CHAPTER V

ANALYSIS OF THIN-LAYER DRYING DATA

5-1 Development of mathematical models

For cereal grains most drying of interest usually takes place in the falling rate periods. Many theories have been proposed to explain the transport of moisture in biological materials but it is now generally recognised that diffusion is the rate controlling mechanism in the drying of grain. Therefore, various developed mathematical models have been based on diffusion. In this dissertation three mathematical models will be developed. One model is modified form of the diffusion model and the other models will be in empirical forms. Finally, the statistical analysis will be used to specify an appropriate model.

Model 1

The single term approximation equation can be expressed from first term of the infinite series solution of diffusion equation (Equation 2.4). If the drying time is long enough, this single term solution will give a quite good approximation. The equation is of the form :

$$\overline{MR}(t) = \frac{6}{\pi^2} \exp(-D\pi^2 t/R) \qquad \text{........(5.1)}$$

where : $\overline{MR}(t)$ = Average Moisture ratio = $(\overline{M}-M_e)/(M_i-M_e)$

M_e = Equilibrium moisture content,%(D.B.)

M_i = Initial moisture content,%(D.B.)

$\overline{M}$ = Average moisture content at time t,%(D.B.)

Nonlinear approach and linearized approach are presented to determine the functional relationship of experimental parameters to test variables which are drying air temperature (T), relative humidity of drying air (H) and initial moisture content of rough rice (M_i).

Nonlinear approach

Equation (5.1) can be modified in another form :

$$\overline{MR} = A \exp(-Kt) \qquad \text{........(5.2)}$$

In this case, the obtained A and K vary with the experimental conditions used. These two values, therefore, can be evaluated by using the NLIN procedure of SAS (Statistical Analysis System) and can be set as the function of test variables. The NLIN procedure produces least-squares or weighted least-squares estimates of the parameters of a nonlinear model, i.e the residual sum of square of moisture content [or $\sum(M-M_p)^2$ where M is the average moisture content obtained from the experiment and M_p is the predicted average moisture content calculated from the above model] will be minimized.

Firstly, this procedure examines the starting value specification of the parameters. If a grid of values is specified, NLIN will evaluate the residual sum of squares at each combination of values to determine the best set of values to start the iterative algorithm. Then NLIN uses one of the following four iterative methods : the modified Gauss-Newton method, the gradient or steepest-descent method, the multivariate secant or false position method and the Marquardt method (see the appendix C for details). In this dissertation the Marquardt method is selected to be used in the analysis. The Marquardt iterative method will regress the residuals on to the partial derivatives of the model with respect to the parameters until the iterations converge.

Equilibrium moisture content, M_e used in this study were experimentally determined by C.Laithong (1987). The results show that as time passes, the moisture content is closed to the estimated M_e. The equation used to determine the M_e is

$$(1-H/100) = \exp[-4.723\times10^{-6}(T+273)M_e^{2.386}] \quad ...(5.3)$$

Appendix D shows the sample SAS program for NLIN procedure and its output for Model 1 at typical condition and Appendix E shows the values of A,K from the criterior mention above, and equilibrium moisture content for each test condition.

Assuming that A and K for each test are dependent variables and are functions which have the following independent variables T, T^2, T^3, $T^{1/2}$, $T^{-1/2}$, T^{-1}, T^{-2}, ln T, H, H^2, H^3, $H^{1/2}$, $H^{-1/2}$, H^{-1}, ln (H), M_i, M_i^2, M_i^3, $M_i^{1/2}$, $M_i^{-1/2}$, M_i^{-1}, $\ln(M_i)$, TH, TH^{-1}, HT^{-1}, TM_i, TM_i^{-1}, M_iT^{-1},

$TH^{1/2}$ and THM_i, the following equations can be shown :

$$\begin{aligned} A = {} & A_o + A_1T + A_2T^2 + A_3T^3 + A_4T^{1/2} + A_5T^{-1/2} + A_6T^{-1} \\ & + A_7T^{-2} + A_8\ln(T) + A_9H + A_{10}H^2 + A_{11}H^3 + A_{12}H^{1/2} \\ & + A_{13}H^{-1/2} + A_{14}H^{-1} + A_{15}\ln(H) + A_{16}M_i + A_{17}M_i^2 \\ & + A_{18}M_i^3 + A_{19}M_i^{1/2} + A_{20}M_i^{-1/2} + A_{21}M_i^{-1} + A_{22}\ln(M_i) \\ & + A_{23}TH + A_{24}TH^{-1} + A_{25}HT^{-1} + A_{26}TM_i + A_{27}TM_i^{-1} \\ & + A_{28}M_iT^{-1} + A_{29}TH^{1/2} + A_{30}THM_i \qquad \text{........(5.4)} \end{aligned}$$

$$\begin{aligned} K = {} & K_o + K_1T + K_2T^2 + K_3T^3 + K_4T^{1/2} + K_5T^{-1/2} + K_6T^{-1} \\ & + K_7T^{-2} + K_8\ln(T) + K_9H + K_{10}H^2 + K_{11}H^3 + K_{12}H^{1/2} \\ & + K_{13}H^{-1/2} + K_{14}H^{-1} + K_{15}\ln(H) + K_{16}M_i + K_{17}M_i^2 \\ & + K_{18}M_i^3 + K_{19}M_i^{1/2} + K_{20}M_i^{-1/2} + K_{21}M_i^{-1} + K_{22}\ln(M_i) \\ & + K_{23}TH + K_{24}TH^{-1} + K_{25}HT^{-1} + K_{26}TM_i + K_{27}TM_i^{-1} \\ & + K_{28}M_iT^{-1} + K_{29}TH^{1/2} + K_{30}THM_i \qquad \text{........(5.5)} \end{aligned}$$

The equations (5.4) and (5.5) are in a general form containing many independent variables. The RSQUARE procedure of SAS is used to select the group of variables having the maximum coefficient of determination (R^2) from the groups containing the same amount of variables. As the limit of computer time,each set of 25 variables chosen from the equations(5.4)and(5.5)is used for RSQUARE procedure. It is impossible to present all models because of space limitation. The selected models that produced the highest R^2 are presented in Table 5.1 and Table 5.2.

The RSQUARE procedure can selects optimal subsets of independent variables but can not determine the coefficients of the variables. A STEPWISE MULTIPLE REGRESSION procedure will be used to determine the coefficient of these variables for each model in Table 5.1 and Table 5.2. The model will be accepted if null hypothesis of all independent variables is rejected at the significance level 0.15. The selected models are shown in Table 5.3. Models containing more than 4 independent variables are not presented in these tables.

Table 5.1 Results of RSQUARE procedure for A in Model 1

No. of variable	Variable/s for A
1	H^{-1}
2	$M_i, M_i^{1/2}$
3	$\ln(H), M_i, M_i^{1/2}$
4	$H^{-1}, M_i, M_i^{1/2}, TH^{-1}$

Table 5.2 Results of RSQUARE procedure for K in Model 1

No. of variable	Variable/s for K
1	TH^{-1}
2	TM_i, THM_i
3	$TH^{-1}, \ln(M_i), M_i^{-1}$
4	$\ln(M_i), M_i^{-1}, M_iT^{-1}, TH^{1/2}$

Table 5.3 Results of STEPWISE MULTIPLE REGRESSION procedure for A and K in Table 5.1 and Table 5.2

$$\overline{M} = A(M_i - M_e)\exp(-Kt) + M_e$$

Model 1.1 : $A = 1.044 - 3.7412H^{-1}$

Model 1.2 : $A = -3.5544 - 0.1661M_i + 1.7421M_i^{1/2}$

Model 1.3 : $A = -3.7342 + 0.0807\ln H - 0.1616M_i + 1.6924M_i^{1/2}$

Model 1.4 : $A = -3.3420 - 6.7229H^{-1} - 0.1616M_i + 1.6942M_i^{1/2} + 0.0523TH^{-1}$

Model 1.5 : $K = 5.533 \times 10^{-3} + 2.7632 \times 10^{-3}TH^{-1}$

Model 1.6 : $K = 5.0306 \times 10^{-3} + 4.3 \times 10^{-6}TM_i - 4 \times 10^{-8}THM_i$

Model 1.7 : $K = -0.3286 + 2.6475 \times 10^{-3}TH^{-1} + 0.0773\ln(M_i) + 2.1075M_i^{-1}$

Model 1.8 : $K = -0.4018 + 0.0834\ln M_i + 2.0894M_i^{-1} + 2.8833M_iT^{-1}$
$+ 0.2793TH^{1/2}$

Note The 16 models are obtained by combination of the 4 models for A and 4 models for K

Linearized Approach

After the equation (5.2) is linearized, the equation can be expressed as,

$$\ln(\overline{MR}) = \ln A - Kt \qquad \text{........(5.6)}$$

If A and K are assumed to be dependent variables and A is an exponential function, K is a polynomial function and independent variables chosen for investigation are T, T^2, T^3, $T^{1/2}$, $T^{-1/2}$, T^{-1}, T^{-2}, ln(T), H, H^2, H^3, $H^{1/2}$, $H^{-1/2}$, H^{-1}, ln(H), M_i, M_i^2, M_i^3, $M_i^{1/2}$, $M_i^{-1/2}$, M_i^{-1}, ln (M_i), TH, TH^{-1}, HT^{-1}, TM_i, TM_i^{-1}, M_iT^{-1}, $TH^{1/2}$ and THM_i. The following equations can be written :

$$\begin{aligned} A = \exp[&A_o + A_1T + A_2T^2 + A_3T^3 + A_4T^{1/2} + A_5T^{-1/2} + A_6T^{-1} \\ &+ A_7T^{-2} + A_8\ln(T) + A_9H + A_{10}H^2 + A_{11}H^3 + A_{12}H^{1/2} \\ &+ A_{13}H^{-1/2} + A_{14}H^{-1} + A_{15}\ln(H) + A_{16}M_i + A_{17}M_i^2 + A_{18}M_i^3 \\ &+ A_{19}M_i^{1/2} + A_{20}M_i^{-1/2} + A_{21}M_i^{-1} + A_{22}\ln(M_i) + A_{23}TH \\ &+ A_{24}TH^{-1} + A_{25}HT^{-1} + A_{26}TM_i + A_{27}TM_i^{-1} + A_{28}M_iT^{-1} \\ &+ A_{29}TH^{1/2} + A_{30}THM_i] \qquad \text{...........(5.7)} \end{aligned}$$

$$\begin{aligned} K = &K_o + K_1T + K_2T^2 + K_3T^3 + K_4T^{1/2} + K_5T^{-1/2} + K_6T^{-1} \\ &+ K_7T^{-2} + K_8\ln(T) + K_9H + K_{10}H^2 + K_{11}H^3 + K_{12}H^{1/2} \\ &+ K_{13}H^{-1/2} + K_{14}H^{-1} + K_{15}\ln(H) + K_{16}M_i + K_{17}M_i^2 + K_{18}M_i^3 \\ &+ K_{19}M_i^{1/2} + K_{20}M_i^{-1/2} + K_{21}M_i^{-1} + K_{22}\ln(M_i) + K_{23}TH \\ &+ K_{24}TH^{-1} + K_{25}HT^{-1} + K_{26}TM_i + K_{27}TM_i^{-1} + K_{28}M_iT^{-1} \\ &+ K_{29}TH^{1/2} + K_{30}THM_i \qquad \text{...........(5.8)} \end{aligned}$$

Substituting the expressions for A and K in equation (5.6)

$$
\begin{aligned}
\ln(\overline{MR}) = {} & A_o + A_1T + A_2T^2 + A_3T^3 + A_4T^{1/2} + A_5T^{-1/2} + A_6T^{-1} \\
& + A_7T^{-2} + A_8\ln(T) + A_9H + A_{10}H^2 + A_{11}H^3 + A_{12}H^{-1/2} \\
& + A_{13}H^{-1/2} + A_{14}H^{-1} + A_{15}\ln(H) + A_{16}M_i + A_{17}M_i^2 \\
& + A_{18}M_i^3 + A_{19}M_i^{1/2} + A_{20}M_i^{-1/2} + A_{21}M_i^{-1} + A_{22}\ln(M_i) \\
& + A_{23}TH + A_{24}TH^{-1} + A_{25}HT^{-1} + A_{26}TM_i + A_{27}TM_i^{-1} \\
& + A_{28}M_iT^{-1} + A_{29}TH^{1/2} \\
& + A_{30}THM_i - t[K_o+K_1T+K_2T^2+.....K_{30}THM_i] \qquad (5.9)
\end{aligned}
$$

The equation (5.9) is a particular case of a general form model containing many independent variables. The list of independent variables can be shorten to 8 independent variables. The RSQUARE procedure of SAS was used to select optimal subsets of these independent variables. As the limit of computer time, each set of 25 variables chosen from the equations(5.9)is used for RSQUARE procedure. The models that produced the highest R^2 values in each group (containing the same number of independent variables) are presented in Table 5.4. The coefficient of these variables were determined by the STEPWISE MULTIPLE REGRESSION procedure of SAS and presented in Table 5.5.

Table 5.4 Results of RSQUARE procedure for equation 5.9

No. of variable	Variable/s in the model.
1	$T^{1/2}t$
2	$H^{-1/2}t, Tt$
3	$H^{-1/2}t, Tt, M_i$
4	$T^{1/2}t, H, M_i, M_i^{-1}$
5	$H^{-1/2}t, M_i, M_i^{-1}, T^3t, T^2t$
6	$T^{1/2}t, H^{-1}, M_i, T^3t, T^2t, Tt$
7	$T^{1/2}t, TH, M_i, M_i^{-1}, T^3t, T^2t, Tt$
8	$H^{-1/2}t, Tt, M_i, M_i^{-1}, TH^{-1}, T^3t, T^2t, T^{1/2}t$

Table 5.5 Results of STEPWISE MULTIPLE REGRESSION procedure for models in Table 5.4

Model 1.9 :

$$A = \exp(-0.0441)$$
$$K = 0.0012T^{1/2}$$

Model 1.10 :

$$A = \exp(-0.0486)$$
$$K = 0.033H^{-1/2} + 7.1 \times 10^{-5}T$$

Model 1.11 :

$$A = \exp(0.2512-0.0099M_i)$$
$$K = 0.0323H^{-1/2} + 7.3 \times 10^{-5}T$$

Model 1.12 :

$$A = \exp(6.7644+0.0058H-0.1277M_i-95.2218M_i^{-1})$$
$$K = 11.86 \times 10^{-4}T^{1/2}$$

Model 1.13 :

$$A = \exp(7.1652-0.1290M_i-96.9295M_i^{-1})$$
$$K = 0.0293H^{-1/2} - 4\text{x}10^{-8}T^3 + 3.72\text{x}10^{-6}T^2$$

Model 1.14 :

$$A = \exp(0.5176-13.7537H^{-1} - 91.54\text{x}10^{-4}M_i)$$
$$K = 0.0858T^{1/2} - 2.12\text{x}10^{-6}T^3 + 3.3536\text{x}10^{-4}T^2 - 0.0234T$$

Model 1.15 :

$$A = \exp(6.6388 + 1.06\text{x}10^{-4}TH - 0.124M_i - 93.6851M_i^{-1})$$
$$K = 0.0838T^{1/2} - 0.023T - 2.1\text{x}10^{-6}T^3 + 3.31\text{x}10^{-4}T^2$$

Model 1.16 :

$$A = \exp(6.8384-0.1213M_i-91.69M_i^{-1}-0.081TH^{-1})$$
$$K = 0.0219H^{-1/2} - 0.0227T - 2.06\text{x}10^{-6}T^3 + 3.2669\text{x}10^{-4}T^2 + 0.0828T^{1/2}$$

Model 2

$$\overline{MR} \quad = \quad \exp(-Kt^N) \qquad ....(5.10)$$

This equation was developed from the exponential model and called Page's model. Page's model will be modified and used in this section. Nonlinear and linearized approach were also presented to determine the functional relationship of experimental parameters to test variables (T, H and M_i).

Nonlinear Approach

NLIN procedure of SAS was used to obtain the parameters K and N for each drying test. The values of K and N which obtained from this procedure with their corresponding equilibrium moisture contents for each test are presented in Appendix F.

Assuming of K and N from NLIN procedure were dependent variables and the independent variables chosen for investigation were T, T^2, T^3, $T^{1/2}$, $T^{-1/2}$, T^{-1}, T^{-2}, $\ln(T)$, H, H^2, H^3, $H^{1/2}$, $H^{-1/2}$, H^{-1}, $\ln(H)$, M_i, M_i^2, M_i^3, $M_i^{1/2}$, $M_i^{-1/2}$, M_i^{-1}, $\ln(M_i)$, TH, TH^{-1}, HT^{-1}, TM_i, TM_i^{-1}, M_iT^{-1}, $TH^{1/2}$ and THM_i, the following equations can be expressed :

$$\begin{aligned} K \quad = \quad & K_o + K_1T + K_2T^2 + K_3T^3 + K_4T^{1/2} + K_5T^{-1/2} + K_6T^{-1} \\ & + K_7T^{-2} + K_8\ln(T) + K_9H + K_{10}H^2 + K_{11}H^3 + K_{12}H^{1/2} \\ & + K_{13}H^{-1/2} + K_{14}H^{-1} + K_{15}\ln(H) + K_{16}M_i + K_{17}M_i^2 \\ & + K_{18}M_i^3 + K_{19}M_i^{1/2} + K_{20}M_i^{-1/2} + K_{21}M_i^{-1} + K_{22}\ln(M_i) \\ & + K_{23}TH + K_{24}TH^{-1} + K_{25}HT^{-1} + K_{26}TM_i + K_{27}TM_i^{-1} \\ & + K_{28}M_iT^{-1} + K_{29}TH^{1/2} + K_{30}THM_i \qquad ....(5.11) \end{aligned}$$

$$N = N_o + N_1T + N_2T^2 + N_3T^3 + N_4T^{1/2} + N_5T^{-1/2} + N_6T^{-1} + N_7T^{-2} + N_8\ln(T) + N_9H + N_{10}H^2 + N_{11}H^3 + N_{12}H^{1/2} + N_{13}H^{-1/2} + N_{14}H^{-1} + N_{15}\ln(H) + N_{16}M_i + N_{17}M_i^2 + N_{18}M_i^3 + N_{19}M_i^{1/2} + N_{20}M_i^{-1/2} + N_{21}M_i^{-1} + N_{22}\ln(M_i) + N_{23}TH + N_{24}TH^{-1} + N_{25}HT^{-1} + N_{26}TM_i + N_{27}TM_i^{-1} + N_{28}M_iT^{-1} + N_{29}TH^{1/2} + N_{30}THM_i \quad ....(5.12)$$

The equations (5.11) and (5.12) are in a general form containing many independent variables. The RSQUARE procedure of SAS is used to select the group of variables having the maximum coefficient of determination (R^2) from the groups containing the same amount of variables. As the limit of computer time, each set of 25 variables chosen from the equations(5.11)and(5.12)is used for RSQUARE procedure. It is impossible to present all models because of space limitation. The selected models that produced the highest R^2 are presented in Table 5.6 and Table 5.7.

A STEPWISE MULTIPLE REGRESSION procedure will be used to determine the coefficient of these variables for each model in Table 5.6 and Table 5.7. The model will be accepted if null hypothesis of all independent variables is rejected at the significance level 0.15. The selected models are shown in Table 5.8. Models containing more than 4 independent variables are not presented in these tables.

Table 5.6 Results of RSQUARE procedure for K in Model 2

No. of variable	Variable/s for K
1	H^{-1}
2	$\ln(M_i), M_i^{-1}$
3	$H^{-1}, M_i, M_i^{1/2}$
4	$HT^{-1}, TM_i^{-1}, T, TM_i$

Table 5.7 Results of RSQUARE procedure for N in Model 2

No. of variable	Variable/s for N
1	M_i
2	$M_i, M_i^{1/2}$
3	$M_i, M_i^{1/2}, \ln(H)$
4	$\ln(H), TM_i^{-1}, TM_i, T$

Table 5.8 Results of STEPWISE MULTIPLE REGRESSION procedure for K and N in Table 5.6 and Table 5.7

$$\overline{M} = (M_i - M_e)\exp(-Kt^N) + M_e$$

Model 2.1 :

$$K = -5.921.10^{-3} + 1.2203H^{-1}$$

Model 2.2 :

$$K = -3.0785 + 0.7173\ln(M_i) + 19.4428M_i^{-1}$$

Model 2.3 :

$$K = 1.2243 + 1.1908H^{-1} + 0.045M_i - 0.4731M_i^{1/2}$$

Model 2.4 :

$$K = 1.3468 + 2.7839HT^{-1} + 0.376TM_i^{-1} + 0.437T - 3.744TM_i$$

Model 2.5 :

$$N = 1.2743 - 0.0117M_i$$

Model 2.6 :

$$N = -12.7706 - 0.5022M_i + 5.2727M_i^{1/2}$$

Model 2.7 :

$$N = -13.1759 - 0.492M_i + 5.1597M_i^{1/2} + 0.1820\ln(H)$$

Model 2.8 :

$$N = 0.1387 + 0.185\ln(H) - 2.1582TM_i^{-1} - 2.926\times10^{-3}TM_i + 0.1634T$$

<u>Note</u> The 16 models are obtained by combination of the 4 models for K and 4 models for N

Linearized Approach

After the equation (5.10) is linearized, the equation can be expressed as,

$$\ln(-\ln \overline{MR}) = \ln K + N \ln t \quad ....(5.13)$$

If K and N are assumed to be dependent variables and K is an exponential function, N is a polynomial function and independent variables chosen for investigation are T, T^2, T^3, $T^{1/2}$, $T^{-1/2}$, T^{-1}, T^{-2}, ln(T), H, H^2, H^3, $H^{1/2}$, $H^{-1/2}$, H^{-1}, ln(H), M_i, M_i^2, M_i^3, $M_i^{1/2}$, $M_i^{-1/2}$, M_i^{-1}, $\ln(M_i)$, TH, TH^{-1}, HT^{-1}, TM_i, TM_i^{-1}, M_iT^{-1}, $TH^{1/2}$ and THM_i. The following equations can be written :

$$\begin{aligned} K = \; & \exp[K_o + K_1T + K_2T^2 + K_3T^3 + K_4T^{1/2} + K_5T^{-1/2} \\ & + K_6T^{-1} + K_7T^{-2} + K_8\ln(T) + K_9H + K_{10}H^2 + K_{11}H^3 \\ & + K_{12}H^{1/2} + K_{13}H^{-1/2} + K_{14}H^{-1} + K_{15}\ln(H) + K_{16}M_i \\ & + K_{17}M_i^2 + K_{18}M_i^3 + K_{19}M_i^{1/2} + K_{20}M_i^{-1/2} + K_{21}M_i^{-1} \\ & + K_{22}\ln(M_i) + K_{23}TH + K_{24}TH^{-1} + K_{25}HT^{-1} + K_{26}TM_i \\ & + K_{27}TM_i^{-1} + K_{28}M_iT^{-1} + K_{29}TH^{1/2} + K_{30}THM_i] ....(5.14) \end{aligned}$$

$$\begin{aligned} N = \; & N_o + N_1T + N_2T^2 + N_3T^3 + N_4T^{1/2} + N_5T^{-1/2} + N_6T^{-1} \\ & + N_7T^{-2} + N_8\ln(T) + N_9H + N_{10}H^2 + N_{11}H^3 + N_{12}H^{1/2} \\ & + N_{13}H^{-1/2} + N_{14}H^{-1} + N_{15}\ln(H) + N_{16}M_i + N_{17}M_i^2 + N_{18}M_i^3 \\ & + N_{19}M_i^{1/2} + N_{20}M_i^{-1/2} + N_{21}M_i^{-1} + N_{22}\ln(M_i) + N_{23}TH \\ & + N_{24}TH^{-1} + N_{25}HT^{-1} + N_{26}TM_i + N_{27}TM_i^{-1} + N_{28}M_iT^{-1} \\ & + N_{29}TH^{1/2} + N_{30}THM_i \quad ....(5.15) \end{aligned}$$

Substituting the expression for K and N in equation (5.13)

$$\ln(-\ln(\overline{MR})) = K_o + K_1T + K_2T^2 + K_3T^3 + K_4T^{1/2} + K_5T^{-1/2} + K_6T^{-1} + K_7T^{-2} + K_8\ln(T) + K_9H + K_{10}H^2 + K_{11}H^3 + K_{12}H^{1/2} + K_{13}H^{-1/2} + K_{14}H^{-1} + K_{15}\ln(H) + K_{16}M_i + K_{17}M_i^2 + K_{18}M_i^3 + K_{19}M_i^{1/2} + K_{20}M_i^{-1/2} + K_{21}M_i^{-1} + K_{22}\ln(M_i) + K_{23}TH + K_{24}TH^{-1} + K_{25}HT^{-1} + K_{26}TM_i + K_{27}TM_i^{-1} + K_{28}M_iT^{-1} + K_{29}TH^{1/2} + K_{30}THM_i - \ln t[N_o + N_1T + K_2T^2 + \ldots\ldots K_{30}THM_i] \quad (5.16)$$

The equation (5.16) is a particular case of a general form model containing many independent variables. The list of independent variables can be shorten to 9 independent variables. The RSQUARE procedure of SAS was used to select optimal subsets of these independent variables. As the limit of computer time, each set of 25 variables chosen from the equations(5.16)is used for RSQUARE procedure. The models that produced the highest R^2 values in each group (containing the same number of independent variables) are presented in Table 5.9. The coefficient of these variables were determined by the STEPWISE MULTIPLE REGRESSION procedure of SAS and presented in Table 5.10.

Table 5.9 Results of RSQUARE procedure for equation 5.16

No. of variable	Variable/s in the model
1	$\ln(T)\ln(t)$
2	$\ln(H)\ln(t), \ln(H)$
3	$\ln(H)\ln(t), \ln(H), M_i^2$
4	$\ln(t), TH^{-1}, M_i, M_i^{1/2}$
5	$M_i^{1/2}\ln(t), M_i\ln(t), TH^{-1}, M_i, M_i^{1/2}$
6	$TM_i, T, TM_i^{-1}, M_i^{1/2}\ln(t), M_i\ln(t), H^{-1}$
7	$M_i^{1/2}\ln(t), M_i\ln(t), H^{-1}, H^{-1}\ln(t), TM_i, T, TM_i^{-1}$
8	$\ln(t), H^{-1/2}, TM_i, H^2\ln(t), T, TM_i^{-1}, M_i\ln(t), M_i^{1/2}\ln(t)$
9	$\ln(t), H, TM_i, H^2\ln(t), \ln(T), TM_i^{-1}, T^2, M_i\ln(t),$ $\ln(M_i)\ln(t)$

Table 5.10 Results of STEPWISE MULTIPLE REGRESSION procedure for models in Table 5.9

Model 2.9 :

$$K = \exp(-4.3310)$$
$$N = 0.2345\ln(T)$$

Model 2.10 :

$$K = \exp(1.5753-1.5226\ln(H))$$
$$N = 0.2345\ln(H)$$

Model 2.11 :

$$K = \exp(1.2675-1.5318\ln(H)+3.701\times10^{-4}M_i^2)$$
$$N = 0.2342\ln(H)$$

Model 2.12 :

$$K = \exp(20.3602+0.3744TH^{-1}+0.9168M_i-9.6473M_i^{1/2})$$
$$N = 0.9142$$

Model 2.13 :

$$K = \exp(24.428+0.3778TH^{-1}+1.1627M_i-11.8622M_i^{1/2})$$
$$N = 0.5045M_i^{1/2}-0.0610M_i$$

Model 2.14 :

$$K = \exp(-4.9671 + 8.729\times10^{-3}TM_i - 0.4552T + 5.682TM_i^{-1} + 21.287H^{-1})$$
$$N = 0.5075M_i^{1/2} - 6.16\times10^{-2}M_i$$

Model 2.15 :

$$K = \exp(- 5.9079 + 65.9305H^{-1} + 8.933\times10^{-3}TM_i - 0.4669T + 5.8393TM_i^{-1})$$
$$N = 0.5910M_i^{1/2}- 0.0693M_i- 10.4467H^{-1}$$

<u>Model 2.16</u> :

$$K = \exp(-1.79-0.3711H^{1/2}+0.0153TM_i-0.8996T +11.0581TM_i^{-1})$$

$$N = -9.121 + 3.855\times10^{-5}H^2 - 0.3735M_i + 3.8746M_i^{1/2}$$

<u>Model 2.17</u> :

$$K = \exp(43.6914-0.0286H+0.016011TM_i-0.1629\ln(T) + 11.5974TM_i^{-1} - 4.685\times10^{-3}T^2)$$

$$N = -11.8465 + 4.422\times10^{-5}H^2 - 0.2070M_i + 5.5696\ln(M_i)$$

Model 3

$$\overline{MR} = 1 + At + Bt^2 \qquad \text{..............(5.17)}$$

This model is of a quadratic model and can could not be expressed in linear form. As a result, the initial regression performed was nonlinear. NLIN procedure of SAS was used to obtain the parameter A and B for each drying test. The results from this step are shown in Appendix G

Assuming A and B were dependent variables and polynomial functions. The independent variables chosen for investigation were the same as in Model 1 and Model 2. Therefore,

$$\begin{aligned} A = \; & A_o + A_1T + A_2T^2 + A_3T^3 + A_4T^{1/2} + A_5T^{-1/2} + A_6T^{-1} \\ & + A_7T^{-2} + A_8\ln(T) + A_9H + A_{10}H^2 + A_{11}H^3 + A_{12}H^{1/2} \\ & + A_{13}H^{-1/2} + A_{14}H^{-1} + A_{15}\ln(H) + A_{16}M_i + A_{17}M_i^2 \\ & + A_{18}M_i^3 + A_{19}M_i^{1/2} + A_{20}M_i^{-1/2} + A_{21}M_i^{-1} + A_{22}\ln(M)_i \\ & + A_{23}TH + A_{24}TH^{-1} + A_{25}HT^{-1} + A_{26}TM_i + A_{27}TM_i^{-1} \\ & + A_{28}M_iT^{-1} + A_{29}TH^{1/2} + A_{30}THM_i \qquad \text{.......(5.18)} \end{aligned}$$

$$\begin{aligned} B = \; & B_o + B_1T + B_2T^2 + B_3T^3 + B_4T^{1/2} + B_5T^{-1/2} \\ & + \ldots\ldots B_{30}THM_i \qquad \text{.......(5.19)} \end{aligned}$$

A RSQUARE procedure was applied to select the best set of independent variables for A and B. As the limit of computer time, each set of 25 variables chosen from the equations(5.18)and (5.19) is used for RSQUARE procedure. The models that produced the highest coefficient of determination (R^2) in each group which contain the same number of independent variables are shown in Table 5.11 and Table 5.12.

A STEPWISE MULTIPLE REGRESSION procedure was then run to estimate the coefficients in each models in Table 5.11 and in Table 5.12. The selected models are presented in Table 5.13

Table 5.11 Results of RSQUARE procedure for A in Model 3

No. of variable	Variable/s for A
1	TH^{-1}
2	$\ln(M_i), M_i^{-1}$
3	$TH^{-1}, M_i^{1/2}, M_i^{-1}$
4	$\ln(M_i), M_i^{-1}, M_iT^{-1}, TH^{1/2}$

Table 5.12 Results of RSQUARE procedure for B in Model 3

No. of variable	Variable/s for B
1	TH^{-1}
2	$\ln(M_i), M_i^{1/2}$
3	$TH^{-1}, M_i^{1/2}, \ln(M_i)$
4	$\ln(M_i), M_i^{1/2}, M_iT^{-1}, TH^{1/2}$

Table 5.13 Results of STEPWISE MULTIPLE REGRESSION procedure for A and B in Table 5.11 and Table 5.12

$$\overline{M} = (1+At+Bt^2)(M_i-M_e)+M_e$$

Model 3.1 :

$$A = -5.082x10^{-3}-1.621x10^{-3}TH^{-1}$$

Model 3.2 :

$$A = 0.2729-0.0645ln(M_i)-1.773M_i^{-1}$$

Model 3.3 :

$$A = 0.1149-1.538x10^{-3}TH^{-1}-0.0151M_i^{1/2}-1.0918M_i^{-1}$$

Model 3.4 :

$$A = 0.3179-0.0778ln(M_i)-2.0185M_i^{-1}+7.446x10^{-3}M_iT^{-1}$$
$$+1.057x10^{-5}TH^{1/2}$$

Model 3.5 :

$$B = 7.166x10^{-6}+5.0308x10^{-6}TH^{-1}$$

Model 3.6 :

$$B = 5.3160x10^{-4}-3.928x10^{-4}ln(M_i)+1.4861x10^{-4}M_i^{1/2}$$

Model 3.7 :

$$B = 5.0135x10^{-4}+4.8477x10^{-6}TH^{-1}+1.4218x10^{-4}M_i^{1/2}$$
$$-3.7509x10^{-4}ln(M_i)$$

Model 3.8 :

$$B = 5.9142x10^{-4}-4.4033x10^{-4}ln(M_i)+1.7211x10^{-4}M_i^{1/2}$$
$$-2.411x10^{-5}M_iT^{-1}-3.507^{-8}TH^{1/2}$$

<u>Note</u> The 16 models are obtained by combination of the 4 models for A and 4 models for B

5-2 Selection of a most suitable model

From the section 5.1, various models will be selected to use in the deep bed simulation. There are 48 models come from the nonlinear approach (combination of Model 1.1 through Model 1.8, Model 2.1 through Model 2.8 and Model 3.1 through 3.8) and 17 models come from the linearized approach (Model 1.9 through Model 1.16 and Model 2.9 through Model 2.17)

In this section, the most promising models were selected on the basis of goodness of fit. Two criteria were used for this purpose that are 1) Residual Analysis, and 2) Analysis of the coefficient of determination (R^2) for each model.

1) For the residual analysis, the distribution of residual of each model and the sum of square of residual (SSE) will be considered for minimum SSE.

2) For the analysis by R^2 value, the model produced higher R^2 value will be selected, R^2 value for each model was determined by using the following expression :

$$R^2 = \frac{CSS-SSE}{CSS} \qquad ....(5.20)$$

CSS is Corrected sum of square of average moisture content or $\sum M^2 - n(M_m)^2$

where n is number of observation

M is average moisture content from the experiment (% D.B.)

M_m is mean value of average moisture content from all data (% D.B.)

SSE is Sum of square of residual or $\sum(M - M_p)^2$ where M_p is average moisture content from the calculation (predicted value)

The conclusions using two above criteria are,

a) By consideration from sum of square of residual (SSE) for nonlinear approach, in general Model 2 gives SSE less than Model 1 and Model 1 gives SSE less than Model 3. Figure 5.1 and Figure 5.2 show the residual plots of Model 1, Model 2 and Model 3 at specific conditions. Almost all of data set, Model 1 and Model 2 give residual less than Model 3 but Model 2 give residual less than Model 1.

b) In the linearized approach, Model 2 gives R^2 values higher than Model 1, Model 1 gives R^2 values higher than Model 3.

c) The models containing more independent variables give higher R^2 values and lower residuals.

d) The models that come from the nonlinear approach produce R^2 value less than the models that come from the linearized approach.

Based on above conclusions and computer time, the models that came from the linearized models will be concentrated. The linearized approach used the computers time less than nonlinear approach. These models and the coefficient of determination shown in Table 5.14. Model 2.16 was chosen to be the most suitable model. It contains all the experimental variables, gives a high R^2 value and produces small residuals.

Table 5.14 Summary of the coefficient of determination from various models to describe thin-layer drying rate.

CSS = 69460.603

Model	SSE	(R^2)
1.9	10064.84	0.8551
1.10	9189.63	0.8677
1.11	8967.36	0.8709
1.12	8640.89	0.8756
1.13	8376.94	0.8794
1.14	7529.53	0.8916
1.15	7098.87	0.8978
1.16	6904.38	0.9006
2.9	8564.49	0.8767
2.10	6536.24	0.9059
2.11	5813.85	0.9163
2.12	4855.29	0.9301
2.13	4396.85	0.9367
2.14	3910.63	0.9437
2.15	3563.33	0.9487
2.16	3257.70	0.9531
2.17	3229.91	0.9535

Note

CSS = Corrected sum of square of average moisture content

SSE = Sum of square of residual

R^2 = Coefficient of determination

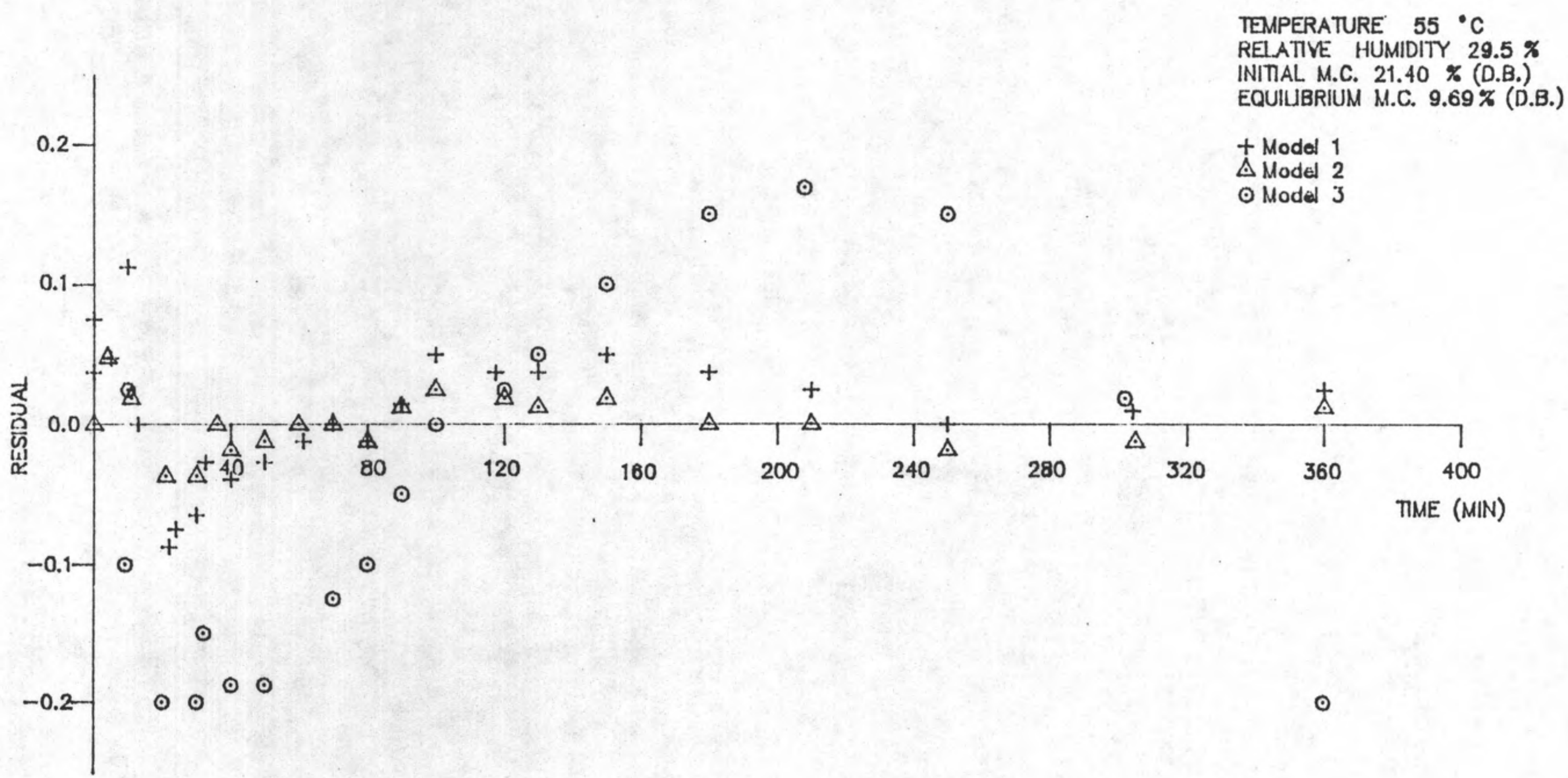

Figure 5.1 Residual plot of Model 1, Model 2 and Model 3 at $T = 55\ ^{\circ}C$, $RH = 29.5\ \%$, $M_i = 21.40\ \%$ (D.B.)

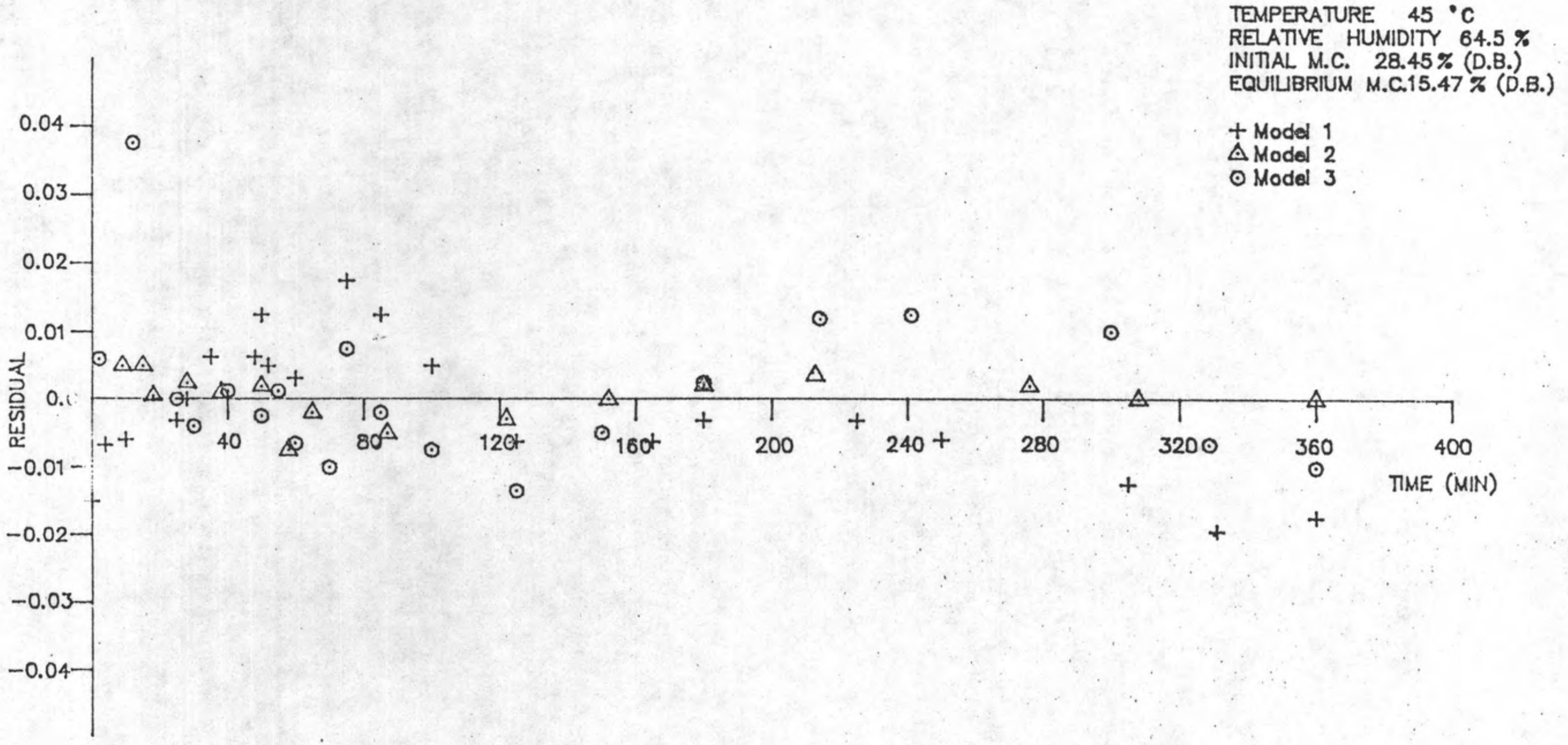

Figure 5.2 Residual plot of Model 1, Model 2 and Model 3 at $T = 45$ °C, RH = 64.5 %, M_i = 28.45 % (D.B.)