CHAPTER I

INTRODUCTION

1. <u>The Lorentz Transformation</u> is

$$\left.\begin{aligned} x' &= \frac{1}{(1 - v^2/c^2)^{\frac{1}{2}}}(x - vt), \\ y' &= y, \\ z' &= z, \\ t' &= \frac{1}{(1 - v^2/c^2)^{\frac{1}{2}}}(t - vx/c^2), \end{aligned}\right\} \quad (1)$$

where v is the coordinate velocity of S' relative to S, and c is the constant velocity of light. (Tolman, reference 4) The coordinate velocity v is the value of dx(o')/dt, where x(o') is the x-coordinate in S of the origin of S'.

The assumptions that are used in the usual derivation of the Lorentz transformation are as follows:

1. The transformation is linear.
2. The velocity of light is constant.
3. The transformation is symmetric.

In the following chapters, instead of assuming that the velocity of light is constant we shall assume that the form of the transformation is such that the composition of two transformations produces a new transformation of the same form. The transformation will be found to contain an arbitrary constant that must be determined by a physical experiment. Experiment shows that this constant is the coordinate velocity of light.

2. The Transformations for Velocity and Acceleration.

From the Lorentz transformation above we may derive the following transformation for velocity using coordinate velocities

$$\left.\begin{aligned} u'_x &= \frac{u_x - v}{1 - \frac{u_x v}{c^2}}, \\ u'_y &= \frac{u_y}{1 - \frac{u_x v}{c^2}} (1 - v^2/c^2)^{\frac{1}{2}}, \\ u'_z &= \frac{u_z}{1 \frac{u_x v}{c^2}} (1 - v^2/c^2)^{\frac{1}{2}}, \end{aligned}\right\} \quad (2)$$

where $u'_x = dx'/dt'$, etc., $u_x = dx/dt$, etc., v is the coordinate velocity of S' in S, and c is the velocity of light.

The transformations for acceleration using coordinate accelerations are

$$\left.\begin{aligned} a'_x &= (1 - u_x v/c^2)^{-3}(1 - v^2/c^2)^{3/2}\, a_x, \\ a'_y &= (1 - u_x v/c^2)^{-2}(1-v^2/c^2)a_y + u_y v/c^2(1-u_x v/c^2)^{-3}(1-v^2/c^2)a_x \end{aligned}\right\} (3),$$

$$a'_z = (1-u_x v/c^2)^{-2}(1-v^2/c^2)a_z + u_z v/c^2(1-u_x v/c^2)^{-3}(1-v^2/c^2)a_x ,$$

where $a'_x = d^2x'/dt'^2$, etc., $a_x = d^2x/dt^2$, etc.,

These formulas are very complicated and inconvenient. It would be better to do calculations involving velocities and accelerations by a simpler method. In the following chapters we shall express the Lorentz transformation, and the resulting transformations for velocity and acceleration in terms of the proper velocities and proper accelerations defined below.

3. <u>The Definition of Proper Velocity</u>

The components of the proper velocity of a body relative to S are defined to be $u_x = dx/dr$, $u_y = dy/dr$, and $u_z = dz/dr$, where r is the invariant time measured by a clock carried with the body.

The three components of the coordinate velocity are defined to be $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$, where t is the time measured by a system of clocks fixed in S.

The relationship between the coordinate velocity and the proper velocity is as follows :

$$\left.\begin{aligned} u_x &= v_x/(1 - v^2/c^2)^{\frac{1}{2}}, \\ u_y &= v_y/(1 - v^2/c^2)^{\frac{1}{2}}, \\ u_z &= v_z/(1 - v^2/c^2)^{\frac{1}{2}}, \end{aligned}\right\} \quad (4)$$

where v is the coordinate velocity of the body in S.

The proper acceleration of the body relative to S is defined thus

$$\left.\begin{aligned} a_x &= du_x/dr , \\ a_y &= du_y/dr , \\ a_z &= du_z/dr. \end{aligned}\right\} \quad (5)$$