CHAPTER III

DERIVATION OF GUO'S FOUR-PHASE FLOW MODEL

3.1 General equation of multiphase flow

In a production well, the fluid pressure at the inlet minus the fluid pressure at the outlet is called the pressure drop in the total system. This pressure drop is the sum of the pressure drops occurring in all of the components of the system. The flowing fluid is compressible, or slightly compressible and; therefore, the pressure drop in a particular component depends not only on the flow rate through the component, but also on the average pressure that exists in the component. The amount of oil and gas flowing from the reservoir into the well depends on the pressure drop in the piping system, and the pressure drop in the piping system depends on the amount of fluid flowing through it.

Therefore, it is necessary to be able to calculate the pressure losses in all components in the system. Most producing oil or gas wells operate under multiphase conditions. There is usually some free gas produced along with the oil in an oil well, and most gas wells produce either water or condensate along with the gas.

The present of both liquid and gas in the component complicates the pressure loss calculations immensely. As average pressure existing in a component changes, phase changes occur in the fluids. This causes changes in densities, viscosities, volumes of each phase, and fluid properties. Also, temperature changes occur in the piping system and restrictions. This is not a problem in calculating the reservoir performance since reservoir temperature remains constant. Calculation of the pressure change with distance, or pressure gradient, at any point in the system requires knowledge of the temperature at that point. Therefore, procedures to estimate heat or temperature losses must be available.

The pressure gradient (or rate of change in pressure with respect to unit of flow length) for vertical multiphase flow is the sum of three contributing factors: hydrostatic pressure gradient, friction pressure gradient, and acceleration pressure gradient. The general pressure gradient equation for flowing any fluid in a pipe at any inclination angle is as follows:

$$\frac{dp}{dL}=\frac{\rho\, g\sin\theta}{g_c}+\frac{f\rho\, v^2}{2g_c d}+\frac{\rho\, v\, dv}{g_c dL} \tag{3.1}$$

where

$\frac{\rho\, g\sin\theta}{g_c}$ = potential energy change or hydrostatic pressure gradient

$\frac{f\rho\, v^2}{2g_c d}$ = friction pressure gradient

$\frac{\rho\, v\, dv}{g_c dL}$ = kinetic energy change or acceleration pressure gradient

There are many correlations to predict a vertical multiphase flow pressure traverse. No methods presently exist for analytically evaluating either liquid holdup H_L or friction factor. Therefore, it has been necessary to develop empirical correlations for these two parameters as functions of variables that will be known or can be calculated from known data. This requires an experimental facility from which values of H_L and two-phase friction factor f_{TP} can be measured under a wide range of flow conditions and flow geometries.

3.2 Derivation of a four-phase flow model

Boyun Guo[1] has developed a systematic approach to modeling of gas, water, oil, and solid particle four-phase flow in boreholes. This four-phase flow model was first published in drilling literature and coded in a spreadsheet program.

In this section, the derivation of this model is expressed. The flowing borehole pressure can be formulated on the basis of the first law of thermodynamics for a vertical hole. The change in pressure can be written as

$$dP=\gamma_m\left(1+\frac{fv^2}{2gd_H}\right)dh \tag{3.2}$$

or

$$\frac{dP}{dh}=\gamma_m+\frac{f\gamma_m v^2}{2gd_H}$$

where dP = pressure incremental, lbf/ft^2,

γ_m = specific weight of mixture, lbf/ft^3,

f = Moody friction factor, dimensionless,

v = fluid velocity, ft/s,

d_H = hydraulic diameter, ft, and

dh = depth incremental, ft.

The specific weight of mixture is expressed as

$$\gamma_m = \frac{\dot{W}_s + \dot{W}_l + \dot{W}_g}{\dot{Q}_s + \dot{Q}_l + \dot{Q}_g} \qquad (3.3)$$

where $\dot{W}_s$ = weight flow rate of solid, lb/sec,

$\dot{W}_l$ = weight flow rate of liquid, lb/sec,

$\dot{W}_g$ = weight flow rate of gas, lb/sec,

$\dot{Q}_s$ = volumetric flow rate of solid, ft^3/sec,

$\dot{Q}_l$ = volumetric flow rate of liquid, ft^3/sec, and

$\dot{Q}_g$ = volumetric flow rate of gas, ft^3/sec.

The volumetric flow rate of solid $\dot{Q}_s$ in Equation 3.3 can be expressed as

$$\dot{Q}_s = \frac{Q_s}{86400}$$

or

$$\dot{Q}_s = 1.16 \times 10^{-5} Q_s \qquad (3.4)$$

where

Q_s = volumetric flow rate of solid in ft^3/day

The weight rate of solid depends on volumetric flow rate and specific gravity of solid (S_s) and can be written as

$$\dot{W}_s = (62.4 S_s)(1.16 \times 10^{-5} Q_s)$$

or

$$\dot{W}_s = 7.2 \times 10^{-4} S_s Q_s \qquad (3.5)$$

where

S_s = specific gravity (related to water) of solid.

The volumetric flow rate of liquid $\dot{Q}_l$ in Equation 3.3 can be expressed as

$$\dot{Q}_l = \frac{5.615Q_w}{86400} + \frac{5.615Q_o}{86400}$$

or

$$\dot{Q}_l = 6.5 \times 10^{-5}(Q_w + Q_o) \tag{3.6}$$

where $Q_w =$ water production rate, bbl/d

$Q_o =$ oil production rate, stb/d

The weight rate of liquid depends on injection flow rate and formation fluid influx rate and can be expressed as

$$\dot{W}_l = 62.4S_w\left(\frac{5.615Q_w}{86400}\right) + 62.4S_o\left(\frac{5.615Q_o}{86400}\right)$$

or

$$\dot{W}_l = 4 \times 10^{-3}(S_wQ_w + S_oQ_o) \tag{3.7}$$

where $S_w =$ specific gravity of produced water

$S_o =$ specific gravity of produced oil

The in-situ volumetric flow rate of gas Q_g in Equation 3.3 is expressed in terms of gas flow rate at standard condition through gas law for ideal gas as

$$\dot{Q}_g = \frac{(14.7)(144)TQ_{gs}}{(86400)(520)P}$$

or

$$\dot{Q}_g = \frac{4.7 \times 10^{-5}TQ_{gs}}{P} \tag{3.8}$$

where $Q_{gs} =$ gas rate in standard condition (14.7 psia, 60° F), scf/d

$P =$ pressure, lbf/ft^2

$T =$ temperature, °R

The weight rate of gas depends on volumetric gas flow rate (Q_{gs}) and specific gravity of gas (S_g) and can be expressed as

$$W_g = 0.0765 S_g \left(\frac{Q_{gs}}{86400} \right)$$

or

$$W_g = 8.85 \times 10^{-7} S_g Q_{gs} \tag{3.9}$$

where S_g = specific gravity of gas, air = 1.

Substituting Equations 3.4 through 3.9 into Equation 3.3 becomes

$$\gamma_m = \frac{7.2\times10^{-4} S_s Q_s + 4\times10^{-3}(S_w Q_w + S_o Q_o) + 8.85\times10^{-7} S_g Q_{gs}}{1.16\times10^{-5} Q_s + 6.5\times10^{-5}(Q_w + Q_o) + \dfrac{4.7\times10^{-5} T Q_{gs}}{P}}$$

Dividing both sides by $\dfrac{4.7\times10^{-5} T Q_{gs}}{P}$,

$$\gamma_m = \frac{(7.2\times10^{-4} S_s Q_s + 4\times10^{-3}(S_w Q_w + S_o Q_o) + 8.85\times10^{-7} S_g Q_{gs}) / (\dfrac{4.7\times10^{-5} T Q_{gs}}{P})}{(1.16\times10^{-5} Q_s + 6.5\times10^{-5}(Q_w + Q_o) + \dfrac{4.7\times10^{-5} T Q_{gs}}{P}) / (\dfrac{4.7\times10^{-5} T Q_{gs}}{P})}$$

$$\gamma_m = \frac{[\ 15.32 S_s Q_s + 85.1(S_w Q_w + S_o Q_o) + 0.019 S_g Q_{gs}\ ] \dfrac{P}{T Q_{gs}}}{[\ 0.247 Q_s + 1.38(Q_w + Q_o)\ ] \dfrac{P}{T Q_{gs}} + 1}$$

$$\gamma_m = \frac{a'' P}{b'' P + 1} \tag{3.10}$$

where

$$a'' = \frac{15.32 S_s Q_s + 85.1(S_w Q_w + S_o Q_o) + 0.019 S_g Q_{gs}}{T Q_{gs}} \qquad (3.11)$$

and

$$b'' = \frac{\left[0.247 Q_s + 1.38(Q_w + Q_o)\right]}{T Q_{gs}} \qquad (3.12)$$

Flow velocity can be formulated based on volumetric gas rate given by Equation 3.8, liquid flow by Equation 3.6, and flow path cross sectional area as

$$v = \frac{144}{A}\left[6.5\times10^{-5}(Q_w + Q_o) + \frac{4.7\times10^{-5} T Q_{gs}}{P}\right]$$

which is rearranged to be

$$v = \frac{6.77\times10^{-3} T Q_{gs}}{AP} + \frac{9.4\times10^{-3}(Q_w + Q_o)}{A}$$

or

$$v = \frac{c''}{P} + d'' \qquad (3.13)$$

where

$$c'' = \frac{6.77\times10^{-3} T Q_{gs}}{A} \qquad (3.14)$$

and

$$d'' = \frac{9.4\times10^{-3}(Q_w + Q_o)}{A} \qquad (3.15)$$

Substituting Equations 3.10 and 3.13 into Equation 3.2 yields

$$dP = \left(\frac{a''P}{b''P+1}\right)\left[1+\frac{f}{2gd_H}\left(\frac{c''}{P}+d''\right)^2\right]dh \qquad (3.16)$$

or

$$dP = \left(\frac{a''P}{b''P+1}\right)\left[1+e''\left(\frac{c''}{P}+d''\right)^2\right]dh \qquad (3.17)$$

where

$$e'' = \frac{f}{2gd_H} \qquad (3.18)$$

In Equation 3.17 the hydrostatic pressure component and the frictional pressure component may be decoupled as

$$dP_{hy} + dP_{fr} = \left(\frac{a''P_{hy}}{b''P_{hy}+1}\right)dh + e''\left(\frac{a''P_{fr}}{b''P_{fr}+1}\right)\left(\frac{c''}{P_{fr}}+d''\right)^2 dh \qquad (3.19)$$

so that

$$dP_{hy} = \left(\frac{a''P_{hy}}{b''P_{hy}+1}\right)dh \qquad (3.20)$$

and

$$dP_{fr} = e''\left(\frac{a''P_{fr}}{b''P_{fr}+1}\right)\left(\frac{c''}{P_{fr}}+d''\right)^2 dh \tag{3.21}$$

Rearranging Equation 3.20 as follows:

$$b''P_{hy}dP_{hy} + dP_{hy} = a''P_{hy}dh$$

Dividing both sides by P_{hy}, we obtain

$$b''dP_{hy} + \frac{1}{P_{hy}}dP_{hy} = a''dh$$

Integrating both sides from wellhead pressure P_{wh} to hydraulic pressure P_{hy} at vertical depth H,

$$\int_{P_{wh}}^{P_{hy}} b''dP_{hy} + \int_{P_{wh}}^{P_{hy}} \frac{1}{P_{hy}}dP_{hy} = a''\int_{O}^{H} dh$$

$$b''[\ dP_{hy}\ ]_{P_{wh}}^{P_{hy}} + \ln[\ dP_{hy}\ ]_{P_{wh}}^{P_{hy}} = a''[\ h\ ]_{O}^{H}$$

$$b''(P_{hy} - P_{wh}) + \ln\left(\frac{P_{hy}}{P_{wh}}\right) = a''H \tag{3.22}$$

from which P_{hy} can be solved using a numerical method such as Newton- Raphson's algorithm.

Recalling Equation 3.21,

$$dP_{fr} = e''\left[\frac{a''P_{fr}}{b''P_{fr}+1}\right]\left[\frac{c''}{P_{fr}}+d''\right]^2 dh$$

then, rearranging the above equation as

$$dP_{fr} = e''\left[\frac{a''P_{fr}}{b''P_{fr}+1}\right]\left[(\frac{c''}{P_{fr}})^2 + 2(\frac{c''}{P_{fr}})d'' + d''^2\right] dh$$

$$dP_{fr} = e''\left[\frac{a''P_{fr}}{b''P_{fr}+1}\right]\left[\frac{c''^2 + 2c''d''P_{fr} + d''^2P_{fr}^2}{P_{fr}^2}\right] dh$$

$$dP_{fr} = \left[\frac{a''c''^2e''}{b''P_{fr}{}^2 + P_{fr}} + \frac{2a''c''d''e''}{b''P_{fr}+1} + \frac{a''d''^2e''P_{fr}}{b''P_{fr}+1} \right] dh$$

The above frictional pressure component may consist three components ($P_{fr1}, P_{fr2}, P_{fr3}$) that can be written in the following equations:

$$dP_{fr1} = \frac{a''c''^2e''}{b''P_{fr1}^2 + P_{fr1}} dh \tag{3.23}$$

$$dP_{fr2} = \frac{2a''c''\ d''e''}{b''P_{fr2}+1} dh \tag{3.24}$$

$$dP_{fr3} = \frac{a''d''^2e''^2P_{fr3}}{b''P_{fr3}+1} dh \tag{3.25}$$

Integration of each component from wellhead to depth of interest at measured depth L in order to get the total pressure loss.

Integrating Equation 3.23 on both sides, we obtain

$$\int_O^{P_{fr1}} b''P_{fr1}^2 dP_{fr1} + \int_O^{P_{fr1}} P_{fr1} dP_{fr1} = \int_O^L a''c''^2e''dh$$

$$b''\left[\frac{P_{fr1}^3}{3}\right]_O^{P_{fr1}} + \left[\frac{P_{fr1}^2}{2}\right]_O^{P_{fr1}} = a''c''^2e''[h]_O^L$$

$$\frac{b''}{3}P_{fr1}^3 + \frac{1}{2}P_{fr1}^2 = a''c''^2e''L \tag{3.26}$$

Integrating Equation 3.24 both sides, the equation becomes

$$\int_O^{P_{fr2}} b''P_{fr2} dP_{fr2} + \int_O^{P_{fr2}} dP_{fr2} = \int_O^L 2a''c''d''e''dh$$

$$b''\left[\frac{P_{fr2}^2}{2}\right]_O^{P_{fr2}} + \left[P_{fr2}\right]_O^{P_{fr2}} = 2a''c''d''\ e''\left[h\right]_O^L$$

$$\frac{b''}{2}P_{fr2}^2 + P_{fr2} = 2a''c''d''\ e''L \tag{3.27}$$

Integrating Equation 3.25 both sides, we have

$$\int_O^{P_{fr3}} \frac{b''P_{fr3}dP_{fr3}}{P_{fr3}} + \int_O^{P_{fr3}} \frac{1}{P_{fr3}} dP_{fr3} = \int_O^L a''d''^2e''dh$$

$$b''\left[P_{fr3}\right]_O^{P_{fr3}} + \ln\left[P_{fr3}\right]_O^{P_{fr3}} = a''d''^2e''\left[h\right]_O^L$$

$$b''P_{fr3} + \ln(P_{fr3}) = a''d''^2e''L \tag{3.28}$$

A numerical method can be used to solve Equations 3.26, 3.27, and 3.28 for the frictional pressure of each equation. The sum of these friction pressures gives the total friction pressure P_{fr}.

Equation 3.22 is utilized to solve the hydrostatic pressure P_{hy} using a numerical method such as Newton-Raphson's algorithm. Then, the pressure at the bottom of the well is obtained from the sum of the hydrostatic pressure P_{hy} and the total friction pressure P_{fr}. It can be expressed in the following equation as

$$P_{wf} = P_{hy} + P_{fr} \tag{3.29}$$

where

P_{wf} = bottomhole flowing pressure

P_{hy} = hydrostatic pressure

P_{fr} = friction pressure

ไช โจว โจวออง : การประยุกต์ใช้แบบจำลองการไหลสี่สถานะของ Guo ในหลุมผลิตที่มีสอง และสามสถานะ (APPLICTION OF GUO'S FOUR-PHASE FLOW MODEL TO TWO-PHASE AND THREE-PHASE FLOW WELLS) อาจารย์ที่ปรึกษา : ผศ.ดร. สุวัฒน์ อธิชนากร, อาจารย์ที่ปรึกษาร่วม : ดร.วินิตย์ หาญสมุทร, จำนวน 123 หน้า, ISBN 974-14-3896-6

การศึกษานี้มีวัตถุประสงค์เพื่อตรวจสอบการประยุกต์ใช้แบบจำลองการไหลสี่สถานะของGuo กับแบบจำลองสองสถานะและสามสถานะของการไหลภายใต้สภาวะต่างๆเพื่อเพิ่มความแม่นยำของแบบจำลองดังกล่าว ข้อมูลจำนวน 208 ชุดถูกนำมาใช้ในการคำนวนความดันขณะไหลของก้นหลุมจากความดันขณะไหลของปากหลุมที่ทราบค่าโดยใช้แบบจำลองการไหลสี่สถานะของ Guo และความสัมพันธ์การไหลสหภาคอีก 6 แบบ (Duns and Ros (modified), Hagedorn and Brown, Fancher and Brown, Beggs and Brill, and Duns and Ros (original)) จากนั้นจึงปรับแต่งและประเมินผลเพื่อเพิ่มความแม่นยำของแบบจำลอง ตัวประกอบที่ถูกนำมาปรับแต่งนี้ ได้แก่ ตัวประกอบการอัดตัวได้ของก๊าซ (Z) ,อัตราส่วนของก๊าซในน้ำมันต่อน้ำมัน (R_s), ตัวประกอบปริมาตรของน้ำมัน (B_o) และ การคำนวณความดันสูญเสียย่อยในส่วนต่างๆของท่อ จากนั้นแบบจำลอง Guo ที่ถูกปรับแต่งบนพื้นฐานของการปรับแต่งหลายๆแบบรวมกันถูกนำมาเปรียบเทียบกับความสัมพันธ์อื่นอีก 6 ความสัมพันธ์ที่กล่าวมา จากการเปรียบเทียบพบว่าค่าเฉลี่ยสัมบูรณ์ผิดพลาดของแบบจำลองGuoที่ถูกปรับแต่งด้วยตัวประกอบการอัดตัวได้ของก๊าซสำหรับการไหลของก๊าซจะดีกว่าแบบจำลองของGuoที่ถูกปรับแต่งแบบอื่นๆและความสัมพันธ์การไหลสหภาคทั้ง 6ความสัมพันธ์ดังกล่าว ยกเว้นความสัมพันธ์ Duns and Ros (ปรับแต่ง) และความสัมพันธ์ Duns and Ros(ดั้งเดิม) ดังนั้นแบบจำลองที่ถูกปรับแต่งในการศึกษานี้สามารถนำมาใช้ได้ด้วยความแม่นยำสูง

ภาควิชาวิศวกรรมเหมืองแร่และปิโตรเลียม	ลายมือชื่อนิสิต..
สาขาวิชาวิศวกรรมปิโตรเลียม	ลายมือชื่ออาจารย์ที่ปรึกษา....................................
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KEY WORD : /GUO'S MODEL/MODIFICATIONS/MULTIPHASE FLOW

SAI KYAW KYAW AUNG. THESIS TITLE : APPLICATION OF GUO'S FOUR-PHASE FLOW MODEL TO TWO-PHASE AND THREE-PHASE FLOW WELLS. THESIS ADVISOR : ASST. PROF. SUWAT ATHICHANAGORN, Ph.D. THESIS CO-ADVISOR : VINIT HANSAMUIT, Ph.D. 123pp. ISBN 974-14-3896-6

This study is intended to investigate the applicability of Guo's four-phase flow model to two-phase and three-phase flow wells under different conditions and improve its accuracy. Two hundred and eight data sets were used to calculate the bottomhole flowing pressure (BHFP) from the known wellhead flowing pressure (WHFP) by Guo's four-phase flow model and six other multiphase flow correlations (Duns and Ros (modified), Hagedorn and Brown, Fancher and Brown, Beggs and Brill, and Duns and Ros (original)). Then, modifications were tried and evaluated in order to improve the model's accuracy. These modifications are improving the tuning factor, inclusion of gas compressibility factor (Z), solution gas-oil ratio (R_s), and oil formation volume factor (B_o), and incremental calculation of pressure losses. Then, the modified Guo's models based on different combinations of modifications were compared against the other six multiphase flow correlations. In the comparison, it can be seen that the average absolute error of Guo's model modified with the Z factor tuning for gas flow rate is better than the other modified Guo's models and multiphase flow correlations except Duns and Ros (modified) correlation and Duns and Ros (original) correlation. Therefore, the modified model proposed in this study may be used with high accuracy.

Department of Mining and Petroleum Engineering	Student's signature...................
Field of study: Petroleum Engineering	Advisor's signature...................
Academic year: 2006	Co-advisor's signature..............