Chapter II

THE DETERMINATION OF MASSES OF SLOW PARTICLES IN NUCLEAR EMULSIONS

When the slowly moving charged particle passes through matters, its energy loss is almost entirely due to inelastic collisions between the charged particle and the bound electrons in the atoms of the medium. When the energy transferred to an electron is greater than ionization energy, the electron will escape from the atom. This interaction is called ionization. The ionization loss of charged particle can be calculated by using Bethe's formula.

Bethe's formula at non-relativistic velocity is

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mv^2} NZ \ln \frac{2mv^2}{I} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where m is electronic mass,

v is velocity of the charged particle,

z is the charge of the particle coming into the medium,

Z is the average atomic number of the medium,

N is the number of atoms per c.c. of the medium,

I is the mean ionization potential of the medium,

and $-\frac{dE}{dx}$ is the average total energy loss by ionization per cm.

The above formula is valid when the energy of the charged particle satisfies the inequality

$$\frac{M}{m} E_K \ll E \ll Mc^2.$$

Where M and m are the rest masses of the incident particle and electron respectively. E_K is the kinetic energy of electron in the K shell.

When the charged particle passes through the nuclear emulsion, the atoms of AgBr which distribute uniformly in nuclear emulsion, are ionized. After development, the grains of silver along the track of the particle can be seen. The silver grain density of the track of the charged particle is proportional to the energy loss by ionization of the charged particle, i.e.

$$Q\left(-\frac{dE}{dx}\right) = n_g, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where Q is a proportional constant and n_g is the grain density. Hence

$$n_g = Q\,\frac{4\pi z^2 e^4}{mv^2}\,NZ\,\ln\frac{2mv^2}{I}$$

$$= Qz^2F(v), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where

$$F(v) = \frac{4\pi e^4}{mv^2}\,NZ\,\ln\frac{2mv^2}{I}$$

One can see that the grain density is a function of the charge and the velocity of the incident charged particle. Hence the particles of the same charge, when they have the same velocity, the grain densities along their tracks are equal.

The length of the track of a particle at the points where the particle has energies E_o and E can be written as

$$l = \int_{E_o}^{E} \frac{-dE}{\left(-\frac{dE}{dx}\right)}, \text{ where } E_o > E.$$

One defines the range to be the distance from the point that the charged particle comes to rest to the point which is considered. If one considers the point corresponding to the energy E_o, the range at that point is

$$R(E_o) = \int_0^{E_o} \frac{dE}{(-\frac{dE}{dx})}$$

$$= \int_{E_{min}}^{E_o} \frac{dE}{(-\frac{dE}{dx})} + \sigma_R, \quad \ldots\ldots\ldots\ldots\ldots (4)$$

where $\sigma_R = \int_0^{E_{min}} \frac{dE}{(-\frac{dE}{dx})}$.

E_{min} is the minimum energy that Bethe's formula is valid.

For the singly charged particle σ_R is very small comparing to the first term in the above expression. Hence one can write

$$R(E_o) = \int_{E_{min}}^{E_o} \frac{dE}{(-\frac{dE}{dx})}$$

$$= \frac{M}{Z} \frac{m}{4\pi e^4 N} \int_{v_{min}}^{v_o} \frac{v^3 dv}{\ln\,(2mv^2/I)}$$

$$= Mf(v_o), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where

$$f(v_o) = \frac{m}{4\pi e^4 NZ} \int_{v_{min}}^{v_o} \frac{v^3 dv}{\ln\,(2mv^2/I)}$$

is the function of the velocity at the considered point.

If the first singly charged particle has the range R_1 and mass M_1 and the second singly charged particle has the range R_2 and mass M_2. At the same velocities or grain densities, from equation (5), one obtains

$$\frac{M_1}{M_2} = \frac{R_1}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Therefore if one knows the relation between the range and the grain density of the first particle and the range and the grain density of the second particle, one can find the ratio of masses of the two particles. At first this method has been used by Lattes et al.[1] Their observations led to the discovery of π-mesons.

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[1] C.M.G. Lattes et al., Nature, 159(1947), 694.