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APPENDICES

APPENDIX A

Notation and Definition

A.1 Notation

Boldface: Matrix or homogeneous vector, i.e. $\mathbf{x}, \mathbf{X}, \mathbf{K}, \mathbf{P}, \mathbf{R}$. Boldface with ~ up on top refers to an arbitrary vector such as $\tilde{\mathbf{C}}$.

$\hat{\mathbf{x}}$: Estimated value of $\mathbf{x}$

^{T} : Matrix transposition.

$\mathbb{P}^n$: Projective space of dimension n

$\mathbb{R}^n$: Real space of dimension n

π: Plane.

$\|\cdot\|$: 2nd norms.

$\|\cdot\|_\infty$: Infinity norms.

A.2 Definition

Homogeneous Vector

Homogeneous vector is vector which defined up to constant scale factor. Given k is arbitrary constant, $\mathbf{x}$ and $\mathbf{y}$ are homogeneous vectors with the same in dimensions. Thus, $\mathbf{x}$ and $\mathbf{y}$ are equal if and only if $\mathbf{y} = k\mathbf{x}$.

Homography

Homography is $n \times n$ matrix that used to transform between two coordinates. These coordinates are defined by homogeneous n-vector. Homography are also defined up to scale factor.

World Coordinate System (WCS)

WCS is arbitrary 3-spcae coordinate system. It used to reference the coordinates of interested object. The coordinate in WCS defined by homogeneous 4-vector $\mathbf{X_w} = \begin{bmatrix} x_w & y_w & z_w & 1 \end{bmatrix}^T$.

Camera Coordinate System (CCS)

CCS is 3-space coordinate system attached to each camera. The origin of CCS is at optical center and Z axis is coincident with principle axis of the camera. The coordinate in CCS defined by homogeneous 4-vector $\mathbf{X_c} = \begin{bmatrix} x_c & y_c & z_c & 1 \end{bmatrix}^T$.

Image Coordinate System (ICS)

ICS is 2-space coordinate system of image captured by camera. The coordinate in ICS defined by homogeneous 2-vector $\mathbf{x} = \begin{bmatrix} u & v & 1 \end{bmatrix}^T$.

APPENDIX B

Levenberg-Marquardt Algorithm

B.1 Introduction

The Levenberg-Marquardt (LM) algorithm is an iterative technique that locates the minimum of a multivariate function that is expressed as the sum of squares of non-linear real-valued functions. It has become a standard technique for non-linear least-squares problems, widely adopted in a broad spectrum of disciplines. LM can be thought of as a combination of steepest descent and the Gauss-Newton method. When the current solution is far from the correct one, the algorithm behaves like a steepest descent method: slow, but guaranteed to converge. When the current solution is close to the correct solution, it becomes a Gauss-Newton method.

B.2 The Levenberg-Marquardt Algorithm

In the following, vectors and arrays appear in boldface and ^{T} is used to denote transposition. Also, $\|\cdot\|$ and $\|\cdot\|_\infty$ denote the 2nd and infinity norms respectively. Let f be an assumed functional relation which maps a parameter vector $\mathbf{p} \in \mathbb{R}^m$ to an estimated measurement vector $\hat{\mathbf{x}} = f(\mathbf{p})$, $\hat{\mathbf{x}} \in \mathbb{R}^n$. An initial parameter estimate $\mathbf{p}_0$ and a measured vector $\mathbf{x}$ are provided and it is desired to find the vector $\mathbf{p}^+$ that best satisfies the functional relation f, i.e. minimizes the squared distance $\varepsilon^T\varepsilon$ with $\varepsilon = \mathbf{x} - \hat{\mathbf{x}}$. The basis of the LM algorithm is a linear approximation to f in the neighborhood of $\mathbf{p}$. For a small $\|\delta_\mathrm{p}\|$, a Taylor series expansion leads to the approximation

$$f(\mathbf{p}+\delta_\mathrm{p}) \approx f(\mathbf{p}) + \mathbf{J}\delta_\mathrm{p} \tag{B.1}$$

where $\mathbf{J}$ is the Jacobian matrix $\frac{\partial f(\mathbf{p})}{\partial \mathbf{p}}$. Like all non-linear optimization methods, LM is iterative: Initiated at the starting point $\mathbf{p}_0$, the method produces a series of vectors $\mathbf{p}_1, \mathbf{p}_2, \ldots,$ that converge towards a local minimizer $\mathbf{p}^+$ for f. Hence, at each step, it is required to find the δ_p that minimizes the quantity:

$$\|\mathbf{x} - f(\mathbf{p}+\delta_\mathrm{p})\| \approx \|\mathbf{x} - f(\mathbf{p}) - \mathbf{J}\delta_\mathrm{p}\| = \|\varepsilon - \mathbf{J}\delta_\mathrm{p}\| \tag{B.2}$$

The sought δ_{p} is thus the solution to a linear least-squares problem: the minimum is attained when $\mathbf{J}\delta_{\mathrm{p}}-\varepsilon$ is orthogonal to the column space of $\mathbf{J}$. This leads to $\mathbf{J}^T\left(\mathbf{J}\delta_{\mathrm{p}}-\varepsilon\right)=0$, which yields δ_{p} as the solution of the so-called normal equations:

$$\mathbf{J}^T\mathbf{J}\delta_{\mathrm{p}}=\mathbf{J}^T\varepsilon \tag{B.3}$$

The matrix $\mathbf{J}^T\mathbf{J}$ in the left hand side of equation (B.3) is the approximate Hessian, i.e. an approximation to the matrix of the second order derivatives. The LM method actually solves a slight variation of equation (B.3), known as the augmented normal equations

$$\mathbf{N}\delta_{\mathrm{p}}=\mathbf{J}^T\varepsilon \tag{B.4}$$

where the off-diagonal elements of $\mathbf{N}$ are identical to the corresponding elements of $\mathbf{J}^T\mathbf{J}$ and the diagonal elements are given by $\mathbf{N}_{ii}=\mu+\left[\mathbf{J}^T\mathbf{J}\right]_{ii}$ for some $\mu>0$. The strategy of altering the diagonal elements of $\mathbf{J}^T\mathbf{J}$ is called damping and μ is refered to as the damping term. If the updated parameter vector $\mathbf{p}+\delta_{\mathrm{p}}$ with δ_{p} computed from equation (B.4) leads to a reduction in the error ε, the update is accepted and the process repeats with a decreased damping term. Otherwise, the damping term is increased, the augmented normal equations are solved again and the process iterates until a value of δ_{p} that decreases error is found. The process of repeatedly solving equation (B.4) for different values of the damping term until an acceptable update to the parameter vector is found corresponds to one iteration of the LM algorithm.

In LM, the damping term is adjusted at each iteration to assure a reduction in the error ε. If the damping is set to a large value, matrix $\mathbf{N}$ in equation (B.4) is nearly diagonal and the LM update step δ_{p} is near the steepest descent direction. Moreover, the magnitude of δ_{p} is reduced in this case. Damping also handles situations where the Jacobian is rank deficient and $\mathbf{J}^T\mathbf{J}$ is therefore singular. In this way, LM can defensively navigate a region of the parameter space in which the model is highly nonlinear. If the damping is small, the LM step approximates the exact quadratic step appropriate for a fully linear problem. LM is adaptive because it controls its own damping: it raises the damping if a step fails to reduce ε; otherwise it reduces the damping. In this way LM is capable to alternate between a slow descent approach when being far from the minimum and a fast convergence when at least on of the following conditions is met:

i) The magnitude of the gradient of $\varepsilon^T \varepsilon$, i.e. $\mathbf{J}^T \varepsilon$ in the right hand side of equation (B.3), drops below a threshold ε_1.

ii) The relative change in the magnitude of δ_{p} drops below a threshold ε_2.

iii) The error $\varepsilon^T \varepsilon$ drops below a threshold ε_3.

iv) A maximum number of iterations $k_{\max}$ is completed.

If a covariance matrix $\sum_{\mathbf{x}}$ for the measured vector $\mathbf{x}$ is available, it can be incorporated into the LM algorithm by minimizing the squared $\sum_{\mathbf{x}}^{-1}$ -norm $\varepsilon^T \sum_{\mathbf{x}}^{-1} \varepsilon$ instead of the Euclidean $\varepsilon^T \varepsilon$. Accordingly, the minimum is found by solving a weighted least squares problem defined by the weighted normal equations

$$\mathbf{J}^T \sum_{\mathbf{x}}^{-1} \mathbf{J} \delta_{\mathrm{p}} = \mathbf{J}^T \sum_{\mathbf{x}}^{-1} \varepsilon \tag{B.5}$$

B.3 Pseudocode

Input: A vector function $f:\mathbb{R}^m \to \mathbb{R}^n$ with $n \geq m$, a measurement vector $\mathbf{x} \in \mathbb{R}^n$ and an initial parameters estimate $\mathbf{p}_0 \in \mathbb{R}^m$.

Output: A vector $\mathbf{p}^+ \in \mathbb{R}^m$ minimizing $\left\|\mathbf{x} - f(\mathbf{p})\right\|^2$.

Algorithm:

$k := 0; \nu := 2; \mathbf{p} := \mathbf{p}_0;$

$\mathbf{A} := \mathbf{J}^T\mathbf{J}; \varepsilon_{\mathbf{p}} := \mathbf{x} - f(\mathbf{p}); \mathbf{g} := \mathbf{J}^T\varepsilon_{\mathbf{p}};$

$\text{stop} := \left(\left\|\mathbf{g}\right\|_\infty \leq \varepsilon_1\right); \mu := \tau * \max_{i=1,\ldots,m}\left(A_{ii}\right);$

while $(\text{non stop})\,\text{and}\,(k < k_{\max})$

  $k := k+1;$

  repeat

    Solve $(\mathbf{A} + \mu\mathbf{I})\delta_{\mathbf{p}} = \mathbf{g};$

    if $\left(\left\|\delta_{\mathbf{p}}\right\| \leq \varepsilon_2 \left\|\mathbf{p}\right\|\right)$

      stop:=true;

    else

      $\mathbf{p}_{new} := \mathbf{p} + \delta_{\mathbf{p}};$

      $\rho := \left(\left\|\varepsilon_{\mathbf{p}}\right\|^2 - \left\|\mathbf{x} - f(\mathbf{p}_{new})\right\|^2\right) \Big/ \left(\delta_{\mathbf{p}}^T\left(\mu\delta_{\mathbf{p}} + \mathbf{g}\right)\right);$

      if $\rho > 0$

        $\mathbf{p} := \mathbf{p}_{new};$

        $\mathbf{A} := \mathbf{J}^T\mathbf{J}; \varepsilon_{\mathbf{p}} := \mathbf{x} - f(\mathbf{p}); \mathbf{g} := \mathbf{J}^T\varepsilon_{\mathbf{p}};$

        $\text{stop} := \left(\left\|\mathbf{g}\right\|_\infty \leq \varepsilon_1\right) \text{ or } \left(\left\|\varepsilon_{\mathbf{p}}\right\|^2 \leq \varepsilon_3\right);$

        $\mu := \mu * \max\left(\frac{1}{3}, 1 - (2\rho - 1)^3\right); \nu := 2;$

      else

        $\mu := \mu * \nu; \nu := 2 * \nu;$

      endif

    endif

  until $(\rho > 0)$ or (stop)

endwhile

$\mathbf{p}^+ := \mathbf{p};$

BIOGRAPHY

Kritsana Uttamang was born on July 22, 1977 in Nakornratchasima, Thailand and went to Chulalongkorn University, where he studied and obtained his Bachelor's Degree in Mechanical Engineering in 1998. He continued to attend in the Master of Engineering program with "Development of A 3-D Solid Modeling System Based on the Parasolid Kernel" as his research topic and graduated in 2002. After that he continued to attend in the Doctor of Philosophy program in Mechanical Engineering.