Chapter 4
Transport of Cosmic Rays Across Shock

4.1 Introduction

This work is a partial sequel of the works developed by Ruffolo (1991,1995). We have simulated the interplanetary transport of solar cosmic rays with a varying solar wind speed. The shock generated by solar flare make solar wind speed different at downstream region, the region that had been passed by the shock, and upstream, the region that had not been passed by the shock yet. When cosmic particles cross the shock they will be accelerated, thereafter they go away from that shock. This makes anisotropy at both sides of the shock.

4.2 Reference Frames

Before we go into details some convenient convention about reference frames will be devised, following de Hoffmann and Teller (1950). There are two reference frames used:

The co-moving or local frames These frames are those in which the observer is at rest relative to the medium on the one or other side of the shock.

The shock frame This frame is one in which the observer is at rest relative to the shock front. Viewed from this frame the flow will appear time-independent. This frame from now on is called de Hoffmann–Teller frame.

4.3 Conservation and Lorentz Transformation of Physical Quantities

Because flow in de Hoffmann–Teller frame appears time-independent, we will work in this frame when we consider the conservation and Lorentz Transformation of physical quantities. Consider the shock which stands stationary between two regions, upstream and downstream solar wind. In upstream region the flow velocity V_{su} will be directed toward the shock plane; in downstream region the flow velocity V_{sd} is directed away from that plane.

For convenience we assume that the directions of the flows are parallel to magnetic fields in corresponding region. We denote cosine of pitch angle by μ, the angle between a flowing particle and magnetic field. In de Hoffmann-Teller frame magnetic moment $M = P^2(1-\mu^2)/2mB$ of the particle is conserved (Dekker, 1983), so do its energy and momentum (de Hoffmann and Teller, 1950; Lüst,1960). From these principles we can derive relation about μ's and momenta P's from one region, after crossing the shock, to the other region as following.

1. Particle Crossing from Upstream to Downstream Region: Consider a particle with momentum P_u and cosine of pitch angle μ_u in upstream local frame. The particle is directed toward the shock. Using Lorentz Transformation from upstream local frame to de Hoffmann–Teller frame for μ and P we do the following:

   In any frame μ can be determined from P by the relation

   $$\mu = \frac{P_{||}}{P} = \frac{P_{||}}{\sqrt{P_{||}^2 + P_{\perp}^2}} \tag{4.1}$$

   where

   $P_{||}$ is the component of P parallel to the magnetic field

   $P_{\perp}$ is the component of P perpendicular to the magnetic field

   then in upstream de Hoffmann–Teller frame we obtain

   $$\begin{aligned} P_{su,||} &= \gamma_{su}(P_{u,||} - \beta_{su}E_u) \\ P_{su,\perp} &= P_{u,\perp} = P_u\sqrt{1-\mu_u^2} \end{aligned} \tag{4.2}$$

   where $\beta_{su} = V_{su}/c$ and $\gamma_s u = 1/\sqrt{1-\beta_{su}^2}$ and

   $$\mu_{su} = \frac{P_{su,||}}{P_{us}} = \frac{P_{su,||}}{\sqrt{P_{su,||}^2 + P_{su,\perp}^2}}. \tag{4.3}$$

and energy of the particle was calculated from relation

$$E_{us} = \sqrt{P_{us}^2 c^2 + m^2 c^4}. \tag{4.4}$$

When a particle crosses the shock in this frame, its magnetic moment and momentum are conserved, so

$$P_{us} = P_{ds} \tag{4.5}$$

and

$$\frac{P_{us}^2(1-\mu_{us}^2)}{2B_u m} = \frac{P_{ds}^2(1-\mu_{ds}^2)}{2B_d m} \tag{4.6}$$

From eqs (4.5) and (4.6) we can find a relation between μ of particle before and after crossing the shock as follow:

$$\mu_{ds} = -\sqrt{1 - \frac{B_d}{B_u}(1-\mu_{us}^2)} \tag{4.7}$$

minus sign is used because the particle is directed toward the shock. Now we know momentum P_{ds} and μ_{ds} in downstream de Hoffmann–Teller frame, then we transform these physical quantities from this frame to downstream local frame:

$$P_{d,\|} = \gamma_{sd}(P_{ds,\|} + \beta_{du} E_{du}) \tag{4.8}$$

$$P_{d,\perp} = P_{ds,\perp} = P_{ds}\sqrt{1-\mu_{ds}^2} \tag{4.9}$$

where $\beta_{du} = V_{du}/c$ and $\gamma_{sd} = 1/\sqrt{1-\beta_{du}^2}$ and because of conserving energy, we obtain

$$E_{du} = E_{us} \tag{4.10}$$

then we can calculate μ in downstream de Hoffmann–Teller frame:

$$\mu_d = \frac{P_{d,\|}}{P_d} = \frac{P_{d,\|}}{\sqrt{P_{d,\|}^2 + P_{d,\perp}^2}} \tag{4.11}$$

2. Particle Crossing from Downstream to Upstream Region: known in downstream local frame are μ_d and P_d , now consider Lorentz transformation from downstream local frame to downstream de Hoffmann–Teller frame:

$$P_{ds,\|} = \gamma_{ds}(P_{d,\|} - \beta_{ds}E_d) \tag{4.12}$$

$$P_{ds,\perp} = P_{d,\perp} = P_d\sqrt{1-\mu_d^2} \tag{4.13}$$

$$\mu_{ds} = \frac{P_{ds,\|}}{P_{ds}} = \frac{P_{ds,\|}}{\sqrt{P_{ds,\|}^2 + P_{ds,\perp}}} \tag{4.14}$$

where $\beta_{ds} = V_{ds}/c$ and $\gamma_{ds} = 1/\sqrt{1-\beta_{ds}^2}$ when a particle crosses the shock from downstream to upstream, it conserves magnetic moment and momentum:

$$P_{us} = P_{ds} \tag{4.15}$$

$$\frac{P_{us}^2(1-\mu_{us}^2)}{2mB_u} = \frac{P_{ds}^2(1-\mu_{ds}^2)}{2mB_d} \tag{4.16}$$

then we obtain

$$\mu_{us} = \sqrt{1 - \frac{B_u}{B_d}(1-\mu_d^2)} \tag{4.17}$$

we then transform P_{us} and μ_{us} to upstream local frame:

$$P_{u,\|} = \gamma_{us}(P_{us,\|} + \beta_{us}E_{us}) \tag{4.18}$$

$$P_{u,\perp} = P_{us,\perp} \tag{4.19}$$

where $\beta_{us} = V_{us}/c$ and $\gamma_{us} = 1/\sqrt{1-\beta_{us}^2}$ and $E_us = \sqrt{P_{us}^2c^2 + m^2c^4}$ and

$$\mu_u = \frac{P_{u,\|}}{P_u} = \frac{P_{u,\|}}{\sqrt{P_{u,\|}^2 + P_{u,\perp}^2}} \tag{4.20}$$

In the case a particle crosses shock from upstream to downstream, it is possible to be reflected by the shock front. When this is the case its momentum is conserved in the de Hoffmann–Teller frame, then we can calculate its value in upstream local frame by Lorentz transformation between de Hoffmann–Teller frame and upstream local frame.

Before collision with shock, the particle has momentum and cosine of pitch angle P_u and μ_u respectively. Transforming to upstream de Hoffmann–Teller frame we obtain

$$P_{us,\|} = \gamma_{us}(P_{us,\|} - \beta_{us} E_u) \tag{4.21}$$

$$P_{us,\perp} = P_{u,\perp} = P_u\sqrt{1-\mu_u^2} \tag{4.22}$$

when reflecting at the shock front, the particle has new P_{rs} and μ_{rs}:

$$P_{rs,\|} = P_{us,\|} \tag{4.23}$$

$$\mu_{rs} = -\mu_{us} \tag{4.24}$$

where $P_{rs,\|}$ is momentum component parallel B_u in de Hoffmann–Teller frame $P_{rs,\perp}$ is momentum component perpendicular B_u in de Hoffmann–Teller frame. We then transform back from de Hoffmann–Teller frame to upstream local frame again:

$$P_{r,\|} = \gamma_{us}(P_{rs,\|} + \beta_{us} E_{rs}) \tag{4.25}$$

$$P_{r,\perp} = P_{rs,\perp} \tag{4.26}$$

where $E_{rs} = \sqrt{P_{rs}^2 c^2 + m^2 c^4}$.

4.4 Numerical Method The transport equation in this work is in the form

$$\begin{aligned}\frac{\partial F(t,\mu,z,p)}{\partial t} &= -\frac{\partial}{\partial z}\mu v F(t,\mu,z,p) \qquad \text{(streaming)}\\ &-\sec(\psi)\frac{\partial}{\partial z}\left(1-\mu^2\frac{v^2}{c^2}\right)v_{sw}\\ &.F(t,\mu,z,p) \qquad \text{(convection)}\\ &+\frac{\partial}{\partial \mu}\frac{\phi(\mu)}{2}\frac{\partial}{\partial \mu}\left(1-\mu\frac{v_{sw}}{c^2}v\sec\psi\right)\\ &.F(t,\mu,z,p) \qquad \text{(scattering)}\end{aligned} \tag{4.27}$$

The above transport equation was solved by means of finite-difference method. Each term in the right was treated as if it stands without the other two terms, then it was updated with the left term in each time step. This method of updating $F(t, \mu, z, p)$ is called operator or time splitting:

1. Update the distribution function F with the first half the effect of pitch-angle scattering.

2. From the result above compute new F due to streaming and convection effect.

3. Update F due to the scattering effect for the second half. The result of this step is F at new time.

Above are general procedures to update distribution function F under appropriate assumptions. We have the following assumptions in mind for transport across an oblique shock front:

1. Transmission of flux can be classified into two cases

   a If the flux in entire cell is transmitted, we perform z-transport by assuming p, μ, V_{sw} are at initial grid point.

   b If partial flux was transmitted we perform z-transport by assuming p, V_{sw} are at initial grid point but use μ at center of transmitted portion of cell.

2. Reflection of flux. In this case we perform z-transport by assuming P, μ, V_{sw} are at initial grid point as in the first case except that when particles reach z of shock, we work in the de Hoffmann–Teller frame for change $\mu \rightarrow -\mu$, and thereafter transform it to local frame.

3. The new value of P is determined based on $P_{old} = P[w]$ and μ_{old} was as in the first case.

4. For the effect of acceleration we compute $F_{new} = (dP_{old}/dP_{new})F_{old}$ where F_{old} is the flux at the old grid point, but interpolated or extrapolated to $2P_{old} - P_{new}$ where $P_{old} = P[w]$, i.e. $F(P[w] - \Delta P)$, as if a particle started at $P[w] - \Delta P$ to end up at $P[w]$.

5. Divide F_{new} among new cell as if $F(\mu)$ were uniform in the old cell i.e. we investigated what fraction of the old μ interval is mapped to each cell.

4.5 Numerical Results

After computer program was coded from the procedure above it was run until steady state was reached. The output was plotted between μ, P, and density of particle as figure below.

Figure 4.1 is the initial distribution of particles before simulation. The flux at upstream region is higher than at downstream one. Figure 4.2a,b,c are results of simulation to investigate only propagation effect. Three figures are shown at different angles of views. We find that the flux near the shock at positive values of μ is sharply higher than other region. This is due to reflection of particles at the shock front. Figure 4.3a,b,c are result of simulation to investigate both propagation and acceleration effects. We find that when acceleration effect is included, regions not only at positive values of μ but also at negative values of μ are curved up near the shock. However, those curved distribution are still less than the sharp peak. The curve distributions occur from acceleration effect of shock to particles i.e. the shock accelerates particles into higher momentum state. Normally the number of particles at high momentum are less than the number of particles at low momentum states. So the acceleration of shock to particles increases a number of particles near the shock front.

Figure 4.1 Initial distribution of particles

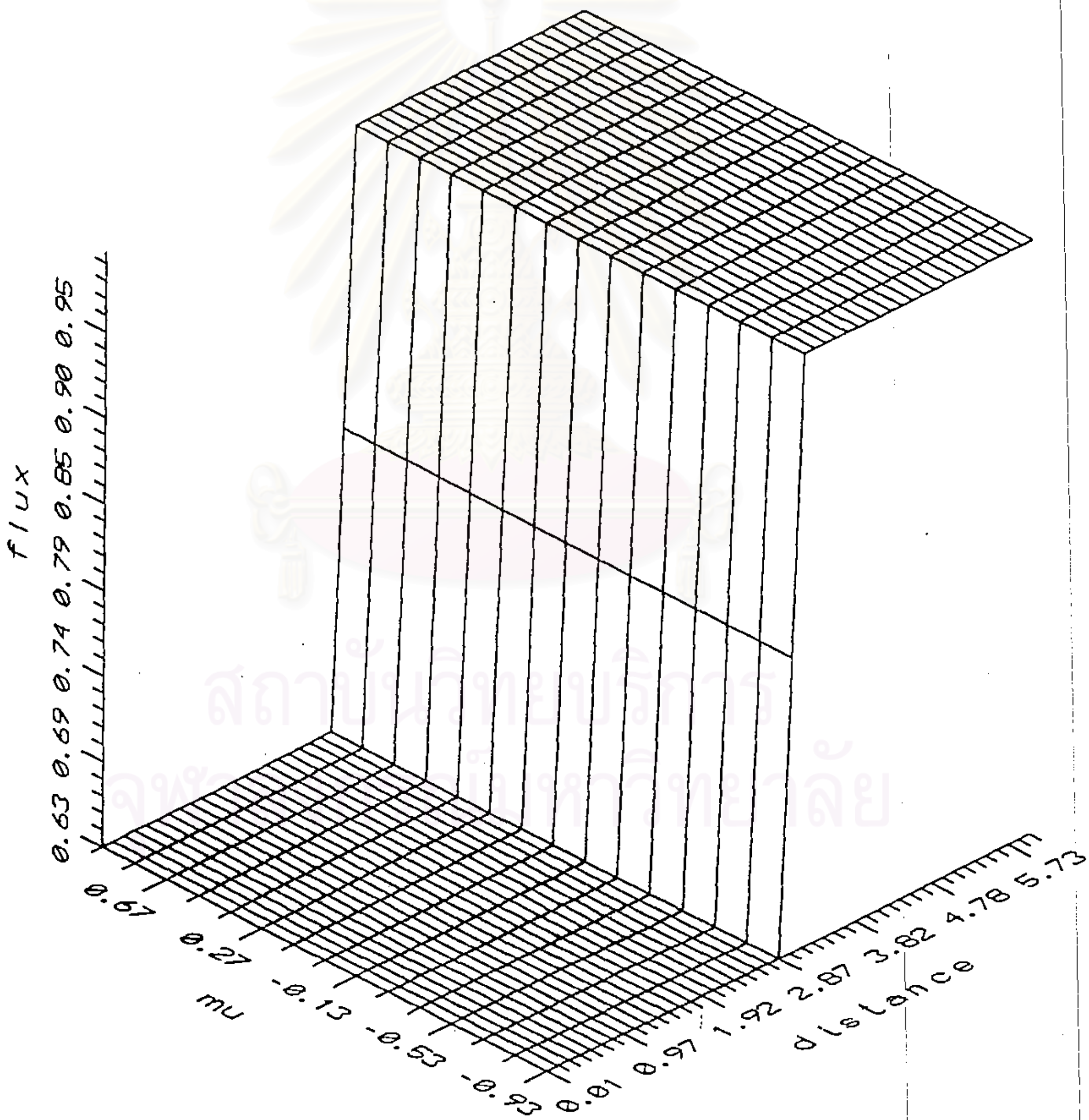


Figure 4.2a Distribution due to propagation effect

[a), b), and c) show different views]

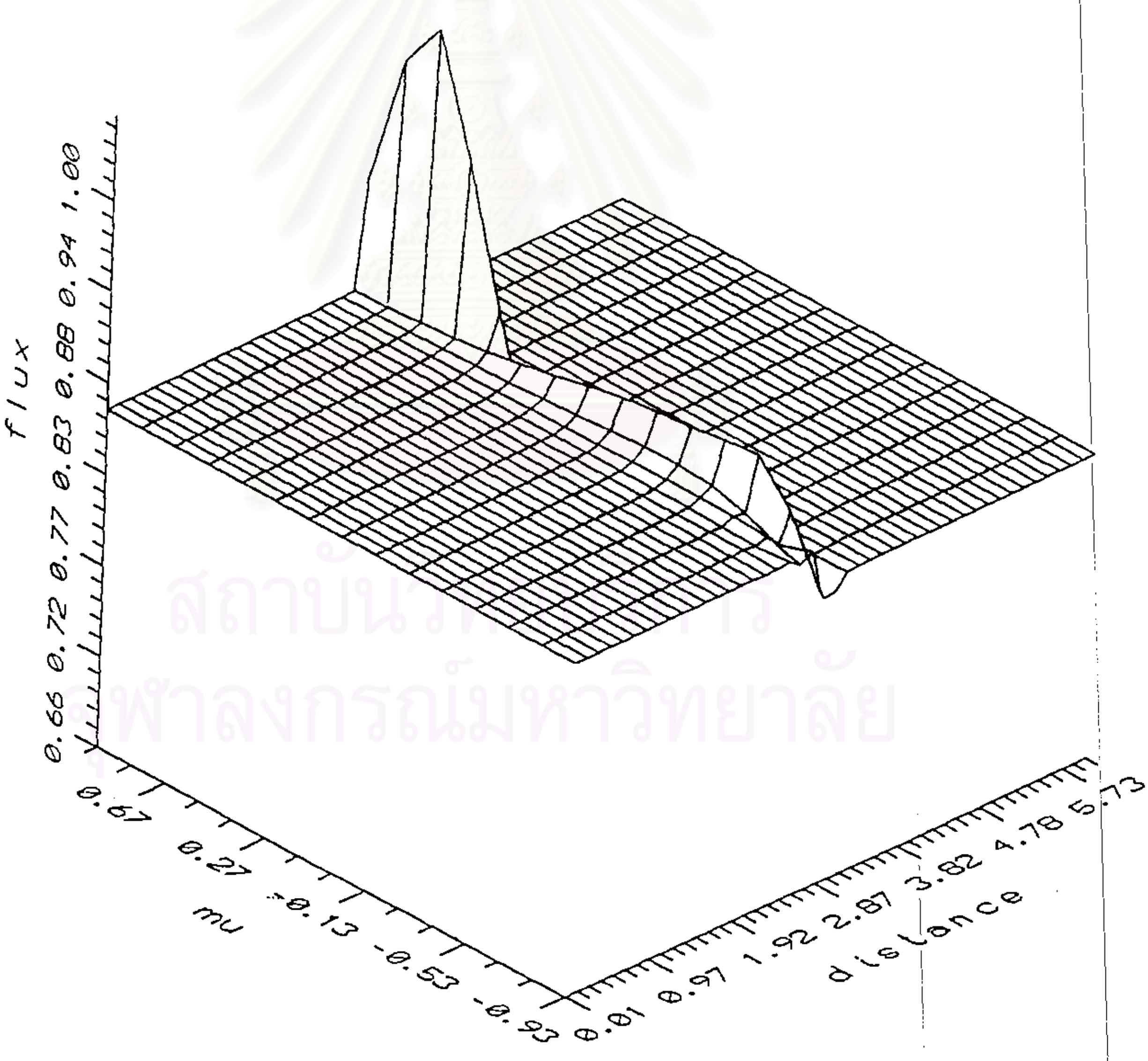


Figure 4.2b

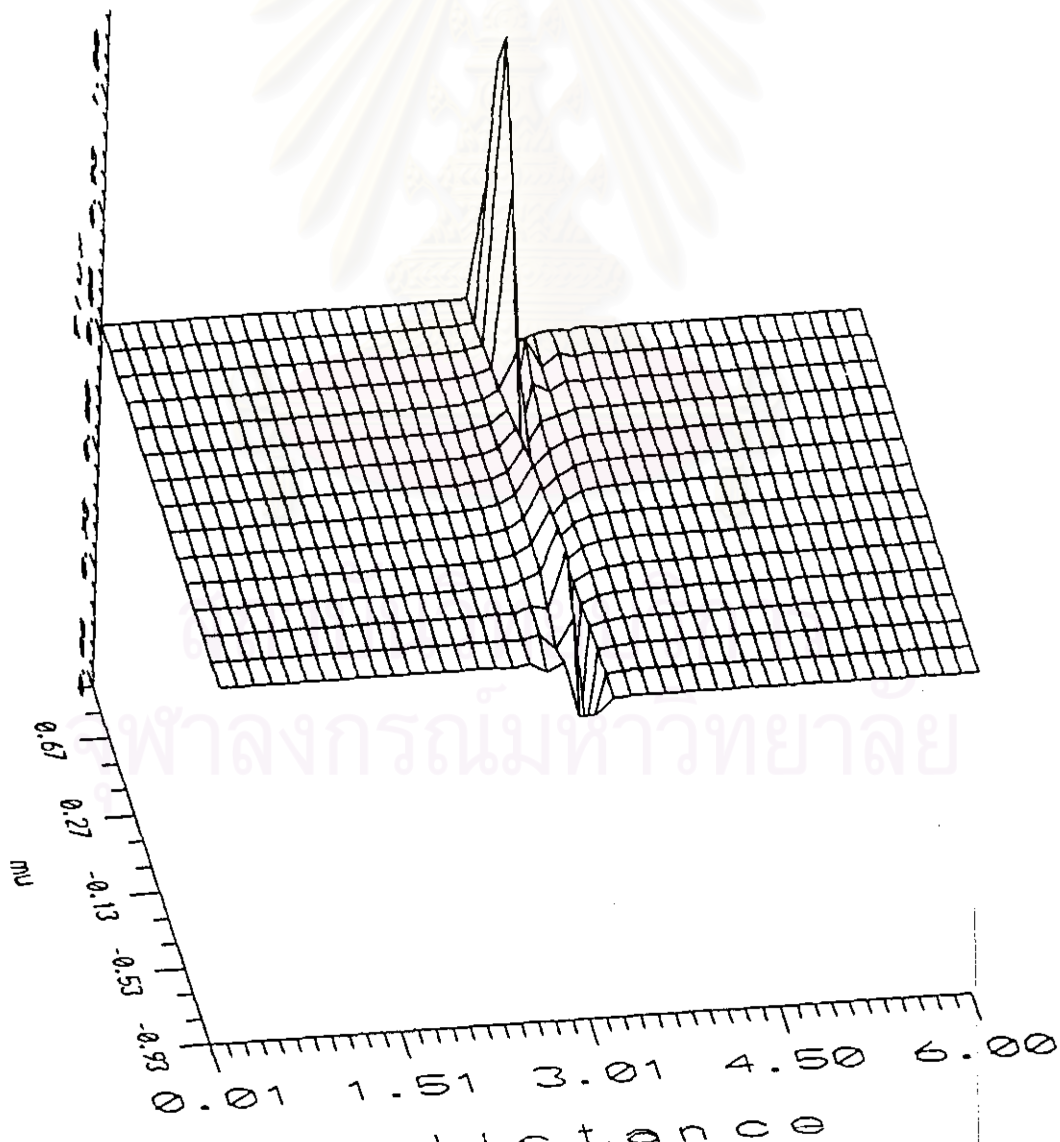


Figure 4.2c

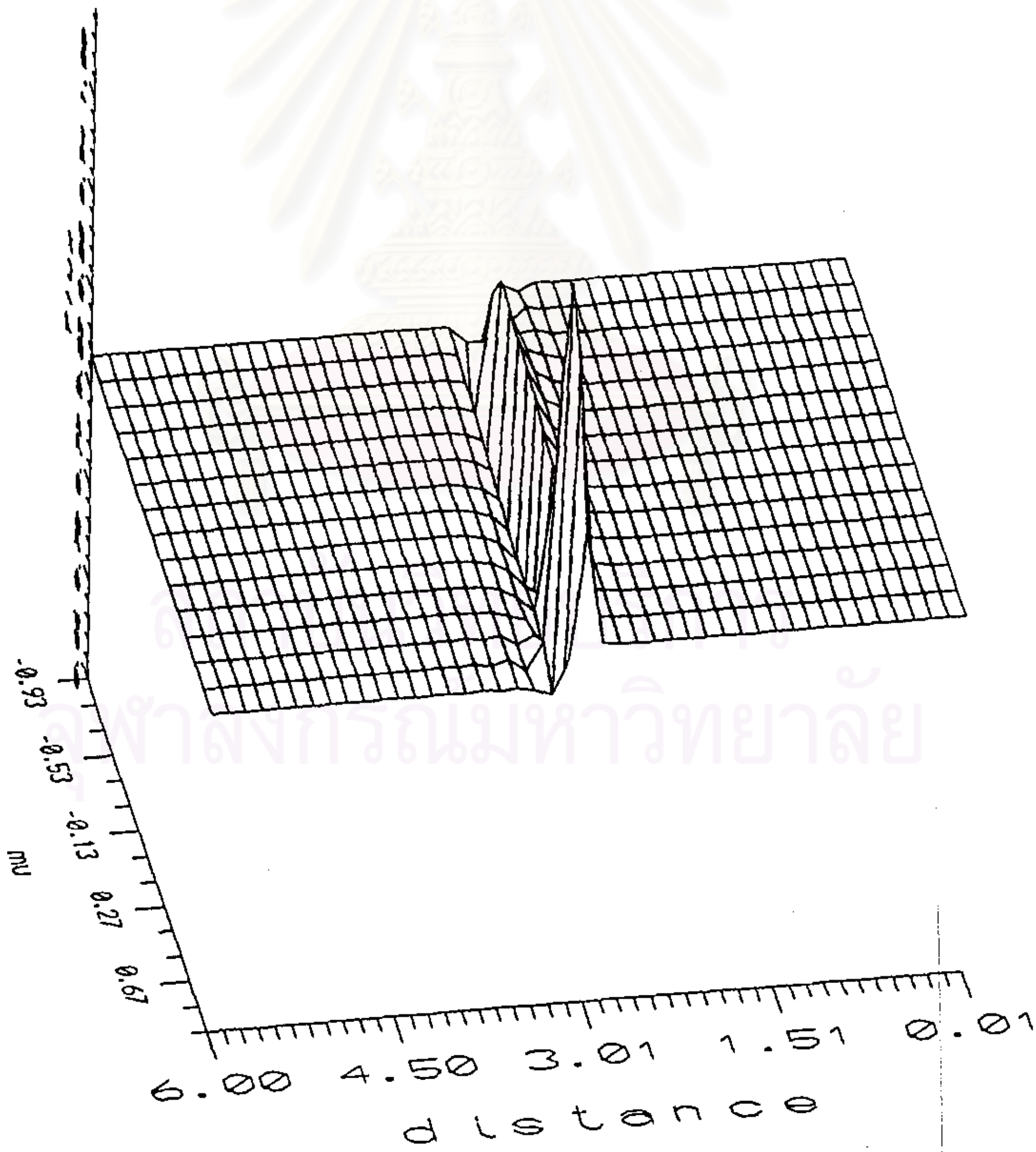


Figure 4.3a Distribution due to both propagation and acceleration effects

[a), b), and c) show different views]

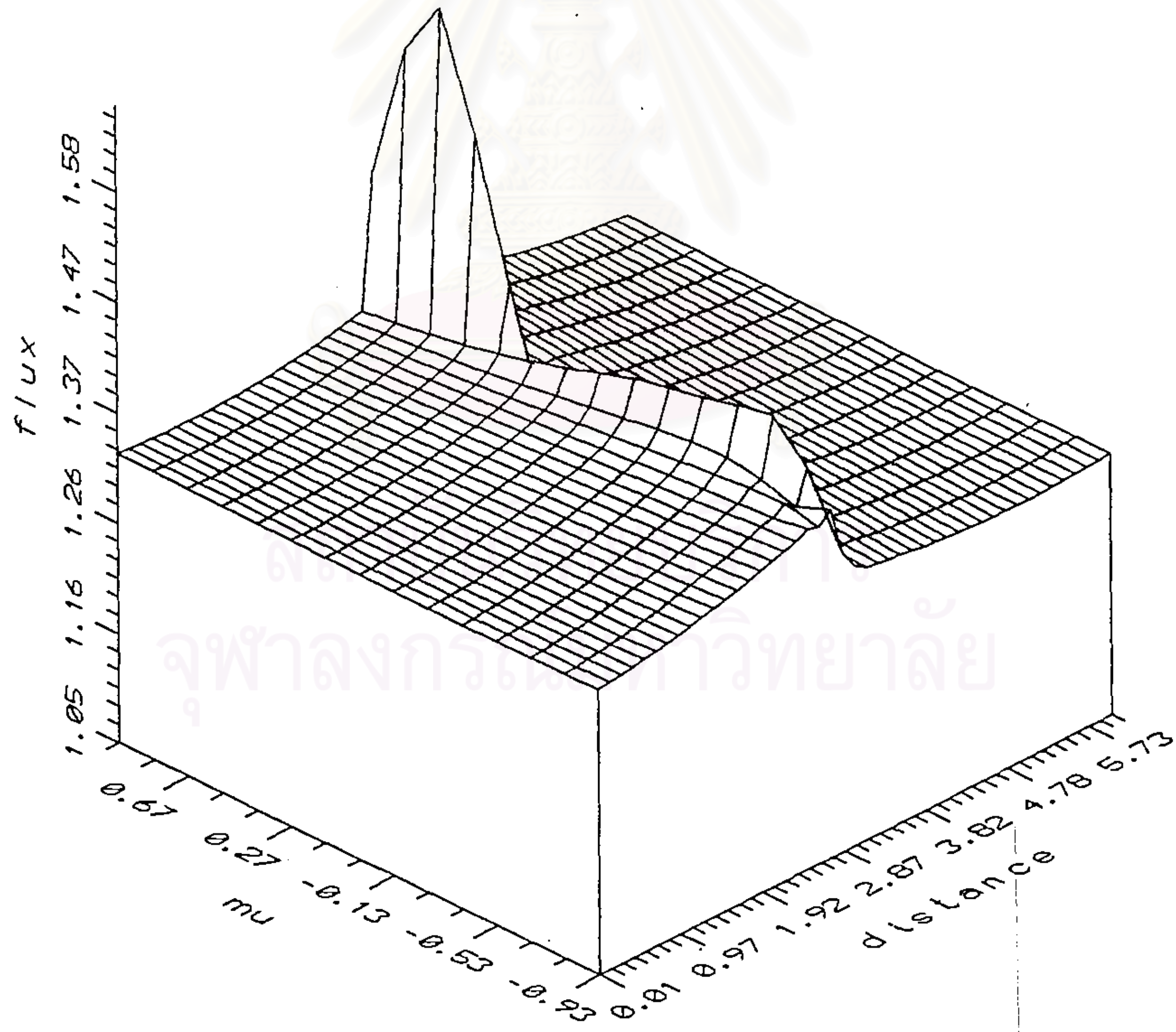


Figure 4.3b

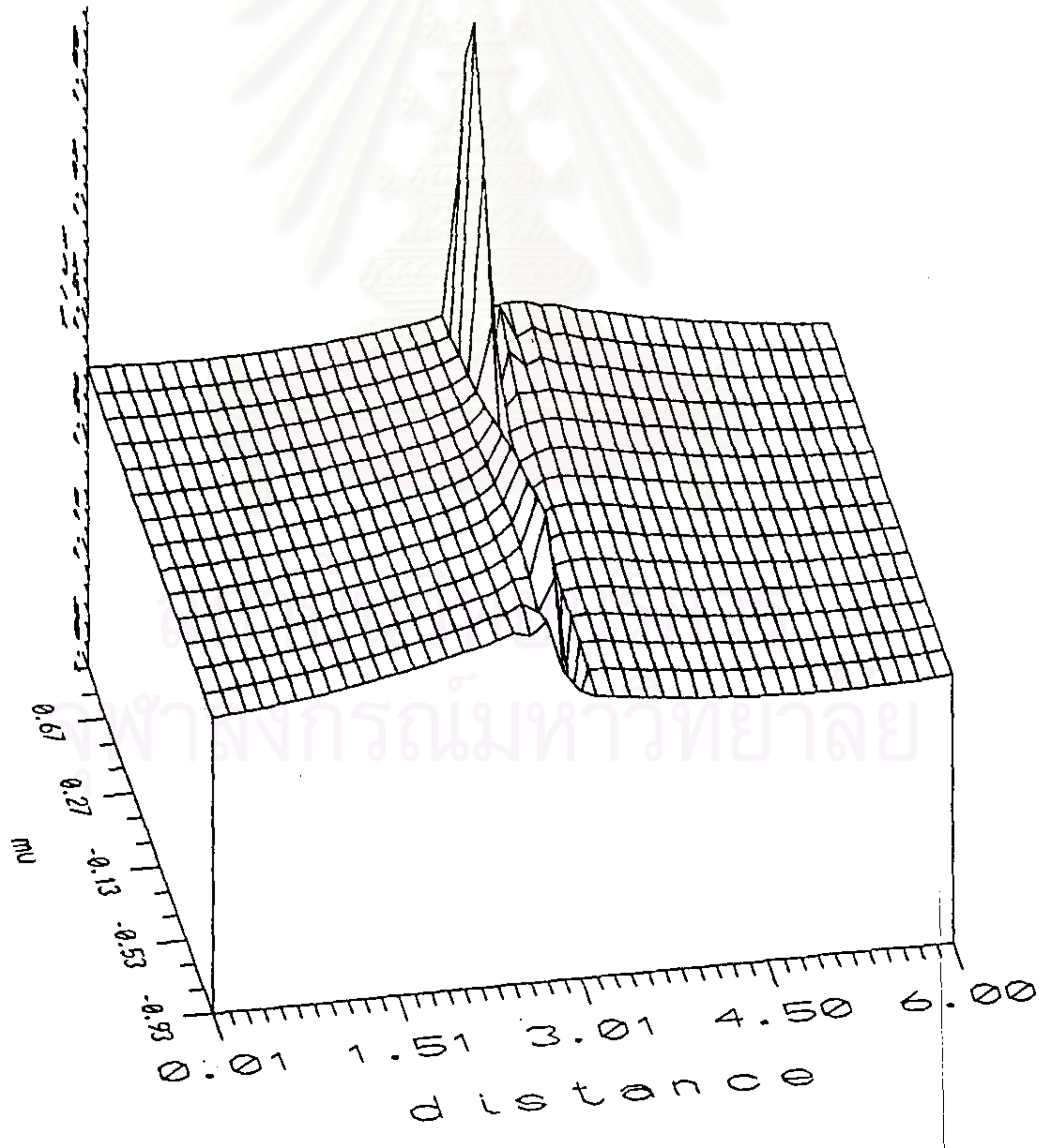
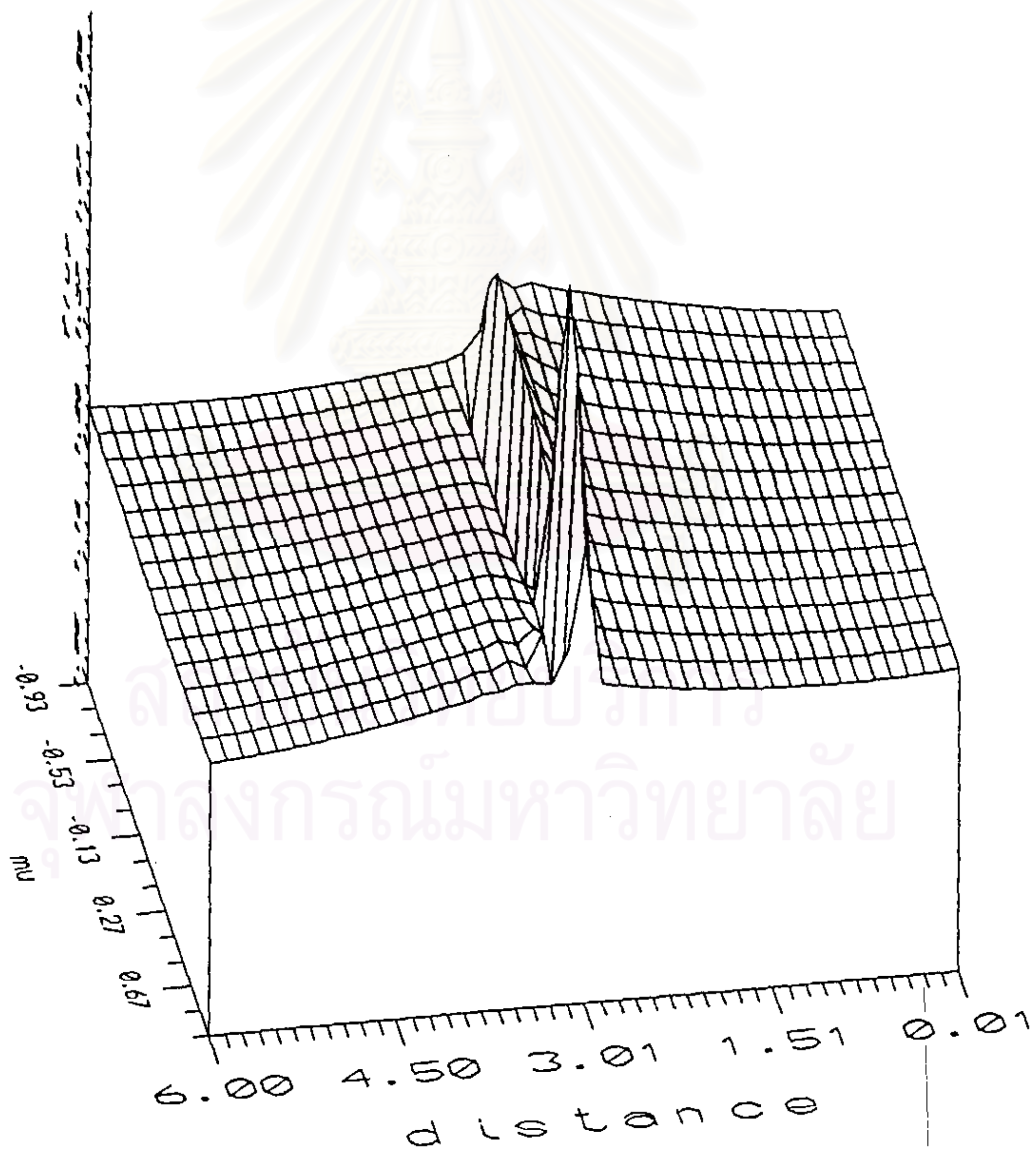


Figure 4.3c



4.6 Data Analysis

We use assumption about $F(\mu)$ depending on δ as Ruffolo and Khumlumlert(1995):

$$F(\mu) = F_0(1 + \delta\mu) \tag{4.28}$$

we find the average of μ from

$$\begin{aligned} <\mu> &= \int_{-1}^{1} \mu F d\mu / \int_{-1}^{1} F d\mu \\ &= \int_{-1}^{1} \mu F_0(1+\delta\mu) d\mu / \int_{-1}^{1} F_0(1+\delta\mu) d\mu \\ &= \frac{\delta}{3} \end{aligned} \tag{4.29}$$

then we can find anisotropy from

$$\delta = 3 <\mu> \tag{4.30}$$

i.e., we found relation between anisotropy and average of μ. Above is a procedure to find analytically average of μ and assumed the relation $\delta = 3 <\mu>$ to hold numerically too. But in our simulation we determined $<\mu>$ by numerical method, i.e. integral sign was replaced by summation of discrete values of μ as follow

$$<\mu> = \sum_{\mu=-1}^{1} \mu F(\mu) / \sum_{\mu=-1}^{1} F(\mu). \tag{4.31}$$

Then we can quantitatively determine δ from eq.(4.23) for each value of z. From figure 4.1 make us know qualitatively anisotropy of particles in region near the shock front both upstream and downstream local frame. In downstream region density of particle decreases while μ increases, such distribution of particles due to μ is called negative anisotropy, while in the upstream region the density of particles has a positive anisotropy. We found that density is very high at positive values of μ. Those results are due to propagation effects. Particles move from upstream region toward the shock and are thereafter reflected at the shock front back to upstream region. The reason we are sure that this is the effect of reflection is because when we dropped the effect of acceleration we found that this high density does exist while other results disappear. The other effect we have studied is the acceleration effect, i.e., acceleration of particles by the shock while crossing it. This effect excites particles to a higher state of momentum, then the high-momentum particles move away from the shock. This accounted for the anisotropy of particles mentioned above.

The results of simulations are compared qualitatively with those from observations from the ISEE-3 spacecraft (Richardson et al, 1990). The agreement was found from both results, i.e., in the upstream region there is positive anisotropy while in the downstream region there is a negative anisotropy.