CHAPTER II

Electric Fields of a Corner Reflector Antenna

2.1 <u>An Antenna and its Image</u>. Before facing the investigation of characteristic of corner reflector, let's consider the electric field from an array at any orientation.

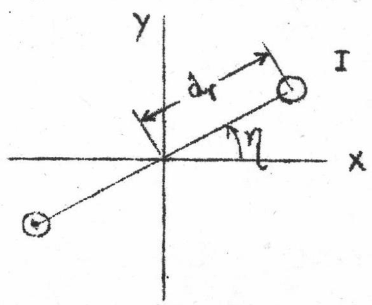
<u>Case</u> 1 Two short dipoles and inphase are placed parallel to Z axis. The electric field at distant point in xz plane is

$$E_H = 2kI\cos\theta\cos(d_r\cos\eta\cos\theta)$$

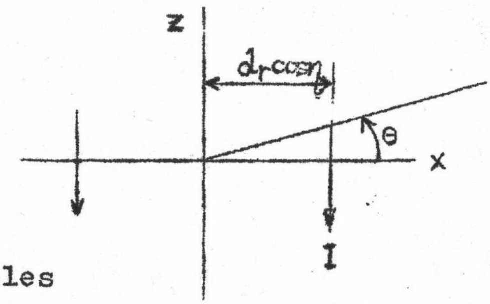
If η is 0 and 90°

$$E_H = 2kI\cos\theta\cos(d_r\cos\theta) \quad \Rightarrow \eta = 0^\circ$$

$$= 2kI\cos\theta \quad \Rightarrow \eta = 90^\circ$$



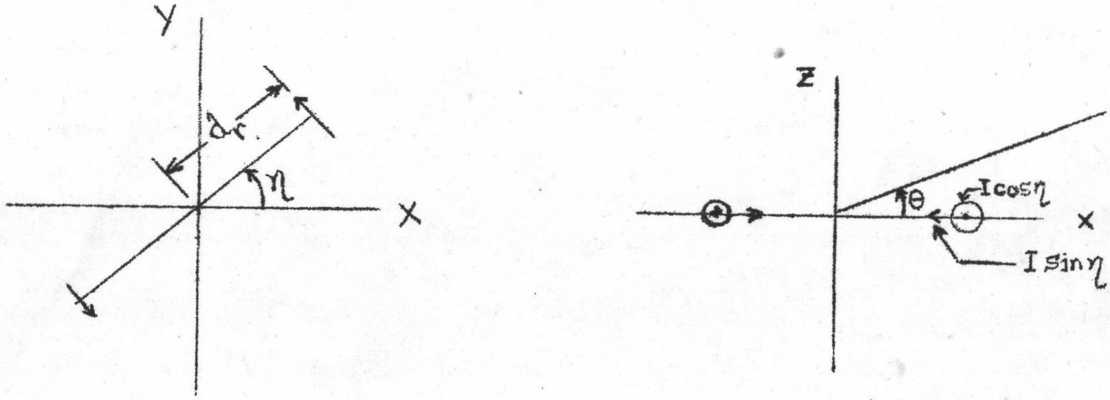
Array of dipoles



View in Z-direction.

View in Y-direction.

<u>Case</u> 2 Two short dipoles and opposite phase are placed in XY plane and parallel to each other . To solve the field at distant point in XZ plane, the dipoles should be resolved in X-direction and Y-direction. The fields at far point will compose of field in X-direction and field in Y-direction.



View in Z-direction. View in Y-direction.

Array of dipoles

$$E_{Vx} = j2kI\sin\eta\,\sin\theta\,\sin(d_r\cos\eta\cos\theta)$$

$$E_{Vy} = j2kI\cos\eta\,\sin(d_r\cos\eta\cos\theta)$$

We conclude that the arrays of inphase-short dipoles placed parallel to Z axis, radiate the horizontal field which is

$$E_H = 2kI\cos\theta\cos(d_r\cos\eta\cos\theta)$$

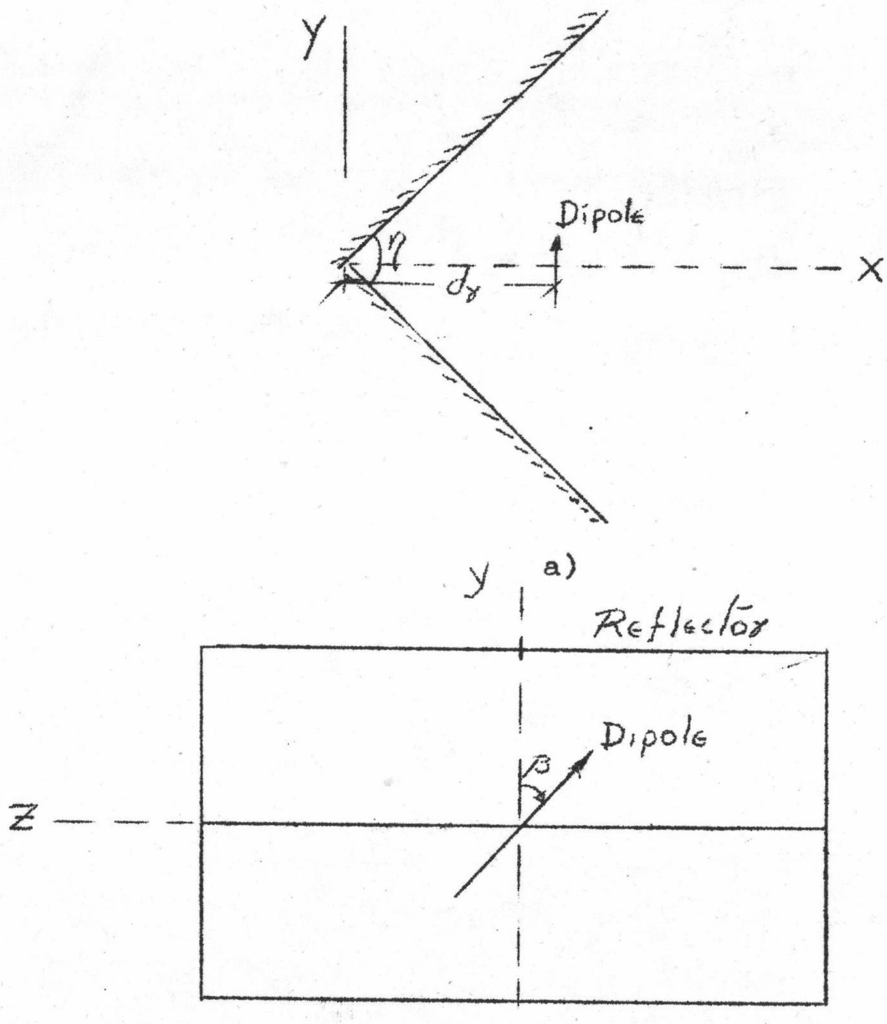
and the arrays of opposite-phase-short dipoles placed in XY plane radiate fields in Y-direction and X-direction which are

$$E_{Vy} = j2kI\cos\eta\sin(d_r\cos\eta\cos\theta)$$

$$E_{Vx} = j2kI\sin\eta\sin\theta\sin(d_r\cos\eta\cos\theta)$$

2.2 The Corner Reflector Antenna.

2.2.1 Dipole Orientation. The dipole is placed in Y-Z plane and makes angle β with Y-axis. The corner reflector angle is η, d_r is the space between dipole and corner angle, which is shown in Fig. 2.1



b)

Fig. 2.1 Arrangement of Dipole and Reflector
a) Side view, b) Front view.

2.2.2 Field Analysis. The dipole in Fig. 2.1b can be resolved into a horizontal and vertical component, as shown in Fig. 2.2. Each component has its images as shown in Fig. 2.3 and Fig. 2.4.

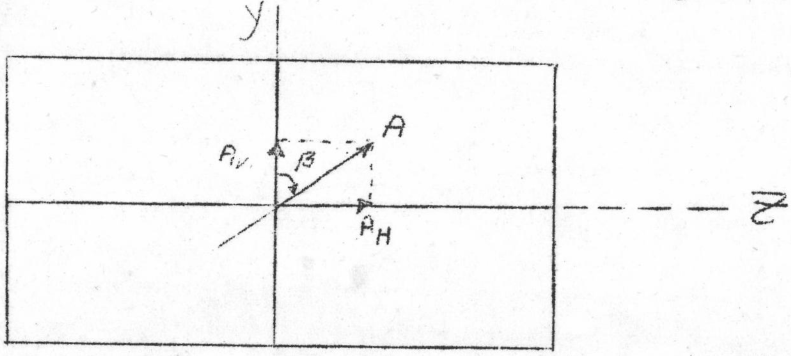


Fig. 2.2 Components of dipole.

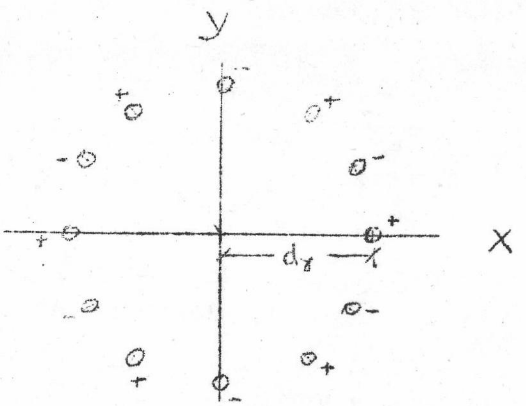


Fig. 2.3 Horizontal images of dipole.

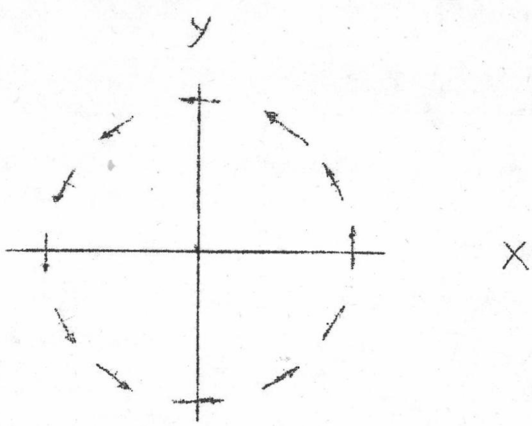


Fig. 2.4 Vertical images of dipole.

The number of images are depend on corner angle of reflector, which can be calculated from equation below.

$$N = \frac{360^\circ}{\eta} - 1 \qquad (31)$$

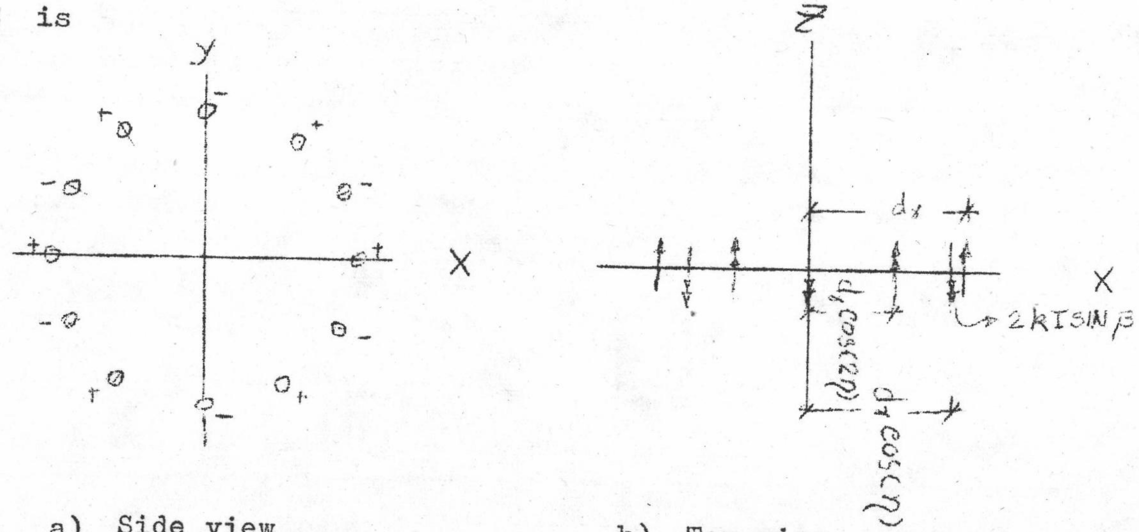
where N = number of images

η = corner angle

We are now interested in the corner angle which forms an even numbers of dipoles, because a principle of arrays can be applied to derive electric field at distant point.

A horizontal dipole produces a horizontal electric field. The total field at far point is the sum of fields due to all arrays of horizontal dipoles which orients around the corner. The phases of dipoles which form anarray are inphase or opposite phase according to corner angle. If it forms even numbers of arrays, dipoles which form an array are inphase, and if odd numbers of arrays are formed, Phases of dipoles which form anarray are opposite.

Let corner angle be η which produces K arrays of the horizontal dipoles oriented around corner as in Fig. 2.5. The total field E is



a) Side view b) Top view

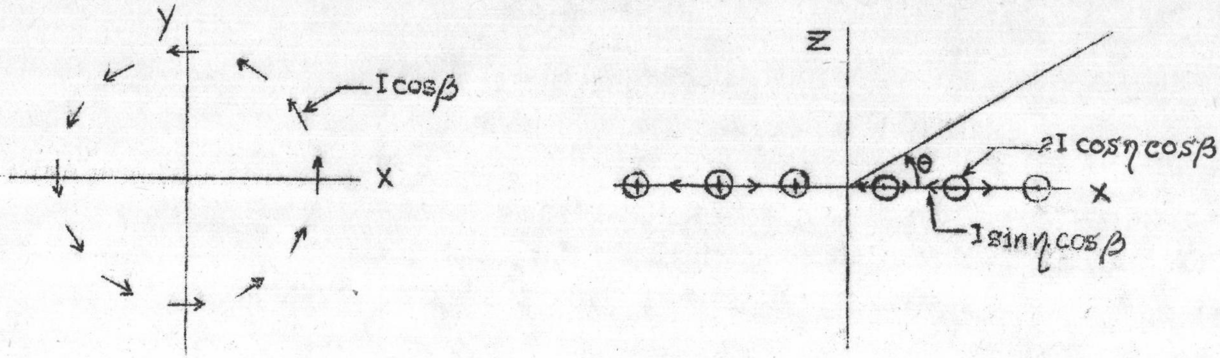
Fig. 2.5 Images of horizontal dipole

$$
\begin{aligned}
E_H = \; & 2kI\sin\beta\cos\theta\cos(d_r\cos\theta) \\
& -2\times 2kI\sin\beta\cos\theta\cos(d_r\cos\eta\cos\theta) \\
& +2\times 2kI\sin\beta\cos\theta\cos(d_r\cos(2\eta)\cos\theta) \\
& -2\times 2kI\sin\beta\cos\theta\cos(d_r\cos(3\eta)\cos\theta)+\ldots
\end{aligned}
\qquad (32)
$$

$$E_H = 2kI\sin\beta\Big[\cos\theta\,\cos(d_r\cos\theta) + 2\sum_{n=1}^{K-1}(-1)^n\cos(d_r\cos(n\eta)\cos\theta) + (-1)^K\Big] \quad (33)$$

where K = 90/ , Kis integer. (34)

At the same time a vertical dipole produced a vertical field. Total fields at distant point is the sum of fields radiated from dipole and its images which are shown in Fig. 2.6. Dipoles which form an array are opposite phase. Thus electric field from an array is



a) View in Z-direction. b) View in Y-direction.

Fig. 2.6 Images of vertical dipole

To solve the fields, the dipoles are splited into X-axis and Y-axis as shown in Fig. 2.6b. The electric fields in X axis will be vanished Therefore, the electric field due to vertical dipoles is

$$\begin{aligned} E_V = {} & j2kI\cos\beta\sin(d_r\cos\theta) + 2j2kI\cos\beta\sin(d_r\cos\eta\cos\theta)\cos\eta \\ & + 2j2kI\cos\beta\sin(d_r\cos\theta\cos(2\eta))\cos(2\eta) \\ & + 2j2kI\cos\beta\sin(d_r\cos\theta\cos(3\eta))\cos(3\eta) + \dots \end{aligned} \quad (35)$$

$$E_V = j2kI\cos\beta\Big[\sin(d_r\cos\theta) + 2\sum_{n=1}^{K-1}\cos(n\eta)\sin(d_r\cos(n\eta)\cos\theta)\Big] \quad (36)$$

where $K = 90/\eta$, K is integer (37)

At $\theta = 0$ equation (36) becomes

$$E_V = j2kI\cos\beta\left[\sin(d_r) + 2\sum_{n=1}^{K-1}\cos(n\eta)\sin(d_r\cos(n\eta))\right] \tag{38}$$

It is seen from equation (33) and (36) that E_H is inphase quadrature with E_V. The elliptical polarization in YZ-plane is obtained. The circular polarization appears at some directions. But the circular polarization in X axis is interesting. To receive this condition requires that the amplitudes of E_H and E_V are equal. From the equation (33) and (38), and at $\theta = 0^\circ$

$$2kI\sin\beta\left[\sum_{n=1}^{K-1} 2(-1)^n\cos(d_r\cos(n\eta)) + \cos(d_r) + (-1)^K\right]$$
$$= 2kI\cos\beta\left[\sum_{n=1}^{K-1} 2\cos(n\eta)\sin(d_r\cos(n\eta)) + \sin(d_r)\right]$$

$$\tan\beta = \frac{\sum_{n=1}^{K-1} 2\cos(n\eta)\sin(d_r\cos(n\eta)) + \sin(d_r)}{\sum_{n=1}^{K-1} 2(-1)^n\cos(d_r\cos(n\eta)) + \cos(d_r) + (-1)^K} \tag{39}$$

This equation shows that circular polarization in the plane normal to X-axis, will occur or not, depends on the dipole orientation β. For a given corner angle and dipole distance d_r, angle β can be calculated directly from equation (39). Not that equation (39) is valid only in the case that corner angle must satisfy equation (34). If the value of β calculated from equation (39) are equal to 0° or 90°, they mean that no fields in both vertical and horizontal. Because total fields in each component are zero. The curves of Fig. 2.7 to Fig. 2.10 shows the relation of dipole angle β and dipole distance d_r.

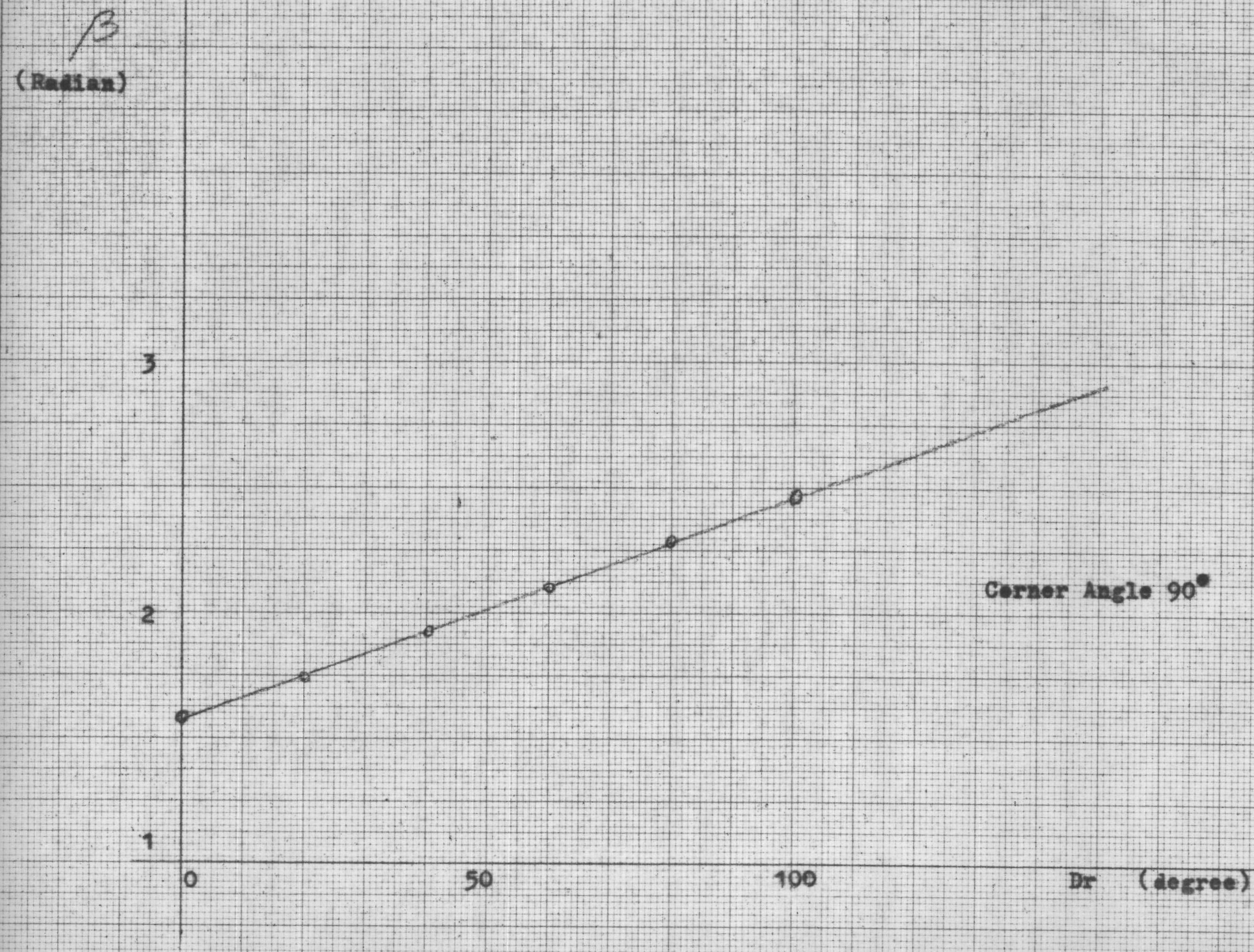


Fig. 2.7 The relation of dipole angle and dipole distance.

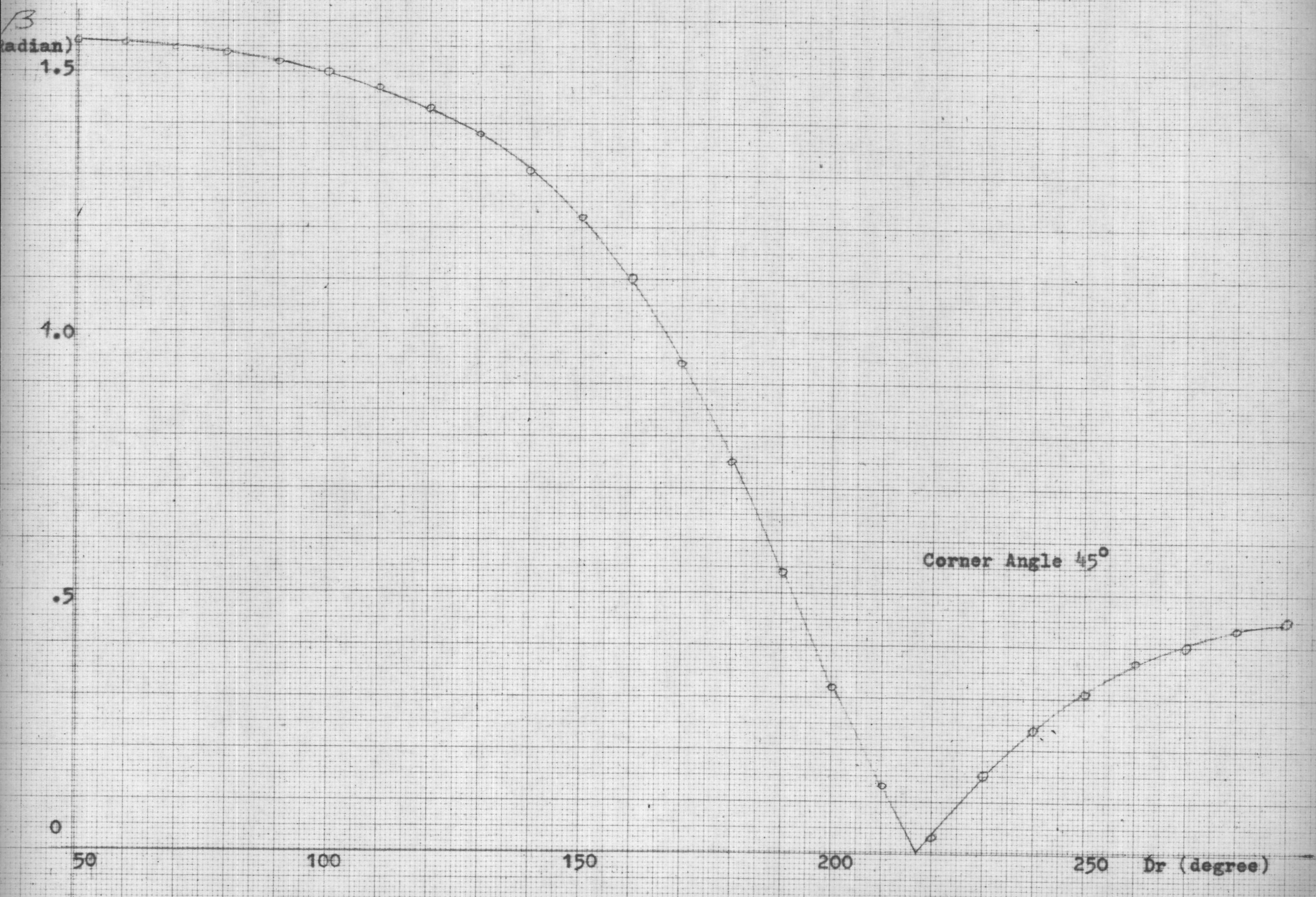


Fig. 2.8 The relation of dipele angle and dipole distance.

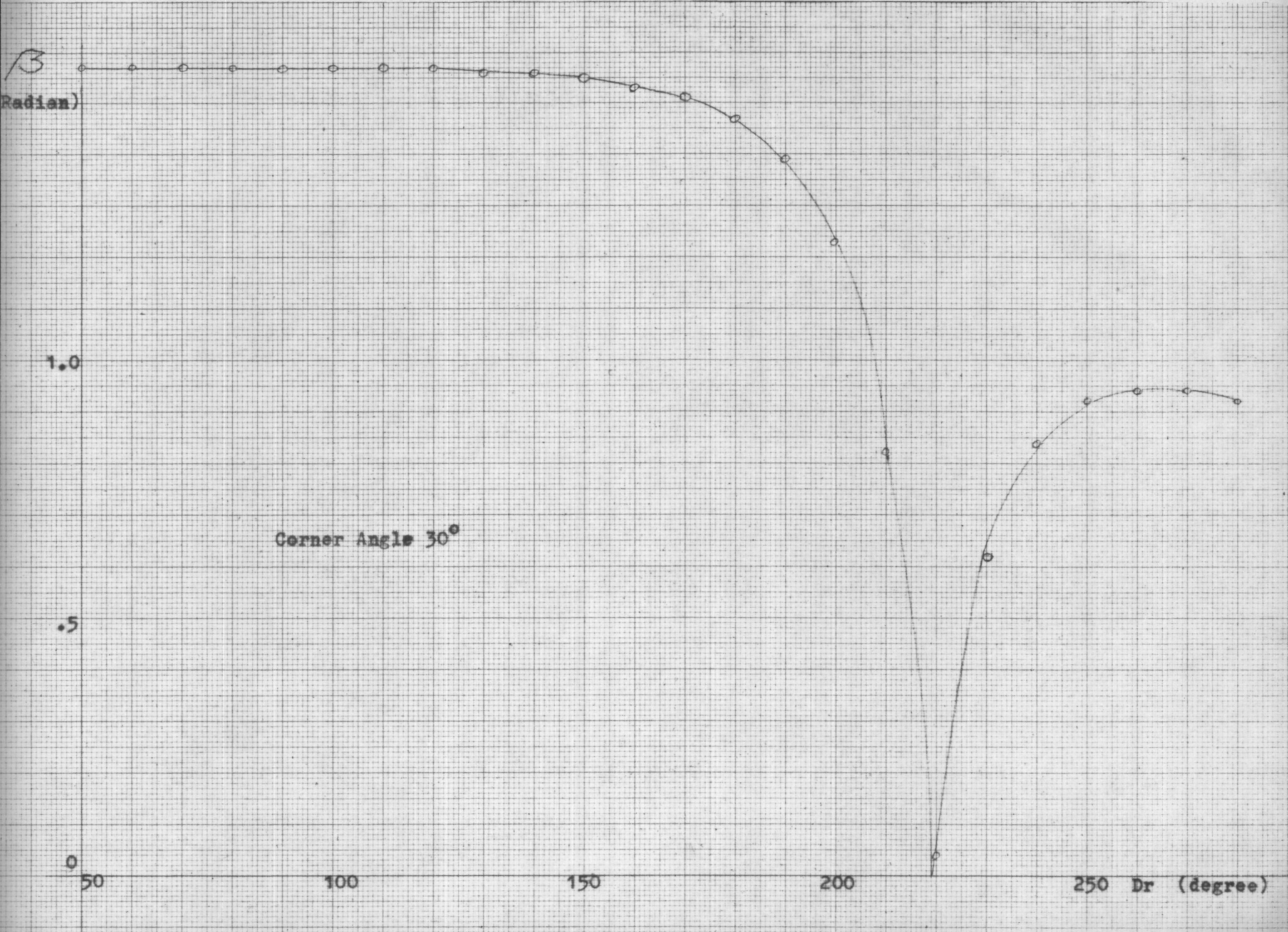


Fig. 2.9 The relation of dipole angle and dipole distance.

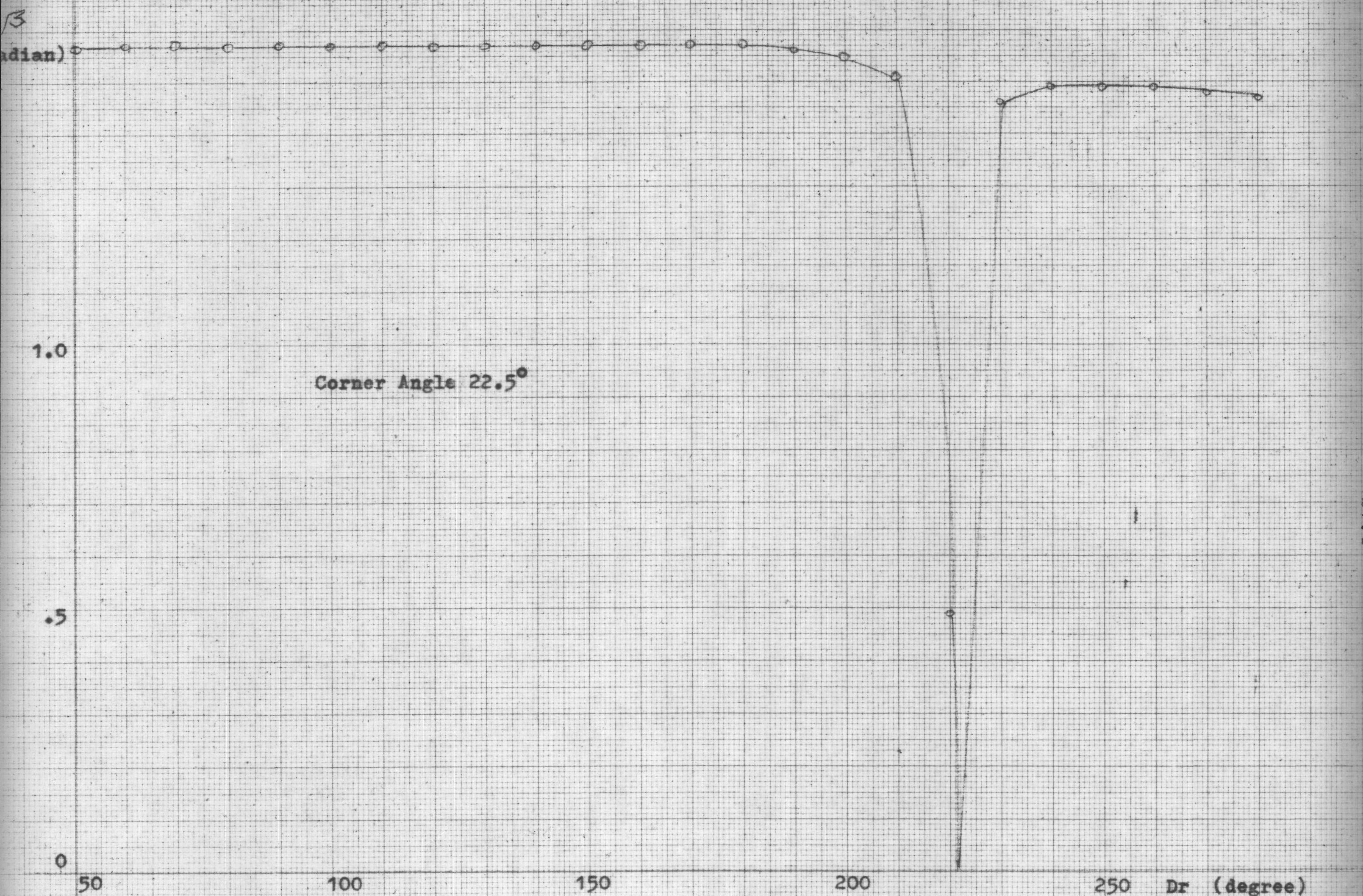


Fig. 2.10 The relation of dipole angle and dipole distance.

27.4

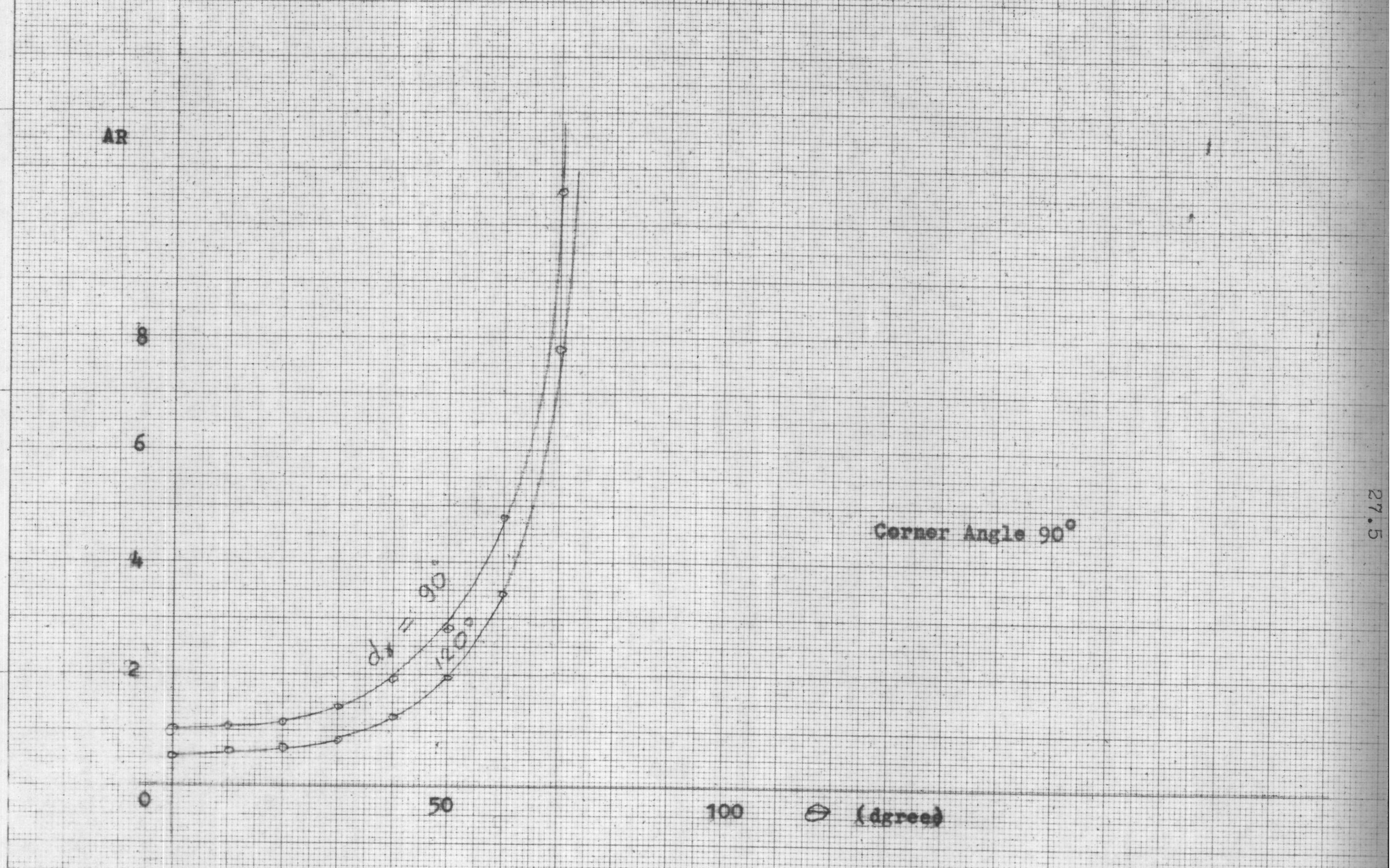


Fig. 2.11 The variation of axial ratio as a function of off angle.

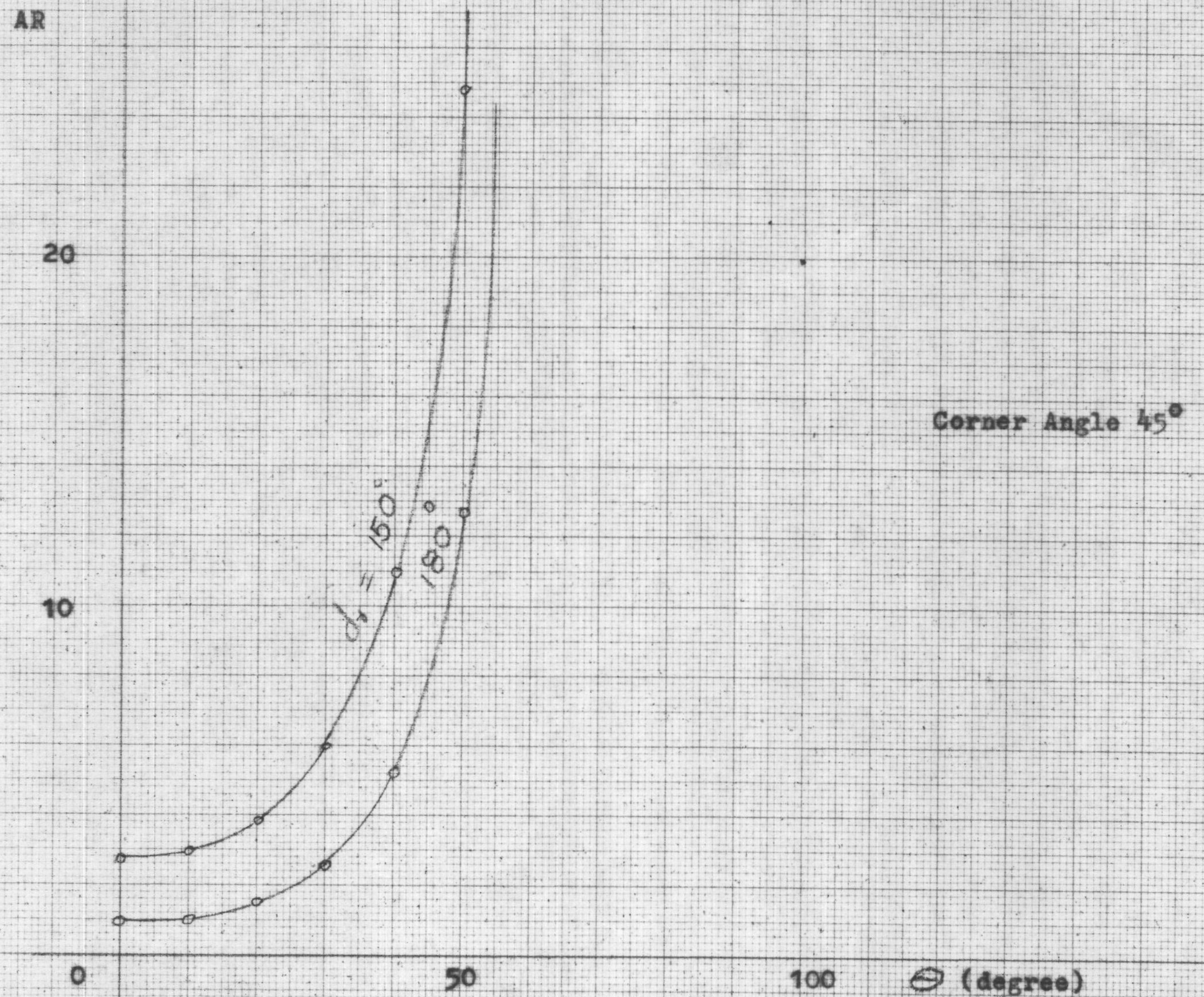


Fig. 2.12 The variation of axial ratio as a function of off angle.

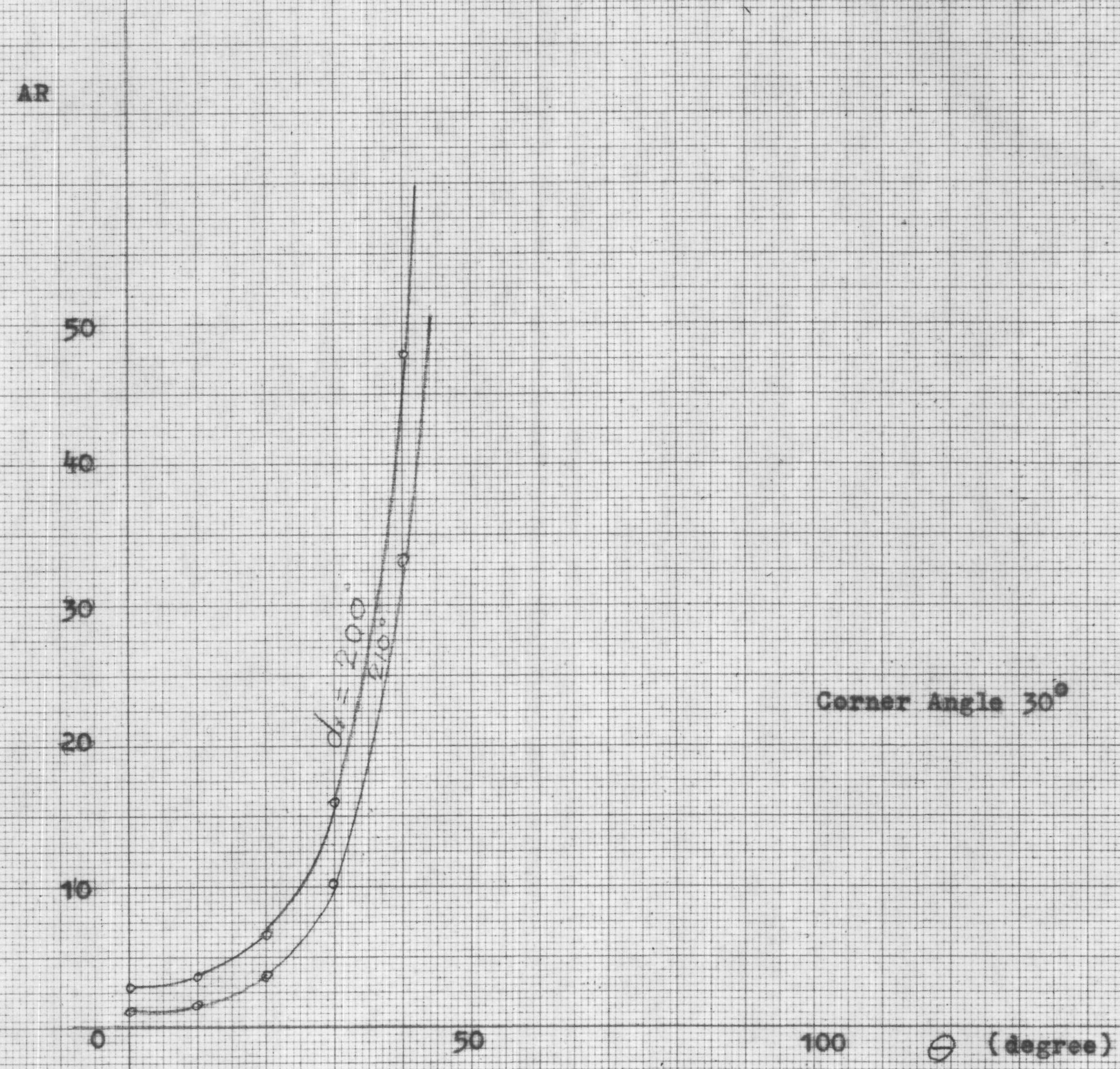


Fig. 2.13 The variation of axial ratio as a function of off angle.

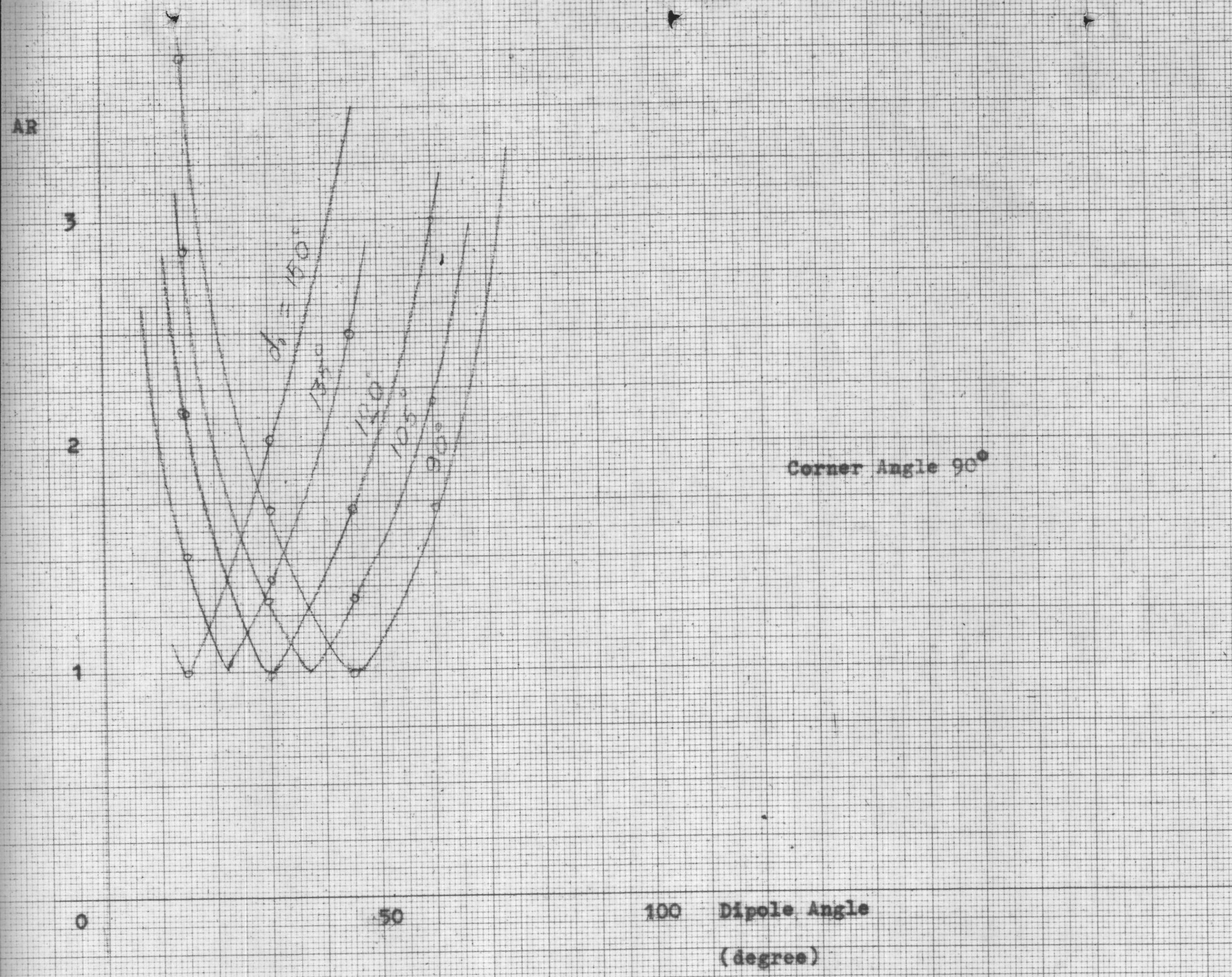


Fig. 2.14 The variation of axial ratio as a function of dipole angle.

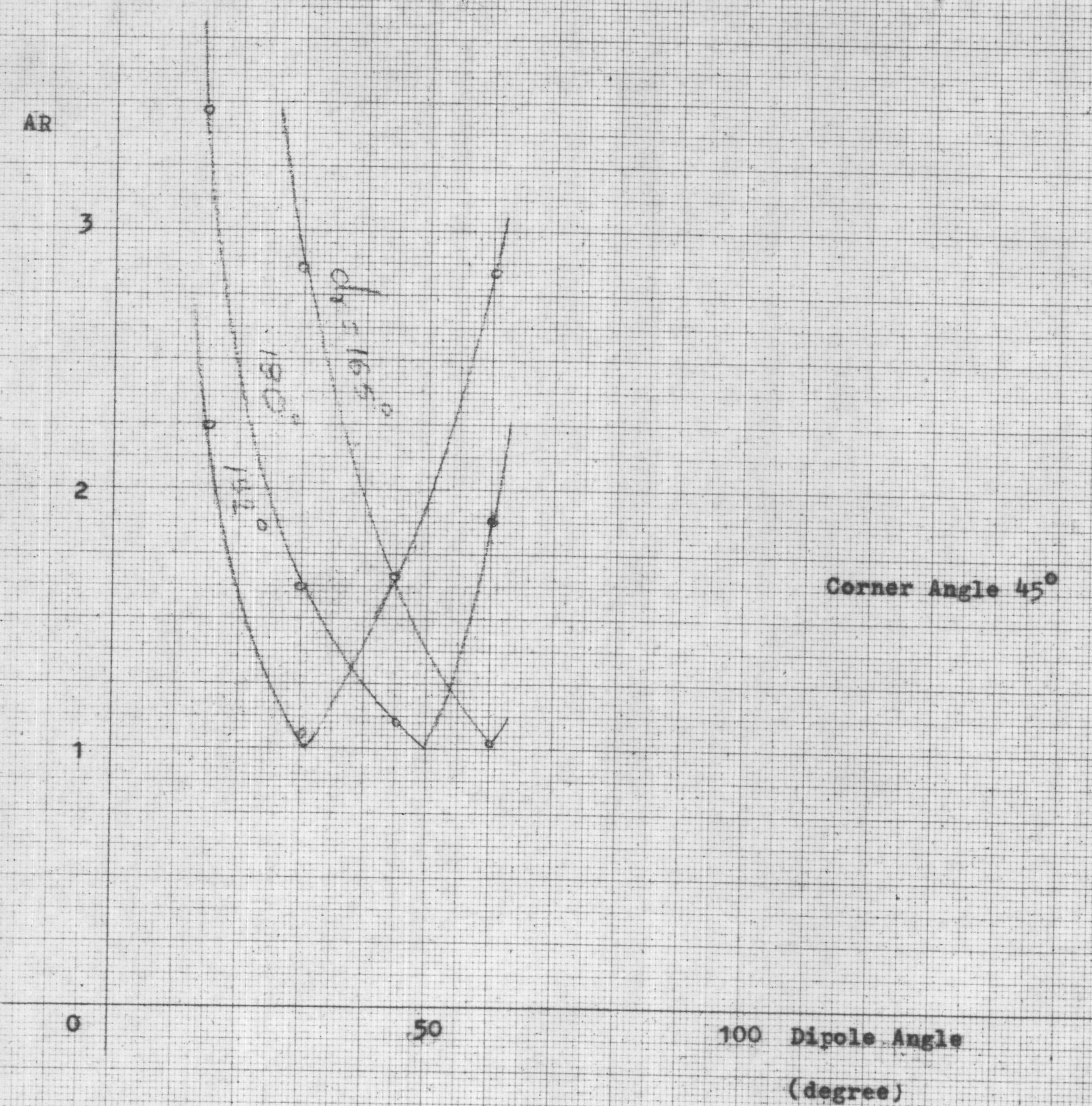


Fig. 2.15 The variation of axial ratio as a function of dipole angle.

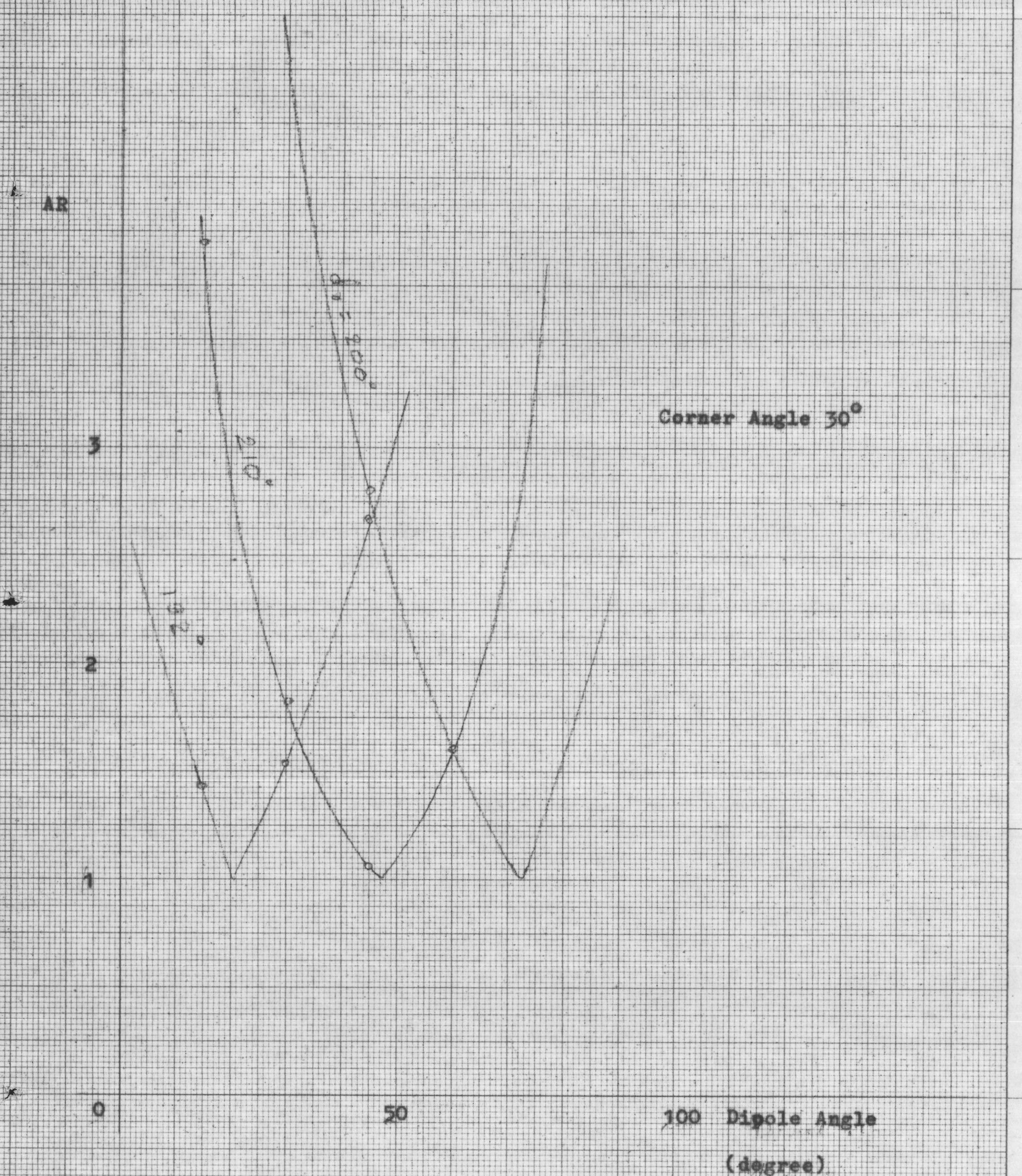


Fig. 2.16 The variation of axial ratio as a function of dipole angle.

The equation (39) is derived with the condition that corner angle of reflectors must provide even numbers of dipoles. Now let's derive the same equatio n but is useful for the case that corner angle of reflectors provide odd numbers of dipoles.

Let corner angle of reflectors be η which provides odd numbers of dipoles as shown in Fig. 2.17. The electric field due to all horizontal dipoles are

$$E_H = kI\sin\beta\cos\theta e^{jd_r\cos\theta} - 2kI\sin\beta\cos\theta e^{jd_r\cos\eta\cos\theta}$$

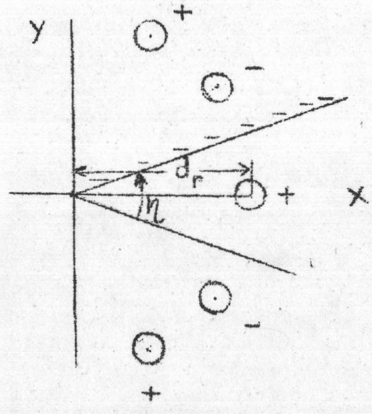
$$+2kI\sin\beta\cos\theta e^{jd_r\cos(2\eta)\cos\theta}$$

$$-2kI\sin\beta\cos\theta e^{jd_r\cos(3\eta)\cos\theta} + \ldots\ldots$$

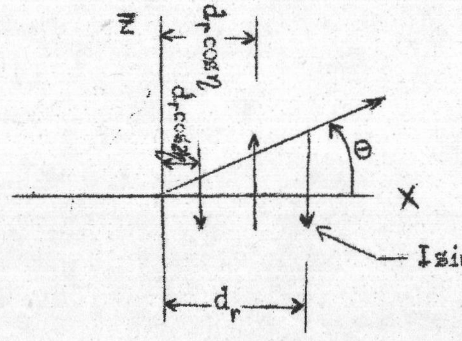
$$= kI\sin\beta\cos\theta\left(e^{jd_r\cos\theta} + 2\sum_{n=1}^{K-1}(-1)^n e^{jd_r\cos(n\eta)\cos\theta}\right) \tag{40}$$

where $K = \left(\frac{360^\circ}{\eta} - 1\right)/2$ K is integer. (41)

The first term of (40) is field due to dipole on X axis, the second term due to all dipoles above X axis and below X axis .



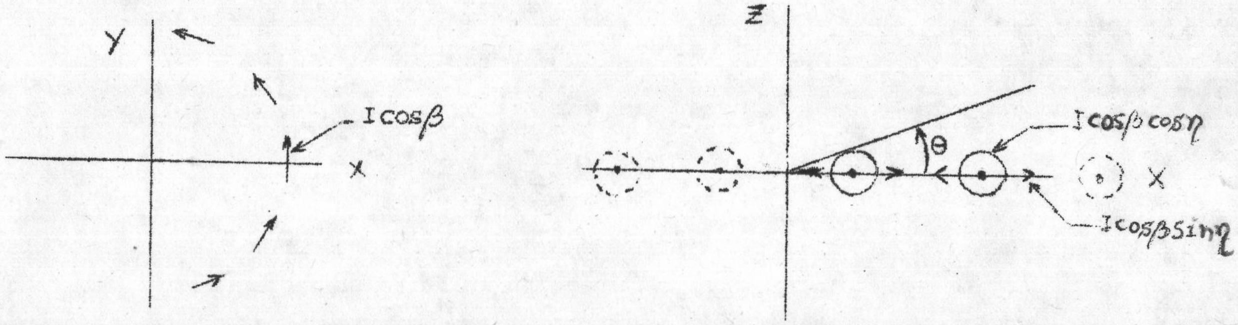
a) View in Z-direction.



b) View in Y-direction.

Fig. 2.17 images of horizontal dipole

The electric fields from vertical dipole and its images shown in Fig. 2.18 are



a) View in Z-direction. b) View in Y-direction.

Fig. 2.18 Images of vertical dipole.

$$E_V = kI\cos\beta e^{jd_r\cos\theta} + 2kI\cos\beta\cos\eta e^{jd_r\cos\eta\cos\theta}$$

$$+ 2kI\cos\beta\cos(2\eta)e^{jd_r\cos(2\eta)\cos\theta}$$

$$+ 2kI\cos\beta\cos(3\eta)e^{jd_r\cos(3\eta)\cos\theta} + \dots; \quad (42)$$

$$= kI\cos\beta\left[e^{jd_r\cos\theta} + 2\sum_{n=1}^{K}\cos(n\eta)e^{jd_r\cos(n\eta)\cos\theta}\right] \quad (43)$$

where $K = (\frac{360^\circ}{\eta} - 1)/2$ and is integer. (44)

E_V is the electric field in Y-direction, the field in X-direction is vanished.

At $\theta = 0^\circ$ equation (40) and (43) become

$$E_H = kI\sin\beta e^{jd_r}\left[1 + 2\sum_{n=1}^{K}(-1)^n e^{jd_r(\cos(n\eta)-1)}\right] \quad (45)$$

$$E_W = kI\cos\beta e^{jd_r}\left[1 + 2\sum_{n=1}^{K}\cos(n\eta)\, e^{jd_r(\cos(n\eta)-1)}\right] \quad (46)$$

Equations (45) and (46) are the fields due to dipoles equispaced on a circle of radius d_r and center at origin. Both equations do not show exactly the phase shift of E_H and E_V.

Let ψ_1 = phase of E_H

ψ_2 = phase of E_V

From equation (45)

$$\tan(\psi_1) = \frac{2\sum_{n=1}^{K}(-1)^n\sin(d_r(\cos(n\eta)-1))}{1+2\sum_{n=1}^{K}(-1)^n\cos(d_r(\cos(n\eta)-1))}$$

$$\tan(\psi_2) = \frac{2\sum_{n=1}^{K}\cos(n\eta)\sin(d_r(\cos(n\eta)-1))}{1+2\sum_{n=1}^{K}\cos(n\eta)\cos(d_r(\cos(n\eta)-1))}$$

For 90° phase difference $\tan(\psi_1).\tan(\psi_2) = -1$. Thus

$$\frac{2\sum_{n=1}^{K}(-1)^n\sin(d_r(\cos(n\eta)-1))}{1+2\sum_{n=1}^{K}(-1)^n\cos(d_r(\cos(n\eta)-1))} \times \frac{2\sum_{n=1}^{K}\cos(n\eta)\sin(d_r(\cos(n\eta)-1))}{1+2\sum_{n=1}^{K}\cos(n\eta)\cos(d_r(\cos(n\eta)-1))} = -1$$

$$4\sum_{m=1}^{K}\sum_{n=1}^{K}(-1)^m\sin(d_r(\cos(m\eta)-1))\cos(n\eta)\sin(d_r(\cos(n\eta)-1))$$

$$= -\Big[1+2\sum_{n=1}^{K}\Big[(-1)^n\cos(d_r(\cos(n\eta)-1))+\cos(n\eta)\cos(d_r(\cos(n\eta)-1))\Big]$$
$$+4\sum_{m=1}^{K}\sum_{n=1}^{K}(-1)^m\cos(d_r(\cos(m\eta)-1)).\cos(n\eta)\cos(d_r(\cos(n\eta)-1))\Big]$$

$$2\sum_{n=1}^{K}\Big[(-1)^n\cos(d_r(\cos(n\eta)-1))+\cos(n\eta)\cos(d_r(\cos(n\eta)-1))\Big]$$
$$+4\sum_{m=1}^{K}\sum_{n=1}^{K}(-1)^m\cos(n\eta)\Big\{\cos(d_r(\cos(m\eta)-1))\cos(d_r(\cos(n\eta)-1))$$
$$+\sin(d_r(\cos(m\eta)-1))\sin(d_r(\cos(n\eta)-1))\Big\} = -1$$

$$2\sum_{n=1}^{K}\left\{(-1)^n\cos(d_r(\cos(n\eta)-1))+\cos(n\eta)\cos(d_r(\cos(n\eta)-1))\right\}$$
$$+4\sum_{m=1}^{K}\sum_{n=1}^{K}(-1)^m\cos(n\eta)\cos(d_r(2\sin\frac{m+n}{2}\eta\cdot\sin\frac{n-m}{2}\eta))=-1 \quad ----(47)$$

Above equation is used to solve for d_r for a given corner angle of reflectors to obtain 90^o phase difference of E_H and E_V. For a circular polarization, amplitudes of E_H and E_V must be equal.

From equation (45) and (46)

$$\sin^2\beta\left[(1+2\sum_{n=1}^{K}(-1)^n\cos(d_r(\cos(n\eta)-1)))^2+(2\sum_{n=1}^{K}(-1)^n\sin(d_r(\cos(n\eta)-1)))^2\right]$$
$$=\cos^2\beta\left[(1+2\sum_{n=1}^{K}\cos(n\eta)\cos(d_r(\cos(n\eta)-1)))^2\right.$$
$$\left.+(2\sum_{n=1}^{K}\cos(n\eta)\sin(d_r(\cos(n\eta)-1)))^2\right] \quad (48)$$

It is seen that circularly polarized wave will occur by means of rotating dipole about axis of reflectors to an angle β which is calculated from equation (48).