CHAPTER III

METHOD OF ANALYSIS

3.1 Introductory Remarks

To study the behaviour of pile caps by analytical methods, following assumptions will be made:

1) The column load is located at the centroid of pile group so that equal share of load is transmitted to each pile.

2) The pile reaction is concentrated at the center of each pile.

3) No part of the column load is carried by the soil beneath the cap and no external bending moment applied.

4) No lateral rigidity of the pile.

It was recommended by a working party of the Concrete Society (10) for standardization of design and detailing of isolated pile caps for commonly used groups of pile supporting single axially loaded column that two major considerations must be concerned ,i.e., (a) the best shape of a pile cap for a giving number of piles, and (b) the most economical depth of the pile caps.

(a) Shape of pile cap;

The minimum spacing of piles permitted is controlled by the soil condition, and will normally lie between two and three times the pile diameter. As shown in Fig. 3.1, the spacing of piles will be made in terms of a spacing factor, k, times the pile diameter

An overhang of 150 mm.to the edge is considered sufficient.

(b) Depth of pile cap

A cost analysis was made for a large number of pile caps and the most economical depth for each pile size are presented in the Table 1 .

In this study, a pile diameter of 350 mm. will be used throughout with spacing of three times the pile diameter. The column size will be 350x350 mm. for single-pile cap and two-pile cap, and 450x450 mm. for three-pile cap and four-pile cap. The total depth of pile caps are 600 mm. for single-pile cap, and fixed at 850 mm. for two-pile, three-pile, and four-pile cap. For all the pile caps, the distance from centroid of reinforcement to bottom extreme fiber will be taken as 100 mm.. The allowable reaction at working load in each pile will be 50 tons.

3.2 Analytical Method for Pile Caps

In this research, three methods of analysis will be employed for the strength evaluation of the standard pile caps.

3.2.1 Finite Element Method

As the pile caps always been designed with large depth compared to the span to ensure the stiffness of the cap to transmit an equal load to the piles, thus the non-linear stress distribution would be induced in the pile caps and the simple beam theory does not apply. An exact solution of the stress analysis problem can be obtained from the theory of elasticity by formulating this problem in terms of partial differential equations. However, the loading and boundary conditions are such that a solution is very difficult to obtain. Thus, approximate solutions often involve by replacing the continuum with a substitute structure having a finite number degrees of freedom. It might expect that as the subdivision is made finer, the substitute structure models the original stucture more closely. The substitute structure is a finite element structure, and each seperate area is a finite element. Points where the elements are connected to one another are called nodal points.

The shape of the finite element used in the stress analysis of each pile cap are three dimensional rectangular element with 8 nodes called BRICK 8 element. The typical description of a BRICK 8 is shown in Fig. 3.2 .

As the geometry of pile caps and loading condition are symmetry at the column center, there are two planes of symmetry in single-pile, two-pile, and four-pile cap, only one quarter of the cap can be represented by the substituted finite element structure. For three-pile cap, one half of the actual structure is represented by finite element structure. The details of each finite element structure are shown in Fig. 3.3 .

The boundary conditions and loading for the nodal points are as follow:

1) The nodal points on the plane of symmetry XZ, are free to move in the direction X and Z-axis, and fixed in Y-axis.

2) The nodal points on the plane of symmetry YZ, are free to move in the direction Y and Z-axis, and fixed in X-axis.

3) Other nodal points are free to move in any direction

and load may be applied.

4) Reaction of the pile is taken as the uniformly distributed load acted to the corresponding face of the specified element. In all cases the uniform pressure acted on face 6. (see Fig. 3.2)

The three dimensional Stress Analysis Program based on the Finite Element Method will be used in this research to obtain information on stress distributions within pile caps.

3.2.2 Beam Analogy

In beam analogy of pile caps, according to the ACI code (318-63) [1], the critical stress resultants are determined along prescribed section in the pile cap as shown in Fig. 3.4 . Based on the allowable shearing and flexural stresses, the depth of the pile cap and the amount of steel reinforcement are determined from the following requirements;

a) Punching Shear

The depth of the cap is determined as

$$d = \frac{V}{vb_o} \tag{3.1}$$

where V = the column load or piles reactions

v = the allowable shearing stress along the periphery of the section considered, ksc., $0.53\sqrt{f'_c}$ for WSD and $1.06\phi\sqrt{f'_c}$ for USD where ϕ is 0.85

b_o = the perimeter of the pseudocritical section taken at a distance d/2 from the face of column

b) <u>Beam Shear</u>

The depth of the cap is determined as

$$d = \frac{V}{v_c b} \tag{3.2}$$

where V = the net shear force due to the pile reactions exist outside of the critical section; piles located near or at the critical section are considered to produce shear proportion to the distance of their centers from this section;

v_c = allowable shearing stress in a beam, ksc., $0.29\sqrt{f'_c}$ for WSD and $0.58\phi\sqrt{f'_c}$ for USD where ϕ is 0.85

c) <u>Flexure</u>

From flexural requirement, the depth of the cap can be determined as;

$$d = \sqrt{\frac{M}{Rb}} \tag{3.3}$$

where M = the critical bending moment at the section along a face of the column base

R = a coefficient in concrete beam design

From allowable flexure and bond stresses, the amount of reinforcement is determined as

$$A_s = \frac{M}{f_s jd} \tag{3.4}$$

or

$$\Sigma_o = \frac{V}{ujd} \tag{3.5}$$

where V = shear force at critical section for bond

3.2.3 Truss Analogy

a) Basis of Design[3]

The load is assumed to take the shortest line to the supports, and to transmit to the piles by inclined compression in the cap. Assuming that the piles have no lateral rigidity, the reinforcement is required to resist these inclined thrust, as illustrated in Fig. 3.6-3.8 .

b) Design of Two-Pile Cap

Fig. 3.6 shows the distributed load from the base of column. Considering the effect of an infinitesimal load pdx acting a distance x from the center line, causing the pile reactions and tie-tension. When integrated over the width of the base the tension in the steel reinforcement becomes

$$T = 2\int_{0}^{\frac{c}{2}} \left(\frac{p}{Ld}\right)\left(\frac{L^2}{4}-x^2\right)dx$$

$$= \frac{P(3L^2-c^2)}{12Ld} \qquad (3.6)$$

where

P = total column load

c = width of a square column

L = spacing of the piles

If the effect of the column size was neglect by assuming a concentrated column load at the center line, Eq. 3.6 becomes

$$T = \frac{PL}{4d} \qquad (3.7)$$

c) Design of Three-Pile Cap

The assumed load trajectories are shown in Fig. 3.7 . For a square column base and the symmetry of the spacing of piles, the tension in steel between two adjacent supports can be determined as

$$T = \left(\frac{p}{Ld}\right)\int_{-\frac{c}{2}}^{\frac{c}{2}}\int_{\frac{L}{2\sqrt{3}}-\frac{c}{2}}^{\frac{L}{2\sqrt{3}}+\frac{c}{2}}\left[\left(\frac{L}{2}-\frac{y}{3}\right)^2-x^2\right]dxdy$$

$$= \frac{P(2L^2-c^2)}{18Ld} \qquad (3.8)$$

For the tie-tension in reinforcement between centroid of the cap and support,

$$T = \frac{\sqrt{3}P(2L^2-c^2)}{18Ld} \qquad (3.9)$$

Neglecting the effect of the column size, eq. 3.8 becomes

$$T = \frac{PL}{9d} \qquad (3.10)$$

and eq.3.9 becomes

$$T = \frac{PL}{3\sqrt{3}d} \qquad (3.11)$$

d) Design of Four-Pile Cap

As before, the column load transmitted to the supporting piles as shown in Fig. 3.8 . Since the four horizontal ties do not form a stable lattice system, an extra tie is introduced in diagonal direction. From the equilibrium of forces and geometrical symmetry, the tension in reinforcement between two adjacent supports can be obtained as

$$T = \left(\frac{p}{2Ld}\right)\int_{-\frac{c}{2}}^{\frac{c}{2}}\int_{-\frac{c}{2}}^{\frac{c}{2}}\left(\frac{L}{2}-x\right)\left(\frac{L}{2}+x+y\right)dxdy$$

$$= \frac{P(3L^2-c^2)}{24Ld} \qquad (3.12)$$

From the equilibrium of forces, the tie in diagonal direction is zero.

For the type of reinforcement arrangement in diagonal direction, tension in reinforcement can be obtained as

$$T = \frac{\sqrt{2}P(3L^2-c^2)}{24Ld} \qquad (3.13)$$

Neglecting the effect of the column size, eq 3.12 becomes

$$T = \frac{PL}{8d} \qquad (3.14)$$

and eq. 3.13 becomes

$$T = \frac{PL}{4\sqrt{2}d} \qquad (3.15)$$

3.3 Proposed Reinforcement Arrangements in Pile Cap

It is difficult to assess explicitly the elastic behaviour of such short and relatively deep pile cap. Finite Element Method however, can furnish the required stress trajectories, which describe best the mechanism of load disposal. When the load intensity is sufficiently large, cracks will form approximately at right angles to these principle tension trajectories. After cracking of the concrete, the reinforcement will operate most efficiently if it is located at least approximately along such tension trajectories.

The distribution of principal stress trajectories in single-pile, two-pile, three-pile, and four-pile cap at cross sections of interest are shown in Fig. A.4-A.7 . An evaluation of these results reveal the existence of the following;

1) The vertical compressive stresses are almost uniform between the column base and the distributed reaction from pile top. This indicated that with the increase in depth of pile cap the component of the column load transmitted directly to the supporting piles would gradually increase, thus resulting in the reduction of bending stresses.

2) The tensile stresses in plane section near the bottom of pile cap tend to flow from centroid of the pile cap to the supporting piles with the components of tensile stresses between two adjacented piles.

3) The curvature of tensile stresses in vertical section at the supports are vary from 10 to 20 degrees.

To the above observed stress pattarns, proper arrangements of reinforcement can be made regardless of the conventional practices where reinforcements are uniformly distributed across the cross section. Only two forms of pile caps, i.e., three-pile and four-pile cap, will be considered in the experimental program. The proposed reinforcement arrangements in three-pile cap and four-pile cap are shown in Fig. 3.11-3.12 . P3-1 and P4-1 denote pile caps with conventional reinforcement arrangement according to beam analogy, whilst P3-2 and P4-2 based on truss analogy.

P3-3, P3-4, P4-3, and P4-4 denote pile caps with proposed reinforcement arrangements conforming to the observed stress trajectory patterns and must be tested to verify the assumptions.

3.4 Cracking and Ultimate Loads

In reinforced concrete design, the cracking and ultimate loads must be predicted to ensure that the structure would service the design working load and the limit ultimate load at failure of the structure. Two methods in design are formulated as follow;

3.4.1 Beam Analogy

According to the ACI code, the cracking moment of the section is determined from flexure formula[2]

$$M_{cr} = \frac{I_g f_r}{y_t} \qquad (3.16)$$

where M_{cr} = cracking moment

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement

f_r = modulus of rupture of concrete

y_t = distance from centroidal axis to the extreme fiber of tension

The ultimate strength is determined from the ultimate moment at critical section resisted by concrete and reinforcement as

$$M_u = \phi(bd^2 f'_c q(1-0.59q)) \qquad (3.17)$$

or

$$M_u = \phi A_s f_y (d-\frac{a}{2}) \qquad (3.18)$$

where M_u = ultimate moment

$q = \dfrac{A_s f_y}{bdf'_c}$

$a = \dfrac{A_s f_y}{0.85 f'_c b}$

$\phi = 0.90$

Fig. 3.9 shows the stress distribution in concrete on beam cross section under bending moment at cracking and ultimate conditions.

3.4.2 Truss Analogy

In finite element procedure, it was observed that the thrust in concrete is in the form of a cone which must be supported along the edges of the cap. The edge supports then transmit the load to the piles. In this study, only elastic behaviour is valid in finite element procedure.

After the cracks are formed, the internal force system will conformed to the basic of truss analogy.[10]

Figure 3.10(a) shows the assumed internal force system in pile caps within elastic range. The shaded area are assumed as the tension area in concrete, thus the cracking load can be expressed as follow,

For two-pile cap;

$$P_{cr} = \frac{12T_c L_c d_c}{(3L_c^2-c^2)} + \frac{12T_s Ld}{(3L^2-c^2)} \qquad (3.19)$$

For three-pile cap;

a) tie-tension between two adjacent piles,

$$P_{cr} = \frac{18T_c L_c d_c}{(2L_c^2-c^2)} + \frac{18T_s Ld}{(2L^2-c^2)} \qquad (3.20)$$

b) tie-tension between centroid of the cap and pile,

$$P_{cr} = \frac{18T_c L_c d_c}{\sqrt{3}(2L_c^2-c^2)} + \frac{18T_s Ld}{\sqrt{3}(2L^2-c^2)} \qquad (3.21)$$

For four-pile cap;

a) tie-tension between two adjacent piles,

$$P_{cr} = \frac{24T_c L_c d_c}{(3L_c^2-c^2)} + \frac{24T_s Ld}{(3L^2-c^2)} \qquad (3.22)$$

b) tie-tension between centroid of the cap and pile,

$$P_{cr} = \frac{24T_c L_c d_c}{\sqrt{2}(3L_c^2-c^2)} + \frac{24T_s Ld}{\sqrt{2}(3L^2-c^2)} \qquad (3.23)$$

where the following assumptions will be made;

T_c = tie-tension in concrete = $(hc/2)(f_r/2)$

T_s = tie-tension in steel = $nf_r A_s$

d_c = $3h/4$ for all pile caps

$$L_c = L \quad \text{for two-pile caps}$$

$$= 3L/4 \quad \text{for three-pile and four-pile caps}$$

Figure 3.10(b) shows the internal force system in pile cap at ultimate condition. Since the reinforcement in all pile caps are subjected to direct tension, the ultimate loads can be expressed as follow

For two-pile cap;

$$P_u = \frac{12A_s f_y Ld}{(3L^2-c^2)} \tag{3.24}$$

For three-pile cap;

a) tie-tension between two adjacent piles,

$$P_u = \frac{18A_s f_y Ld}{(2L^2-c^2)} \tag{3.25}$$

b) tie-tension between centroid of the cap and pile,

$$P_u = \frac{18A_s f_y Ld}{\sqrt{3}(2L^2-c^2)} \tag{3.26}$$

For four-pile cap;

a) tie-tension between two adjacent piles,

$$P_u = \frac{24A_s f_y Ld}{(3L^2-c^2)} \tag{3.27}$$

b) tie-tension between centroid of the cap and pile,

$$P_u = \frac{24A_s f_y Ld}{\sqrt{2}(3L^2-c^2)} \tag{3.28}$$