CHAPTER IV

ASSIGNMENT – LOT SIZE HEURISTIC (A-LS)

As problem statement in chapter 1, the integrating the decisions involved in multi-workstation problems seem to be a preferable approach. Unfortunately, integrated approach may result in a symmetric structure of the integrated model and a majority of integer variables and constraints. Even though for small-scale problem instances, finding exact solutions of MLCLSP-M does not appear to be a practical consideration, let alone the integrated model with industrial size. On the basis of the above description, a heuristic method (A-LS) is proposed. This method based on decomposition the MLCLSP-M into two phases consisting of an assignment with given lot size phase and a lot size with given assignment phase. Iteration in either phase, the sub problem mathematical model is solved with AMPL/CPLEX 8.0.0 solver. Computational test results are analyzed and discussed on performance and running time. The result indicates that the proposed Assignment – Lot size heuristic (A-LS) give a satisfactory solution with very fast solving time on comparing with the original mathematical model solving. The remainder of this chapter is organized as follows. The introduction is presented in section 4.1. The heuristics are described in Section 4.2. In Section 4.3, we present the heuristics procedures that were used. The computational results are given in Section 4.4. Finally, some concluding remarks are provided in Section 4.5.

4.1 Introduction

Even several studies on lot sizing problems could applicable for lot size planning, the algorithm with the optimal solution yet remain challenging. Furthermore, this multi-item multi-period decision has affected various costs in the planning system (Armagan and Kingsman, 2004; Robinson and Lawrence, 2004). To find the minimum cost, the mixed integer programming model that can represent all aspects as aforementioned is proposed in chapter 3. As our problem is an extension from the CLSP problem, we conjecture that the multi-level capacitated lot size problem with set-up costs is also NP-hard. Based on the literature, solution methods of the problem can be classified into three main categories. The first is an exact methods or mathematical modeling (Grubbstrom and Thu Thuy Huynh, 2006; Wolsey, 2002), the second category is common-sense or specialized heuristics (Berretta et al., 2005; Chaudhry and Luo, 2005; Comelli et al., 2007; Hung and Chien, 2000; Kimms, 1996; Lalas et al., 2005; Pitakaso et al., 2006) and the third category belongs to mathematical programming-based heuristics (Berretta et al., 2005; Sambasivan and Yahya, 2005; Tempelmeier and Derstroff, 1996). Based on these results, it is unlikely that one can develop any effective optimal algorithm for this problem. Therefore, research on developing effective algorithms has been a profitable research area for a long time.

The purpose of this chapter is to propose a solution for the aforementioned problem. The holistic view model (consideration purchasing planning situation together with production planning situation while feasible warehouse capacity constraint) is appropriated and needed to find optimal plan (Karimi et al., 2003; Pochet, 2001). Such problem can be defined as Multi-level Multi-item Capacitated Lot Sizing Problem with Multi-workstation (MLCLSP-M). That the developed model might be classified as a capacitated lot sizing with setup time model (Chinprateep and Boondiskulchok, 2007; Karimi et al., 2003; Kimms, 1996; Pochet, 2001; Zangwill, 1966), which is NP-hard problem, it can be solved to optimality only with a huge computational effort. Besides, it's well known that the lot-sizing MIP models are often very large in practice even advanced solvers such as CPLEX are unable to identify provably-optimal solutions in acceptable computational time (Clark, 2003; Silvio et al., 2008). Clearly it takes an impracticable amount of computer time and memory, motivating the development of the alternative approaches. Therefore, this research proposed a heuristic algorithm that can solve large-scale problems to near-optimality with a reasonable computational time. Although there is more than one way to tackle the problem, the effective one is decomposition (Aardal and Larsson, 1990; See-Toh et al., 2006; Vercellis, 1999; Wu et al., 2003; Zapfel, 1996), consequently; the problem can be solved more efficiently.

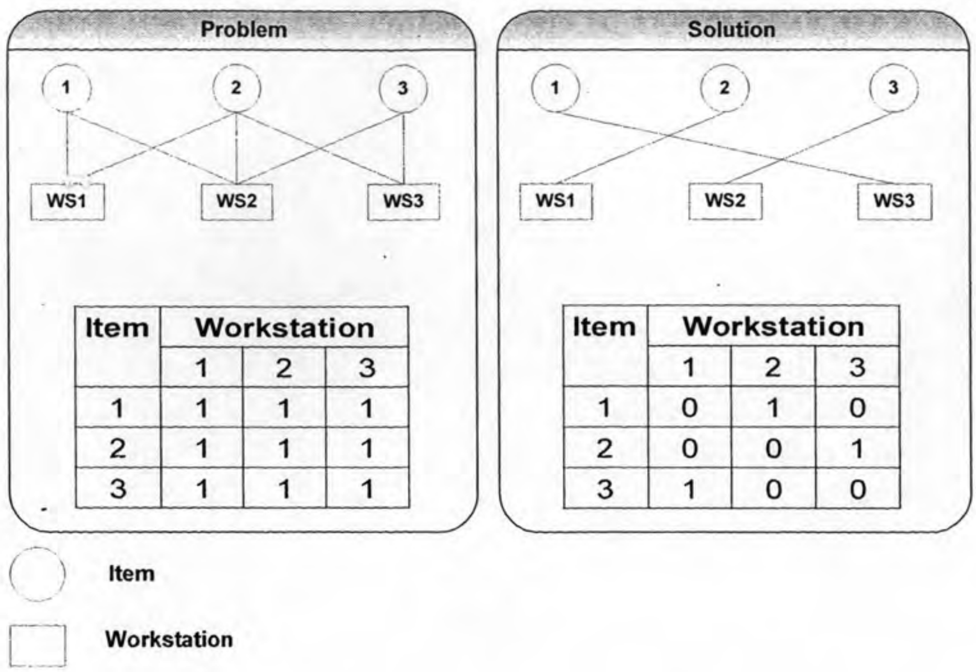
4.2 Heuristic Description

This heuristics was motivated by observing that there are obvious differences between single-workstation and multi-workstation problem. If we can find the efficient way of assigning an item on a workstation while not violating all constraints; the multi-workstation will be simplified to the single-workstation problem. As a result, the size of the problem is reduced, and it is relatively easier to solve. Thus, it tends to reduce the computation time significantly. Furthermore, as we observed later, the heuristics based on this simplification can generate a satisfactory solution with a reasonable computation time.

On the other hand, this approach is still suitable for a large problem simplified into a smaller problem. The limited size allows using exact methods for their solution, which would be impossible for the overall problem. The number and the size of the problem define the computational burden and the solution quality of the heuristic procedures.

4.3 Heuristic Procedures

The proposed heuristic consists of two sub procedures: an assignment solution with given lot size (P1) and a lot size solution with given assignment (P2). This is because the characteristic of this problem as shown in Figure 4.1.



Item	Workstation		
	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1

Item	Workstation		
	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

Figure 4.1 The demonstration of the problem item-workstation matrix and the solution item workstation matrix

In Figure 4.1, the situation is 3-item-workstation with NT planning horizon. In the problem, all items have a capability to be operated on all workstations and all workstation also have a capability to operate all items. However, in the solution, each period only an item can be operated on one machine and a machine can operate only one item. In the way of illustration, this problem has a characteristic of an assignment model. Other findings are that

- The amount production of the lot size can be provided for demand in this period or later periods.
- In each sub-problem, on given lot size, the assigning can be done. Likewise, on given assignment, the lot sizing can be done.
- With echelon demand of item i in period t (e_t^i), the demand of each item in each period can be calculated from end-item demand with the equation (e)

$$e_t^i = d_t^i + \sum_{j \in S(i)} u^{ij} e_t^j, \forall i \in I, \forall t \in T \qquad \text{(e)}$$

- There is an assumption initiating the present heuristic approach, namely, all echelon demand lot size are feasible for P2. In the other word, Lot-for-Lot policy is feasible for this problem.

With given assignment, the lot size will be found from the smaller model as it was reduced to similar to one-one item-workstation model. As aforementioned, this research proposes a heuristic method which decomposes the MLCLSP-M into the following two sub-problems:

4.3.1 Phase 1: Assignment problem with given lot-size.

In assigning part, the mathematical model is formulated. The problem was considered here is to find the assigning of an item to a workstation under minimized total cost criteria. The situation is the same as the MLCLSP-M problem but the decision is not to find what or when lot size should be. On given demand, the goal of this part is to find the assigning matrix which makes the minimum total cost.

4.3.1.1 Problem description

The decision variables are the assigning item on a workstation in a period. The constraints are the forcing constraints of an item to be used only one workstation, the forcing constraints of a workstation to be operated only one item. However, in each period, there is not necessary that all workstations will be used or all items will be operated. Except demand parameters, the data of this model is similar to the parameters of the full problem (MLCLSP-M). The demands in this step are calculated from the echelon demands or the previous lot size solution. The objective is the minimized total costs of the demand assigned to the workstation for all items and all periods.

4.3.1.2 Assumption

The assumption is same as the MLCLSP-M model. However, the decision variable is the assigning matrix and decision on the given lot sizes which are the echelon demands for the first iteration and the result from the lot sizing part for the other iterations.

4.3.1.3 Assignment Model

Let $w_t^{i,k}$ represents the status of assignment ($w_t^{i,k}=1$ if in period t item i is assigned to be operated on workstation k, and $w_t^{i,k}=0$ for otherwise) and D_t^i represents the given lot size. The formulated model can be represented as follows

$$Min \ \left(p_t^{i,k}\cdot D_t^i + f_t^{i,k}\right)\times w_t^{i,k} \tag{P1}$$

Subject to

$$D_t^i = d_t^i + \sum_{j\in S(i)} u^{ij} D_t^j \ , \ \forall i\in I, \forall t\in T \tag{4.1}$$

$$\left(\sum_{i\in I(k,t)} D_t^i\right) w_t^{i,k} \le Avail_t^k \ , \ \forall k\in WR, \forall t\in T \tag{4.2}$$

$$\left(\sum_{i\in I(k,t)} \left(o_t^{i,k}\cdot D_t^i + g_t^{i,k}\right)\right) w_t^{i,k} \le cap_t^k \ , \ \forall k\in WP, \forall t\in T \tag{4.3}$$

$$\sum_{k\in WS} w_t^{i,k} \le 1, \ \forall i\in I, \forall t\in T \tag{4.4}$$

$$\sum_{i\in I} w_t^{i,k} \le 1 \ , \ \forall k\in WS, \forall t\in T \tag{4.5}$$

$$w_t^{i,k}\in\{0,1\}, \ \forall i\in I, \forall k\in WS, \forall t\in T \tag{4.6}$$

The result of this part is the assigning matrix or the pair of item-workstation to be used as input data for the lot sizing part.

4.3.2 Phase 2: Lot sizing problem with given assignment matrix.

Let set $A(t)=\{(i,k)\mid w_t^{i,k}=1\}$ is a set of the assignment item-workstation in period t.

$$Min \sum_{t\in T}\sum_{(i,k)\in A(t)}\left(p_t^{(i,k)}\cdot x_t^{(i,k)}+f_t^{(i,k)}\cdot y_t^{(i,k)}\right)+\sum_{t\in T}\sum_{i\in I}\left(h_t^i\cdot s_t^i\right) \qquad \text{(P2)}$$

Subject to

$$s_{t-1}^i+\sum_{(i,k)\in A(t)}x_t^{(i,k)}=d_t^i+s_t^i,\forall i\in E,\forall t\in T \qquad (4.7)$$

$$s_{t-1}^i+\sum_{(i,k)\in A(t)}x_t^{(i,k)}=\sum_{j\in S(i)}\left(u^{i,j}\cdot\sum_{(j,k)\in A(t)}x_t^{(j,k)}\right)+s_t^i,\ \forall i\in I-E,\forall t\in T \qquad (4.8)$$

$$\sum_{(i,k)\in A(t)}x_t^{(i,k)}\le Avail_t^k,\ \forall k\in WR,\forall t\in T \qquad (4.9)$$

$$\sum_{(i,k)\in A(t)}\left(o_t^{(i,k)}\cdot x_t^{(i,k)}+g_t^{(i,k)}\cdot y_t^{(i,k)}\right)\le cap_t^k,\ \forall k\in WP,\forall t\in T \qquad (4.10)$$

$$\sum_{i\in I^*}s_t^i\le V,\ \forall t\in T \qquad (4.11)$$

$$s_0^i=s_{NT}^i=0,\ \forall i\in I \qquad (4.12)$$

$$\sum_{(i,k)\in A(t)}y_t^{(i,k)}\le 1,\ \forall i\in I,\forall t\in T \qquad (4.13)$$

$$\sum_{(i,k)\in A(t)}y_t^{(i,k)}\le 1,\ \forall k\in WS,\forall t\in T \qquad (4.14)$$

$$x_t^{(i,k)}\le M_t^{(i,k)}\cdot y_t^{(i,k)},\ \forall t\in T,\forall (i,k)\in A(t) \qquad (4.15)$$

$$x_t^{(i,k)}\ge 0,\ \forall (i,k)\in A(t),\forall t\in T \qquad (4.16)$$

$$s_t^i\ge 0,\ \forall i\in I,\forall t\in T \qquad (4.17)$$

$$y_t^{(i,k)}\in\{0,1\},\ \forall (i,k)\in A(t),\forall t\in T \qquad (4.18)$$

As each sub-problem is solved in sequentially and iteratively manner, we call this heuristic as Assignment-Lot size heuristic (A-LS).

In the first iteration, the assignment problem will be solved with echelon demand ($D_t^i = e_t^i$). The solution of this phase (the assignment matrix) will be given to the lot sizing problem and then we will get the initial solution. This procedure can be demonstrated as Figure 4.2.

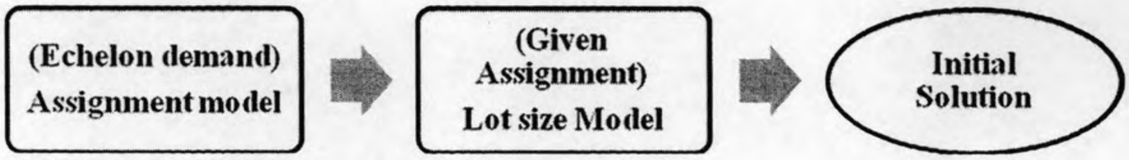


Figure 4.2 A demonstration of the first iteration procedure

As Lot-for-Lot has no consideration about holding or the produce for demand in other periods, the better solution will be established with bigger lot size in each period. At this moment the lot size solution of the phase 2 is generally bigger than the echelon demand. Therefore, in the next iteration, the lot size solution will be used as input data (given lot size) for the phase 1 for improving solution. This procedure is repeated until meet the number of iteration limited or the lot size from this iteration equal to the lot size of the previous solution. The flow of this heuristic can present as in Figure 4.3.

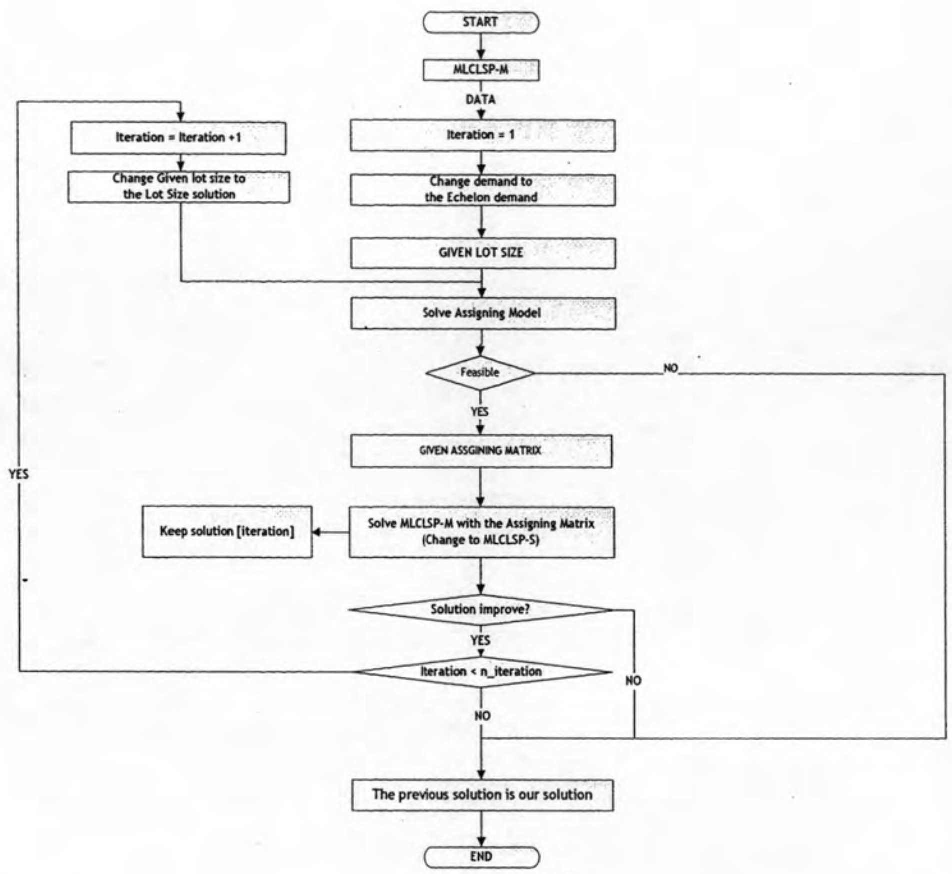


Figure 4.3 The flow of the proposed heuristic

4.3.3 Termination

The termination of this heuristics occurs in two ways. First, the maximum iteration is met. Second, there is no improvement of the solution. In these iterations, we use the AMPL/CPLEX 8.0.0 to solve the problems.

4.4 Computational Results

In this section, we solved the model using the A-LS heuristic as presented. We consider a scenario with ten-item ten-workstation over a planning horizon of four periods. The product structure is shown in Figure 4.4 whereas the capability matrix is given in Table 4.1.

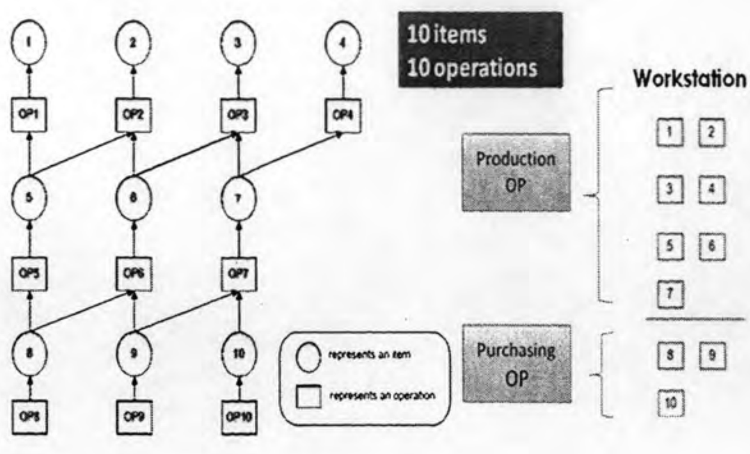


Figure 4.4 An example of BOM

Table 4.1 Capability table.

Operations	Capable Workstation	Required Item	Immediate Successor
OP1	WS1/WS3	Item 5	Item 1
OP2	WS1/WS3	Item 5, Item 6	Item 2
OP3	WS1/WS2	Item 6, Item 7	Item 3
OP4	WS2	Item 7	Item 4
OP5	WS3	Item 8	Item 5
OP6	WS1	Item 8, Item 9	Item 6
OP7	WS2/WS3	Item 9, Item 10	Item 7
OP8	WS4/WS5	-	Item 8
OP9	WS4	-	Item 9
OP10	WS5	-	Item 10

In order to analyze the mathematical model's characteristics, test instances were generated. The pattern of the generation was introduced in standard library (Stadtler and Surie, 2000). The demand series are defined by the number of periods in the planning horizon, the length and amplitude of the seasonal demand pattern and different coefficients of variation. The products' structure profiles are general and assembly. However, this research problem is an extension version of their problem with multi-workstation and one warehouse consideration. Therefore, the production cost, setup cost/ordering cost, operation time, setup time and capacity/availability were initially generated for this research as shown in Table 4.2.

Table 4.2 All parameters initially generated for this research

Parameters	Data Generation
$p_t^{i,k}$	For production : NORM(12,5) , For purchasing : NORM (45,9)
$f_t^{i,k}$	For production : NORM(570,190) , For purchasing : NORM (820,250)
$o_t^{i,k}$	For production : NORM(15,3) , For purchasing : 0
$g_t^{i,k}$	For production : NORM(70,14) , For purchasing : 0
V	10% of the summation of all workstations' capacity for all period divided by product of number of workstations and number of items
$Avail_t^k$	Maximum value of summation of all demand divided by %utilization for all items in period
cap_t^k	Maximum value of summation of all demand (subtracted with setup time then divided by operation time) divided by %utilization for all items in period (Utilization profile are (1) constant 90%, (2) constant 70%, (3) constant 50%, (4) 90/70/50 reduced relatively with production level, (5) 50/70/90 increased relatively with production level, (6) increased by period time, and (7) decreased by period time)

We use problem size 10-item 10-workstation with eight periods (the product structure is same as the standard library) to show this problem. To solve this Mixed Integer Programming (MIP) problem, we used AMPL/CPLEX 8.0.0 solver. Figure 4.5 shows example of one problem instance AG01130 showing improvement between iterations. Second, we compare the solution from solving MLCLSP-M and A-LS in three aspects which are %different of total cost, %different of running time and %running time between the heuristics solution and mathematical model solution. The results are classified by the capacity profiles of the problem instance.

It can be observed from Figure 4.5 for the problem instance AG01130 example that solution is improved dramatically in the early iterations and then stay constantly until end. The comparison between MLCLSP-M and A-LS is shown in Table 4.3.

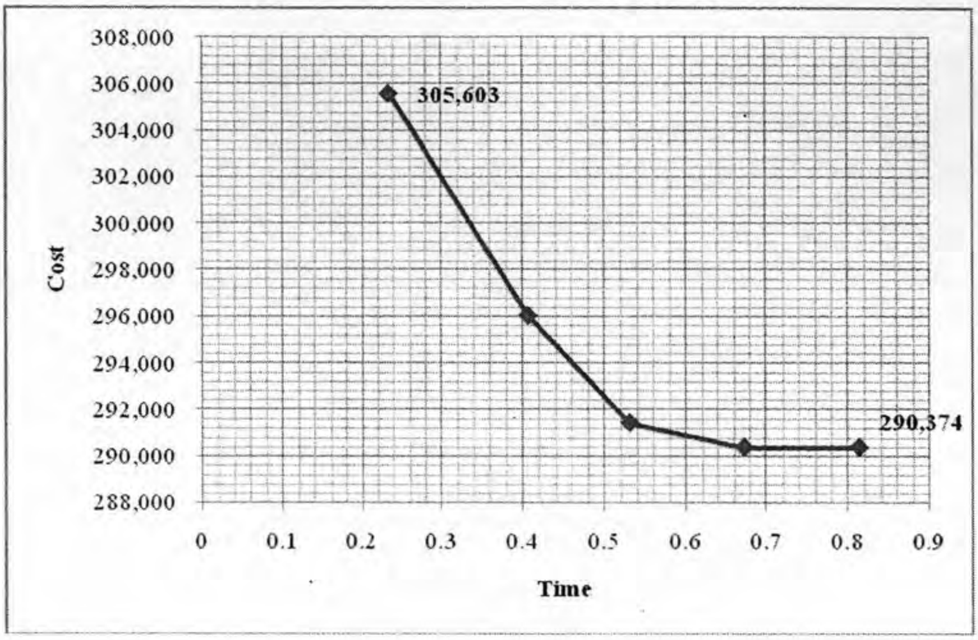


Figure 4.5 The improvement between iterations sample for instance AG01130

Table 4.3 The comparison between MLCLSP-M and A-LS for problem AG01130

	MLCLSP-M	A-LS Heuristic	%diff.
Cost($)	286,623	290,374	1%
Time(sec.)	8,232.16	1.5	100%

The capacity profiles of the test instances are constant utilization, varying utilization by product structure level or BOM, and varying by periods. Comparison of results for these capacity profiles between MLCLSP-M and A-LS are presented in Table 4.4, Table 4.5 and Table 4.6, respectively. For more clearly, we present the summary of A-LS for different capacity profiles in Figure 4.6.

As seen in Table 4.4, Table 4.5 and Table 4.6, the solutions from MLCLSP-M and A-LS are closed. The maximum different of total cost between MLCLSP-M and A-LS is only 5.02%. The average different is 1%, while minimum different is only

0%. The results also show that the A-LS heuristic results with higher different when solved the problems which have a higher capacity utilization profile and seasonal demand profile for end-item. However, the solving time of the heuristic usually range between one to two seconds. Therefore, the problem profiles have no effect on solving time but have some effects on performance of the solution. To sum up, the heuristic give almost optimal solution (average 1% different) within very fast solving time (100% different).

Table 4.4 The comparison between different capacity profiles with given constant utilization capacity profiles

Capacity Profile	%Diff Cost			%Diff Time			% Running Time		
	Min	Avg.	Max	Min	Avg.	Max	Min	Avg.	Max
Constant 90%	1.58%	2.78%	5.18%	99.12 %	99.74 %	99.99 %	0.01%	0.26%	0.88%
Constant 70%	1.20%	2.62%	4.77%	99.82 %	99.95 %	100%	0.00%	0.05%	0.18%
Constant 50%	0.01%	0.78%	1.73%	99.18 %	99.72 %	99.97 %	0.03%	0.28%	0.82%

Table 4.5 The comparison between different capacity profiles with given varying by level capacity profiles

Capacity Profile	%Diff Cost			%Diff Time			% Running Time		
	Min	Avg.	Max	Min	Avg.	Max	Min	Avg.	Max
Vary 90/70/50 by level	0.14%	0.81%	2.09%	98.35 %	99.45 %	99.90 %	0.10%	0.55%	1.65%
Vary 50/70/90 by level	0.00%	0.56%	1.37%	99.04 %	99.63 %	100%	0.00%	0.37%	0.96%

Table 4.6 The comparison between different capacity profiles with given varying by period capacity profiles

Capacity Profile	%Diff Cost			%Diff Time			% Running Time		
	Min	Avg.	Max	Min	Avg.	Max	Min	Avg.	Max
Increasing by period	0.00%	0.48%	0.93%	99.07 %	99.52 %	100%	0.00%	0.48%	0.93%
Decreasing by period	0.00%	0.29%	0.81%	99.44 %	99.81 %	100%	0.00%	0.19%	0.56%

Table 4.7 The average solving time comparison between MLCLSP-M and A-LS for all problems

	MLCLSP-M	A-LS Heuristic	%diff.
Average Time(sec.)	10,239.09 (2.80 hr.)	1.53	100%

In Table 4.7, the cost from the heuristic is slightly different from the MLCLSP-M but uses dramatically lesser computational time in all problems. The solving time comparison Illustration of summary of A-LS for different capacity profiles is shown in Figure 4.6.

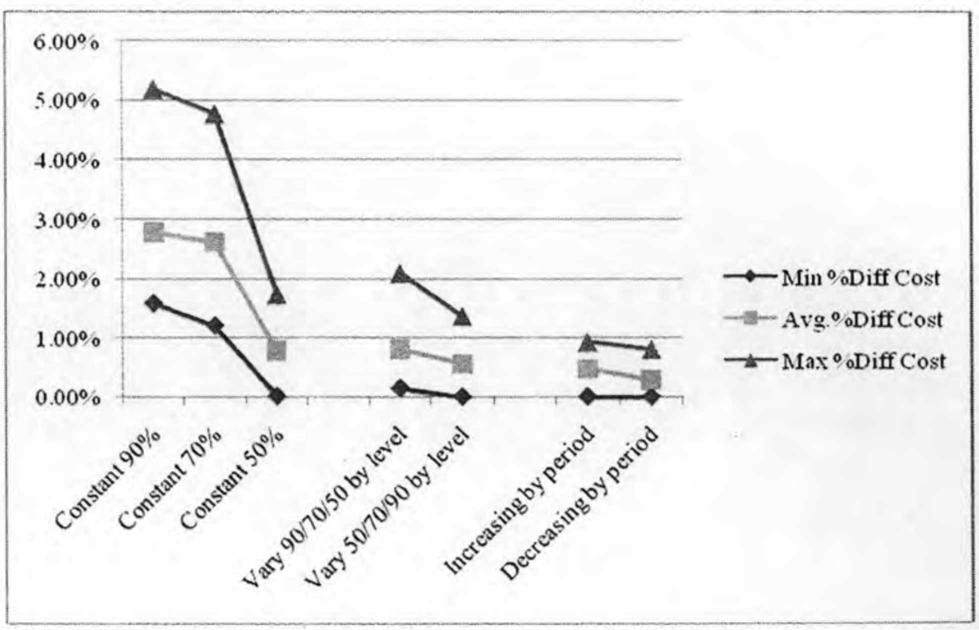


Figure 4.6 Illustration of summary of A-LS for different capacity profiles

As seen in Figure 4.6, although over all the solution performance is good with maximum 5.2% different cost form optimal solution, the solution performance of problem instances with constant 90% capacity profile is lowest.

4.5 Conclusion

This paper develops a mathematical model for an integrated purchasing and production lot-sizing problem with multi-workstation, which is very frequently confronted in real life and still more interesting to the operations researchers. The determinations of the multi-period multi-item multi-level lot-sizing under the consideration of the availability of raw materials and capacity of workstations to minimize cost are studied in this paper. The dynamic purchasing cost, production cost and holding cost are examined in the problem. Because the problem is obviously NP-hard, there is a need to find alternative approaches that can be used in practical. Therefore, this research proposes a new decomposition heuristic based on all assumptions of the MLCLSP-M model that lot-for-lot policy is a feasible solution. The heuristics composes of two phases which are assignment with given lot size phase and lot size with given assignment phase. These phases are solved iteratively and sequentially with AMPL/CPLEX 8.0.0 solver. The assignment solution of the first phase is used as input data for the lot size phase and the lot size solution will be used as input for the assignment phase in the next iteration. The termination of this heuristic will happen whenever the iteration numbers reach the maximum iteration limited or the lot solution of the previous iteration equal to following iteration.

Numerical experiment demonstrates significant characteristics of the A-LS heuristic in two aspects, i.e. the problem profiles have no effect on solving time but have some effects on performance of the solution and the heuristic give near optimal solution with very fast solving time.