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นายโอภาส ชาญพงศ์พิสุทธิ์

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CORRELATIONS BETWEEN GOLD SPOT PRICES AND FUTURES PRICES

Mr. Opat Chanpongpisud

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โอภาส ชาญพงศ์พิสุทธิ์ : สหสัมพันธ์ระหว่างราคาซื้อขายทันทีและราคาซื้อขายล่วงหน้าของทองคำ. (CORRELATIONS BETWEEN GOLD SPOT PRICES AND FUTURES PRICES) อ.ที่ปรึกษาวิทยานิพนธ์หลัก : อ.ดร.กิตติพัฒน์ วอง, อ.ที่ปรึกษาวิทยานิพนธ์ร่วม : อ.ดร.เสน่ห์ รุจิวรรณ, 33 หน้า.

ในวิทยานิพนธ์นี้เราได้ศึกษาพฤติกรรมของราคาทองคำโดยตัวแบบสโทแคสติก (stochastic model) ที่ใช้จำลองราคาซื้อขายทันทีของทองคำของนายนราธิป (2553) โดยนำข้อมูลราคาซื้อขายทันทีของทองคำย้อนหลังซึ่งมาจาก London Gold Market Fixing Limited มาประมาณค่าพารามิเตอร์โดยวิธีความควรจะเป็นสูงสุด (maximum likelihood method) และคำนวณราคาซื้อขายล่วงหน้าแบบยุติธรรม (no-arbitrage prices)เพื่อนำมาวิเคราะห์สหสัมพันธ์ระหว่างราคาซื้อขายล่วงหน้าแบบยุติธรรม และราคาซื้อขายล่วงหน้าในตลาดสัญญาซื้อขายล่วงหน้าจากบริษัท ตลาดสัญญาซื้อขายล่วงหน้า (ประเทศไทย) จำกัด (มหาชน) (Thailand Futures Exchange หรือ TFEX) โดยพบว่าค่าสัมประสิทธิ์สหสัมพันธ์มีค่าเป็นบวกทั้งหมด เนื่องจากราคาซื้อขายทันทีและราคาซื้อขายล่วงหน้าแบบยุติธรรมมีสหสัมพันธ์สมบูรณ์ (perfectly correlated) เราจึงสามารถสรุปได้ว่าราคาซื้อขายทันทีและราคาซื้อขายล่วงหน้ามีสหสัมพันธ์แนบแน่น (strongly correlated) ในทางเศรษฐศาสตร์เราอาจกล่าวได้ว่าตลาดล่วงหน้าของทองคำในประเทศไทยเป็นไปอย่างมีประสิทธิภาพ นอกจากนี้เรายังได้ศึกษาการแปรผันของฤดูกาลของราคาทองคำโดยการสังเกตจากผลตอบแทนสะดวก (convenience yields) ของทองคำในประเทศไทย เราพบว่าผลตอบแทนสะดวกได้แสดงให้เห็นถึงฤดูกาลของทองคำ กล่าวคือมีค่าสูงสุดในช่วงปลายเดือนมกราคมและมีค่าต่ำสุดในช่วงปลายเดือนกรกฎาคมซึ่งสอดคล้องกับช่วงเทศกาลตรุษจีนที่คนไทยเชื้อสายจีนนิยมซื้อทองคำเป็นของขวัญมอบให้กันในช่วงปลายเดือนมกราคมของทุกปี และในช่วงกลางปีไม่มีเหตุการณ์ที่สำคัญที่เกี่ยวข้องกับทองคำจนกระทั่งประเพณีการแต่งงานของชาวอินเดียซึ่งต้องใช้ทองคำจำนวนมากเป็นสินสอดซึ่งเกิดขึ้นในช่วงปลายปีของทุกปี

ภาควิชา...คณิตศาสตร์และวิทยาการคอมพิวเตอร์... ลายมือชื่อนิสิต...................................

สาขาวิชา..คณิตศาสตร์ประยุกต์และวิทยาการคณนา.ลายมือชื่อ อ.ที่ปรึกษาวิทยานิพนธ์หลัก.......

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OPAT CHANPONGPISUD : CORRELATIONS BETWEEN GOLD SPOT PRICES AND FUTURES PRICES. ADVISOR : KITTIPAT WONG, Ph.D., CO-ADVISOR : SANAE RUJIVAN, Dr.rer.nat. 33 pp.

In this thesis, we study behavior of gold prices by using the stochastic model proposed by Issaranusorn (2010). We use gold spot price data obtained from the LGMF (London Gold Market Fixing Limited) to estimate the model parameters based on the maximum likelihood method. Utilizing the gold spot prices data with the estimated parameters, we compute the correlations between the no-arbitrage futures prices and futures prices obtained from TFEX (Thailand Futures Exchange). We find that all of the correlation coefficients are positive. The spot prices and the no-arbitrage futures prices are perfectly correlated. This implies that the gold spot prices and their futures prices are strongly correlated. In the economic point of view, these results suggest that the futures market in Thailand is efficient. Moreover, we study seasonal variation in gold prices by investigating the convenience yields of gold in Thailand. We find that the convenience yields of gold exhibited seasonality in which are highest in the end of January and the lowest in the end of July. These results have supported the idea that many Thai people, especially Thai-Chinese people, purchase gold as a gift on Chinese new year's day during the end of January annually and no special event until the traditional Indian wedding that needs a large amount of gold as a dowry in the end of year.

Department : ...Mathematics and Computer Science... Student's Signature ....
Field of Study : ....Applied Mathematics and............ Advisor's Signature ....
...Computational Science............... Co-advisor's Signature.............
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CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

Page

Page

CHAPTER I
INTRODUCTION

A valuable metal that can be used in many applications especially jewelry industry is gold. In addition, it has been used as the standard value for money of each country that determines the rate of exchange. Moreover, it is used as an assurance when issuing bank notes. Therefore it is used for a guaranteed source of investment funds in many countries. Gold has been considered as an economic asset with high liquidity, low risk, and high returns in the long run. Consequently, many investors are interested in gold futures contracts. Furthermore, hedgers can use gold futures to manage price risk due to uncertainty in gold spot prices. In 2009, Thailand Futures Exchange (TFEX) began trading gold futures and its trading volume has been rising sharply in the last few years [6].

In finance, an efficiency of a futures market of an underlying asset can be measured by the correlation coefficients between the underlying spot prices and the corresponding future prices. Naturally, one would expect that the correlation coefficients should be positive and closed to +1. It implies that the futures market is efficient since the futures prices reflect the behavior of the underlying spot prices. In other words, we can conclude that there are many hedgers in the futures market who need to hedge the price risk of the underlying asset. Sometime, however, the correlation coefficients are less than zero or closed to zero. This indicates that the futures market participants propose the future prices without considering the underlying spot prices. In other words, the future market is full of speculators who wish to get only exposure to price moves. In this case, the futures market is called to be less efficient since the future prices cannot be used to infer the behavior of the underlying spot prices.

In this thesis, we use the gold spot prices obtained from London Gold Market Fixing Limited (LGMF) to estimate the no-arbitrage future prices over the period June, 2009 to December, 2011. Then we compute Pearson's correlation coefficients between the estimated no-arbitrage futures prices and the corresponding futures prices obtained from TFEX. Since the spot prices of an underlying asset are perfectly correlated* to the corresponding no-arbitrage futures prices. Thus the correlation coefficients between the gold spot prices and the futures prices can be referred by the correlation coefficients between the no-arbitrage gold futures prices and the gold futures prices. Moreover, we determine the arbitrage opportunities in TFEX from price difference between the no-arbitrage futures prices and the futures prices obtained from TFEX.

In term of modeling commodity prices such as gold prices, an unobservable factor known as "convenience yield" arising from inventories of storable commodities is important. Most precisely, the convenience yield is the flow of services that accrues to an owner of the physical commodity but not to the owner of a contract for future delivery of the commodity. Most obviously, the owner of the physical commodity is able to choose where it will be stored and when to liquidate the inventory. Recognizing the time lost and the costs incurred in transporting a commodity from one location to another, the convenience yield may be thought of as the value of being able to profit from temporary local shortages of the commodity through ownership of the physical commodity. The profit may arise either from local price variations or from the ability to maintain a production process as a result of ownership of an inventory of raw material.

Besides the convenience yield of commodity prices, an other main empirical characteristic that makes commodities noticeable difference from stocks, bonds and other financial assets is seasonality in prices. Many commodities, such

*The general categories indicate strength of the correlation coefficient values : -0.1 to 0.1 are none or very weak correlations, -0.3 to -0.1 or 0.1 to 0.3 are weak correlations, -0.5 to -0.3 or 0.3 to 0.5 are moderate correlations, -0.9 to -0.5 or 0.5 to 0.9 are strong correlations and -1.0 to -0.9 or 0.9 to 1.0 are perfect correlations.

as agricultural commodities or natural gas, exhibit seasonality in prices, due to harvest cycles in the case of agricultural commodities and change consumptions in the case of natural gas. In addition, gold also have seasonal variation in prices as well. In this thesis, we investigate seasonality in gold prices by estimating convenience yield of gold for every trading day.

In the literature, Brennan-Schwartz [2] proposed a model to describe the dynamics of natural resource prices, like copper, in 1985. By considering gold as one of those commodities, they assumed gold spot prices to follow a Geometric Brownian Motion (GBM) and the convenience yield. It is described in the same way as a dividend yield. However, the model is not suitable because they do not concern the mean-reversion property of commodity prices. Mean reversion is one of the main mathematical methodology. In the general idea, high and low prices are temporary. In other words, the prices almost always converge to a long-run mean asymptotically.

In 1997, Schwartz [5] introduced a variation of the model in which the convenience yield is mean reverting and intervenes in the commodity price dynamics as follows,

$$dS_t = \kappa(r - \ln S_t)S_t dt + \sigma S_t dW_t,$$

where S_t is the commodity price at time $t \in [0,T]$, $T \geq 0$, r is a risk-free interest rate, κ is the speed of adjustment of the commodities prices, σ is the volatility of commodities prices, $\{W_t\}_{t\geq 0}^{(\omega)}$ is a Wiener process. When $\ln S_t$ is an instantaneous convenience yield term at time $t \in [0,T]$.

Issaranusorn [3] developed a one-factor model of stochastic behavior gold prices, which is an extension of Schwartz model as follows,

$$dS_t = (r - \delta(t))S_t dt + \sigma S_t dW_t, \qquad S_0 = s_0 > 0, \tag{1.1}$$

$$\frac{d\delta(t)}{dt} = \kappa(\alpha(t) - \delta(t)), \qquad \delta(0) = \delta_0 \tag{1.2}$$

$$\alpha(t) = \alpha_0 + \alpha_1 \sin(2\pi(t - t_\alpha)) \tag{1.3}$$

where S_t is the gold price at time $t \in [0,T]$, $T \geq 0$, r is a risk-free interest rate, $\delta(t)$ is an instantaneous convenience yield at time $t \in [0,T]$ which follows an ordinary differential equation (1.2) and has a mean-reversion property, S_0 and δ_0 are an initial condition. $\{W_t\}_{t\geq 0}^{(\omega)}$ is a Wiener process, κ is the speed of adjustment of the gold prices, α_0 is a long run mean, σ is the volatility of gold prices, and $\alpha(t)$ represents seasonal variation in convenience yield. The parameters α_1 and t_α denote the annual seasonality parameter representing the amplitude of seasonality and the starting point of seasonality in each year, respectively.

The strong solution of the SDE (1.1) can be expressed as

$$S_t = S_{t_0} \exp\left(\int_{t_0}^{t} \left(r - \frac{\sigma^2}{2} - \delta(s) \right) ds + \sigma \sqrt{(t - t_0)} Z \right), \tag{1.4}$$

for all $0 \leq t_0 \leq t \leq T$, where Z is the standard normal random variable when its closed-form solution is

$$\delta(t) = \alpha_0 + C(t) + (\delta_0 - \alpha_0 - C(0)) \exp(-\kappa t), \quad \delta(0) = \delta_0 \tag{1.5}$$

where

$$C(t) = \frac{-2\alpha_1 \kappa \pi \cos(2\pi(t - t_\alpha)) + \alpha_1 \kappa^2 \sin(2\pi(t - t_\alpha))}{\kappa^2 + 4\pi^2} \quad . \tag{1.6}$$

By considering equation (1.4), we obviously see that $\ln\left(\dfrac{S_t}{S_{t_0}}\right)$ is normal distributed with mean $\int_{t_0}^{t}\left(r-\dfrac{\sigma^2}{2}-\delta(s)\right)ds$ and variance $\sigma\sqrt{(t-t_0)}$ as shown in the following proposition.

Proposition 1.1 (Issaranusorn [3])

The gold spot price S_t *in* (1.4) *is neither negative nor zero for all* $t \geq 0$. *Because its closed-form solution* (1.4) *is an exponential function and its initial values is greater than zero. Moreover, for a fixed* $t_0 \geq 0$, $\ln\left(\dfrac{S_t}{S_{t_0}}\right)$ *is normally distributed with mean* $\int_{t_0}^{t}\left(r-\dfrac{\sigma^2}{2}-\delta(s)\right)ds$ *and variance* $\sigma\sqrt{(t-t_0)}$.

In order to determine the no-arbitrage futures prices, we need the following theorem.

Theorem 1.2 (Issaranusorn [3])

For given and fixed maturity date T, *the closed-form solutions for no-arbitrage gold futures price can be expressed as*

$$F_t(t,S_t) = S_t e^{A(T-t)} \tag{1.7}$$

where

$$A(\tau) = r\tau - \int_0^{\tau} \delta(T-s)ds,$$

for all $\tau \geq 0$.

The aim of this thesis is threefold.

(I) To estimate the model parameters in (1.1) - (1.3) based on the maximum likelihood method using the gold spot prices data obtained from London Gold Market Fixing Limited (LGMF) over the period June 2009 to December 2011 in order to evaluate the no-arbitrage futures prices.

(II) To compute the correlations between the no-arbitrage futures prices and futures prices obtained from TFEX over the period January 11, 2010 to December 29, 2011 in order to analyze efficiency of the futures market (TFEX).

(III) To investigate convenience yields in gold in order to study seasonality in gold prices.

The remainder of this thesis is organized as follows.

In Chapter 2, we review the method of maximum likelihood estimation (MLE) and derive the transition density of the SDE (1.1). Then, we estimate parameters in the model described in (1.1) - (1.3) based on the maximum likelihood method by using the gold spot prices data obtained from London Gold Market Fixing Limited (LGMF). In addition, we simulate sample paths for SDE in (1.1) and illustrate a distribution of gold spot prices on December 31, 2012.

In Chapter 3, we explain about gold futures prices data. Moreover, we compute the Pearson's correlation coefficient between the no-arbitrage futures prices and their corresponding futures prices obtained from historical gold futures prices report of the TFEX over the period January 11, 2010 to December 29, 2011. Furthermore, we discuss the result obtained therein.

In Chapter 4, we study convenience yields of gold and discuss about seasonality in gold prices.

In Chapter 5, we conclude the thesis.

CHAPTER II
RESULTS OF PARAMETER ESTIMATION

Continuous-time models based on stochastic differential equations are widely used to the decision for optimal investment. Other applications are in biology, medicine, econometrics, finance, geophysics and oceanography. Generally, the models contain unknown parameters. Nevertheless, we can estimate unknown parameters by using various parameters estimation methods. Maximum likelihood estimation is the best one of widely recognized among researchers in statistics.

In this chapter we begin with an overview of maximum likelihood estimation. Then, we use gold spot price data obtained from the London Gold Market Fixing Limited (LGMF) over the period June 2009 to December 2011 to estimate the model parameters based on the maximum likelihood method. Utilizing the gold spot price data with the estimated parameters. Finally, we simulate sample paths of gold prices using the SDE in (1.1) with the estimated parameters in order to illustrate a distribution of gold price on December 31, 2012.

2.1 Maximum Likelihood Estimation (MLE)

Consider a stochastic differential equation (SDE) of the form

$$dX_t = \mu(X_t;\theta)dt + \sigma(X_t;\theta)dW_t. \tag{2.1}$$

The deterministic component of the dynamics is described by the drift μ while the random part is driven by Wiener process $\{W_t\}_{t\geq 0}^{(\omega)}$ and diffusion coefficient σ. The SDE in (2.1) is called the *Itô diffusion process* or simply the *Itô diffusion*.

Continuous observation of Itô diffusion is not suitable because the path of the Itô diffusion is very jagged and no measuring device can follow a Itô diffusion trajectory continuously. It's quite to be mathematical idealization. Therefore, the discrete observation is used frequently in practice. Consequently, research on discretely observed Itô diffusion is interesting recently with a powerful theory of statistical inference for Itô diffusion.

Statistical inference for Itô diffusion based on discrete-time observations is based on the likelihood function. The literature has mainly concentrated on the maximum likelihood (ML) approach. Suppose that $X = \{X_t, t \geq 0\}$ is observed at times t_i's with $0 = t_0 < t_1 < t_2 \ldots < t_n = T$ and the *transition probability density function* or the *transition density* of X is denoted by $p_X = p_X(t_i, X_{t_i}, t_{i-1}, X_{t_{i-1}}; \theta)$ where θ is a vector of unknown parameters. Since X is Markov, we use the log likelihood function

$$l_n(\theta) = \sum_{i=1}^{n} \ln p_X(t_i, X_{t_i}, t_{i-1}, X_{t_{i-1}}; \theta) \tag{2.2}$$

to estimate θ.

In the literature [4], p_X satisfies the forward Kolmogorov equation (FKE), also known as the Fokker Planck equation, for every fixed $(t_0, x_0) \in [0, T) \times D_X$

$$\frac{\partial p_X}{\partial t}(t, x, t_0, x_0; \theta) = A_t^* p_X(t, x, t_0, x_0; \theta) \text{ for all } (t, x) \in [t_0, T) \times D_X, \tag{2.3}$$

$$A_t^* p(x) = \frac{1}{2} \frac{\partial^2}{\partial x^2}\left(\sigma^2(t, x) p(x)\right) - \frac{\partial}{\partial x}\left(\mu(t, x) p(x)\right) \text{ for all } p \in C^2(\mathbb{R}),$$

subject to the condition

$$\lim_{t\to t_0}\int_{D_X} p_X(t,x,t_0,x_0;\theta)f(x)dx = f(x_0) \text{ for all } f\in C^0(D_X), \tag{2.4}$$

for a probability space $(\Omega, F, \mathrm{P}_\theta)$, $D_X = \{x\in\mathbb{R} \mid \exists t\in[0,T], \mathrm{P}_\theta(X_t = x) = 1\}$ denotes the domain of X where $C^0(D)$ denotes the set of bounded continuous functions on $D\subseteq\mathbb{R}$.

For instance, if $\sigma(t,x) = a$, and $\mu(t,x) = b$ where $a > 0$, b are constants, the corresponding Itô diffusion is called a *Brownian motion with drift* b, and its transition density is giving by

$$p_X(t,x,t_0,x_0;\theta) = \frac{e^{\frac{-(x-(b(t-t_0)+x_0))^2)}{2a^2(t-t_0)}}}{a\sqrt{2\pi(t-t_0)}} \tag{2.5}$$

where $\theta = (b,a)$. One can verify that p_X as expressed in (2.5) satisfies the FKE (2.3) and the condition (2.4) for which $A_t^* \equiv \frac{1}{2}a^2\frac{\partial^2}{\partial x^2} - b\frac{\partial}{\partial x}$.

The transition density of SDE is known by using Proposition 1.1 as

$$p_X(t,x,t_0,x_0;\theta) = \frac{e^{\frac{-(x-(\mu(t,t_0;\theta)+x_0))^2)}{(2\sigma_1^2(t,t_0;\theta)}}}{\sigma_1(t,t_0;\theta)\sqrt{2\pi}} \tag{2.6}$$

where

$$\mu(t,t_0;\theta) = \int_{t_0}^{t}\left(r - \frac{\sigma^2}{2} - \delta(s)\right)ds,$$

$$\sigma_1(t,t_0;\theta) = \sigma\sqrt{t-t_0}$$

and $\theta = (\kappa, \sigma, \alpha_0, \alpha_1, t_\alpha, r, \delta_0)$.

Maximizing $l_n(\theta)$ with respect to θ over a particular parameter space Θ, one can get a maximum likelihood estimator $\hat{\theta}_n$ which is a solution of the optimization problem

$$\hat{\theta}_n = \arg\max_{\theta\in\Theta} l_n(\theta). \tag{2.7}$$

In this research, we employ a numerical package, NMaximize[$l_n(\theta)$] ,in *Mathematica program*, to maximize equation (2.2).

2.2 London Gold Market Fixing Limited (LGMF)

The London gold fixing or gold fix is the procedure by which the price of gold is determined twice each business day on the London market by the five members of The London Gold Market Fixing Limited consist of Barclays Capital, Scotia Mocatta, Deutsche Bank, Societe Generale, and HSBC Investment Banking Group. It is designed to fix a price for settling contracts between members of the London bullion market, but informally the gold fixing provides a recognized rate that is used as a benchmark for pricing the majority of gold products and derivatives throughout the world's markets. The gold fix is conducted in United States dollars (US$), Pound sterling (GBP), and the euro (€) daily at 10.30am and 3pm, London time, via a dedicated telephone conference facility [7].

2.3 Results of MLE

We use gold spot price data obtained from the LGMF over the period June 1, 2009 to December 31, 2011 to estimate the model parameters based on the maximum likelihood method by given $r = 0.007^{**}$ and $\delta_0 = 0$ are constants. By using package, NMaximize[$l_n(\theta)$]. We get the MLE ($\hat{\theta}_n$), where $n = 802$, that shown in Table 2.1.

Table 2.1: The MLE of the model parameters obtained by using the spot prices data of the LGMF over the period June 1, 2009 to December 31, 2011 and their 95% confidence intervals.

parameters	**MLE($\hat{\theta}_n$)**
κ	13.820 (5.825, 20.895)
σ	0.188 (0.185, 0.189)
α_0	-0.219 (-0.324, -0.100)
α_1	0.116 (0.0488,1.322)
t_α	-9.173 (-16.674, -1.467)

2.4 Distribution of Gold Spot Prices in a Future

We simulate the sample paths of S_t by using Euler-Maruyama method. The Euler-Maruyama approximation runs by using the following formula

$$S_{t_n} = S_{t_{n-1}} + (r - \delta(t_{n-1}))S_{t_{n-1}}(t_n - t_{n-1}) + \sigma S_{t_{n-1}}\sqrt{\Delta t}\, z_{t_{n-1}} \tag{2.8}$$

**The average (Arithmetic Mean) of saving interest rates of Bangkok bank that is a commercial bank registered in Thailand over the period September, 2010 to December, 2011 by using data from Bank of Thailand (BOT).

for $n = 1, 2, \ldots, N$ where $t_i = i\Delta t + t_0$, $\Delta t = \dfrac{T - t_0}{N}$, N is an integer, $z_{t_n} \sim$ normal(0,1) and the initial condition $S_{t_0} = s_0$.

We substitube the values of $\kappa, \sigma, \alpha_0, \alpha_1, t_\alpha$ shown in Table 2.1, $T = 1$, $N = 365$ and $\Delta t = 1/365$. We set initial condition $S_{t_0} = 22{,}390$ baht that is the gold spot prices on December 31, 2011.

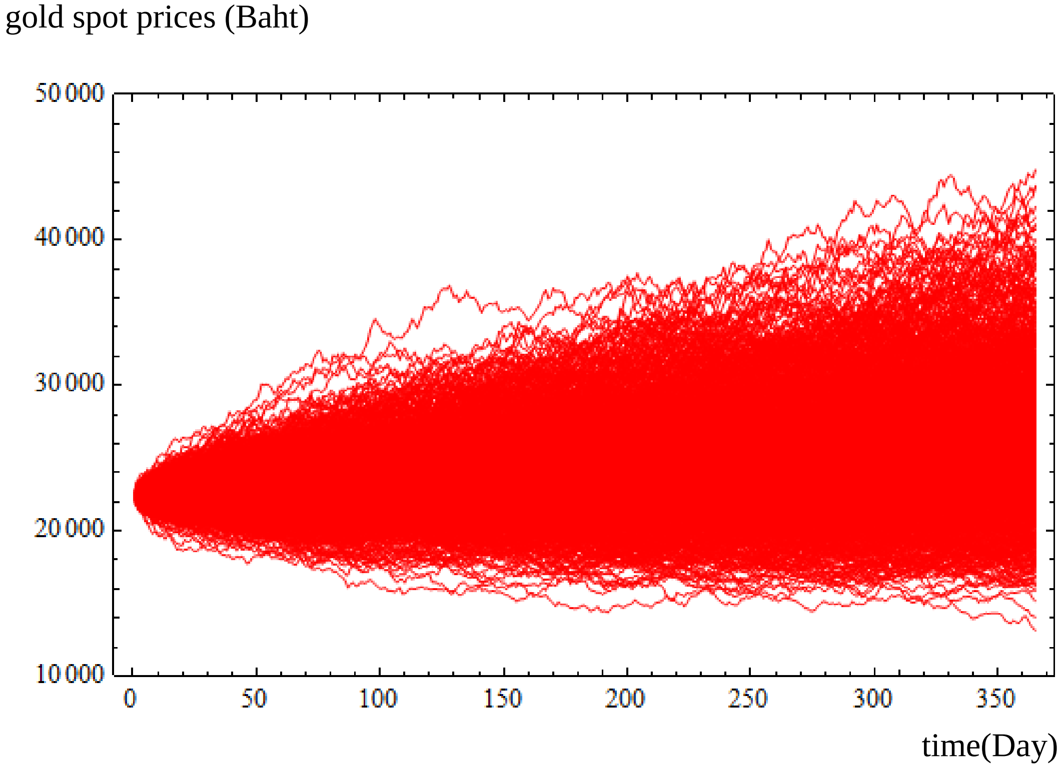

Figure 2.1: The gold future prices sample paths in year 2012.

We illustrate the distribution of gold spot prices on December 31, 2012 in Figure 2.2 that generate by capturing prices in the terminal time of all sample paths in Figure 2.1.

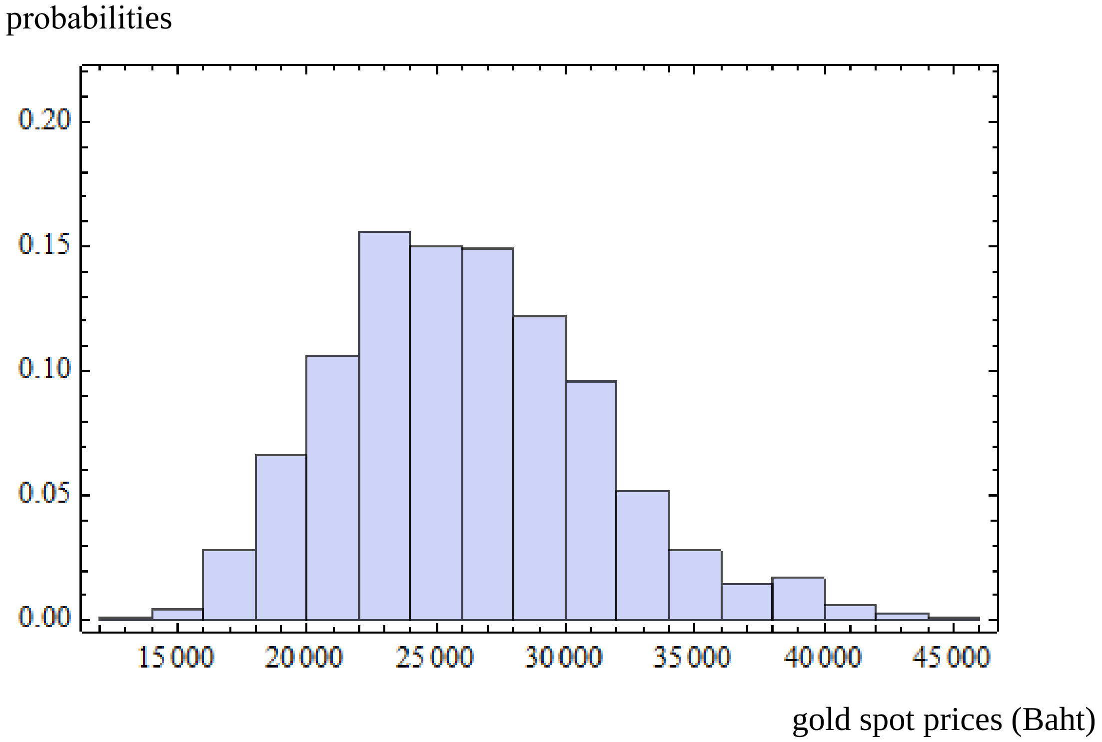

Figure 2.2: Distribution of gold spot prices on December 31, 2012.

In Figure 2.2, the distribution of the gold spot prices is positively skewed that is a tail extending out to the right. Therefore, the probability that highly gold spot prices is more than the probability that lowly gold spot prices. This implies that gold spot prices tend to rise in one year after December 31, 2012.

CHAPTER III

CORRELATIONS BETWEEN GOLD SPOT PRICES AND FUTURES PRICES

As described in Chapter I, the correlation coefficients between the underlying spot prices and the corresponding future prices can measure an efficiency of a futures market of an underlying asset. Using the estimated parameters in Chapter II, we compute the Pearson's correlation coefficients between the no-arbitrage futures prices and their corresponding futures prices obtained from historical gold futures prices from TFEX over the period from January 11, 2010 to December 29, 2011. By using correlation coefficients, we discuss about the efficiency of TFEX.

3.1 Gold Futures Prices Data

In this research, we consider only 50 Baht Gold Futures type with ticker symbol GF. Gold futures have the expiration date in the end of even month that are February, April, June, August, October and December with ticker symbol the abbreviation of each month are G, J, M, Q, V and Z respectively. We collect 10 series of the futures prices obtained from historical gold futures prices report of the TFEX for every trading day as Table 3.1.

Table 3.1: The futures prices obtained from historical gold futures prices report of the TFEX over the period January 11, 2010 to December 29, 2011.

contracts	periods
GFM10	January 11, 2010 - June 29, 2010
GFQ10	February 25, 2010 - August 30, 2010
GFV10	April 29, 2010 - October 28, 2010
GFZ10	June 29, 2010 - December 29, 2010
GFG11	August 30, 2010 - February 25, 2011
GFJ11	October 28, 2010 - April 28, 2011
GFM11	December 29, 2010 - June 29, 2011
GFQ11	February 25, 2011 - August 30, 2011
GFV11	April 28, 2011 - October 28, 2011
GFZ11	June 29, 2011 - December 29, 2011

3.2 Correlations between the Gold Spot Prices and Futures Prices and Discussions

According to Introduction, the correlation coefficients between the no-arbitrage gold futures prices and the gold futures prices can refer the correlation coefficients between the gold spot prices and the futures prices because the spot prices of an underlying asset are perfectly correlated to the corresponding no-arbitrage futures prices. Utilizing the gold spot price data with the estimated parameters, we compute the no-arbitrage prices of gold futures by using equation (1.7). In this research, we investigate correlation coefficients between the no-arbitrage gold futures prices and their corresponding futures prices obtained historical gold futures prices report of the TFEX over the period years 2010 and 2011.

3.2.1 Computing Correlation Coefficients

We compute Pearson's correlation coefficient (ρ) between the no-arbitrage futures prices and their corresponding futures prices using the following formula

$$\rho = \frac{\sum_{i=1}^{n}\left(X_{t_i} - \bar{X}\right)\left(Y_{t_i} - \bar{Y}\right)}{\sqrt{\sum_{i=1}^{n}\left(X_{t_1} - \bar{X}\right)^2}\sqrt{\sum_{i=1}^{n}\left(Y_{t_i} - \bar{Y}\right)^2}}$$

where

$$\bar{X} = \frac{\sum_{i=1}^{n} X_{t_i}}{n} \quad , \; \bar{Y} = \frac{\sum_{i=1}^{n} Y_{t_i}}{n}$$

and $X = \{X_{t_1}, X_{t_2}, \ldots, X_{t_n}\}$ is vector of the no-arbitrage futures prices and $Y = \{Y_{t_1}, Y_{t_2}, \ldots, Y_{t_n}\}$ is vector of their corresponding the futures prices that are observed at times t_i's with $0 = t_0 < t_1 < t_2 \ldots < t_n = T$.

In Figure 3.1-3.10, we plotted the graphs of the no-arbitrage gold futures prices, the gold futures prices that expiration date as Table3.1 over the period January 11, 2010 to December 29, 2011.

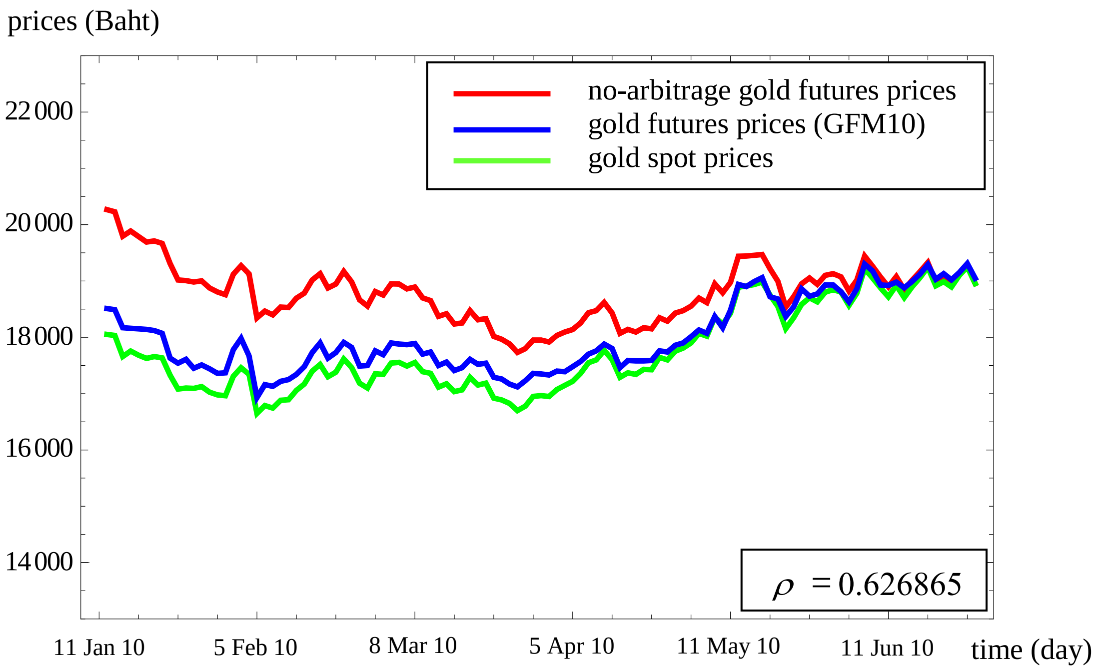

Figure 3.1: The no-arbitrage gold futures prices, the gold futures prices (GFM10) and the gold spot prices over the period January 11, 2010 to June 29, 2010.

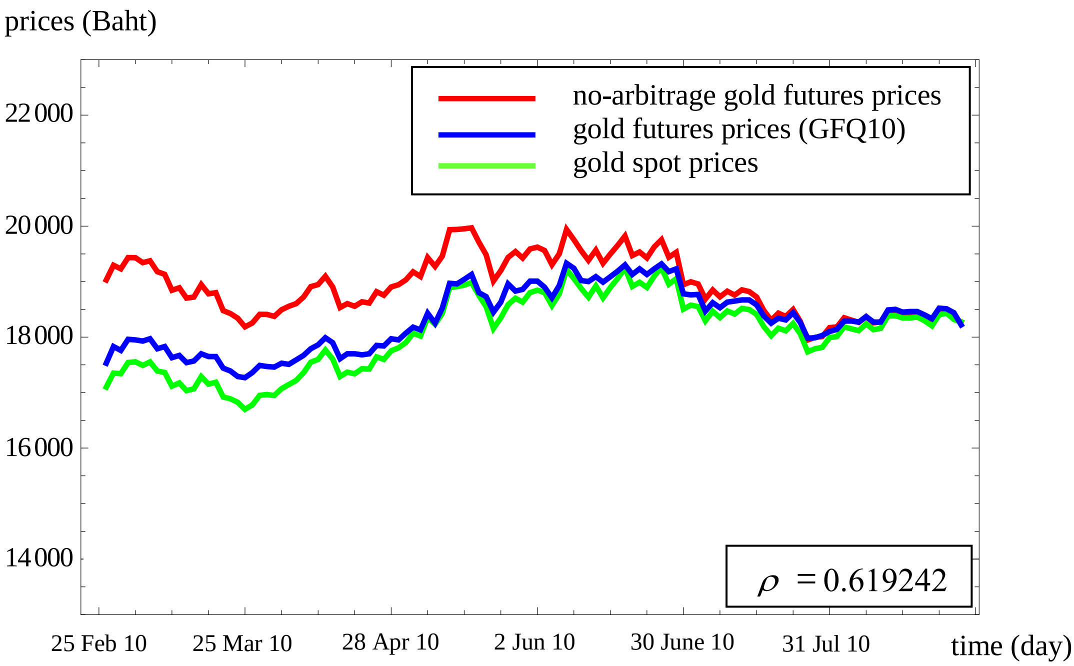

Figure 3.2: The no-arbitrage gold futures prices, the gold futures prices (GFQ10) and the gold spot prices over the period February 25, 2010 to August 30, 2010.

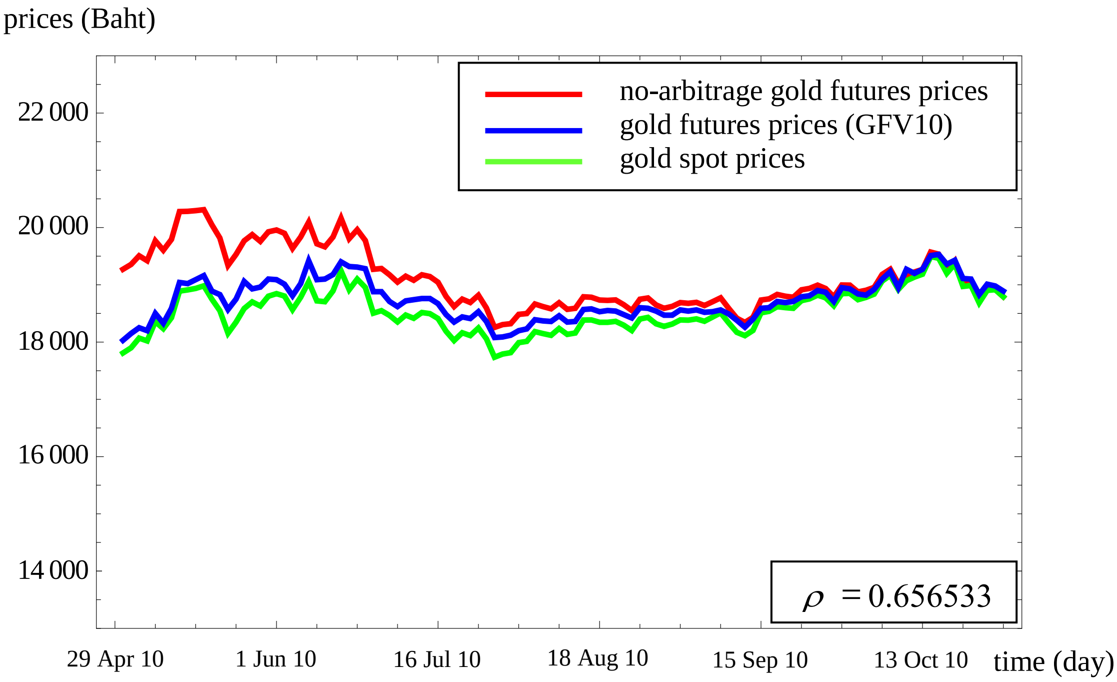

Figure 3.3: The no-arbitrage gold futures prices, the gold futures prices (GFV10) and the gold spot prices over the period April 29, 2010 to October 28, 2010.

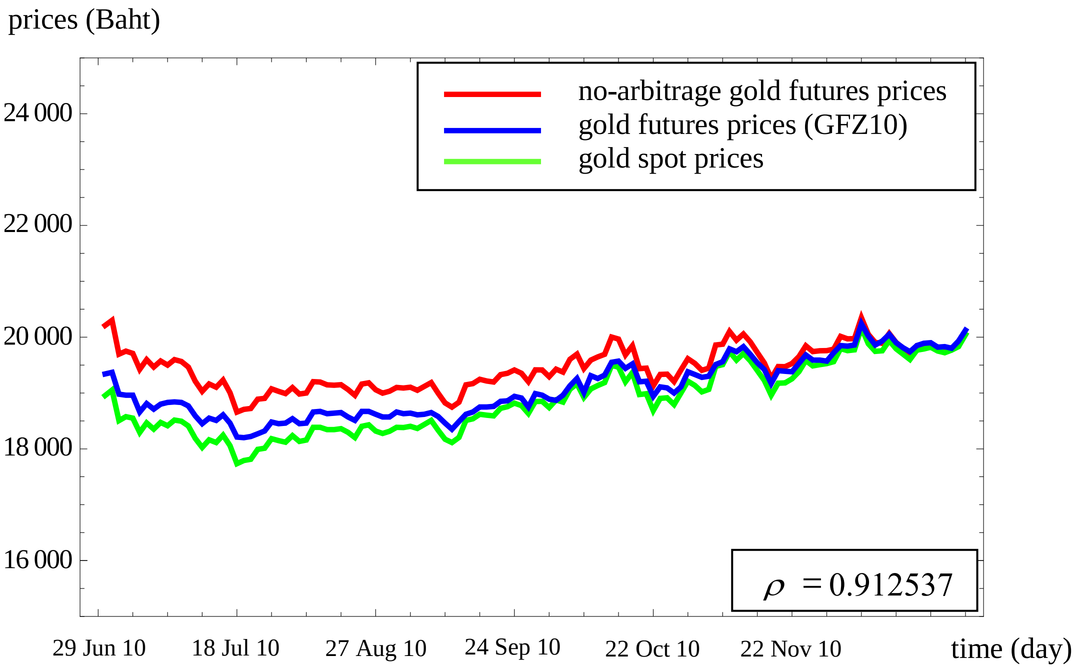

Figure 3.4: The no-arbitrage gold futures prices, the gold futures prices (GFZ10) and the gold spot prices over the period June 29, 2010 to December 29, 2010.

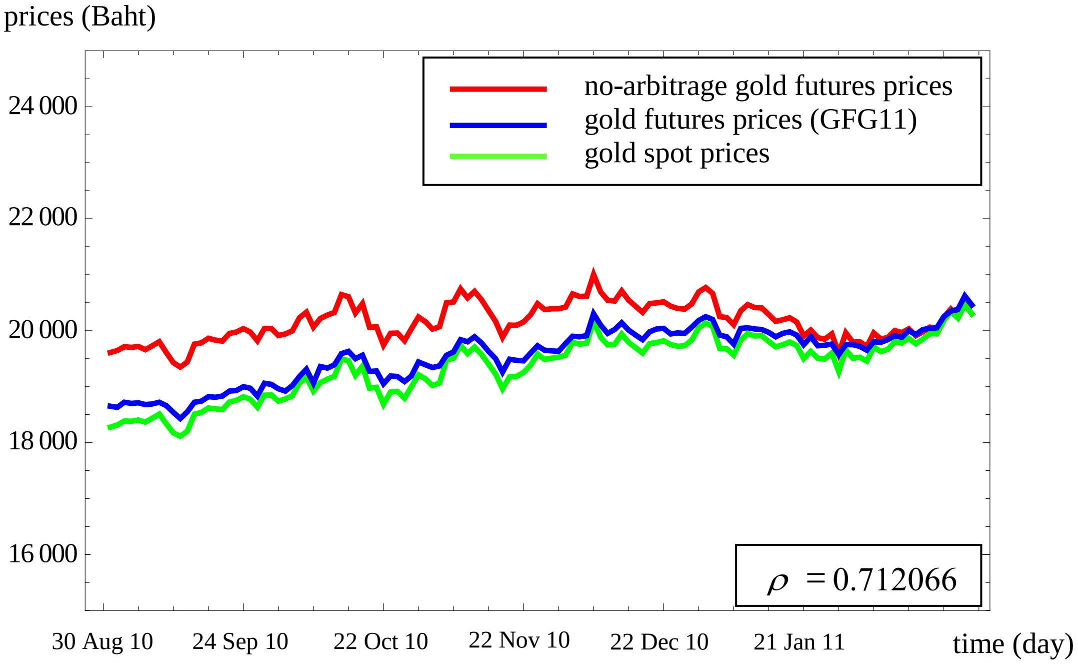

Figure 3.5: The no-arbitrage gold futures prices, the gold futures prices (GFG11) and the gold spot prices over the period August 30, 2010 to February 25, 2011.

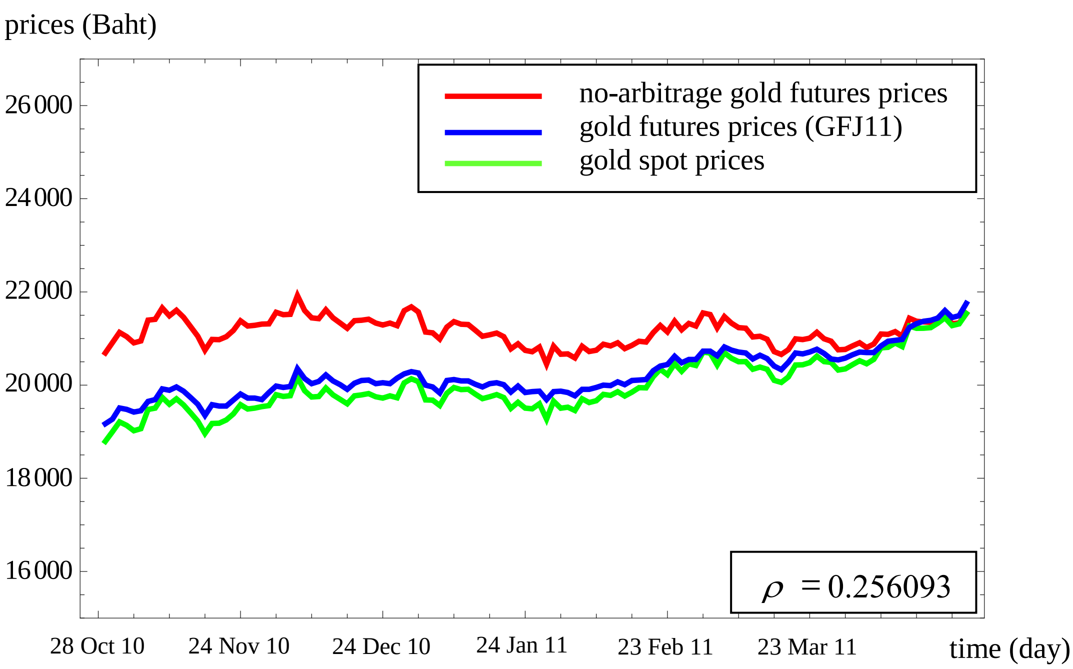

Figure 3.6: The no-arbitrage gold futures prices, the gold futures prices (GFJ11) and the gold spot prices over the period October 28, 2010 to April 28, 2011.

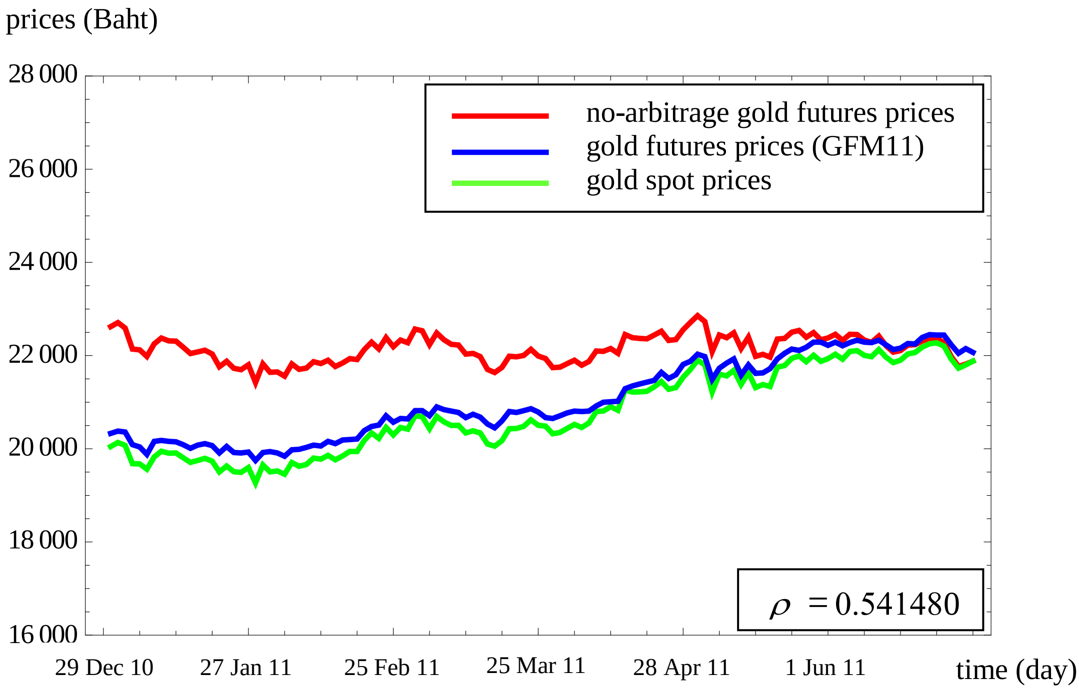

Figure 3.7: The no-arbitrage gold futures prices, the gold futures prices (GFM11) and the gold spot prices over the period December 29, 2010 to June 29, 2011.

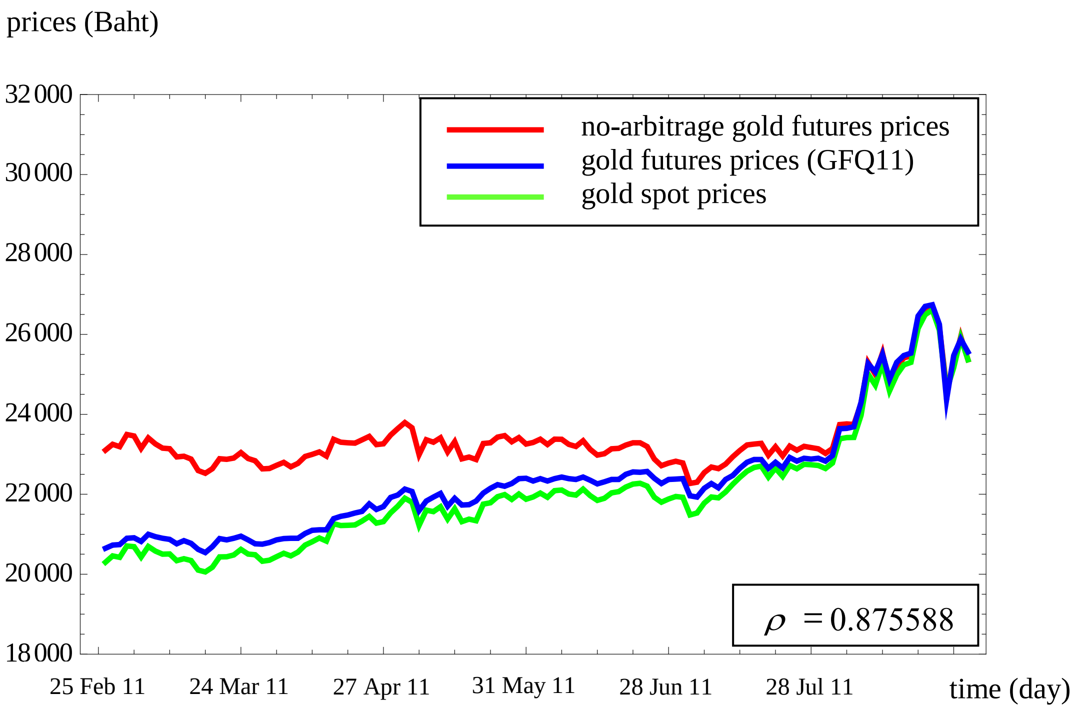

Figure 3.8: The no-arbitrage gold futures prices, the gold futures prices (GFQ11) and the gold spot prices over the period February 25, 2011 to August 30, 2011.

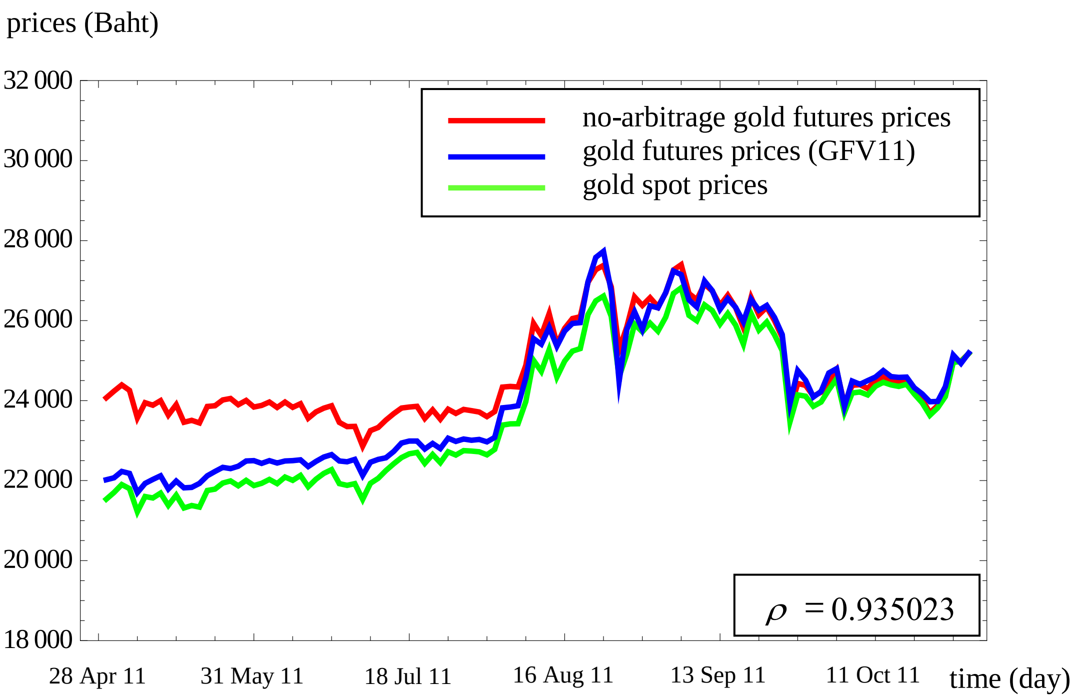

Figure 3.9: The no-arbitrage gold futures prices, the gold futures prices (GFV11) and the gold spot prices over the period April 28, 2011 to October 28, 2011.

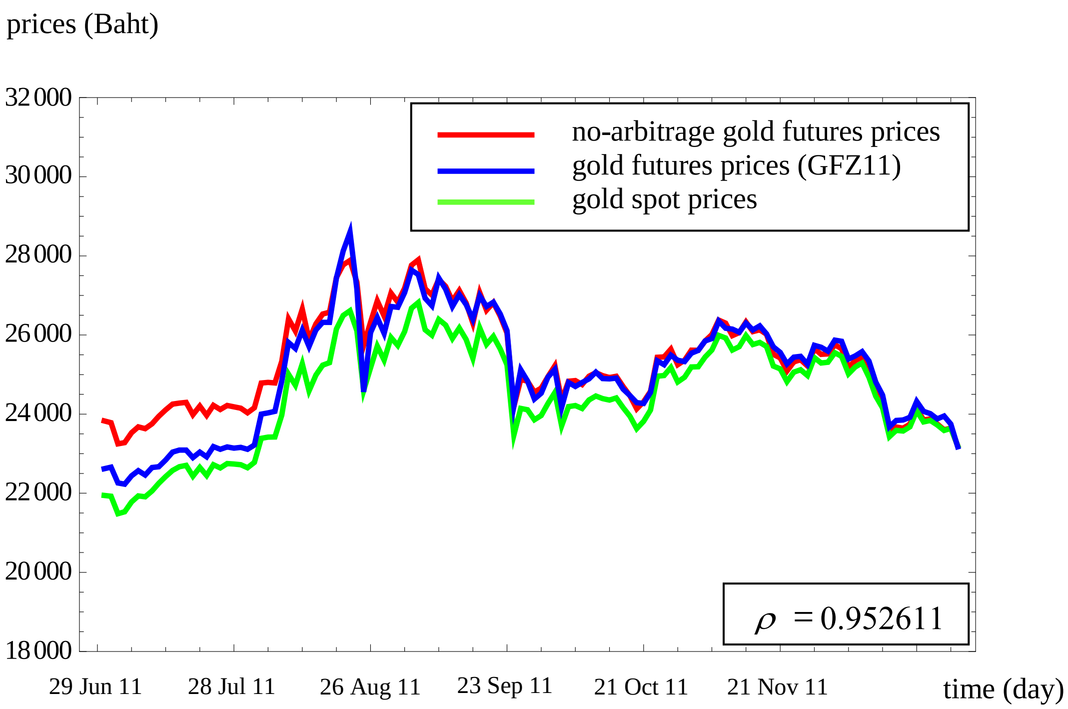

Figure 3.10: The no-arbitrage gold futures prices, the gold futures prices (GFZ11) and the gold spot prices over the period June 29, 2011 to December 29, 2011.

Table 3.2: The correlation coefficients between the no-arbitrage futures prices and their corresponding futures prices over the period January 11, 2010 to December 29, 2011.

contracts	correlation coefficients
GFM10	0.626865
GFQ10	0.619242
GFV10	0.656533
GFZ10	0.912537
GFG11	0.712066
GFJ11	0.256093
GFM11	0.541480
GFQ11	0.875588
GFV11	0.935023
GFZ11	0.952611

3.2.2 Discussion about the Efficiency of TFEX

We have found that the Pearson's correlation coefficients between no-arbitrage futures prices and their corresponding futures prices, except GFJ11, are strongly correlated and perfectly correlated as shown in Table 3.2. There are many reasons that why GFJ11 is weak correlation and nearby series have less correlation coefficients than they should be. First of all, China's inflation rate rose steadily in January 2010. Thus, inventors need safe asset especially gold. In March 2010, problems riots in the Middle East and North Africa caused oil prices and food prices were increasing that lead to inflation. Moreover, in this mouth, the new worst record earthquake and tsunami in Japan affected the growth of the global economy. These results lead to increasing panic of gold investment, so spectacular uses this opportunity to easily make profit.

Overall the Pearson's correlation coefficients between no-arbitrage futures prices and their corresponding futures prices are positively correlated between 0.26 and 0.95. These results support our assertion that TFEX is an efficient futures market since the futures prices reflect the behavior of the underlying spot prices.

3.2.3 Discussion about the No-arbitrage Gold Futures Prices and the Gold Futures Prices

We can easily see from the Figure 3.1-3.10 such that the no-arbitrage gold futures prices are higher than gold futures prices obtained TFEX. These results reflect that TFEX are full of speculators who attempt to make profit from short or medium term fluctuations in gold future prices. Thus, the gold futures prices in TFEX are lower than the theory.

CHAPTER IV

CONVENIENCE YIELDS AND SEASONALITY IN GOLD PRICES

In this chapter, we provide a basic concept of convenience yields. Using the estimated parameters in Chapter II, we determine the convenience yields of gold over the time period January, 2009 to December, 2011. Furthermore, we study seasonality in gold prices which is rarely exhibited in gold markets.

4.1 Inventory, Convenience Yields and Seasonality

In markets for storable commodities such as gold and crude oil, inventories play an important role in determining price. In manufacturing industries, cost of changing production in response to fluctuations in demand is reduced by using inventories. Furthermore, inventories can reduce marketing cost by helping to deliveries scheduling and avoid running out of stock. Producers must decide their production levels jointly with their expected inventory drawdown or buildup. These decisions are made in respect of two prices : a spot price and a price for storage. Though, we cannot directly observe the price for storage, it can be determined from the spread between spot prices and futures prices. This price of storage is equal to the marginal value of storage that is the benefits from a marginal unit of inventory to inventory holders, and is called the *marginal convenience yield*.

For considering gold as a commodity, marginal convenience yield fluctuates considerably over time. This fluctuation can be predictable, in that it correspond to seasonal variations in the convenience yield so we choose Issaranusorn’s model as described in (1.2) - (1.3).

4.2 Seasonality in Gold Prices

We first consider the tendency of gold spot prices data over the period January, 2007 to December, 2011 as Figure 4.1. Since, we cannot directly observe the seasonal variation of gold spot prices in the Figure 4.1. Alternatively, we investigate the seasonal variation of gold spot prices by investigating the convenience yields.

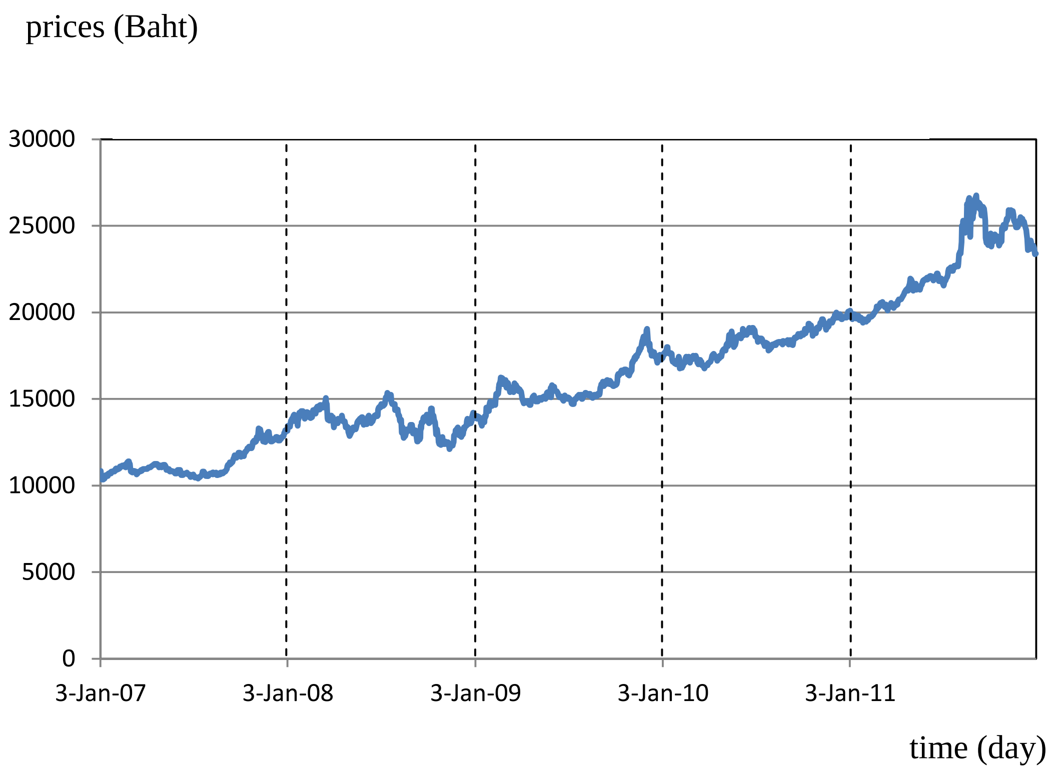

Figure 4.1: The gold spot prices data over the period January 2007 to December 2011.

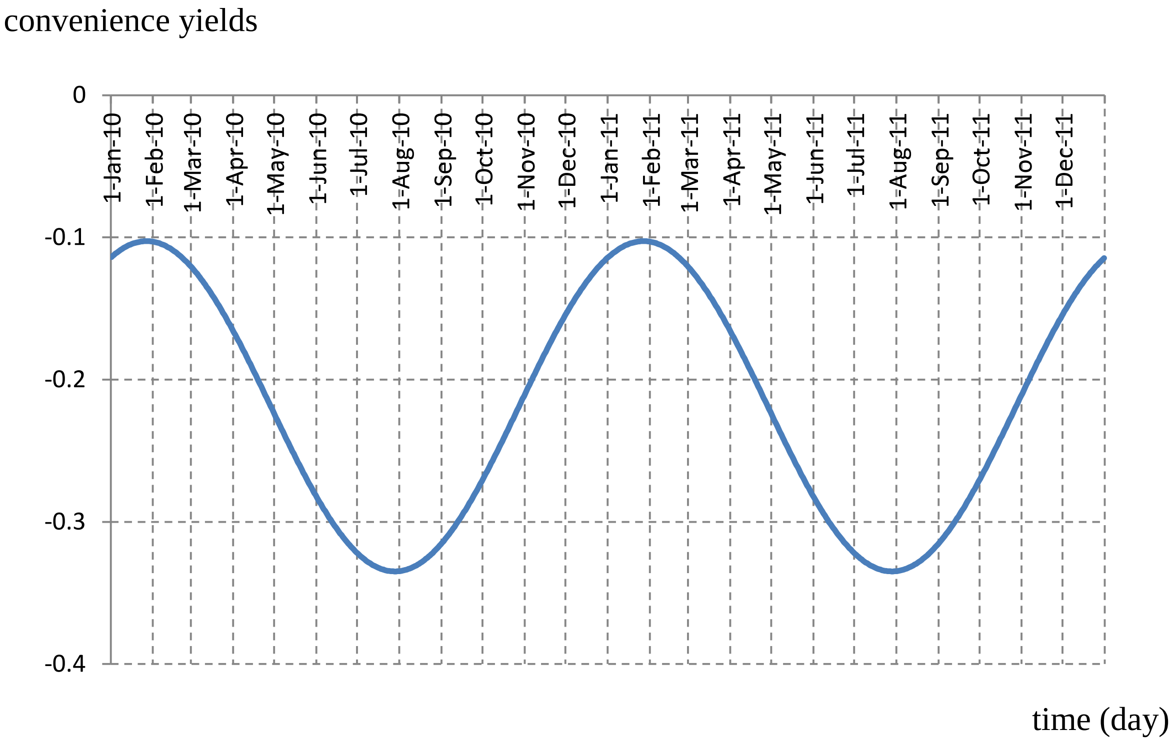

Figure 4.2: The instantaneous convenience yields with seasonal variation of gold spot prices for the model over the period January, 2010 to December, 2011.

In Figure 4.2, we plot the graph of the instantaneous convenience yields in equations (1.5) - (1.6). According to the graph, it shows that the convenience yields of gold spot prices are highest in the end of January. This is could be the case that people especially purchase gold during the end of this month. On the other hands, the convenience yields are the lowest in the end of July. This is could be the case that people less purchase gold during end of this month. These results have supported the idea that many Thai people, especially Thai-Chinese people, purchase gold as a gift on Chinese new year's day during the end of January annually and no special event until the traditional Indian wedding that needs a large amount of gold as a dowry in the end of year. Moreover, we can observe that the convenience yields of gold price are negative over the period January, 2010 to December, 2011. This result implies that the storage cost of gold exceeds the benefit of holding gold.

CHAPTER V
CONCLUSIONS

In this thesis, we have studied behavior of gold prices by using the stochastic model proposed by Issaranusorn [3]. We have used gold spot prices data obtained from the LGMF to estimate the model parameters based on the maximum likelihood method. Utilizing the gold spot prices data with the estimated parameters, we have computed the no-arbitrage prices of gold futures. Then we have considered Pearson's correlation coefficients between the no-arbitrage futures prices and their corresponding futures prices obtained from TFEX. We have found that all the Pearson's correlation coefficients are all positive. Since the spot prices and the no-arbitrage futures prices have been perfectly correlated. This result implies that the gold spot prices and their futures prices are strongly correlated. In the economic point of view, these results have suggested that the futures market, the TFEX, is efficient. In other words, the futures prices reflect behavior of their underlying asset prices. Moreover, we have observed that the no-arbitrage gold futures prices are higher than the gold futures prices obtained TFEX. These result reflect that TFEX is full of speculators who attempt to make profit from short or medium term fluctuations in gold future prices.

Then, we have considered seasonal variation in gold prices by investigating the convenience yields of gold in Thailand. We have found that the convenience yields of gold spot prices are highest in the end of January. This is could be the case that people especially purchase gold during the end of this month. On the other hands, the convenience yields are the lowest in the end of July. This is could be the case that people less purchase gold during end of this month. These results have supported the idea that many Thai people, especially Thai-Chinese people, purchase gold as a gift on Chinese new year's day during the end of January annually and no special event until the traditional Indian wedding that needs a large amount of gold as a dowry in the end of year. Moreover, we can observe that the convenience yields of gold price are negative over the period January, 2010 to December, 2011. This result implies that the storage cost of gold exceeds the benefit of holding gold.

For future work, we may extend the stochastic convenience yields setting that could suitably describe about behavior gold prices including their convenience yields and seasonality. In addition, we may try to apply the model for others Asian gold market or European gold market to measuring an efficiency of a futures market of gold.

REFERENCES

[1] Aït-Sahalia, Y. *Transition Densities for Interest Rate and Other Nonlinear Diffusions*. The Journal of Finance. 1999; 54(2): 1361–1395

[2] Brenan, M.J. & E.S. Schwartz. *Evaluating Natural Resource Investments*. the Journal of Business. 1985; 58(2): 133-155.

[3] Issaranusorn, N. *Stochastic Modeling for Gold Prices and Its Application to gold Derivative Pricing* [dissertation]. Bangkok. Chulalongkorn University. 2010.

[4] Rujivan, S. *Affine Transformations of Itô Diffusions and their Transition Densities*. Walailak Journal April 2011 ; 8(1): 1-9.

[5] Schwartz, E.S. . *The Stochastic Behavior of Commodity Prices*: Implications for Valuation and Hedging. the Journal of Finance. 1997; 52(3): 1297-1338.

[6] Thailand Futures Exchange (TFEX), Knowledge of gold futures investment. [Online] Available: http://www.tfex.co.th/th/investor_corner/ knowledge_p7.html#02, (October 9, 2011).

[7] Wikipedia, Gold fixing. [Online] Available: http://en.wikipedia.org/wiki/Gold_fixing, (December 29, 2012).

APPENDIX

APPENDIX

Code of Mathematica program for maximize the log likelihood function

```
SetDirectory["D:\\"];
Data=Import["GoldSpotPricesData.csv"];
T=Transpose[Data][[1]];
X=Log[Transpose[Data][[2]]];

Ct[t_,kappa_,sigma_,alpha0_,alpha1_,talpha_,r_,delta0_ ]
    := (-2*alpha1*kappa*Pi*Cos[2*Pi*(t-talpha)]
    +alpha1*kappa*kappa*Sin[2*Pi*(t-talpha)])/(kappa*kappa+4*Pi*Pi);

Del[t_,kappa_,sigma_,alpha0_,alpha1_,talpha_,r_,delta0_ ]
    : = alpha0+Ct[t,kappa,sigma,alpha0,alpha1,talpha,r,delta0]
    +(delta0-alpha0-Ct[0,kappa,sigma,alpha0,alpha1,talpha,r,delta0])
    *Exp[-kappa*t];

mu[t_,t0_,kappa_,sigma_,alpha0_,alpha1_,talpha_,r_,delta0_ ]
    := 1/2(t-t0)( (r-sigma^2/2-Del[t,kappa,sigma,alpha0,alpha1,talpha,r,delta0])
    +(r-sigma^2/2-Del[t0,kappa,sigma,alpha0,alpha1,talpha,r,delta0]));

sigma1[t_,t0_,kappa_,sigma_,alpha0_,alpha1_,talpha_,r_,delta0_ ]
   : = sigma*Sqrt[t-t0];

px[t_,t0_,x_,x0_,kappa_,sigma_,alpha0_,alpha1_,talpha_,r_,delta0_ ]
   := Exp[(-(x-(mu[t,t0,kappa,sigma,alpha0,alpha1,talpha,r,delta0]+x0))^2)
   /(2*sigma1[t,t0,kappa,sigma,alpha0,alpha1,talpha,r,delta0]^2)]
   /(sigma1[t,t0,kappa,sigma,alpha0,alpha1,talpha,r,delta0]*Sqrt[2*Pi]);

l[kappa_, sigma_, alpha0_, alpha1_, talpha_, r_, delta0_ ]
```

$$: = \sum_{i=1}^{\mathrm{Length[T]} - 1} \mathrm{Log[px[T[[i + 1]], T[[i]], X[[i + 1]], X[[i]],}$$

```
  kappa, sigma, alpha0, alpha1, talpha, r, delta0]];

Clear[kappa,sigma,alpha0,alpha1,talpha,r,delta0];
r=0.007; delta0=0;

NMaximize[{l[kappa,sigma,alpha0,alpha1,talpha,r,delta0],
   0<kappa≤100 && 0<sigma≤1 && -10≤alpha0≤10 && 0≤alpha1≤10
   &&10≤talpha≤10}, {kappa,sigma,alpha0,alpha1,talpha},
   Method → "DifferentialEvolution"]
```

VITA

Mr. Opat Chanpongpisud was born in June 28, 1987, in Uttaradit. He received a bachelor degree in Mathematics from Department of Mathematics, Faculty of science, ChiangMai University, Thailand 2009. He has been financial supported by the Development and Promotion of Science and Technology talents project (DPST) for doing research.

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