CHAPTER II

PRELIMINARIES

In this thesis, we assume a basic knowledge of group theory. However, this chapter contains a review of some important notations we will be using. Proofs will not be given, and can be found in [3] .

1 Definition. Let G be a group and I an index set. For each α in I, let G_α be a subgroup of G. G is said to be the irredundant union of its subgroups G_α if

(i) $G = \bigcup_{\alpha \in I} G_\alpha$

and (ii) for each β in I $G_\beta \setminus \bigcup_{\substack{\alpha \in I \\ \alpha \neq \beta}} G_\alpha \neq \emptyset$.

2 Notation. Let G be a group and A a subset of G. The notation [A] represents the subgroup of G generated by A. In case A is finite, let $A = \{a_1, a_2, \ldots, a_n\}$, the notations [A] and $[a_1, a_2 \ldots, a_n]$ are used in the same meaning.

3 Definition. A group G is said to be locally cyclic if for each finite subset A of G, there exists an a in G such that [A] = [a] .

4 Theorem. Any factor (or quotient) group of G is a homomorphic image of G and conversely if G' is a homomorphic image of G, then G' is isomomorphic to a factor group of G.

5 Theorem. (Lagranges'Theorem). The order of any subgroup of a finite group G is a factor of the order G.

6 Theorem. Let G_1 and G_2 be subgroups of a group G and G_2 is a normal subgroup of G. Then

(i) $G_1 \cap G_2$ is a normal subgroup of G_1

and (ii) $G_1G_2/G_2 = G_1/G_1 \cap G_2$.

7 Definition. The group defined by the relations

$$a^4 = 1,\ a^2 = b^2,\ ba = a^3b$$

is called the quarternion group and denoted by $\underline{Q}$.

8 Definition. The symmetric group of degree n, denoted by $\underline{S}_n$ is the set of all permutations of n objects.

9 Theorem. If F is a finite field, then $F \smallsetminus \{o\}$ is a cyclic group under multiplication.

10 Definition . Let G be an additive abelian group, and I an index set, G is said to be the direct sum of its subgroups G_α, $\alpha \in I$, denoted by

$$G = \bigoplus_{\alpha \in I} G_\alpha$$

if (i) $G = \left[\bigcup_{\alpha \in I} G_\alpha\right]$

and (ii) $G_\beta \cap \left[\bigcup_{\substack{\alpha \in I \\ \alpha \neq \beta}} G_\alpha\right] = \{o\}$ for all β in I.

We also use one fact in number theory as follows:

11 Theorem. If m and n are relatively prime, then there exist integers k and h such that

$$km + hn = 1.$$