CHAPTER 1

INTRODUCTION

In [1] , Arom consider two-by-two table :

X_1 . . X_P	Y_1 . . Y_Q
Z_1 . . Z_R	W_1 . . W_S

where entries are observations from an experiment in which observations are classified by two factors. A quadruples (X_i, Y_j, Z_k, W_l) in which the relation

$$(1) \qquad \begin{array}{ccc} X_i & > & Y_j \\ \vee & & \wedge \\ Z_k & < & W_l \end{array}$$

or

$$(2) \qquad \begin{array}{ccc} X_i & < & Y_j \\ \wedge & & \vee \\ Z_k & > & W_l \end{array}$$

hold, were used to measure interaction between the two factors.
To count such quadruples, the random variables

$$I_{ijkl} = \begin{cases} 1 & \text{if (1) holds ,} \\ 0 & \text{otherwise ,} \end{cases}$$

and

$$J_{ijkl} = \begin{cases} 1 & \text{if (2) holds ,} \\ 0 & \text{otherwise ,} \end{cases}$$

were introduced.

The work in [1] concerned with the asymptotic behavior of the joint distribution of

$$I = \sum_{l=1}^{S} \sum_{k=1}^{R} \sum_{j=1}^{Q} \sum_{i=1}^{P} I_{ijkl}$$

and

$$J = \sum_{l=1}^{S} \sum_{k=1}^{R} \sum_{j=1}^{Q} \sum_{i=1}^{P} J_{ijkl}$$

when P, Q, R, S become large.

Note that the random variables I_{ijkl} and J_{ijkl} are not mutually independent, so that the classical central limit theorem is not applicable for obtaining the limit distribution of their sums I, J. However, it was shown in [1] that the joint distribution of $\frac{I-E(I)}{\sqrt{var(I)}}$ and $\frac{J-E(J)}{\sqrt{var(J)}}$ tends to a bivariate normal distribution.

In this thesis we consider a more general situation. Let

$$\begin{array}{ccc} X_{11} & \cdots & X_{1n_1} \\ X_{21} & \cdots & X_{2n_2} \\ \cdots & \cdots & \cdots \\ X_{l1} & \cdots & X_{ln_l} \end{array}$$

be independent random variables such that the random variables in the same row have the same distribution.

Let $f_p(x_1, \ldots, x_l)$, $p = 1, \ldots, k$, be real-valued functions which are bounded and integrable. Put

$$U_p = \sum_{i_l=1}^{n_l} \cdots \sum_{i_1=1}^{n_1} f_p(X_{1i_1}, \ldots, X_{li_l}), \; p = 1, \ldots, k .$$

Our purpose is to determine the asymptotic joint distribution of $U_1, \ldots, U_k$. Observe that the random variables I_{ijkl} and J_{ijkl} can be viewed as functions of the random variables X_i, Y_j, Z_k, W_l . Hence the result of [1] will be a special case of this study. To illustrate the usefulness of our result. We show how our main theorem (Theorem 3.3.2) can be applied to obtain an asymptotic joint distribution of Whitney's bivariate U statistic [3].