Chapter II

DESIGN AND ANALYSIS OF TEMPERATURE CONTROL SYSTEM

2.1 Block Diagram of a Temperature Control System

The temperature control system, as shown in Figure 2.1, is a feedback control system (closed-loop control system). The system is divided into two parts as follows :

(1) Oven

(2) Controller

The controller may be represented by three parts as follows :

(1) Temperature/resistance characteristic of NTC thermistors

(2) Bridge

(3) Relay

2.2 Oven[4]

An assumption is made that the temperature of the electric oven shown in Figure 2.2 is uniform throughout the oven at all time (as well stirred). Energy losses due to radiation are neglected.

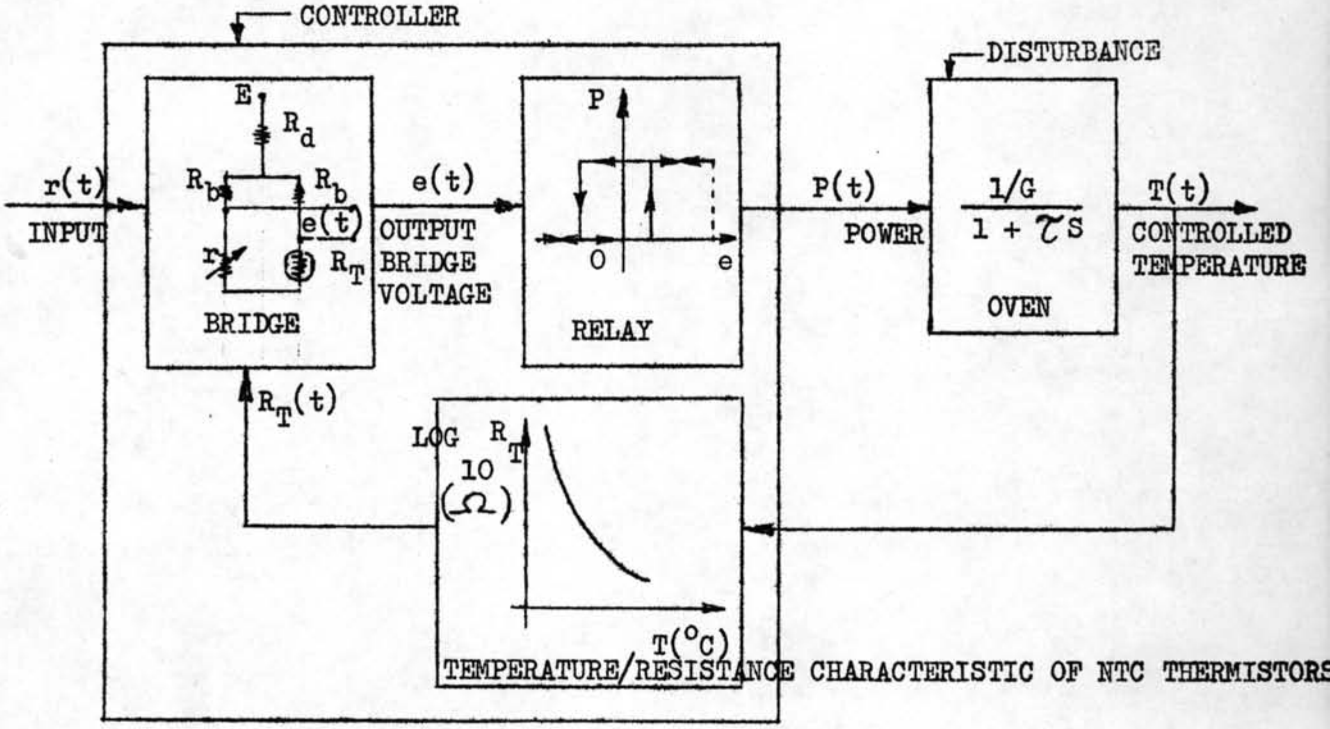

FIGURE 2.1

BLOCK DIAGRAM OF A TEMPERATURE CONTROL SYSTEM.

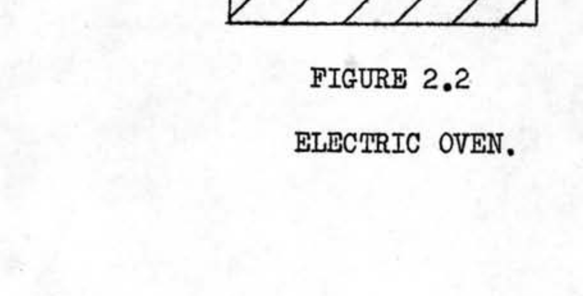

FIGURE 2.2

ELECTRIC OVEN.

The heater which has a very small time constant supplies power P(t) watts to the oven. The mass in the oven is M grams and its average specific heat K joules per gram celsius. Surface losses vary linearly with the difference between oven temperature T(t) and constant ambient temperature T_o . The thermal conductance of insulation is G watts per degree celsius. The fundamental relationship of thermal systems in equilibrium requires that the heat supplied to the oven equals to the heat stored plus the heat lost.

$$P(t) = MK\frac{dT(t)}{dt} + G(T(t) - T_o) \tag{2.1}$$

or

$$\frac{dT(t)}{dt} + \frac{1}{\tau}T(t) = \frac{1}{G\tau}P(t) + \frac{1}{\tau}T_o \tag{2.2}$$

where

$$\tau = \frac{MK}{G}\text{, the thermal time constant} \tag{2.3}$$

Taking the Laplace transform of both sides of eqn. (2.2), we have

$$ST(S) - T_i + \frac{1}{\tau}T(S) = \frac{P(S)}{G\tau} + \frac{T_o}{S\tau}$$

or

$$T(S) = \frac{P(S)}{G(1+\tau S)} + \frac{\tau T_i}{1+\tau S} + \frac{T_o}{S(1+\tau S)} \tag{2.4}$$

where T_i is the initial temperature.

If we consider the temperature above ambient temperature T_o, and the initial temperature equals to zero; from eqn. (2.4), we obtain the transfer function of the oven

$$\frac{T(S)}{P(S)} = \frac{1/G}{1+\tau S} \tag{2.5}$$

2.3 Oven Analysis

The controller as shown in Figure 2.1 is the ON-OFF controller (two-step controller). The power supplied to the oven is a two-stage constant as follows :

$$P(t) = P, \text{ contact closed} \tag{2.6}$$

$$P(t) = 0, \text{ contact open} \tag{2.7}$$

Then, by taking the Laplace transform of both sides of eqn. (2.2), and in the case when the contact is closed, we obtain

$$T(S) = \frac{P}{GS(1 + \tau S)} + \frac{\tau T_i}{1 + \tau S} + \frac{T_o}{S(1 + \tau S)} \tag{2.8}$$

The inverse transform of T(S) is

$$T(t) = \frac{P}{G}\,(1 - e^{-t/\tau}) + T_i\, e^{-t/\tau} + T_o\,(1 - e^{-t/\tau})$$

or

$$T(t) = \frac{P}{G} - \frac{P}{G}\, e^{-t/\tau} + (T_i - T_o)\, e^{-t/\tau} + T_o \tag{2.9}$$

At steady-state, the oven temperatures are

$$T_{ss} = \frac{P}{G} + T_o = \text{constant; contact closed} \tag{2.10}$$

$$T_{ss} = T_o \quad = \text{constant; contact open} \tag{2.11}$$

From eqn. (2.10), the thermal conductance of insulation can be found

$$G = \frac{P}{T_{ss} - T_o} \tag{2.12}$$

By differentiating eqn. (2.9), the rate of change of the oven temperature can be found as follows :

(1) While the temperature of the oven is increasing, i.e., contact is closed and $P(t) = P$

$$\dot{T}_s(t) = \frac{1}{\tau}\frac{P}{G} e^{-t/\tau} - \frac{1}{\tau}(T_i - T_o) e^{-t/\tau} \qquad (2.13)$$

or

$$\dot{T}_s(t) = -\frac{1}{\tau}\left[-\frac{P}{G} e^{-t/\tau} + (T_i - T_o) e^{-t/\tau}\right] \qquad (2.14)$$

where

$$\dot{T}_s(t) = \frac{dT(t)}{dt} \qquad (2.15)$$

Substituting eqn. (2.9) into eqn. (2.14), we obtain

$$\dot{T}_s(t) = -\frac{1}{\tau}\left[T(t) - \frac{P}{G} - T_o\right] \qquad (2.16)$$

(2) While the temperature of the oven is decreasing, i.e., contact is open and $P(t) = 0$

$$\dot{T}_c(t) = -\frac{1}{\tau}(T_i - T_o) e^{-t/\tau} \qquad (2.17)$$

where

$$\dot{T}_c(t) = \frac{dT(t)}{dt} \qquad (2.18)$$

Substituting eqn. (2.9) into eqn. (2.17), we obtain

$$\dot{T}_c(t) = -\frac{1}{\tau}\left[T(t) - T_o\right] \qquad (2.19)$$

Eqns. (2.3), (2.6), and (2.9) show that the oven temperature at time t, T(t), is a function of the followings :

(1) P, the power supply which can be specified.

(2) G, the thermal conductance of insulation which can be improved.

(3) M, the mass, and K, the specific heat of the mass.

(4) T_i, the initial temperature, and T_0, the ambient temperature.

2.4 NTC Thermistors[5]

Negative Temperature Coefficient thermistors are solid semiconductors with a high negative coefficient of resistance. They are prepared from oxides of the iron group of transition elements e.g. Cr, Mn, Fe, Co, or Ni . These oxides have a high resistivity in the pure state, but can be transformed into semiconductors by adding small amounts of foreign ions which have a different valency.

The relation between resistance and temperature of NTC thermistor can be approximated by :

$$R_T = Ae^{B/T} \tag{2.20}$$

where R_T is the resistance value at an absolute temperature T (^oK) .

A and B are constants.

e is the base of the natural logarithm (e = 2.718).

In practice B is not a true constant; with increasing temperature there are small deviations. A better formula for the resistance is :

$$R_T = AT^C \, e^{B/T} \tag{2.21}$$

where C is a small positive or negative number and in some case is zero.

From eqn. (2.20) the temperature coefficient of resistance of a thermistor (which, unlike that of most materials, is negative) may be expressed as :

$$\alpha = \frac{1}{R_T} \cdot \frac{dR_T}{dT}$$

$$\alpha = - \frac{B}{T^2} \quad \text{\% per degree} \tag{2.22}$$

2.5 <u>Bridge</u>

A thermistor is connected as one leg of a bridge as shown in Figure 2.3 .

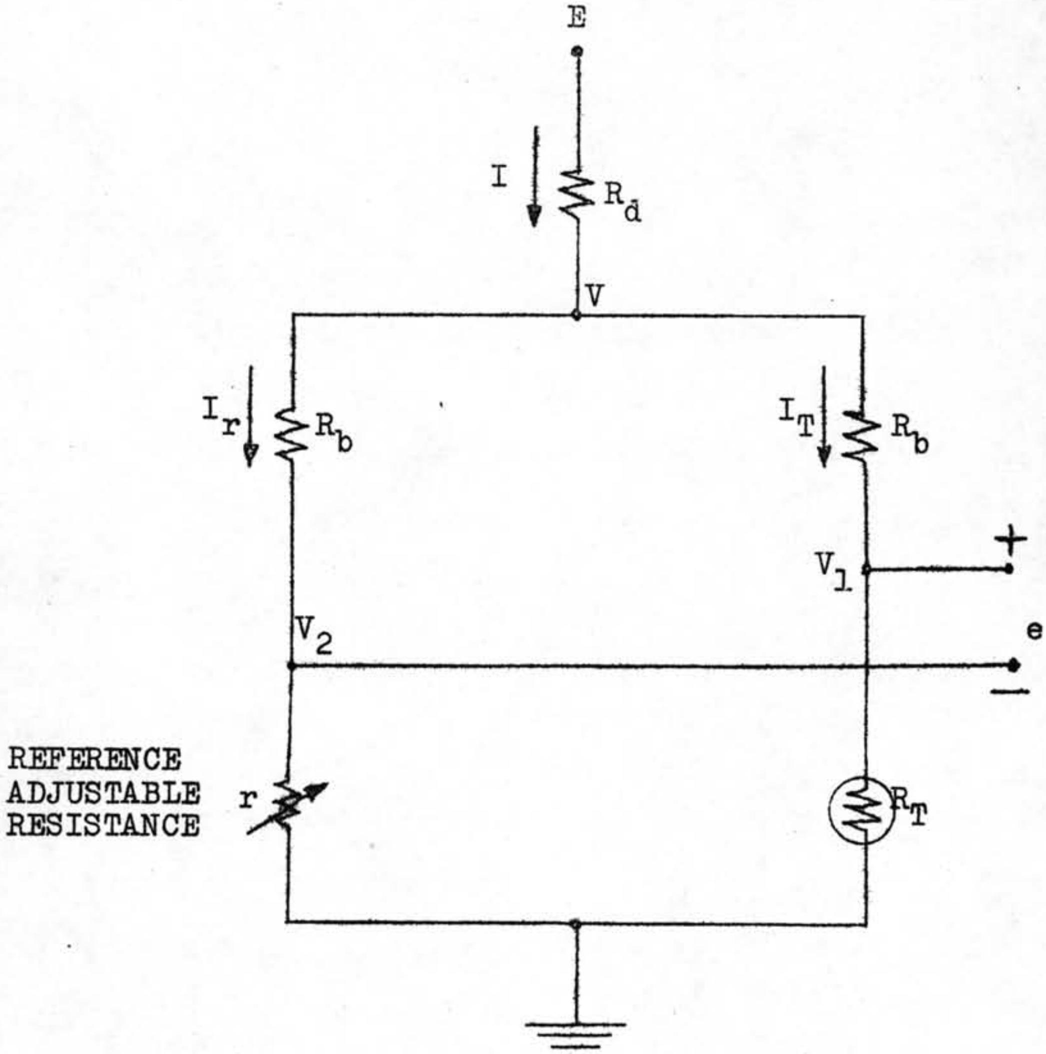

FIGURE 2.3

THERMISTOR BRIDGE CIRCUIT.

From Figure 2.3 and Kirchhoff's laws, we obtain

$$e = V_1 - V_2 \tag{2.23}$$

$$V_1 = I_T R_T \tag{2.24}$$

$$V_2 = I_r r \tag{2.25}$$

$$I_T = V/(R_b + R_T) \tag{2.26}$$

$$I_r = V/(R_b + r) \tag{2.27}$$

$$V = I \left[(R_b + R_T)//(R_b + r) \right] \tag{2.28}$$

$$I = E/\left[R_d + (R_b + R_T)//(R_b + r) \right] \tag{2.29}$$

where

$$(R_b + R_T)//(R_b + r) = \frac{(R_b + R_T)(R_b + r)}{(R_b + R_T) + (R_b + r)}$$

Solving eqns. (2.23) to (2.29), we obtain

$$e = \frac{E\ R_b(R_T - r)}{R_d \left[(R_b + R_T) + (R_b + r) \right] + (R_b + R_T)(R_b + r)} \tag{2.30}$$

2.6 Bridge Analysis

By considering eqn. (2.30), we found that :

(1) When R_T is greater than r, the output bridge voltage e is positive.

(2) When R_T is less than r, the output bridge voltage e is negative.

(3) When the bridge is in a balance condition, $R_T = r$, the output bridge voltage e is zero.

(4) If the two equal-resistance bridge legs, R_b, are increased, the output bridge voltage e will decrease.

(5) If the two equal-resistance bridge legs, R_b, are decreased, the output bridge voltage e will increase.

(6) The drop-out resistance bridge leg, R_d, is not considered because it depends on the design values D, R_b, and E (See Article 3.1).

(7) The output bridge voltage e is a nonlinear function of $(R_T - r)$.

2.7 Relay Characteristic

The relay characteristic is shown in Figure 2.4 . e_1 is the output bridge voltage which causes the relay to open. e_2 is the output bridge voltage which causes the relay to close. e_{11} and e_{22} are the output bridge voltages at maximum and minimum temperature of the oven, respectively, for each temperature setting.

The values of e_1, e_2, e_{11}, and e_{22} indicated in the relay characteristic in Figure 2.4 are not constant for different temperature setting. This can be seen from the

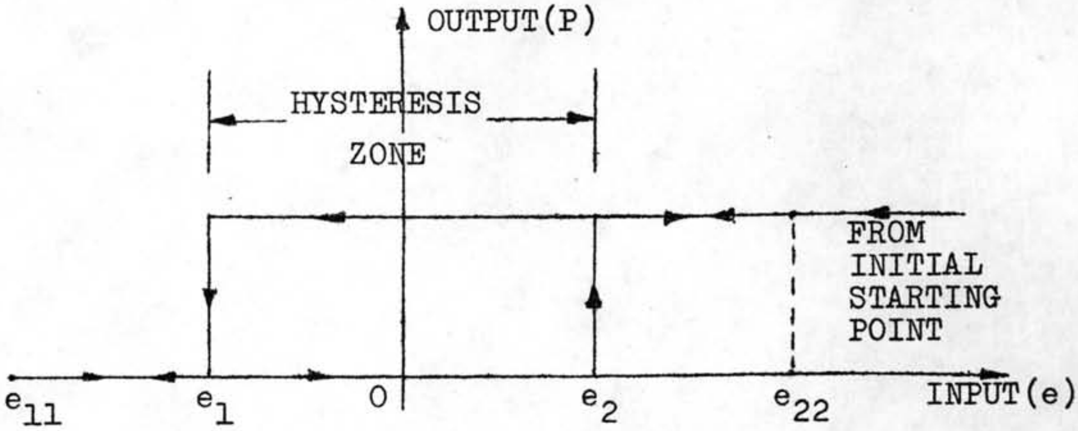

FIGURE 2.4

RELAY CHARACTERISTIC.

experimental data given in Appendix C . In the calculation, the system is assumed to have no time delay and the approximate relay characteristic as shown in Figure 2.5 is used.

2.8 Time Characteristic[1]

After switching of the relay, caused by the voltage e_1 or e_2, see Figure 2.4, the value of the controlled temperature still continues to increase or decrease during the dead time or time delay T_d . Figure 2.6 indicates the resultant fluctuations of the controlled temperature with dead time. The fluctuation of temperature $\triangle T$ is

$$\triangle T = T_{max} - T_{min} \qquad (2.31)$$

where T_{max} is the maximum temperature of the oven.

T_{min} is the minimum temperature of the oven.

The controlled temperature T is

$$T = T_{min} + \frac{\triangle T}{2} \qquad (2.32)$$

The percentage of temperature fluctuation is equal to

$\pm \frac{\triangle T}{2T} \times 100$ per cent.

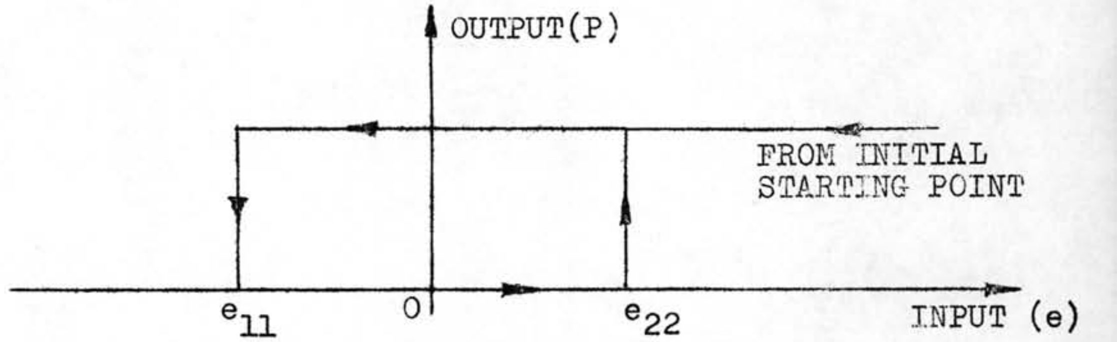

FIGURE 2.5

APPROXIMATE RELAY CHARACTERISTIC.

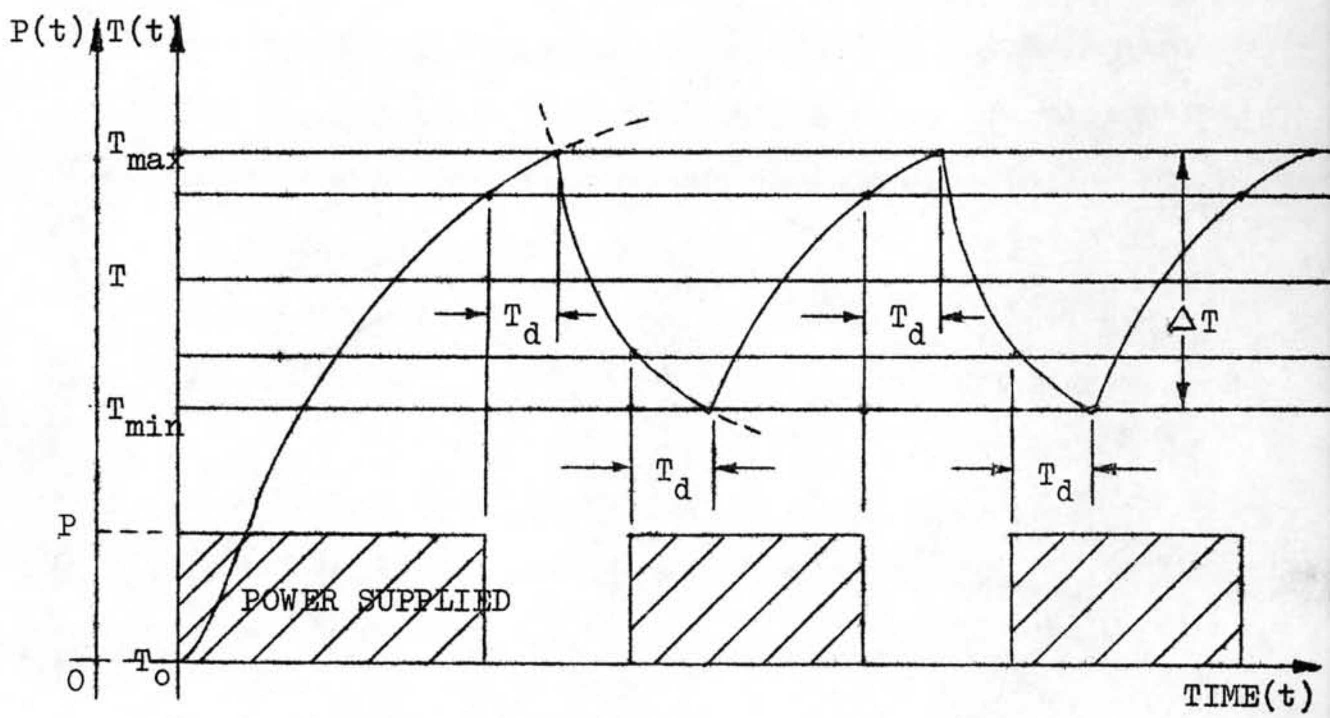

FIGURE 2.6

TIME CHARACTERISTIC OF TEMPERATURE CONTROL SYSTEM.