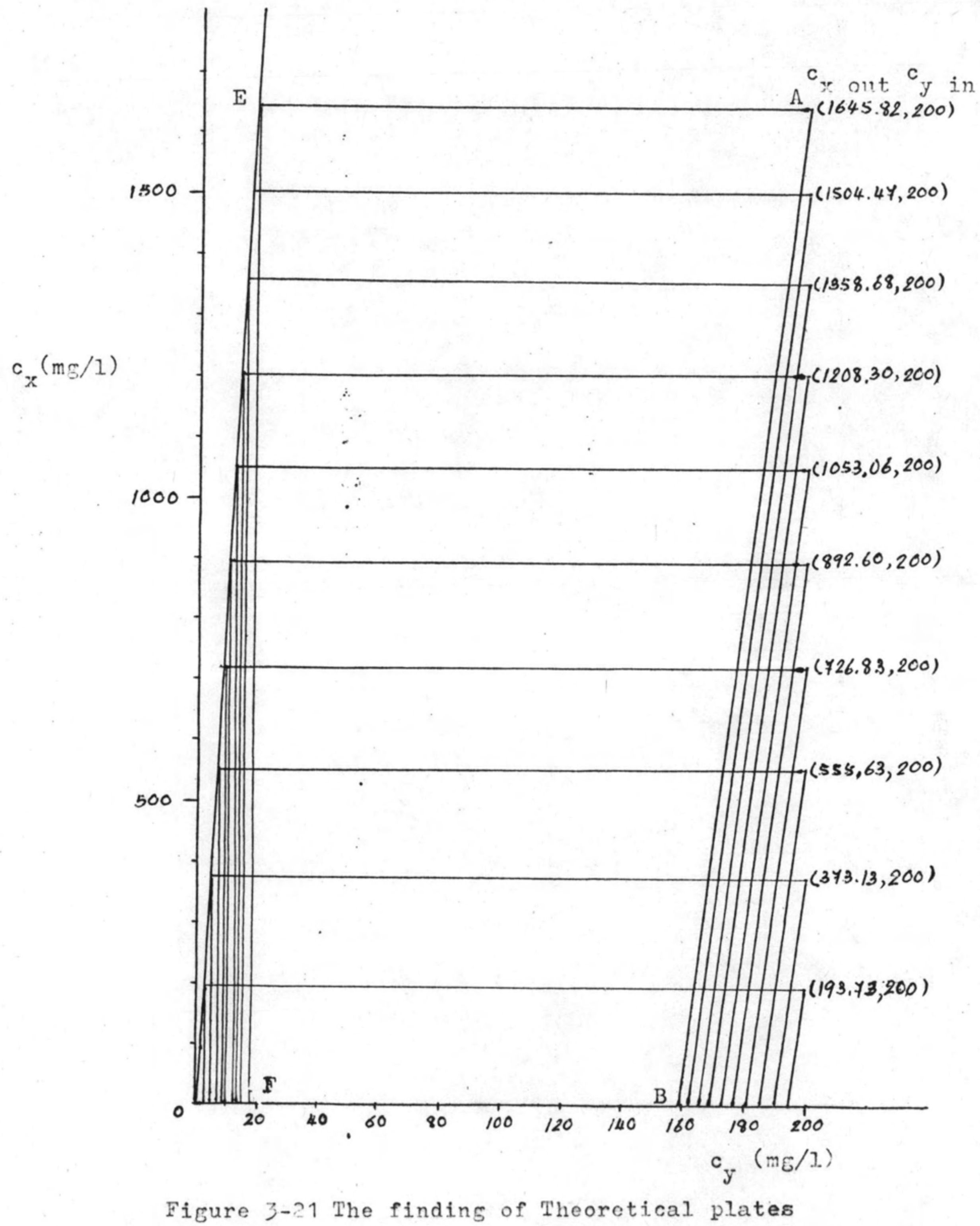


Figure 3-21 The finding of Theoretical plates from relation of c_x and c_y from table 3-24

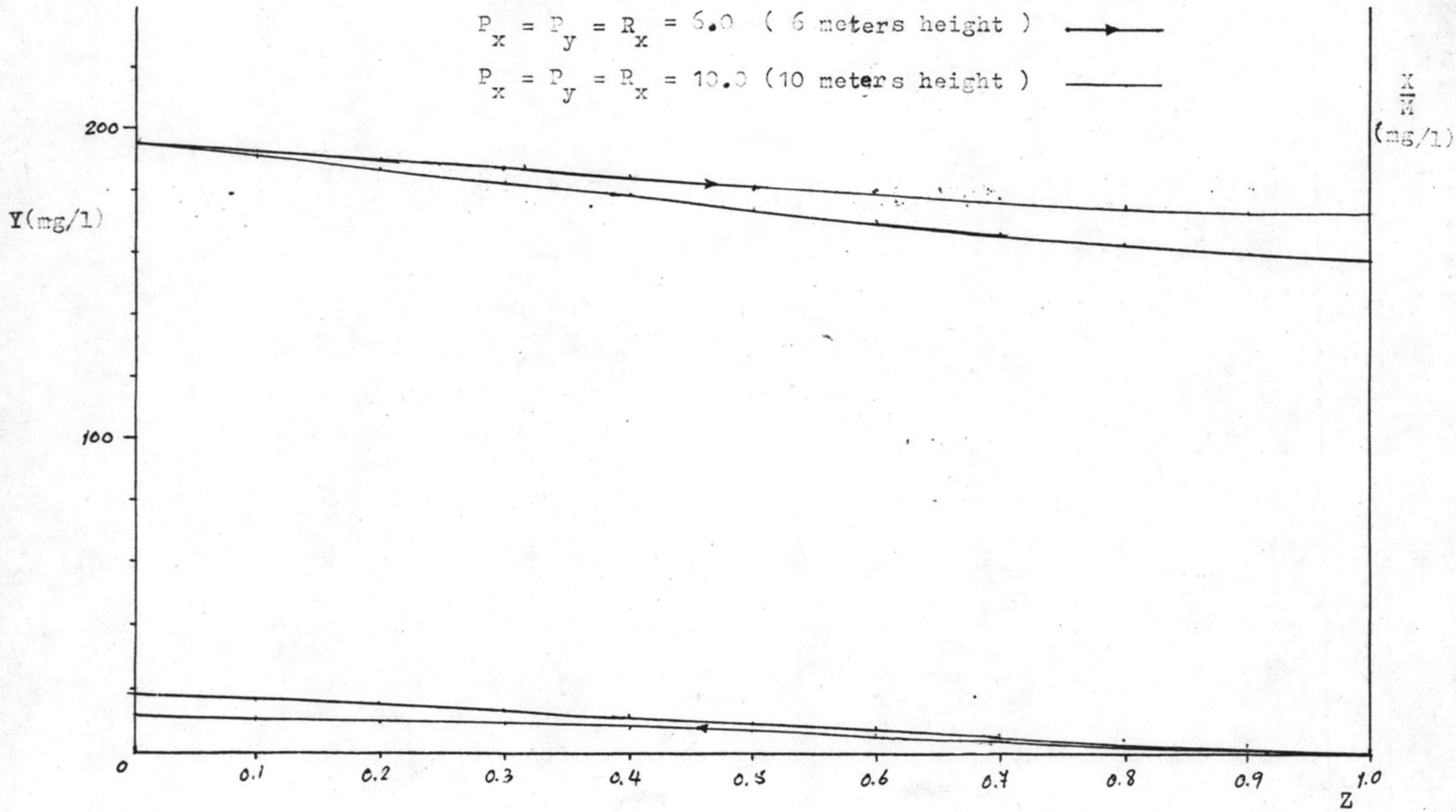


Figure 3-22 Concentration profiles of column at different height

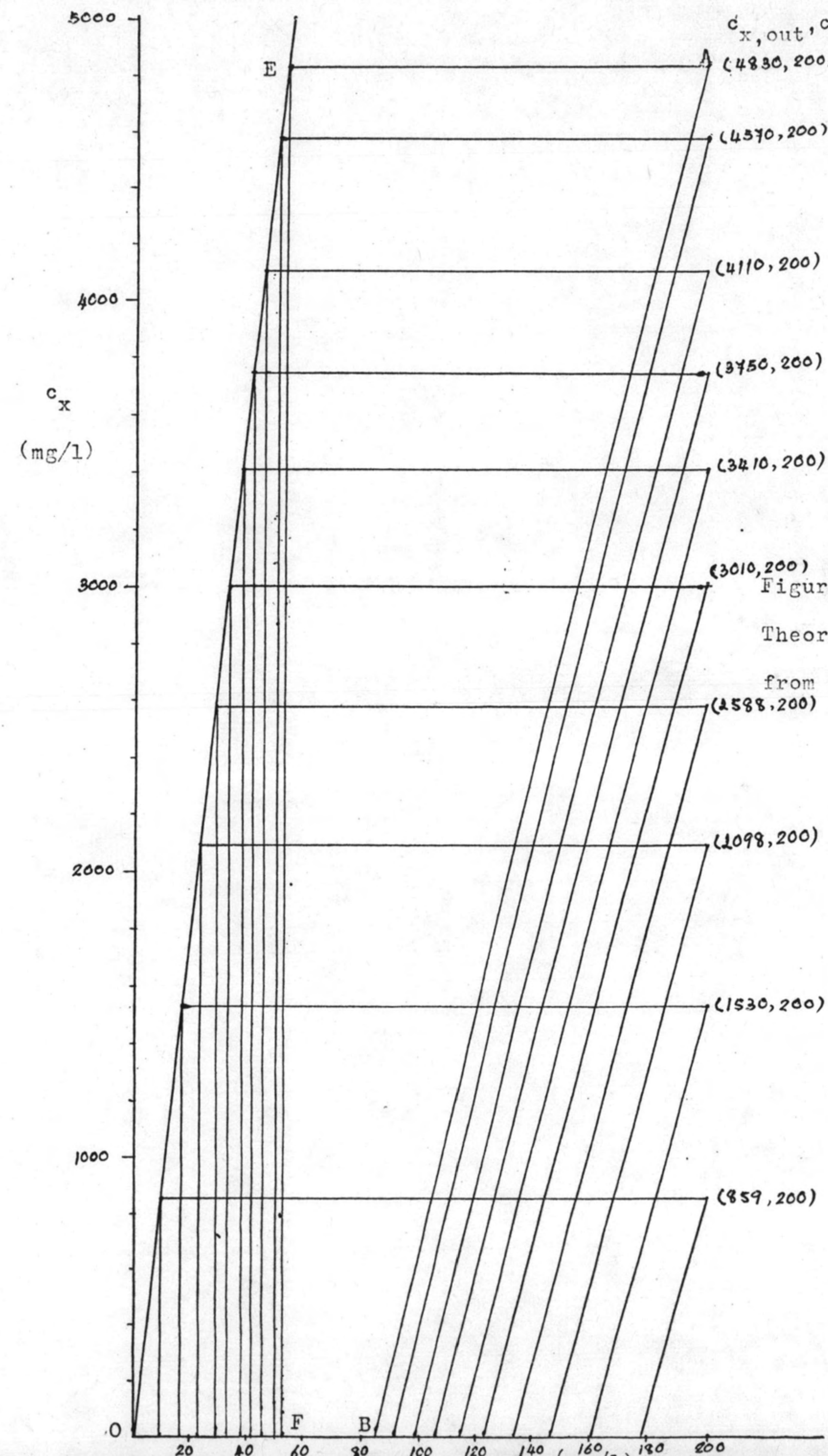


Figure 3-23 To find Theoretical plates from table 3-25

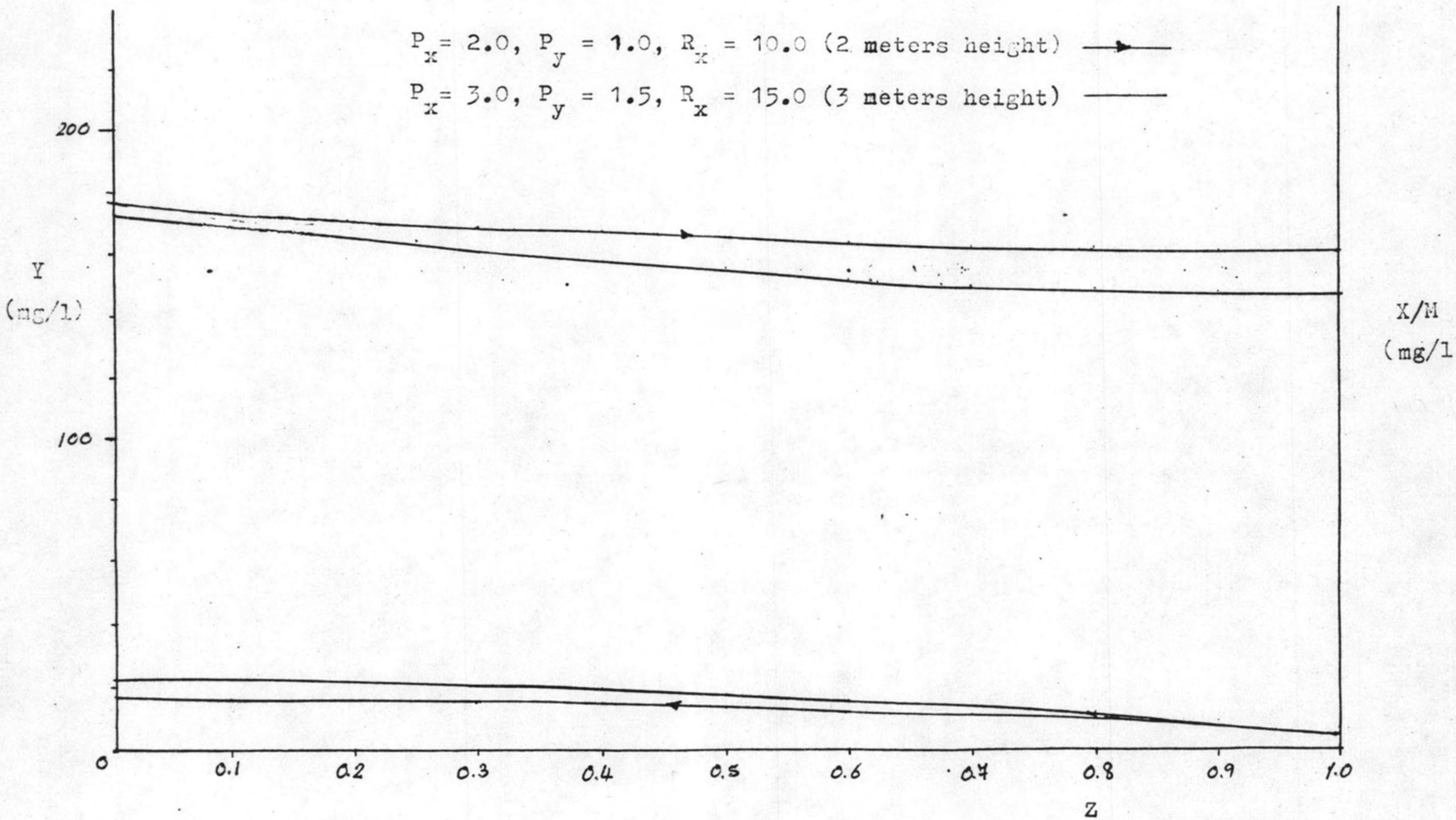


Figure 3-24 Concentration profiles of column at different height

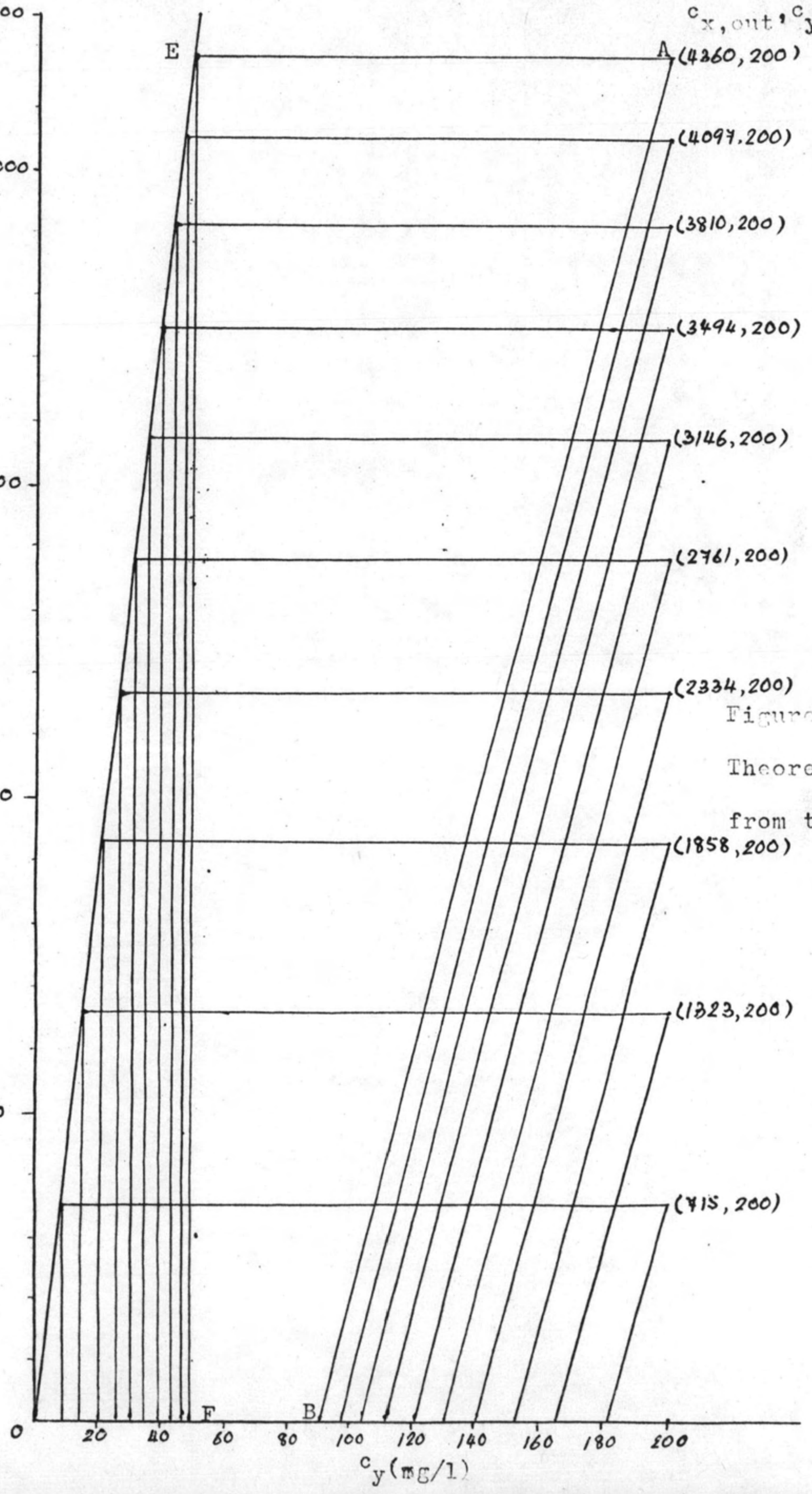


Figure 3-25 To find Theoretical plated from table 3-26

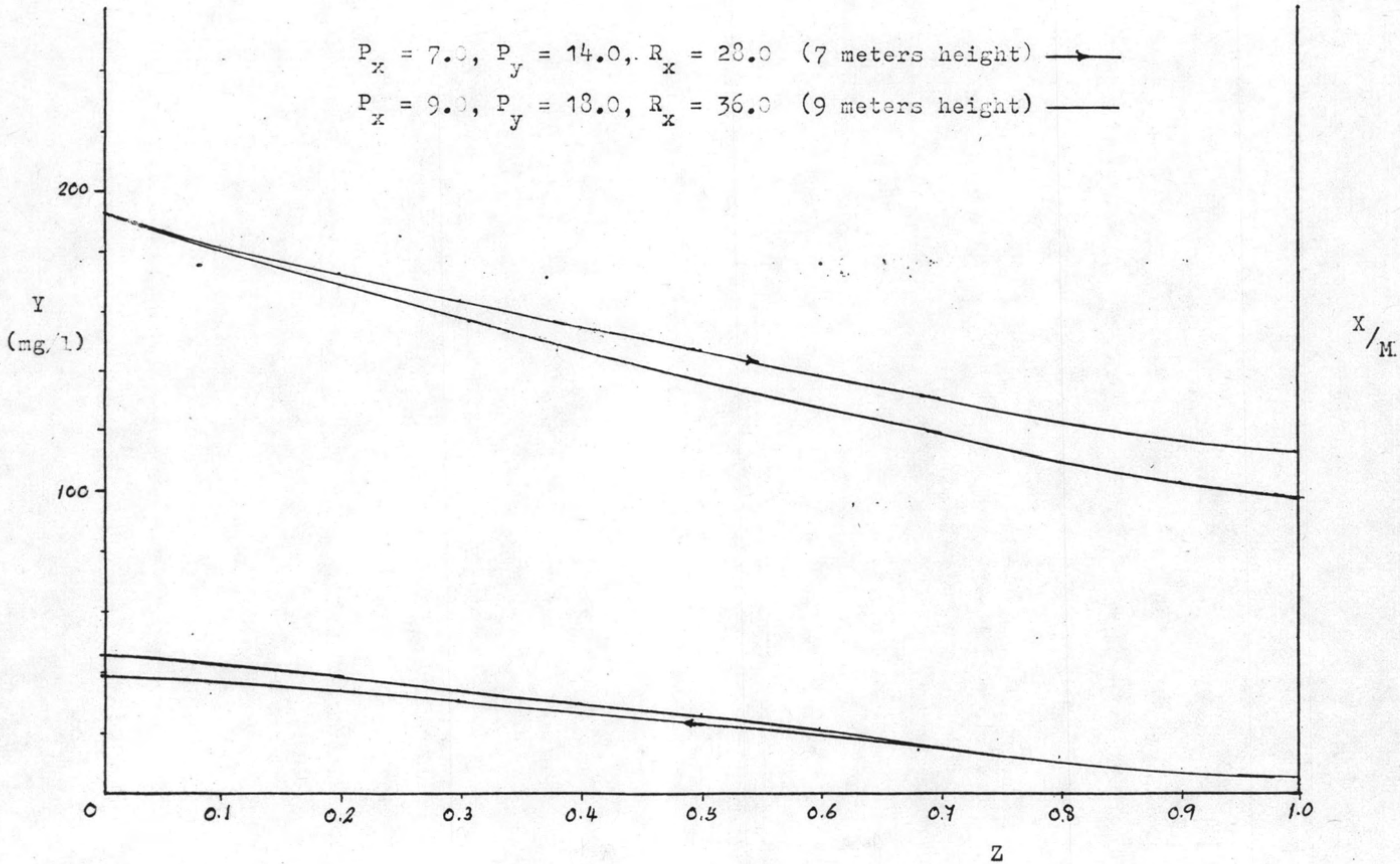


Figure 3-26 Concentration profiles of column at different height

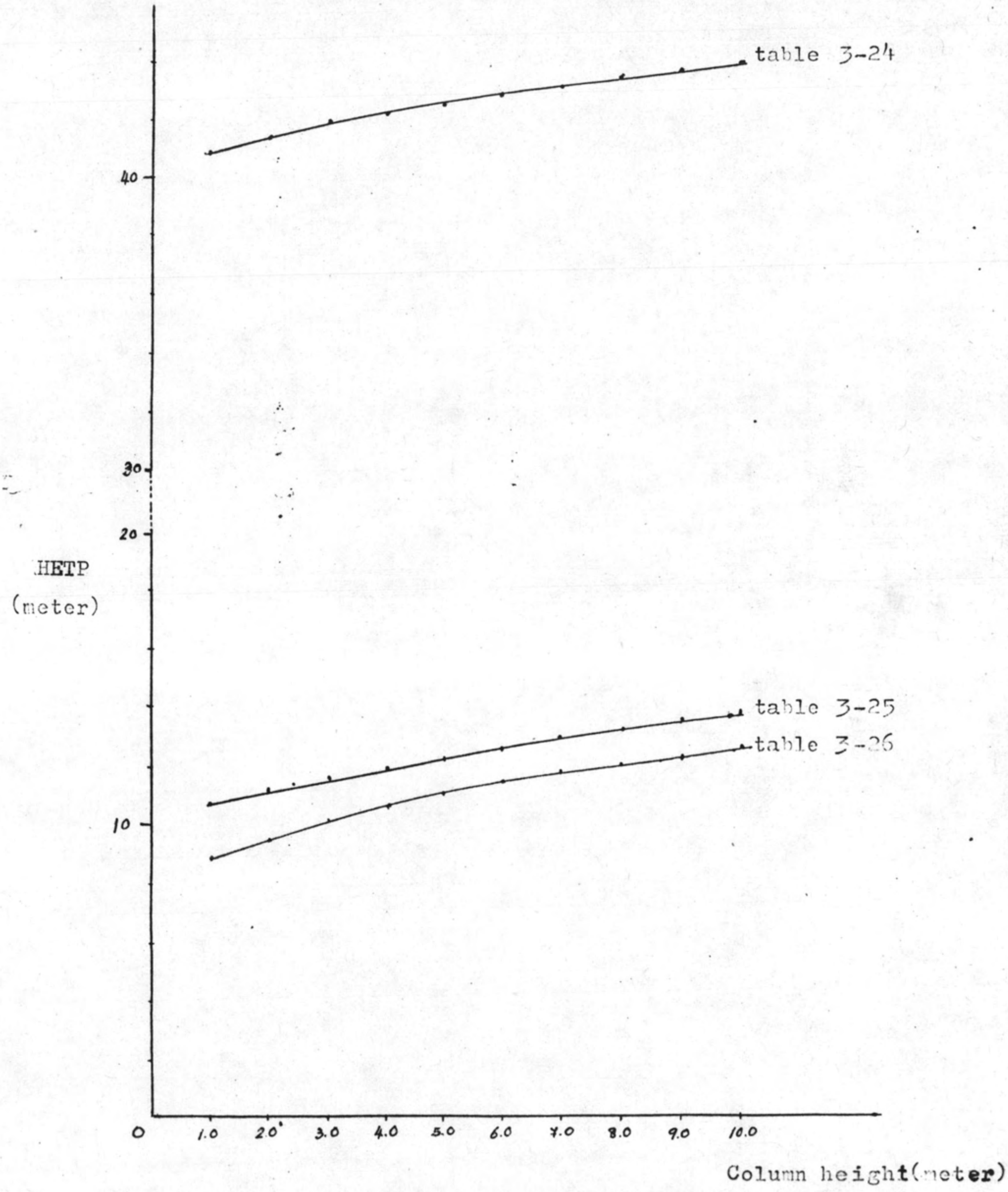


Figure 3-27 The relation of HETP with column height

3-22, 3-24 and 3-26 The other effect is the increase of the value of the Peclet number (the decrease of the degree of dispersion). This effect will give a lower value of HETP when the column is higher. But the first effect has eliminated the favorable influence established by the latter effect.

Therefore the results tend to give support the concencus that the HETP approach cannot be used to scale up column, a result which may Industrialists realized only after many years of experimentations with Industrial Columns. As this theoretical study dealt with only a linear equilibrium line it should be quite natural to expect that the same result would hold true for non-linear equilibrium lines.

3.3 Concentration profiles solutions.

The principal purpose of the work described here was to determine the influence of the Piston-flow assumption on column efficiency of this longitudinal mixing on the extent of interphase mass transfer in a liquid -liquid extraction column. The various sets of values of the parameters correspond to the same performance in evaluation of the analytical solution involved the measurement of concentration profiles. The solutions to the general and linear cases were therefore calculated on a computer. Appendix G. in flow diagram form, shows the computation program that was used. Values of X and Y were developed and tabulated (Appendix I).

In the computation of the longitudinal concentration profiles it was necessary to estimate values of the parameters P_X, P_Y and R_X. Values of P_Y and R_X were set constant at 2,5 and 10. Values for P_X were varied from 1 to 30.

The fractional-length, Z, values were selected to permit construction of the entire concentration profile (Z = 0, 0.05, 0.1, 0.15, 0.2, 0.25,... 1.0).

3.3.1 Criterion of the column efficiency

Various methods [5,9,13] may be used to estimate the effect of the axial dispersion on liquid-liquid extraction column performance. One interesting method will be discussed here, using the correlation of the results of concentrations and Λ parameter and a set of constants. In this method, the column efficiency is calculated by the correlation of the ratio R_{xp}/R_x where R_{xp} is the value of the number of transfer units under piston flow conditions which no axial mixing ($P_x = P_y = \infty$).

The number of transfer units in piston flow conditions may be evaluated from the desired solute concentrations in the outlet streams, from the Colburn Equation [9].

For $\Lambda \neq 1$,

$$R_{xp} = \frac{\operatorname{Ln} \dfrac{C}{C\Lambda + 1 - \Lambda}}{1 - \Lambda} \qquad (3\text{-}1)$$

For $\Lambda = 1$

$$R_{xp} = 1/C - 1 \qquad (3\text{-}2)$$

where

$$C = \frac{X_{in} - X_{out}}{X_{in} - Y_{in}/M} \qquad (3\text{-}3)$$

The number of transfer units is thus based upon a outlet concentration in X phase

3.3.2 Numerical example for column efficiency.

From diffusion model when $P_x = 10.0$, $P_y = 5.0$, $R_x = 2.0$
$\Lambda = MG_x/G_y = 89.9x1/40 = 2.2475$, $X_{in} = 0.0$, $Y_{in} = 200.0$ we obtain

$$X(0) = X_{out} = 372.73193$$ (see page 272)

Then, from equation (3-3)

$$C = \frac{0 - 372.73193}{0 - 200/89.9}$$

$$= 167.543$$

in this case , $\Lambda \neq 1$, so

$$R_{xp} = \frac{\mathrm{Ln}\ \dfrac{167.543}{167.543(2.2475) + 1 - 2.2475}}{1 - 2.2475}$$

$$= 0.646493$$

Column efficiency, E_c, will be calculated from

$$E_c = R_{xp}/R_x$$

$$= 0.646493/2.0$$

$$= 0.3232465$$

which means that the column must be 68% longer than the ideal one in order to overcome the effect of axial Mixing

Other results are shown in table 3-27, 3-28 and figure 3-28, 3-29

3.3.3 Discussion and Conclusion

A Method of defining efficiency is developed by correlating experimental simulations and an efficiency is defined using parameter Λ and a set of outlet concentrations in the X-phase and inlet concentrations in the Y-phase. The results are presented in Tables 3-27, 3-28 and also in Figures 3-28, 3-29. It can clearly be seen that column efficiency tends to increase very rapidly when the Peclet number increases from 1 to 10 and then the increase will be gradual when the Peclet number is above 10. When Tables 3-27 and 3-28 are compared, Column efficiency will be decreased by with an increase in the Number of transfer units.

Table 3-27 Column Efficiency(E_c) for $P_y = 2, R_x = 2$.

P_x	X_{out}	C	R_{xp}	$E_c = R_{xp}/R_x$
1.0	365.10083	164.11282	0.6464373	0.3232187
2.0	367.96631	165.40086	0.6464585	0.3232293
3.0	369.19800	165.95450	0.6464675	0.3232338
4.0	369.81519	166.23193	0.6464720	0.3232360
5.0	370.13745	166.37678	0.6464743	0.3232372
10.0	370.79004	166.67012	0.6464792	0.3232396
15.0	371.01953	166.77328	0.6464808	0.3232404
20.0	371.14258	166.82859	0.6464817	0.3232409
30.0	371.26489	166.88357	0.6464824	0.3232412

<u>Table</u> 3-28 Column Efficiency(E_c) for p_y= 2 ,R_x= 5.

P_x	X_{out}	C	R_{xp}	$E_c=R_{xp}/R_x$
1.0	843.79321	379.28505	0.6479792	0.1295958
2.0	852.22119	383.07343	0.6479907	0.1295981
3.0	856.27393	384.89513	0.6479963	0.1295993
4.0	858.57007	385.92725	0.6479993	0.1295999
5.0	859.97705	386.55968	0.6480013	0.1296003
10.0	863.18042	387.99600	0.6480054	0.1296011
15.0	864.40161	388.54852	0.6480072	0.1296014
20.0	865.04858	388.83934	0.6480080	0.1296016
30.0	865.70898	389.13619	0.6480089	0.1296018

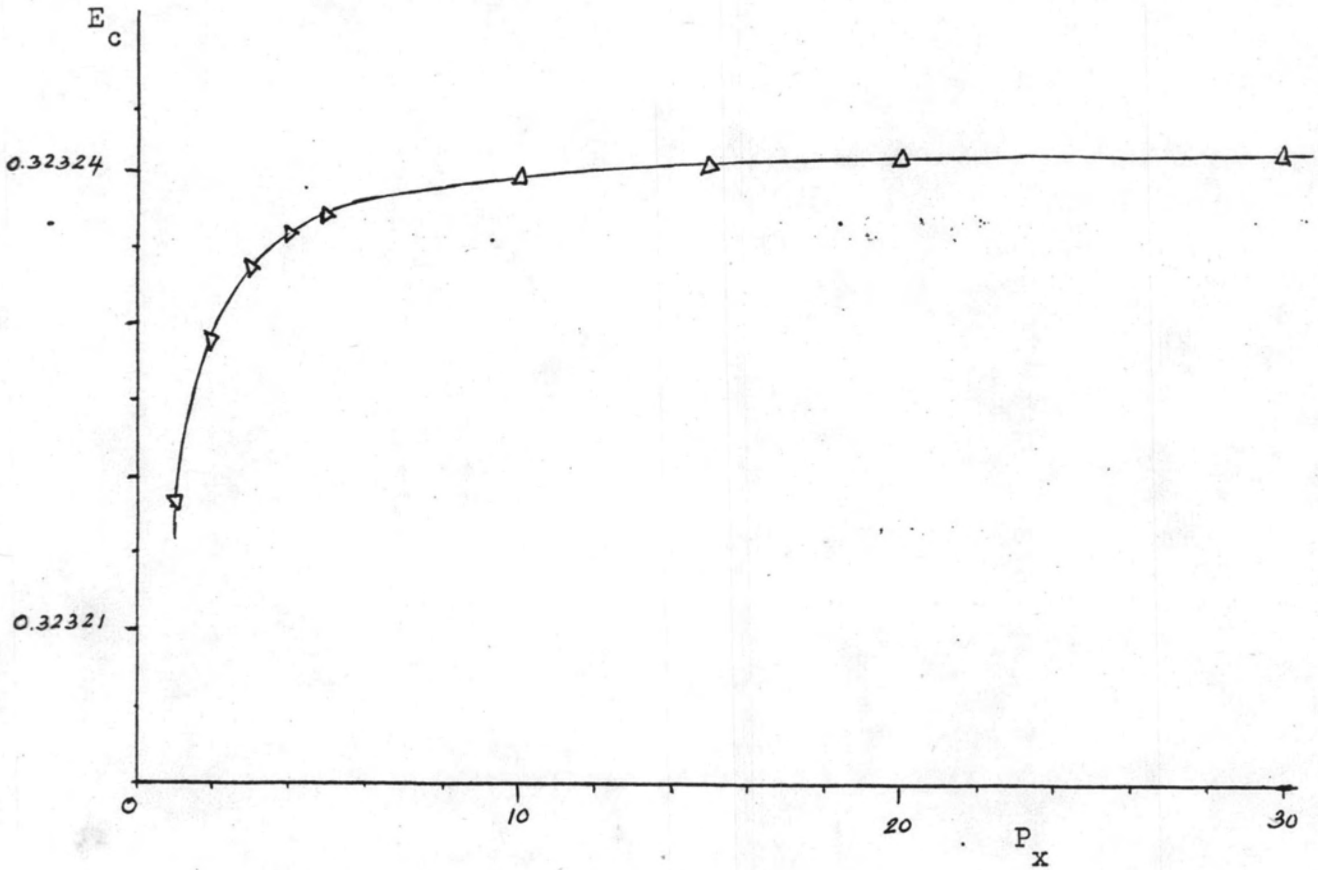


Figure 3-28 Column Efficiency with Peclet number in drop phase for $P_y = 2$, $R_x = 2$

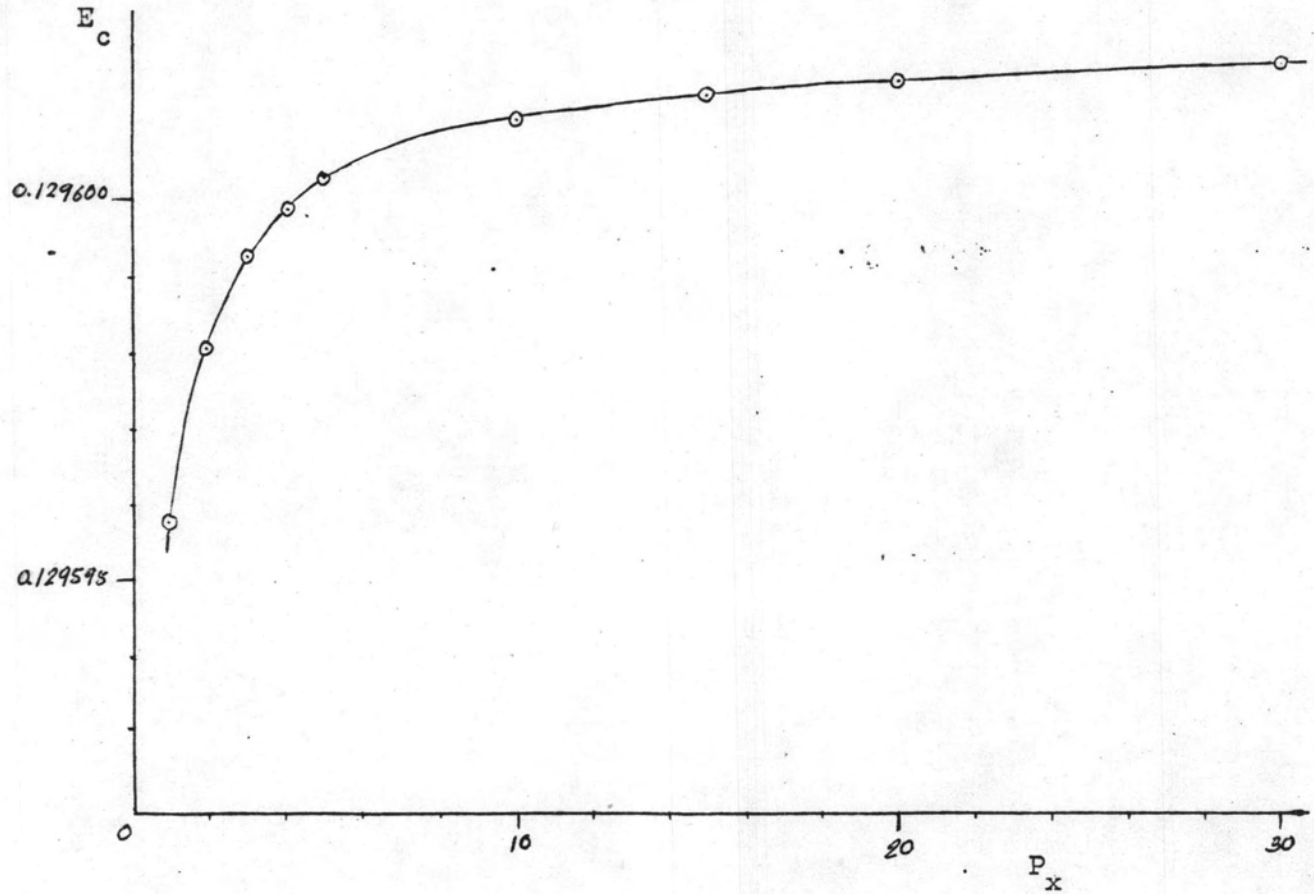


Figure 3-29 Column Efficiency with Peclet number

in drop phase for $P_y = 2$, $R_x = 5$