CHAPTER I

PRELIMINARIES

Throughout this thesis much work will be done in sets of numbers. Our notations using for these sets are:

$\mathbb{N}$ is the set of all natural numbers excluding zero;

$\mathbb{Q}$ is the set of all rational numbers;

$\mathbb{Q}^+$ is the set of all positive rational numbers;

$\mathbb{R}^+$ is the set of all positive real numbers;

$\mathbb{Z}$ is the set of all integers;

$\mathbb{Z}_n$, $n \in \mathbb{N}$ is the set of congruence classes modulo n in $\mathbb{Z}$.

The following definition of semiring will be used in this thesis. This definition is slightly different from the one given in [1].

<u>Definition 1.1.</u> A nonempty set S is said to be a <u>semiring</u> if there are two binary operations, + (addition) and $\cdot$ (multiplication) defined on it such that :

(i)S is a commutative semigroup with respect to addition and multiplication;

(ii) $x(y + z) = xy + xz \quad \forall\ x, y, z \in S$.

Note that $\mathbb{N}$ with the usual addition and multiplication is a semiring.

The following theorems will be used. The proof of these theorems will not be given but can be found in the references.

<u>Theorem 1.2.</u> If p is a prime number, then $\mathbb{Z}_p$ is a field.

See [4] , page 91.

<u>Theorem 1.3.</u> The smallest subfield of a field is either isomorphic to $\mathbb{Q}$ or $\mathbb{Z}_p$ for some prime number p.

See [5] , page 7 - 8.

<u>Theorem 1.4.</u> Every integral domain can be embedded into a field.

See [4] , page 101 - 103.

The field that was constructed in this theorem is called the <u>field of quotients</u> of the given integral domain.

The above theorem also says that if R is an integral domain, then the field of quotients of R is the smallest field containing R.

<u>Theorem 1.5.</u> Any finite abelian group is the direct product (sum) of a finite number of finite cyclic groups.

See [4] , page 162 - 164.