

Chapter II

THEORETICAL CONSIDERATIONS

Oxygen transfer

Aeration is a gas-liquid mass transfer process in which interphase diffusion occurs when a driving force is created by a departure from equilibrium (Eckenfelder, 1966). In the gas phase, the driving force is a partial pressure gradient; in the liquid phase, a concentration gradient; as shown schematically in figure A.

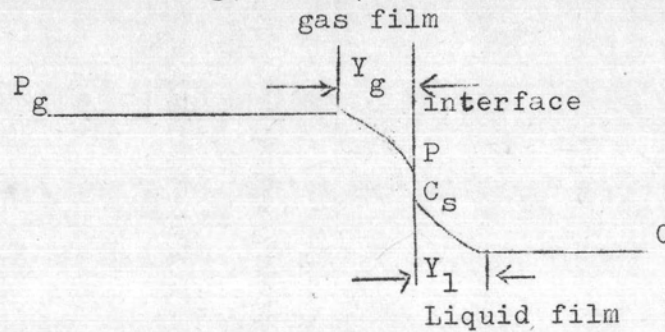


Figure A: Schematic representation of interfacial mass-transfer.

If the gas does not react chemically with water when it is taken into solution (as with oxygen) its solubility is defined in term of Henry's Law (Fair and Geyer, 1954).

Henry's Law states that the concentration of a gas in a liquid such as water, is directly proportional to the concentration or partial pressure of the gas in the atmosphere in contact with liquid or solution, that is:

$$C_s = K_s P$$

where C_s is the saturation concentration of the gas in the water.

P is the partial pressure of the gas in the gas phase

K_s is the proportionality constant, called coefficient of absorption.

Water is saturated with a gas when the proportionality implied in Henry's Law is fully established; rising temperatures decrease the saturation value (Fair and Geyer, 1954).

Even in the absence of mechanical mixing, the concentration of a substance that is dissolved in water will eventually become uniform throughout a given volume of water. The process of equalization of concentration is called molecular diffusion of dissolved gas in a liquid which is defined by Fick's Law and is dependent on the characteristics of the gas and the liquid, the temperature, the concentration gradient and the cross-sectional area across which diffusion occurs (Eckenfelder, 1966). It is analogous to the law of heat conduction (Fair and Geyer, 1954).

Various authors have presented Fick's Law in slightly different forms (notations) but the principle involved is nevertheless the same, thus:

1.
$$N = - D_1 \cdot A \cdot \frac{dc}{dy} \dots\dots\dots \text{Eckenfelder (1966)}$$

where :

N = mass transfer per unit time or $\frac{dm}{dt}$

A = cross-sectional area through which diffusion occurs

$\frac{dc}{dy}$ = concentration gradient perpendicular to cross-sectional area

D_1 = diffusion coefficient.

2.
$$dm = A \cdot K \cdot \frac{dc}{dx} \cdot dt \dots\dots\dots \text{Pasveer (1960)}$$

where :

dm = amount of matter

A = area of section

K = diffusion constant

$\frac{dc}{dx}$ = concentration gradient

dt = brief space of time

3. $\frac{\partial w}{\partial t} = K_d \cdot A \cdot \frac{\partial c}{\partial l}$ Fair and Geyer(1954)

where :

w = weight of dissolved substance

t = time

A = cross-sectional area

c = concentration

l = distance

K_d = coefficient of diffusion



Since the concentration gradient decreases as diffusion take place, Fick's Law is written as a **partial differential equation**. Solution of this equation may be had by the use of Fourior series (Fair and Geyer, 1954). A solution proposed by Back and Phelps (cited by Fair and Geyer, 1954) establishes the following series:

$$C_t = C_s - 0.811(C_s - C_o)(e^{-K_d} + 1/9 \cdot e^{-9K_d} + 1/25e^{-25K_d}..)$$

where : C_s is the saturation concentration of the disselved substance.

C_o and C_t are its concentrations at time zero and time t respectively.

Rate of oxygen transfer

From Eckenfelder, (1966) can be reexpressed in concentration units:

$$\frac{N}{V} = \frac{dc}{dt} = K_L \cdot \frac{A}{V} (C_s - C_l) = K_{La} (C_s - C_l)$$

where :

C_l = the oxygen concentration at a point at a time t (mg/l)

K_L = the overall diffusion coefficient based on liquid film resistance, cu.ft. of volume/sec/sq.ft. of area

A = the total interfacial area, sq.ft.

V = the volume of liquid under consideration (cu.ft.)

C_s = The concentration of O_2 in the liquid when saturated at the existing pressure and temperature.

K_{La} = overall coefficient for oxygen transfer

Naryana, (1971): Stated that : Assume the mixing is such that concentration of oxygen throughout the volume remains the same

and, on integrating the equation $\frac{dc}{dt} = K_L \cdot \frac{A}{V} (C_s - C_1)$

$$K_L \cdot \frac{A}{V} \cdot t = \log_e (C_s - C_1) + \text{constant}$$

when $t = 0$, $C_t = C_o$ and hence

$$K_L \cdot \frac{A}{V} \cdot t = \log_e \frac{C_s - C_o}{C_s - C_1}$$

$$\text{factor } K_L \cdot \frac{A}{V} = K_{La}$$

$$\text{or } K_L \cdot A = 2.3 \frac{V}{t} \cdot \log_{10} \frac{C_s - C_o}{C_s - C_1}$$

$$\text{and } K_{La} = 2.3 \cdot \log_{10} \frac{C_s - C_o}{C_s - C_1} / t$$

if V is in cu.ft.

t in sec.

$K_L \cdot A$ will be in cu.ft./sec.

Factors affecting oxygen-transfer

Eckenfelder, (1970) noted that several factors affect the performance of air aeration devices. Since aeration efficiency is estimated in water under standard conditions (zero dissolved oxygen at $20^\circ C$), corrections must be made for operation in waste water systems. Oxygen Saturation: Oxygen saturation in water

is related to temperature. In waste water, the presence of salts and other substances will affect oxygen saturation, usually decreasing it. Since it is not possible to estimate this value, it is necessary to measure it in the field.

Temperature: The oxygen-transfer coefficient (K_{La}) will increase with increasing temperature. The relationship most commonly used is

$$K_{La}(t) = K_{La}(20^{\circ}C) \times 1.02^{(T-20)}$$

Where T is the temperature in °C. Recently, Landbury et al. (1966) found a temperature coefficient of 1.012 for surface aeration units and Jones and his Co-worker, (1969) found a temperature coefficient range from 1.016 to 1.047.

The Influence of Temperature on Aeration by Pasveer, (1960)

An increase in temperature correlates with an increase in the coefficient of diffusion of oxygen in water. From this follows that for a given deficit the rate of aeration is higher at the higher temperature.

For a given oxygen content, however, an increase in temperature not only correlates with an increase in the coefficient of diffusion of oxygen in water but also with a decrease in deficit. From this it can be calculated that by a change in temperature the rate of oxygenation is influenced to a low extent in the case of a low oxygen content, but to a high extent in the case of a high oxygen content.