Chapter I

Introduction

Moser's worm problem has been one of the interesting mathematical problems for a long time. It asks for the smallest convex set on the Cartesian plane that can cover every unit continuous rectifiable arc. We call those kinds of set "covers". With the aid of Blaschke selection theorem, the existence to the solution of the problem was already guaranteed. Certainly, a disk with unit diameter is a cover.

In 1970, A. Meir showed that a semidisk of unit diameter is a cover. Its area is $\frac{\pi}{8} \approx 0.39270$.

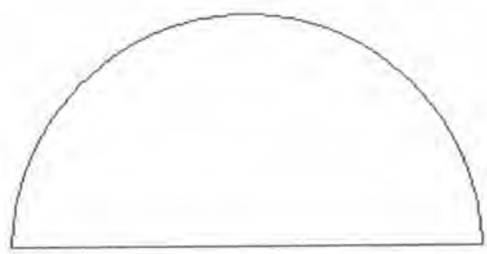

Figure 1.1 : Meir's semidisk of area 0.39270

Three years later, a clipped circular sector described by Wetzel [1] is a cover with an area < 0.34423.

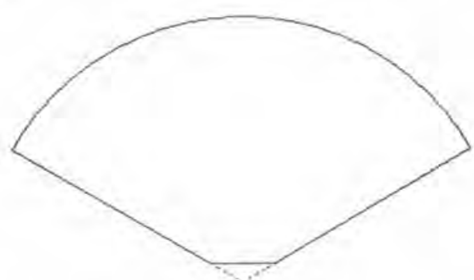

Figure 1.2 : Wetzel's clipped sector of area 0.34423

Later in 1974, Gerriets and Poole [2] showed that the rhombus formed by abutting 2 equilateral triangles of side $\frac{1}{\sqrt{3}}$ is a cover. Moreover, they also proved that the rhombus can be clipped from one of the 120° corners. Thus the area of the cover approximately decreased to 0.28610.

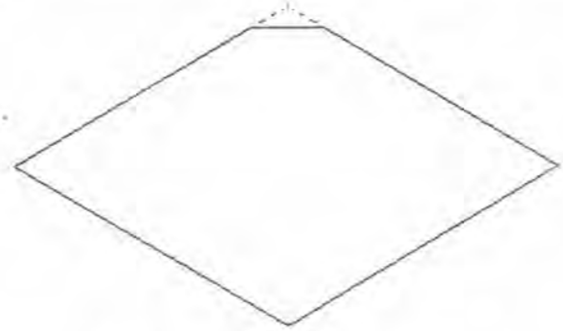

Figure 1.3 : Clipped rhombus of area 0.28610

In 1992, a smaller modified rhombus was discovered by Norwood, Poole, and Laidacker [3]. The region was formed by clipping one of the 120° corners of the former rhombus with a circular arc of radius 0.5. Its area is about 0.27524.

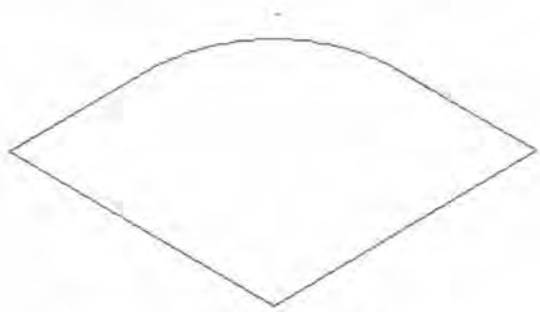

Figure 1.4 : Modified rhombus of area 0.27524

In 2003, Norwood and Poole [4] constructed a non-convex cover of area about 0.26044. The upper boundary is a circular arc with radius 0.5 whose center lies on the vertical axis of symmetry at a point determined numerically as the solution of a geometric minimum problem. The side arcs are parabolic, modified slightly by opposing circular arcs near the ends of the diameter. Almost as an afterthought and virtually without comment, the authors remark that varying the parameters of their construction by only a little gives a new convex cover whose area is about 0.27381

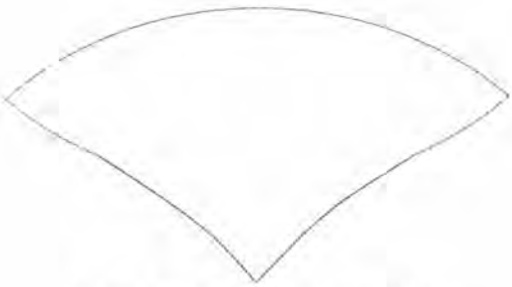

Figure 1.5 : NP's nonconvex cover of area 0.26044

Figure 1.6 : NP's convex cover of area 0.27381

In 2006, Wei Wang [5], a Chinese mathematician, discovered a smaller cover of area 0.270911861

Nowadays, we have known that the cover has area at least 0.21946 and at most 0.27381 squared units for a convex cover and 0.26044 squared units for a cover.

One of the most difficult things dealing with Moser's worm problem is how to prove that the considered set is a cover by using standard geometric methods. In this thesis, we will try to solve this kind of problem with the aid of numerical minimization. The idea is to show that any arc which can not be covered by a considered set is longer than 1 unit.

Certainly, we may use the scaling idea in order to find a cover; for example, if we find that an arc with the length of α units cannot be covered by our considered set, the considered set scaled by $\frac{1}{\alpha}$ will be a cover.

Studying about Moser's worm problem, there are 3 main parameters as follows.

- The class of arcs; such as, convex arcs, 2-secment arcs.
- The characteristic of covers; such as, convex covers, triangular covers.
- How to cover an arc; such as, no flipping, only translations.

In this work, several ways to find the numerical minimization are used. If we consider the problem properly, we will find that the problem is a convex programming. By using Mathematica, the numerical minimization will be seated in 4 general method; Differential Evolution, Nelder Mead, Random Search, and Simulated Annealing. However, none of these methods takes advantage of the problem being a convex programming.