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FUNDAMENTAL LIMITS OF THE TWO-USER GAUSSIAN X CHANNEL WITH LIMITED RECEIVER COOPERATION

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การสื่อสารไร้สายสมัยใหม่ผ่านช่องสัญญาณเอ็กซ์ โดยที่ตัวส่งแต่ละตัวมีข่าวสารที่เป็นอิสระสำหรับตัวรับที่สอดคล้องกัน กลายเป็นหัวข้องานวิจัยที่ได้รับสนใจ เพราะการสื่อสารรูปแบบนี้สามารถรับรองการให้บริการที่หลากหลายซึ่งถูกส่งจากแหล่งกำเนิดไปยังผู้ใช้หรือการได้รับข้อมูลต่างชนิดกันซึ่งถูกส่งจากผู้ใช้ไปยังแหล่งกำเนิด อย่างไรก็ตาม การแทรกสอดเป็นปรากฏการณ์สำคัญที่เกิดขึ้นอย่างหลีกเลี่ยงไม่ได้ในการสื่อสารไร้สายเหล่านี้และมีผลกระทบโดยตรงต่อสมรรถนะของระบบ ความร่วมมือกันเป็นวิธีการหนึ่งที่สามารถบรรเทาผลกระทบของการแทรกสอดลงได้และถูกพิจารณาในช่องสัญญาณการสื่อสารรูปแบบอื่นๆ น่าเสียดายที่ความเข้าใจพื้นฐานของช่องสัญญาณเอ็กซ์ที่มีความร่วมมือกันมีอยู่ค่อนข้างน้อย

ในวิทยานิพนธ์นี้ เราทำการศึกษาช่องสัญญาณช่องสัญญาณเกาส์เอ็กซ์สำหรับผู้ใช้ 2 คนที่มีความร่วมมือของเครื่องรับแบบจำกัดโดยที่เครื่องรับทั้งสองแลกเปลี่ยนข่าวสารระหว่างกันบนการเชื่อมโยงเชิงตั้งฉากกันของเครื่องรับทั้งสอง ผ่านมุมมองของทฤษฎีข่าวสาร ซึ่งแบ่งออกได้เป็น 3 ส่วนดังนี้ ในส่วนแรก เราค้นหาข้อจำกัดพื้นฐานของช่องสัญญาณช่องสัญญาณเกาส์เอ็กซ์สำหรับผู้ใช้ 2 คนที่มีความร่วมมือของเครื่องรับแบบจำกัดโดยการใช้อสมการของฟาโน อสมการการประมวลผลข้อมูล และเทคนิคการได้รับความช่วยเหลือจากจินนี่ ผลลัพธ์ที่ได้ในส่วนแรกนี้ ถูกเรียกว่า ขอบเขตชั้นนอก (Outer bound) ในส่วนที่สอง เราแสดงคุณลักษณะองศาเสรีที่วางนัยทั่วไป (GDoF) ภายใต้ข้อกำหนดความสมมาตรของพารามิเตอร์ช่องสัญญาณโดยการใช้ขอบเขตชั้นนอกที่ถูกนำเสนอ ผลลัพธ์ในส่วนนี้แสดงนัยว่า สมรรถนะของระบบถูกปรับปรุงให้ดีจากการแลกเปลี่ยนข้อมูลระหว่างเครื่องรับทั้งสองที่เพิ่มจำนวนขึ้น อย่างไรก็ตามระบบมาถึงจุดอิ่มตัวของความร่วมมือกันระหว่างเครื่องรับ เมื่อความจุที่ถูกนอร์มัลไลซ์ของการเชื่อมโยงเครื่องรับที่ทำงานร่วมกัน $\kappa \geq$ 1/2, 1, 3/2 และ 2 เมื่อระดับการแทรกสอดที่ถูกนอร์มัลไลซ์ $\alpha \in$ [0, 3/2], (3/2, 2], (2,5/2], และ (5/2, 3] ตามลำดับ ในส่วนที่สาม เรานำเสนอกลยุทธ์ซึ่งประกอบด้วยแบบแผนการส่งสัญญาณบนพื้นฐานของกลยุทธ์แบบฮาน-โคบายาชิ และโปรโตคอลแบบร่วมมือบนพื้นฐานของแบบแผนควอนไตซ์-แมปและฟอร์เวิร์ดและหาค่าบริเวณอัตราข้อมูลที่สามารถได้มา (Achievable rate region) สำหรับทั้งกรณีทั่วไปและกรณีช่องสัญญาณเกาส์เอ็กซ์แบบแข็งแรงชนิดที่ 1 โดยที่ $SNR_1 > INR_2$ และ $SNR_2 > INR_1$ ผลลัพธ์ที่ได้แสดงให้เห็นว่ากลยุทธ์ที่นำเสนอสำหรับกรณีช่องสัญญาณเกาส์เอ็กซ์แบบแข็งแรงชนิดที่ 1 มีค่าน้อยกว่าขอบเขตชั้นนอกที่เราได้นำเสนอไม่เกิน 2 bits/s/Hz/ per message ซึ่งเป็นอิสระจากค่าพารามิเตอร์ช่องสัญญาณ เมื่อค่า β และ γ ในเงื่อนไขอัตราร่วมที่ตัวส่งถูกกำหนดค่าจนกระทั่งค่าขอบเขตต่าง ๆ ในบริเวณอัตราข้อมูลที่สามารถได้มามีค่าสูงสุด

ภาควิชา วิศวกรรมไฟฟ้า ลายมือชื่อนิสิต ....

สาขาวิชา วิศวกรรมไฟฟ้า ลายมือชื่อ อ. ที่ปรึกษาหลัก ....

ปีการศึกษา 2558

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KEY WORDS: CAPACITY REGION / GAUSSIAN X CHANNEL / HAN-KOBAYASHI STRATEGY / GENERALIZED DEGREE OF FREEDOM / RECEIVER COOPERATION

SURAPOL TAN-A-RAM : FUNDAMENTAL LIMITS OF THE TWO-USER GAUSSIAN X CHANNEL WITH LIMITED RECEIVER COOPERATION, ADVISOR : ASSOC. PROF. WATIT BENJAPOLAKUL, D.Eng., 166 pp.

Modern wireless communications via the X channel where each transmitter has an independent message for the corresponding receiver have become an active topic of research since they can support various services transmitted from sources to users or obtain different types of data transmitted from users to sources. However, interference is the important phenomenon that occurs unavoidably in these wireless communications and affects directly on the performance of the system. Cooperation is one of several methods that can mitigate the effect of interference and is considered in several communication scenarios. Unfortunately, the basic comprehension of the X channel with cooperation is somewhat less.

In this dissertation, we study the two-user Gaussian X channel with limited receiver cooperation where both receivers exchange messages over the orthogonal receiver-cooperative links through the perspective of information theory that can be divided into 3 parts as follows: In the first part, we investigate the fundamental limits of the two-user Gaussian X channel with limited receiver cooperation using the Fano's inequality, the data processing inequality, and the genie-aided techniques. The obtained result is called an outer bound. In the second part, we characterize the generalized degrees-of-freedom under the channel symmetric setting by using our proposed outer bound. The results in this part imply that the performance of our system can be improved when the amount of exchanged information between both receivers increases. However, the system reaches the saturation point of the receiver cooperation when the normalized capacity of the receiver-cooperative link $\kappa \geq \frac{1}{2}, 1, \frac{3}{2}$ and 2 for the normalized interference level $\alpha \in [0, \frac{3}{2}], (\frac{3}{2}, 2], (2, \frac{5}{2}]$, and $(\frac{5}{2}, 3]$, respectively. In the final part, we propose the strategies consisting of transmission scheme based on Han-Kobayashi strategy and cooperative protocol based on quantize-map-and-forward scheme and then give achievable rate regions based on the proposed strategies for both the general case and the strong Gaussian X channel type I case where $\mathsf{SNR}_1 > \mathsf{INR}_2$ and $\mathsf{SNR}_2 > \mathsf{INR}_1$. The obtained results show that our proposed strategy in the strong Gaussian X channel type I case achieves the capacity region to within 2 bits/s/Hz per message when β and γ in the common rate constraints at the transmitter are set such that most of the bounds in the achievable rate region reach theirs maximum value. This constant is independent of the channel parameters.

Department: Electrical Engineering Student's Signature ....
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CHAPTER I

INTRODUCTION

1.1 Background

In the past two decades, wireless communications have had significant progresses and made a great impact on human lifestyle, wireless services and wireless industry. Several commercial standards in mobile telecommunication systems, such as the universal mobile telecommunications system (UMTS), wideband code division multiple access (WCDMA) and long-term evolution (LTE), etc., are good examples for these progresses. These standards have been developed from the modern wireless communication techniques, including code division multiple access (CDMA), orthogonal frequency division multiplexing (OFDM), and multiple-input multiple-output (MIMO), etc., that are based on theoretical ideas induced by information theory. In addition to the commercial standards, hardware including smartphones, laptops, tablets, and other mobile nodes are also produced by the advanced technologies. With the rapidly developed wireless technologies, therefore, we obtain the better communication services from mobile operators, create the new solutions for business and then make money from their solutions and have a good health from wireless healthcare systems.

In modern wireless communication systems, communications between two or more transmitter-receiver pairs over a common physical medium cause interference at each receiver see Figure 1.1. Because of the broadcast and the superposition nature of the wireless medium, the intended signal at each receiver is superposed by the unintended signals plus noise. This phenomenon is unavoidable and limits the performance of the wireless system. Currently, most wireless systems cope with interference by either orthogonalizing the communication links in time or frequency so that they do not interfere with each other at all or treating interference as noise. Unfortunately, two approaches are suboptimal because there is an a priori loss of degrees of freedom (DoF) in both links for the orthogonalizing approach and the performance of systems is degraded when the number of interferes grows

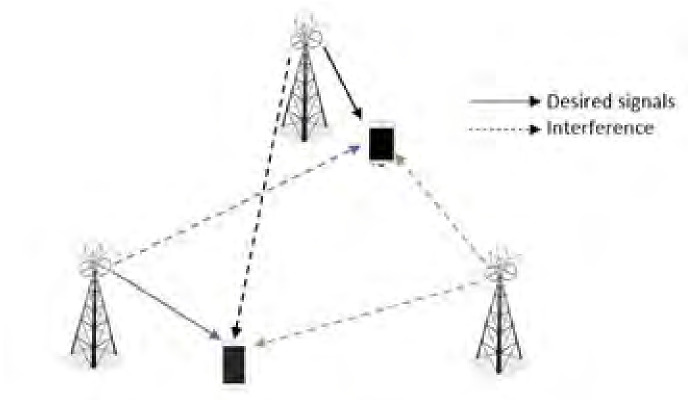


Figure 1.1 Interference in wireless communication network

in the treating interference as noise approach.

1.1.1 Interference Management: Interference Channel and X Channel

To develop the interference management schemes which their performances are better than two well-known approaches above, several researchers have studied the effect of interference and investigated the fundamental limits of communications in wireless channels.

Interference Channel: The simplest information theory model for studying this point is the two-user interference channel. However, the problem of characterizing the capacity region of this channel has been open for over 30 years, except for several special cases such as the strong and very strong interference channel [5–7] and classes of deterministic and semi-deterministic interference channel [3, 8]. Recently, significant progress has been performed by Etkin, Tse and Wang [9] to propose a new approach for approximating the capacity region of the two-user Gaussian interference without cooperation and then characterize this approximate capacity region to within 1 bit/s/Hz for all values of the channel parameters by using Han-Kobayashi strategy in transmission scheme with a simple power split construction. **The important benefit of finding the bounded gap-to-optimality** in [9] is to produce an uniform approximation of the capacity region and to ensure certainly the performance of the proposed scheme [2].

X Channel: Next, the above interference problem is extended to the situation where each transmitter has an independent message for all receivers in the system. This communication scenario is called *the X channel* which is introduced initially by Vishwanath, Jindal and Goldsmith [10]. The X channel is a generalization of the multiuser channels studied in information theory, such as the multiple access channel (MAC), the broadcast channel (BC), the interference channel (IC), the Z-channel (ZC) and the Z-interference channel (ZIC), etc. It is easily seen that, in the X channel, each receiver obtains several interference and desired messages at the same time. Hence, the effect of interference on the receiver in the X channel is more crucial than one in the interference channel due to several desired and undesired messages.

The practical examples for the X channel such as, in a cellular system, each base station provides two different services to two users (see Figure 1.2) and, in modern wireless sensor network for agriculture, each source node can send several measured parameters including air temperature, relative humidity, soil moisture, and picture of product to the servers (see Figure 1.3).

Figure 1.2 The two-user Gaussian X channel in cellular network

In the simplest case, it is seen that the two-user Gaussian X channel is physically the same as the two-user Gaussian interference channel. However, the difference between these two channels is the message set which leads to encode and decode messages differently at transmitter and receiver, respectively.

1.1.2 Related works for X channel

The research works about X channel have been studied widely in [11–17] to characterize the achievable rate region, sum capacity, upper bounds, generalized degrees of freedom (GDoF), approximate capacity region, and degrees of freedom (DoF) region, etc., as the following details:

Figure 1.3 The X channel in agriculture

- *Characterizing the Achievable Rate Region:* Koyluoglu, Shahmohammadi and Gamal [11] give the best known achievable region for the X channel based on the combination of the Marton's binning technique for the broadcast channels [18, 19] and the message splitting for interference channel [20] with joint decoding at receivers. However, characterization of this rate region is *extremely complicated* [12]. In the work [13], Sridhar and Bhashyam considered the two-user Gaussian X channel using superposition coding [20]. They derived achievable rates of the 6 messages where each transmitter consists of two private messages and one common message and determined that these messages were useful in maximizing the sum rate for the various interference conditions.

- *Characterizing the Sum Capacity, Upper bounds and Generalized Degrees of Freedom (GDoF):* Huang, Cadambe and Jafar [14] characterized the sum capacity[1] of the deterministic and Gaussian X channel under a symmetric channel setting. They also proposed the upper bounds[2] for the deterministic and Gaussian X channel. Furthermore, they explored the GDoF of the symmetric Gaussian X channel from their sum capacity of the Gaussian X channel. Recently, Niesen and Maddah-Ali [15] gave the

[1]The sum capacity in [14] means that the sum of 4 rates, i.e., $R_{11} + R_{12} + R_{21} + R_{22}$.

[2]Huang, Cadambe and Jafar [14] gave the upper bounds for the sum of 3 rates, i.e., $R_{11}+R_{12}+R_{21}$, $R_{11}+R_{12} + R_{22}$, $R_{11} + R_{21} + R_{22}$, $R_{12} + R_{21} + R_{22}$ and the sum of 4 rates, i.e., $R_{11} + R_{12} + R_{21} + R_{22}$.

new upper bounds[3] for the deterministic X channel and Gaussian X channel.

- *Characterizing the Approximate Capacity Region:* Prasad and Chockalingam [12] provide firstly the approximate capacity region of the X channel which is within the intersection of 4 outer bounds to the capacity region of the X channel where each outer bound is derived by removing only one message to obtain the corresponding Z-channel. Furthermore, they characterized the outer bounds on the capacity region of the Gaussian X channel for two main classes, i.e., strong and mixed X channel, from their approximate capacity region. In the general case, however, their outer bound does not have *the closed form* to use easily. **Hence, characterizing the capacity region of the general two-user X channel has still been open**.

- *Characterizing the Capacity to within a Constant Gap:* Niesen and Maddah-Ali [15] used the interference alignment (IA) technique to characterize capacity of the two-user Gaussian X channel. They proposed a new communication scheme and showed that it achieved capacity of the Gaussian X channel to within a constant gap. This is the first constant-gap result for a general fully-connected X network requiring interference alignment.

- *Characterizing the DoF:* Jafar and Shamai [16] show that there are $\frac{4}{3}$ DoF when the channel coefficients are time-varying or frequency-selective and drawn from a continuous distribution. Cadambe and Jafar [21] extend the work [16] into the case of M transmitters and N receivers of wireless X networks and then showed that the total DoF of the $M \times N$ X networks is equal to $\frac{MN}{M+N+1}$ per orthogonal time and frequency dimension when all nodes have a single antenna and all channel coefficients vary in time or frequency. Very recently, Motahari *et.al.* [17] developed the idea of real interference alignment (IA) that is extremely powerful in achieving the sum DoF of single antenna systems. They showed that the total DoF of the $M \times N$ X network with real and time invariant channel coefficients is equal to $\frac{MN}{M+N-1}$ for almost all channel realizations.

[3]Niesen and Maddah-Ali [15] proposed the upper bounds for the sum of 5 rates, i.e., $2R_{11} + R_{12} + R_{21} + R_{22}, R_{11} + 2R_{12} + R_{21} + R_{22}, R_{11} + R_{12} + 2R_{21} + R_{22}, R_{11} + R_{12} + R_{21} + 2R_{22}$.

1.1.3 Cooperation

In the above X channel set-up [11–17], there are no communications between transmitters (receivers). Hence, each receiver has to handle interference on its own messages. Nowadays, cooperation between transmitters (receivers) which is allowed by exchanging a certain amount of information at the limited rate due to physical constraints is becoming the essential part of modern communication systems. It is known that cooperation can alleviate interference by forming *distributed multiple antenna arrays* or called *distributed multiple-input multiple-output (MIMO) systems* in [2, 22, 23] and help to achieve large performance gains in wireless networks [24]. For example, the base stations in a cellular network can be connected via wireline backhaul links [25] or the mobile nodes are close enough to each other to be able to establish reliable cooperation links.

1.1.4 Related Works for Cooperation

Conferencing among encoders/decoders, a special case of out-of-band cooperation as classified in [26, 27] has been investigated in [2, 28–33]. Willems [28] introduced initially the conferencing among encoders and then characterized the capacity region of multiple access channels (MAC). In the work [29], the capacity region of the two-user compound MAC with a common message and unidirectional conferencing between decoders was characterized. Next, the two-user one-sided Gaussian interference channels with unidirectional and bidirectional rate-limited conferencing between decoders were considered in [30, 31], respectively. Zhou and Yu characterized the capacity region in strong interference regimes and the asymptotic sum capacity at high SNR in [30] and an achievable rate region was shown to be optimal under certain conditions in [31]. In the work [2], Wang and Tse considered the two-user Gaussian interference channel with rate-limited receiver cooperation and characterized its entire capacity region to within a constant gap. Very recently, Ashraphijuo, Aggarwal and Wang [32] characterized the approximate capacity region of the two-user MIMO interference channel with limited receiver cooperation within the total number of receive antennas of both receivers. In addition, they gave the proposed GDoF region when all nodes have equal number of antennas. Do, Oechtering and Skoglund [33] gave a new inner bound for the capacity region of the discrete memoryless two-user interference channel with receiver cooperation and an inner bound for the Gaussian interference channel with orthogonal conference links at the receivers. The latter was equivalent to the one-round quantize-bin-and-forward inner bound specially designed for the channel model in [2].

1.2 Two-user Gaussian X channel with limited receiver cooperation

In this section, we introduce two main interesting issues, i.e., fundamental limits and strategy of communications, for the two-user Gaussian X channel with limited receiver cooperation and provide the importance for studying these two topics in this dissertation.

1.2.1 Fundamental limits

With the advantages of cooperation in Section 1.1.3, interference management in wireless X networks using cooperation is the interesting topic. As mentioned above, the previous research works of the X channel [11–17] focus on the case of non-cooperation. Therefore, knowledge of the X channel with cooperation, especially the fundamental limits of the X channel with cooperation which are the important issue in the perspective of information theory, has not been well known even in the two-user case. The better understanding of the fundamental limits leads us to know communication limits and to propose new techniques based on cooperation in practice efficiently for managing interference in the X channel. In this dissertation, we consider the two-user Gaussian X channel and focus on the case of limited receiver cooperation. *We give an attempt to understand the fundamental limits of this channel in terms of an outer bound (or called an approximate capacity region) and then characterize the generalized degrees of freedom (GDoF) of sum capacity obtained from the proposed outer bound to further comprehend the effect of limited receiver cooperation on the two-user Gaussian X channel.*

1.2.2 Strategy for Communications

In addition to find the fundamental limits, the strategy for communications in the two-user Gaussian X channel with limited receiver cooperation is also the interesting issue. Although research involving this issue has been quite rare, there is the work that can be a guideline for this dissertation, i.e., Wang and Tse's work [2]. Based on the work [2], this dissertation proposes strategies also composing of two parts: 1) transmission scheme based on the Han-Kobayashi (HK) strategy [20] which is used widely in several communication scenarios [2, 9, 11, 13, 20, 29–31, 33–36], etc., and 2) cooperative protocol based on quantize-map-and-forward (QMF) scheme [2], for the general case and the strong Gaussian X channel type I case where $\mathsf{SNR}_1 > \mathsf{INR}_2$ and $\mathsf{SNR}_2 > \mathsf{INR}_1$. Furthermore, there are following *two important constraints* which are considered in this dissertation: 1) two different messages are sent simultaneously from each transmitter to both receivers and 2) both receivers are allowed to exchange a certain amount of information between them. Finally, we evaluate the performance of our proposed strategy in the strong Gaussian X channel type I case by comparing its achievable rate region with our proposed outer bound.

1.3 Objectives of the Dissertation

The objectives of this dissertation are

1. To propose an outer bound on the capacity region for the two-user Gaussian X channel with limited receiver cooperation.
2. To characterize the generalized degrees of freedom (GDoF) from our proposed outer bound region under a symmetric channel setting.
3. To propose the strategy for delivering messages in the two-user Gaussian X channel with limited receiver cooperation.
4. To evaluate the performance of the proposed strategy in the case of strong Gaussian X channel type I by comparing its achievable rate region with the proposed outer bound.

1.4 Scope of the Dissertation

1. We consider the two-user Gaussian X channel with out-of-band (orthogonal) limited receiver cooperation, that is, signals in transmitter-receiver links do not interfere with ones in receiver-cooperative links.
2. We focus on the general case and the strong Gaussian X channel type I case, where the direct channels are stronger than the corresponding cross channels, i.e., $\mathsf{SNR}_1 > \mathsf{INR}_2$ and $\mathsf{SNR}_2 > \mathsf{INR}_1$.

3. In our system, each transmitter has a single antenna and each receiver has also a single antenna.

4. In exchanging information between both receivers, we assign

   (a) Both receiver-cooperative links are noiseless with finite capacity from the receiver *i* to *j*, for $i, j = 1, 2$ and $i \neq j$.

   (b) Information at each receiver is encoded causally in the sense that cooperation signal from the receiver 1 to 2, $u_{12}[n]$, is only a function of $\{y_1[1], \ldots, y_1[n-1], u_{21}[1], \ldots, u_{21}[n-1]\}$, for $n = 1, \ldots, N$. Similarly, $u_{21}[n]$, is only a function of $\{y_2[1], \ldots, y_2[n-1], u_{12}[1], \ldots, u_{12}[n-1]\}$.

1.5 Organization and Contributions of the Dissertation

The rest of this dissertation is organized as follows:

In Chapter II, we give the basic knowledge of information theory and techniques which are useful profitably for deriving an outer bound, the generalized degrees-of-freedom (GDoF) under symmetric channel setting, and achievable rate regions in Chapter IV−VI, respectively. In addition, we provide the example based on the two-user Gaussian multiple-access channel (Gaussian MAC) for better understanding the capacity region and the example from [14] to present how to use the genie-aided techniques for finding the sum-rate in the two-user Gaussian X channel without cooperation that is the important basis to comprehend our derived upper bounds in Chapter IV.

In Chapter III, we introduce the channel model for the two-user Gaussian X channel with out-of-band (orthogonal) limited receiver cooperation. Next, the definitions of strategies, achievable rates, and capacity region are provided. Third, the classification of the two-user Gaussian X channel from [12] is mentioned and the modified version for the strong Gaussian X channel type I is introduced. Finally, the notations are given for using throughout in the rest of this dissertation.

In Chapter IV, we provide the knowledge to better comprehend the fundamental limits of the two-user Gaussian X channel with limited receiver cooperation.

- The contribution of this chapter is to propose an outer bound on the capacity region for the two-user Gaussian X channel with limited receiver cooperation and also give the details for deriving all upper bounds which are contained in our proposed outer bound.

In Chapter V, the effect of receiver cooperation on the two-user Gaussian X channel can be further understood by using the GDoF.

- The contribution of this chapter is to characterize the GDoF with our proposed outer bound from Chapter IV under a symmetric channel setting.

In Chapter VI, we propose the strategies for communications in the two-user Gaussian X channel with limited receiver cooperation for the general case and the strong Gaussian X channel type I case.

- The contributions of this chapter are
  1. To provide the achievable rate regions for the two-user Gaussian X channel with limited receiver cooperation in both the general case and the strong Gaussian X channel type I case.
  2. To characterize the capacity region of the two-user Gaussian X channel with limited receiver cooperation in the strong Gaussian X channel type I to within 2 bits/b/s per message to the proposed outer bound in Chapter IV.

Finally, the conclusion of this dissertation is given in Chapter VII.

CHAPTER II

BASIC KNOWLEDGE OF INFORMATION THEORY AND TECHNIQUES

In this chapter, we provide the basic knowledge of information theory and techniques which are used profitably for finding an outer bound, the generalized degrees-of-freedom (GDoF) under symmetric channel setting, and achievable rate regions in Chapter IV−VI, respectively. Next, we provide the definition of the capacity region and give the example based on the two-user Gaussian multiple-access channel (Gaussian MAC) for better understanding our result in Chapter IV. In addition, we introduce the two-user Gaussian X channel and give the example from [14] to present how to use the genie-aided techniques for finding the sum-rate. Finally, we show one of the results from [2] that specifies two regions considering the gain from limited receiver cooperation.

2.1 Entropy

This section gives the definition of entropy that is a measure of the uncertainty of a random variable [1].

Let X be a discrete random variable with alphabet $\mathcal{X}$ and probability mass function (pmf) $p(x) = Pr\{X = x\}, x \in \mathcal{X}$. *The entropy* $H(X)$ *of a discrete random variable* X *is defined as*

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log p(x) = -E \log p(x). \tag{2.1}$$

where E denotes expectation.

The important properties of the entropy $H(X)$ are nonnegative and concave function in $\log p(x)$.

2.2 Differential Entropy, Joint Differential Entropy, Conditional Differential Entropy and Mutual information

In this section, we provide the basic definitions of the differential entropy, joint differential entropy, conditional differential entropy and mutual information for continuous random variables [1, 37] that are useful to derive an outer bound in Chapter IV and an achievable rate region in Chapter VI.

First, we present the differential entropy. Let X be a random variable and its cumulative distribution function is $F(x) = \Pr(X \leq x)$. X is a continuous random variable if $F(x)$ is continuous. Let *f*(*x*) be the derivative of $F(x)$, i.e., $f(x) = F'(x)$ and is called the probability density function with $\int_{-\infty}^{\infty} f(x) = 1$.

- *The differential entropy $h(X)$ of a continuous random variable X with probability density function (pdf) $f(x)$ is defined as*

$$h(X) = -\int f(x) \log f(x) dx \tag{2.2}$$

For example,

1. Uniform distribution: If $X \sim \text{Unif}[a, b]$, then

$$h(X) = \log(b - a)$$

2. Normal distribution: If $X \sim N(\mu, \sigma^2)$, then

$$h(X) = \frac{1}{2} \log(2\pi e \sigma^2)$$

The maximum differential entropy of a continuous random variable $X \sim f(x)$ with the average power constraints $E(X^2) \leq P$ is

$$\max_{f(x): E(X^2) \leq P} h(X) = \frac{1}{2} \log(2\pi e P)$$

and if $X \sim N(0, P)$, then we obtain

$$h(X) = h(X - E\{X\}) = \frac{1}{2} \log(2\pi e \text{Var}(X)).$$

Next, the definition of differential entropy of one random variable is extended to several random variables.

- *The differential entropy of jointly distributed random variables* $X_1, X_2, ..., X_n$ *is defined as*

$$h(X_1, X_2, ..., X_n) = -\int f(x^n)\log f(x^n)dx^n \tag{2.3}$$

where $f(x^n) = f(x_1, x_2, ..., x_n)$ is the joint pdf.

- Let X, Y be two random variables that have a joint density function $f(x, y)$. *The conditional differential entropy* $h(X|Y)$ *is defined as*

$$h(X|Y) = -\int f(x, y)\log f(x|y)dx\,dy \tag{2.4}$$

From the relationship $f(x|y) = f(x, y)/f(y)$, we obtain

$$h(X|Y) = h(X, Y) - h(Y)$$

- *Mutual information* $I(X; Y)$ *between two random variables with joint density* $f(x; y)$ *is defined as*

$$I(X; Y) = \int f(x, y)\log\frac{f(x, y)}{f(x)f(y)}dxdy. \tag{2.5}$$

$$= h(X) - h(X|Y) \tag{2.6}$$

$$= h(Y) - h(Y|X) \tag{2.7}$$

$$= h(X) + h(Y) - h(X, Y) \tag{2.8}$$

Properties of Differential Entropy and Mutual Information

- $h(X + a) = h(X)$ when a denotes any constant (Translation).
- $h(bX) = h(X) + \log|b|$ when b denotes any nonzero constant (Scaling).
- $h(X|Y) \leq h(X)$ with equality iff X and Y are independent.
- $h(X_1, X_2, ..., X_n) = \sum_{i=1}^{n} h(X_i|X_1, X_2, ..., X_{i-1})$ (Chain rule for differential entropy)
- $h(X_1, X_2, ..., X_n) \leq \sum_{i=1}^{n} h(X_i)$ with equality iff $X_1, X_2, ..., X_n$ are independent.
- $I(X; Y) \geq 0$ with equality iff X and Y are independent.

2.3 Asymtotic Equipartition Property for Continuous Random Variables

This section defines the typical set and characterizes the behavior of typical sequences for a continuous random variable [1]. The typical set is used to derive the error probability in Section 6.4.2.

Let $X_1, X_2, ..., X_n$ be a sequence of i.i.d random variables according to the probability density function $f(x)$. Therefore

$$-\frac{1}{n}\log f(X_1, X_2, ..., X_n) \to E[-\log f(X)] = h(X) \text{ in probability} \tag{2.9}$$

For $\epsilon > 0$ and any n, the typical set $A_\epsilon^{(n)}$ with respect to $f(x)$ is defined as follows:

$$A_\epsilon^{(n)} = \left\{ (x_1, x_2, ..., x_n) \in S^n : \left| \frac{1}{n}\log f(x_1, x_2, ..., x_n) - h(X) \right| \leq \epsilon \right\}, \tag{2.10}$$

where $f(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} f(x_i)$.

The volume Vol (A) *of the typical set for continuous random variables $A \subset \mathcal{R}^n$ is defined as follows:*

$$\text{Vol}(A) = \int_A dx_1 dx_2 \cdots dx_n. \tag{2.11}$$

Properties of the typical set $A_\epsilon^{(n)}$ consist of

1. $\Pr(A_\epsilon^{(n)}) > 1 - \epsilon$ for *n* adequately large.
2. $\text{Vol}(A_\epsilon^{(n)}) \leq 2^{n(h(X)+\epsilon)}$ for all n.
3. $\text{Vol}(A_\epsilon^{(n)}) \geq (1-\epsilon)2^{n(h(X)+\epsilon)}$ for all n adequately large.

2.4 Fano's Inequality

Suppose that we wish to estimate a random variable X with with a distribution $p(x)$. We observe a random variable Y that is related to X by the conditional distribution $p(y|x)$. From Y, we calculate a function $g(Y) = \hat{X}$, where $\hat{X}$ is an estimate of X and takes on value in $\hat{\mathcal{X}}$. We will not restrict the alphabet $\hat{\mathcal{X}}$ to be equal to $\mathcal{X}$, and we will also allow the function $g(Y)$ to be random. We wish to bound the probability that $\hat{X} \neq X$. We observe that $X \rightarrow Y \rightarrow \hat{X}$ forms a Markov chain. Define the probability of error

$$P_e = \Pr\{\hat{X} \neq X\}. \tag{2.12}$$

Theorem 2.1 (*Fano's Inequality) For any estimator $\hat{X}$ such that $X \rightarrow Y \rightarrow \hat{X}$, with $P_e = \Pr(X \neq \hat{X})$, we have*

$$H(P_e) + P_e \log |\mathcal{X}| \geq H(X|\hat{X}) \geq H(X|Y). \tag{2.13}$$

where $H(\cdot)$ denotes entropy.
This inequality can be weakened to

$$1 + P_e \log |\mathcal{X}| \geq H(X|Y). \tag{2.14}$$

or

$$P_e \geq \frac{H(X|Y) - 1}{\log |\mathcal{X}|}. \tag{2.15}$$

Proof: See all details in [1]. ∎

Remark 2.2 *Note that from (2.13) $P_e = 0$ implies that $H(X|Y) = 0$, as intuition suggests.*

2.5 Data Processing Inequality

Definition 2.3 *Random variables X, Y, Z are said to form a Markov chain in that order (denoted by $X \rightarrow Y \rightarrow Z$) if the conditional distribution of Z depends only on Y and is conditionally independent of X. Specifically, X, Y and Z from a Markov chain $X \rightarrow Y \rightarrow Z$ if the joint probability mass function can be written as*

$$p(x, y, z) = p(x)p(y|x)p(z|y). \tag{2.16}$$

Some simple consequences are as follows:

- $X \rightarrow Y \rightarrow Z$ if and only if X and Z are conditionally independent given Y.
- $X \rightarrow Y \rightarrow Z$ implies that $Z \rightarrow Y \rightarrow X$. Thus, the condition is sometimes written $X \leftrightarrow Y \leftrightarrow Z$.

- if $Z = f(Y)$, then $X \to Y \to Z$.

The next theorem shows that no processing of Y, deterministic or random, can increase the information that Y contains about X.

Theorem 2.4 (*Data Processing Inequality [1]) If $X \to Y \to Z$, then $I(X;Y) \geq I(X;Z)$.*

Proof: By the chain rule, we can expand mutual information in two different ways:

$$I(X;Y,Z) = I(X;Z) + I(X;Y|Z) \tag{2.17}$$

$$= I(X;Y) + I(X;Z|Y) \tag{2.18}$$

Since X and Z are conditionally independent given Y, we have $I(X;Z|Y) = 0$. Since $I(X;Y|Z) \geq 0$, we have

$$I(X;Y) \geq I(X;Z) \tag{2.19}$$

We have equality if and only if $I(X;Y|Z) = 0$ (i.e., $X \to Z \to Y$ forms a Markov chain). Similarly, one can prove that $I(Y;Z) \geq I(X;Z)$ ■

2.6 Genie-Aided Techniques

Genie-aided techniques are used to derive the upper bounds in various communication scenarios, i.e., interference channel without cooperation [9], interference channel with cooperation [2], X channel without cooperation [14, 15].

The key feature of these techniques is to provide the side information by the genie to the receivers for compensating the damage from interference on the other link.

2.7 Han-Kobayashi Strategy

The Han-Kobayashi (HK) strategy [20] involves splitting the transmitted information at each transmitter into two parts as shown in Figure 2.1: private message m_{ip} which can be decoded only at the intended receiver and common message m_{ic} which can be decoded at both receivers, for $i = 1, 2$. Each transmitter generates a common codeword x_{ic}^N and a private codeword x_{ip}^N using messages m_{ic} and m_{ip}. The power for codewords x_{ic}^N and x_{ip}^N

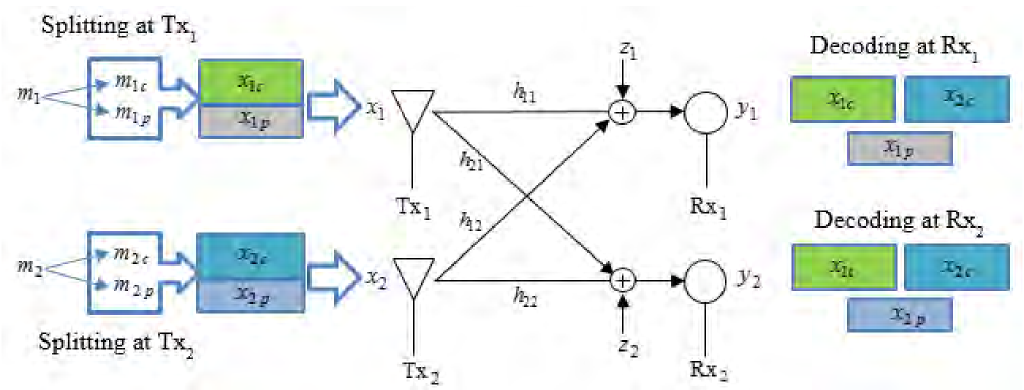


Figure 2.1 The HK strategy in the two-user interference channel

are Q_{ic} and Q_{ip}, respectively. Then, we obtain the transmitted codeword x_1^N with its power = 1 = Q_{ic}+Q_{ip}. Finally, each receiver decodes only its corresponding private information x_{ip}^N and two common information x_{1c}^N, x_{2c}^N, i.e., obtaining m_{ip}, m_{1c}, m_{2c}.

The original HK strategy [20] allows arbitrary splits of each user's transmit power into the private and common information portions (as well as time sharing between multiple such splits). However, there are the problems for finding the HK region as follows:

1. Optimizing a great number of possibilities for each user's transmit power is not well-understood.

2. Time-sharing over many choices of each user's transmit power may be required.

In the work [9], Etkin, Tse and Wang propose a simple HK type scheme achieving rates within 1 bit/s/Hz of the capacity of the channel, independent of channel parameters.

The key concept of their scheme [9] is to set the power of the private information of each user such that it is received at the level of the Gaussian noise at the other receiver as depicted in Fig. 2.2.

This strategy is used widely in several communication scenarios, i.e., Gaussian interference channel [9, 20], X channel [11, 13], Gaussian interference channel with cooperation [2, 22, 33], Compound multiple access channel with cooperation [29], Z interference channel with cooperation [30, 31], Gaussian interference relay channel [34], Z-channel with cooperation [35, 36], etc.

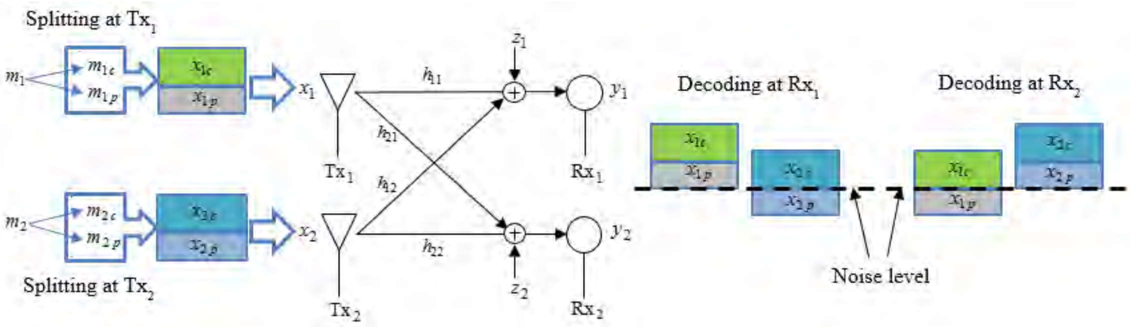


Figure 2.2 A simple Han-Kobayashi type scheme in the two-user interference channel

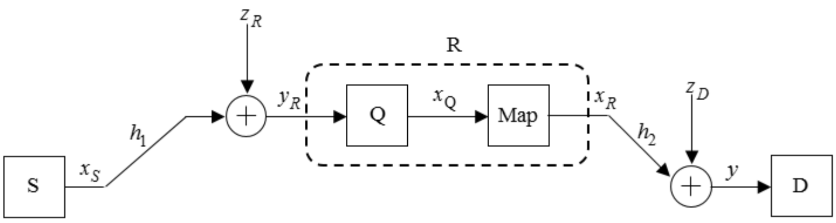


Figure 2.3 Quantize-map-and-forward (QMF) scheme in the relay model

2.8 Cooperative Protocol

This section describes the concept of quantize-map-and-forward (QMF) scheme [38] and then indicates its features. After that, cooperative protocol based on QMF scheme proposed by the work [2] is reported.

2.8.1 Quantize-Map-and-Forward (QMF) Scheme

The QMF scheme [38] is a recently proposed scheme that allows to approximately achieve the capacity of arbitrary wireless relay networks.

In this scheme, the relay (R) quantizes its received signal at noise level, randomly maps it to a codeword and forwards it to the destination (see in Figure 2.3).

2.8.2 Features of QMF scheme

- The quantization and mapping are performed without regard to quality of forward channel at the relay. This reduces the channel estimation and feedback overhead for the link.

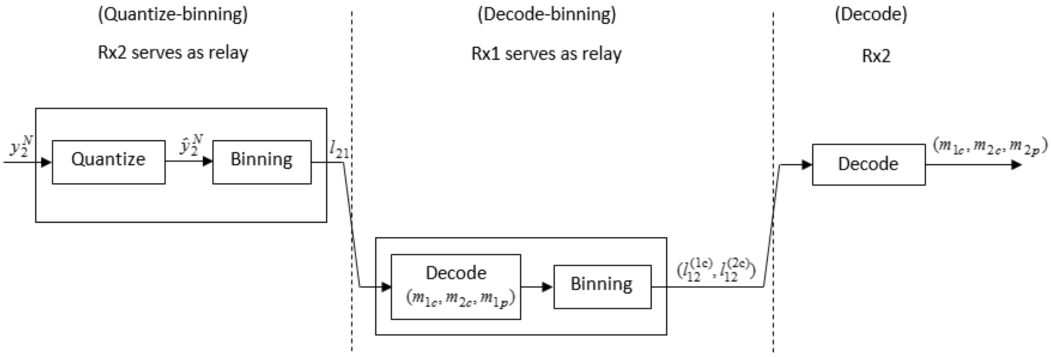


Figure 2.4: Two-round cooperative protocol base on QMF scheme in the two-user Gaussian interference channel

- QMF uses joint decoding of the message (from the transmitter) and side information (from the relay) because mapping at relay is performed without any knowledge of forward channel strength and side information from relays cannot be decoded at the destination independently.

- QMF performs within bounded gap from capacity for networks having an arbitrary number of relays [38].

2.8.3 Cooperative protocol based on QMF scheme

In the work [2], Wang and Tse proposed the two-round QMF strategy for cooperating between two receivers in the two-user Gaussian interference channel (see in Figure 2.4). Remind that [2] uses HK strategy in the transmission scheme.

For simplicity, we describe this strategy with the processing order $\mathrm{Rx}_2 \rightarrow \mathrm{Rx}_1 \rightarrow \mathrm{Rx}_2$ consisting of three stages as follows:

1. *Quantize-Binning:* Receiver 2 first quantizes its received signal y_2^N into $\hat{y}_2^N$ by a pregenerated Gaussian quantization codebook with certain distortion which equal to the aggregate power level of the noise and user 2's private signal and then sends out a bin index determined by a pregenerated binning function $l_{21} = b_2(\hat{y}_2^N)$.

2. *Decode-Binning:* After receiver 1 retrieves the receiver-cooperative side information, that is, the bin index l_{21}, it decodes two common messages and its own private message (m_{1c}, m_{2c}, m_{1p}), by searching in transmitters' codebooks for a codeword triple (indexed by user 1 and user 2's common messages and user 1's own private

message) that is jointly typical with its received signal and some quantization point (codeword) in the given bin. After receiver 1 decodes, it uses two pregenerated binning functions to bin the two common messages ($l_{12}^{(ic)} = b_1^{(ic)}(m_{ic}),\ \text{for}\ i = 1, 2$) and sends out these two bin indices to receiver 2.

3. *Decoding:* After receiving these two bin indices, (l_{12}^{1c},l_{12}^{2c}), receiver 2 decodes two common messages and its own private message (m_{1c}, m_{2c}, m_{2p}), by searching in the corresponding bins (containing common messages) and user 2's private codebook for a codeword triple that is jointly typical with its received signal y_2^N.

2.8.4 Advantage of cooperative protocol based on QMF scheme

In the work [2], Wang and Tse reveal that strategies based on the compress-forward or decode-forward scheme which are used in [29, 30] are not proper for receiver cooperation to mitigate interference in certain regimes because both schemes do not achieve the optimal GDoF universally. However, they show that their proposed cooperative protocol which consists of an improved compress-forward and decode-forward scheme achieves the optimal number of GDoF for all value of the normalized interference (α) and the normalized capacity of the receiver-cooperative link (κ).

Therefore, from the key advantage above, we adopt the cooperative protocol of the work [2] in our cooperative protocol for the two-user Gaussian X channel with limited receiver cooperation.

In the next section, we introduce the capacity region based on the Gaussian multiple-access channel.

2.9 Capacity Region

To understand capacity region more clearly, this section gives the well-known simple example, i.e., the capacity region for the two-user Gaussian multiple-access channel (Gaussian MAC) [1].

The two-user Gaussian multiple-access channel consisting of two transmitters and one receiver can be modeled as follows (see Figure 2.5):

$$y_i = x_{1i} + x_{2i} + z_i \tag{2.20}$$

where $y \in \mathbb{C}$ is the channel output at receiver, x_1 and $x_2 \in \mathbb{C}$ are the channel input at transmitter 1 and 2, respectively, the additive noise processes $\{z_i\}$ are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ over time and i denotes the index of time.

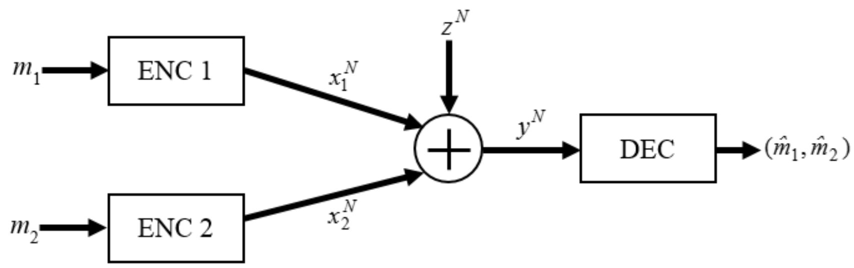


Figure 2.5 Gaussian multiple-access channel [1]

Assume that there is an average power constraint P_j at transmitter j, i.e., for each transmitter, we have

$$\frac{1}{N}\sum_{i=1}^{N} x_{ji}^2(m_j) \le P_j, \quad m_j \in \{1, 2, \cdots, 2^{NR_j}\}, \quad j = 1, 2. \tag{2.21}$$

Next, we provide the definitions for achievable rates and the capacity region based on the two-user Gaussian MAC.

Definition 2.5 *A rate pair (R_1, R_2) is said to be achievable for the Gaussian MAC if there exists a sequence of $(2^{NR_1}, 2^{NR_2}, N)$ codes with $P_e^{(N)}$ approaches to 0, where the average probability of error*

$$P_e^{(N)} := \frac{1}{2^{N(R_1+R_2)}} \times \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} \Pr\left\{d(y^N) \ne (m_1, m_2) \mid (m_1, m_2) \text{ are sent}\right\}$$

Definition 2.6 *The capacity region of the Gaussian MAC is the closure of the set of achievable rate pairs (R_1, R_2).*

From [1], the capacity region is the closure of the convex hull of the set of rate pairs satisfying

$$R_1 \le I(x_1; y|x_2) \tag{2.22}$$

$$R_2 \le I(x_2; y|x_1) \tag{2.23}$$

$$R_1 + R_2 \le I(x_1, x_2; y) \tag{2.24}$$

for some input distribution $f_1(x_1)f_2(x_2)$ satisfying the average power constraints $E[x_1^2] \leq P_1$ and $E[x_2^2] \leq P_2$.

Next, finding the mutual information $I(x_1; y|x_2)$ as follows:

$$I(x_1; y|x_2) = h(y|x_2) - h(y|x_1, x_2) \tag{2.25}$$

$$= h(x_1 + x_2 + z|x_2) - h(x_1 + x_2 + z|x_1, x_2) \tag{2.26}$$

$$= h(x_1 + z|x_2) - h(z|x_1, x_2) \tag{2.27}$$

$$\overset{(a)}{=} h(x_1 + z) - h(z) \tag{2.28}$$

$$\overset{(b)}{\leq} \log(2\pi e)(P_1 + N) - \log(2\pi e)N \tag{2.29}$$

$$= \log\Big(1 + \frac{P_1}{N}\Big), \tag{2.30}$$

where (a) is due to the fact that z is independent of x_1 and x_2 and x_1 is also independent of x_2, (b) is due to the fact that the normal distribution maximizes the entropy for a given second moment.

By choosing $x_1 \sim \mathcal{CN}(0, P_1)$ and $x_2 \sim \mathcal{CN}(0, P_2)$, therefore, bounds (2.31)−(2.33) are maximized with these distributions.

Definition 2.7 *The channel capacity function is defined as* $C(x) \triangleq \log(1 + x)$

With the definition above, the capacity of two-user Gaussian multiple-access channel (2.31)−(2.33) can be rewritten as follows:

$$R_1 \leq C\Big(\frac{P_1}{N}\Big) \tag{2.31}$$

$$R_2 \leq C\Big(\frac{P_2}{N}\Big) \tag{2.32}$$

$$R_1 + R_2 \leq C\Big(\frac{P_1 + P_2}{N}\Big) \tag{2.33}$$

Note that the sum of rates in (2.33) can be as large as $C(\frac{P_1+P_2}{N})$. This value is obtained by a single transmitter sending with a power equal to the sum of the powers.

Using (2.31)−(2.33), the region of the capacity region for Gaussian multiple-access channel is shown in Figure 2.6.

From Figure 2.6, the corner points have the following interpretation.

- Point A corresponds to the maximum rate achievable from transmitter 1 to the receiver when transmitter 2 is not transmitting any information.

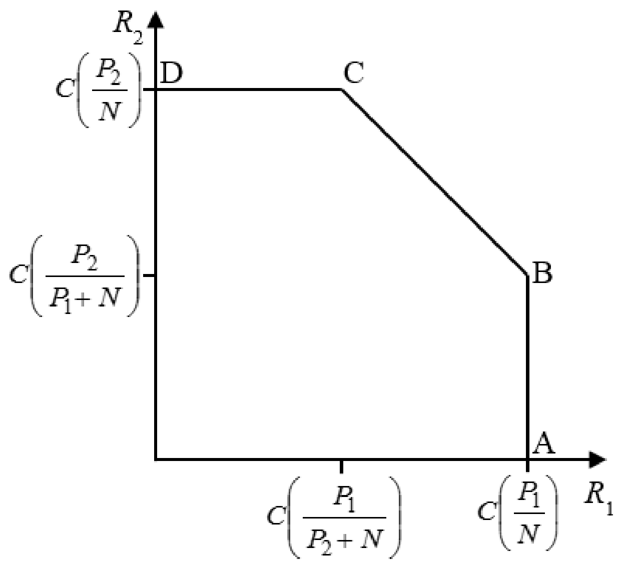


Figure 2.6 Gaussian multiple-access channel capacity [1]

- Point B corresponds to the maximum rate at which transmitter 2 can send as long as transmitter 1 sends at its maximum rate. This is the rate that is obtained if x_1 is considered as noise for the channel from x_2 to y.
- Point C corresponds to point B with the role of the transmitter reversed.
- Point D corresponds to point A with the role of the transmitter reversed.

Remark 2.8 *Decoding in the Gaussian multiple-access channel corresponding point B consists of a two-stage process:*

1. *In the first stage, the receiver decodes the second transmitter by treating the first transmitter as part of the noise. This decoding will have low probability of error if* $R_2 < C(\frac{P_2}{P_1+N})$.
2. *After information of the second transmitter has been decoded successfully, it can be subtracted out and the first transmitter can be decoded correctly if* $R_1 < C(\frac{P_1}{N})$.

Remark 2.9 *If* (R_1, R_2) *is in the capacity region given above, the probability of error goes to* 0 *as* N *tends to infinity.*

2.10 Gaussian X channel

In this section, we introduce the simple two-user Gaussian X channel without cooperation and show that how to use the genie-aided techniques with the example for deriving the sum-rate upper bound from the result in [14] which is a useful guideline to understand our proposed upper bounds in Chapter IV.

The two-user Gaussian X channel that is a communication scenario where each transmitter has an independent message for both receivers can be modeled as follows (see Figure 2.7) [14]:

$$y_1 = h_{11}x_1 + h_{12}x_2 + z_1$$
$$y_2 = h_{21}x_1 + h_{22}x_2 + z_2,$$

where y_i is the channel output at receiver i, x_i is the channel input at transmitter i and the additive noise processes $\{z_i[n]\}$ are independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ over time for $i, j = 1, 2$ and $i \neq j$.

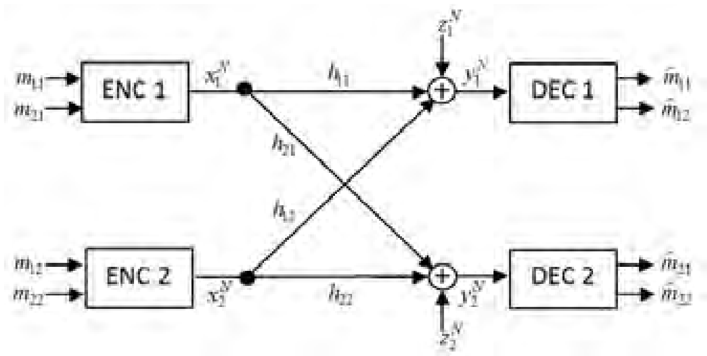


Figure 2.7 Gaussian X Channel

In the X channel, there are four independent messages, $m_{11}, m_{12}, m_{21}, m_{22}$, where message m_{ji} is sent from transmitter i to receiver j. Hence, transmitter i encodes message m_{ii} and m_{ji} into a block codeword $\{x_i[n]\}_{n=1}^{N}$ with an average transmit power constraint

$$\frac{1}{N}\sum_{n=1}^{N}|x_i[n]|^2 \leq P_i, \quad i = 1, 2,$$

for arbitrary block length N. The size of the message m_{ij} is given by $|m_{ij}|$. For codewords spanning N symbols, rates $R_{ij} = \frac{\log|m_{ij}|}{N}$ are achievable if the probability of error for all messages are made arbitrarily small when N is large sufficiently.

Defining the capacity region $\mathcal{C}$ of the X channel is the set of all achievable rate tuples $\mathbf{R} = (R_{11}, R_{12}, R_{21}, R_{22})$ and the sum capacity of the X channel is denoted by $C_{\sum}$.

Next, we give the example for deriving the upper bound of the X channel from [14].

Example: Deriving the upper bound (103) in [14] using the genie-aided technique as follows:

First, setting $m_{21} = \phi$. Letting a genie gives side information y_1^N, m_{11} and m_{12} to receiver 2. Hence, we can upper bound the sum-rate $R_{11} + R_{12} + R_{22}$ as follows:

$$
\begin{aligned}
N(R_{11} + R_{12} + R_{22}) &= \mathrm{H}(m_{11}, m_{12}) + \mathrm{H}(m_{22}) \\
&= I(m_{11}, m_{12}; y_1^N) + \mathrm{H}(m_{11}, m_{12}|y_1^N) + I(m_{22}; y_2^N) + \mathrm{H}(m_{22}|y_2^N) \\
&\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N) + \epsilon_N^1 + I(m_{22}; y_2^N) + \epsilon_N^2 \\
&\overset{(b)}{=} I(m_{11}, m_{12}; y_1^N) + I(m_{22}; y_2^N) + \epsilon_N \qquad (2.34)
\end{aligned}
$$

where $\epsilon_N^1 \to 0$ and $\epsilon_N^2 \to 0$ as $N \to 0$. (a) follows from Fano's inequality, i.e.,

$$
\begin{aligned}
\mathrm{H}(m_{11}, m_{12}|y_1^N) \leq N(R_{11} + R_{12})P_{e1}^{(N)} + \mathrm{H}(P_{e1}^{(N)}) \triangleq N\epsilon_N^1 \\
\mathrm{H}(m_{22}|y_2^N) \leq N(R_{22})P_{e2}^{(N)} + \mathrm{H}(P_{e2}^{(N)}) \triangleq N\epsilon_N^2
\end{aligned}
$$

(b) is due to the fact that $\epsilon_N = \epsilon_N^1 + \epsilon_N^2$.

Then, we rewrite (2.34) as (2.35) and then find the solution as follows:

$$
\begin{aligned}
N(R_{11} + R_{12} + R_{22} - \epsilon_N) &= I(m_{11}, m_{12}; y_1^N) + I(m_{22}; y_2^N) && (2.35) \\
&\overset{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{22}; y_2^N, y_1^N, m_{11}, m_{12}) && (2.36) \\
&\overset{(d)}{=} I(m_{11}, m_{12}; y_1^N) + I(m_{22}; y_2^N, y_1^N|m_{11}, m_{12}) && (2.37) \\
&= h(y_1^N) - h(y_1^N|m_{11}, m_{12}) + h(y_2^N, y_1^N|m_{11}, m_{12}) \\
&- h(y_2^N, y_1^N|m_{11}, m_{12}, m_{22}) \\
&= h(y_1^N) + h(y_2^N|y_1^N, m_{11}, m_{12}) - h(y_2^N, y_1^N|m_{11}, m_{12}, m_{22}) \\
&\overset{(e)}{=} h(y_1^N) + h(y_2^N|y_1^N, m_{11}, m_{12}, x_1^N) && (2.38) \\
&- h(y_2^N, y_1^N|m_{11}, m_{12}, m_{22}, x_2^N, x_1^N) && (2.39) \\
&\overset{(f)}{=} h(y_1^N) + h(s_{22}^N|s_{12}^N) - h(z_2^N, z_1^N) && (2.40) \\
&\leq \sum_{n=1}^{N} h(y_1[n]) + \sum_{n=1}^{N} h(s_{22}[n]|s_{12}[n]) - h(z_2^N, z_1^N) && (2.41)
\end{aligned}
$$

$$\stackrel{\text{(g)}}{\leq} N\log(1 + h_{11}^2 P_1 + h_{12}^2 P_2) + \log\Big(1 + \frac{h_{22}^2 P_2}{1 + h_{12}^2 P_2}\Big) \tag{2.42}$$

where $\epsilon_N \to 0$ as $N \to 0$, $s_{22}^N = h_{22}x_2^N + z_2^N$ and $s_{12}^N = h_{12}x_2^N + z_2^N$. (c) is due to the genie providing y_1^N, m_{11} and m_{12} to receiver 2. (d) is due to the chain rule and the independence of all messages m_{11}, m_{12}, m_{22}. (e) In the first summand on the right-hand side, we use the fact that given m_{11}, x_1^N is known at receiver 2 because $m_{21} = \phi$. In the second term, using the fact that conditioning on x_1^N and x_2^N does not reduce entropy. (f) is due to the fact that conditioning does not reduce entropy. (g) is due to the fact that i.i.d. Gaussian distribution maximizes conditional differential entropy subject to conditional variance constraints.

2.11 Generalized Degrees of Freedom

The GDoF introduced by Etkin, Tse and Wang [9] is a natural generalization of the notion of the DoF in point-to-point communication to multiuser scenarios. This notion provides a useful tool to approximate interference-limited performance in the high-SNR regime.

For simplicity, we consider in the symmetric channel case of the two-user Gaussian interference channel where $\mathsf{SNR}_1 = \mathsf{SNR}_2 = \mathsf{SNR}$, $\mathsf{INR}_1 = \mathsf{INR}_2 = \mathsf{INR}$. The GDoF of the sum capacity is defined as

$$d(\alpha) := \lim_{\substack{\text{fix } \alpha \\ \mathsf{SNR}\to\infty}} \frac{C_\Sigma(\mathsf{SNR}, \mathsf{INR})}{\log \mathsf{SNR}} \tag{2.43}$$

where $C_\Sigma(\mathsf{SNR}, \mathsf{INR})$ is the sum capacity of the two-user Gaussian interference channel, i.e., $C_\Sigma(\mathsf{SNR}, \mathsf{INR}) = R_1 + R_2$, and

$$\lim_{\mathsf{SNR}\to\infty} \frac{\log \mathsf{INR}}{\log \mathsf{SNR}} = \alpha \tag{2.44}$$

More precisely, we use the following approximations [9] such as

$$\log(1 + \mathsf{SNR} + \mathsf{INR}) \approx \max(\log(\mathsf{SNR}), \log(\mathsf{INR})) \tag{2.45}$$

$$\log\Big(1 + \frac{\mathsf{SNR}}{1 + \mathsf{INR}}\Big) \approx \Big(\log\Big(\frac{\mathsf{SNR}}{\mathsf{INR}}\Big)\Big)^+ \tag{2.46}$$

to give an expansion of the capacity region of the Gaussian interference channel which is accurate to the first order terms. Denote that $(a)^+ := \max(0, a)$.

2.12 Gain from the limited receiver cooperation

In the work [2], Wang and Tse give a numerical example to show the gain from the limited receiver cooperation by plotting cooperation rate versus user data rate at the fixed signal-to-noise ratios $(\text{SNR}) = 20$ dB and interference-to-noise ratios $(\text{INR}) = 15$ dB as depicted in Figure 2.8 .

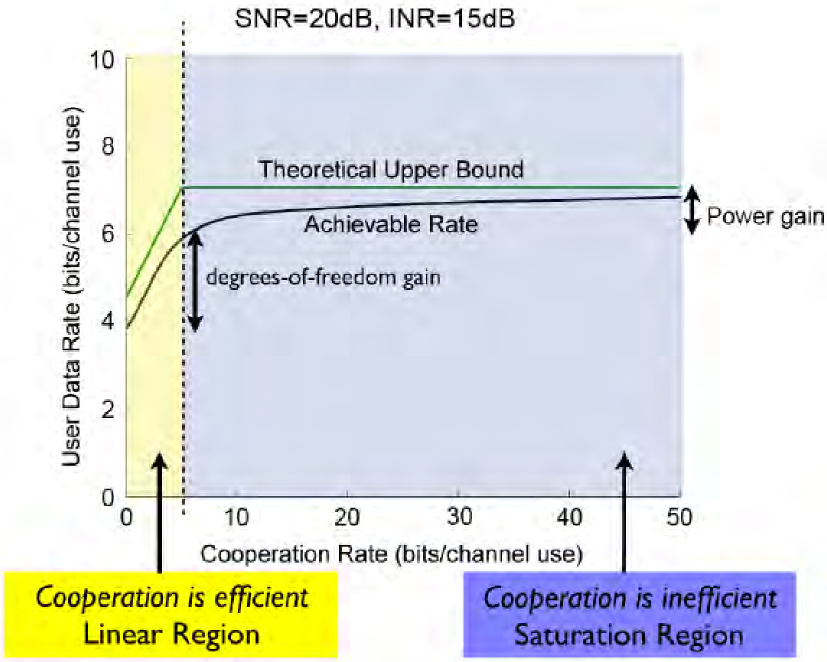


Figure 2.8 Gain from limited receiver cooperation [2]

From Figure 2.8, they classify their result into two regions: linear and saturation regions.

- In the linear region, it can be stated that receiver cooperation is efficient because each user's data rate and the capacity of receiver-cooperative link are approximately linear. The gain in this region is provided by distributed MIMO systems and is called the degrees-of-freedom gain.

- In the saturation region, it can be stated that receiver cooperation is inefficient since each user's data rate does not change anymore even though the capacity of receiver-cooperative link increases. The gain in this region is called the power gain and is bounded independent of the cooperative rate.

CHAPTER III

THE PROBLEM FORMULATION

This chapter provides the channel model for the two-user Gaussian X channel with limited receiver cooperation and formulate the problem.

3.1 Channel Model

Since the two-user Gaussian X channel is physically the same as the two-user Gaussian interference channel [14], therefore, we can describe the two-user Gaussian X channel with limited receiver cooperation using the channel model of the two-user Gaussian interference channel with limited receiver cooperation in [2] as shown in Figure 3.1. This model consists of two transmitters and two receivers, where each transmitter has an independent message for each receiver. They communicate each other via two main non-interference links[1] as follows:

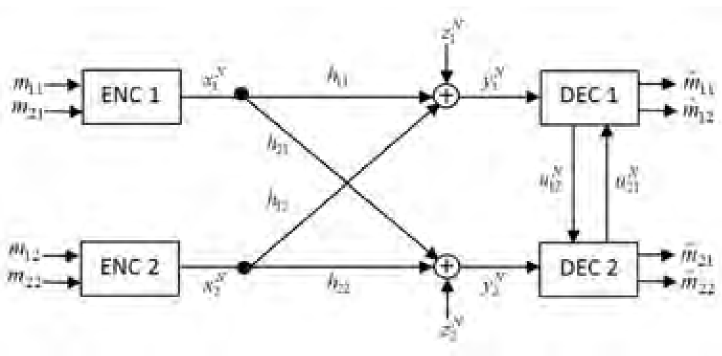


Figure 3.1: Channel model of the two-user Gaussian X channel with limited receiver cooperation

[1] We consider the two-user Gaussian X channel with out-of-band (orthogonal) limited receiver cooperation, that is, signals in transmitter-receiver links do not interfere with ones in receiver-cooperative links.

Transmitter-Receiver Links: These links are modeled as the *normalized* Gaussian X channel

$$y_1 = h_{11}x_1 + h_{12}x_2 + z_1$$
$$y_2 = h_{21}x_1 + h_{22}x_2 + z_2,$$

where the additive noise processes $\{z_i[n]\}, (i = 1, 2)$, are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ over time. For $i, j = 1, 2$ and $i \neq j$, suppose that there are four independent messages, $m_{11}, m_{12}, m_{21}, m_{22}$, where message m_{ji} is sent from transmitter i to receiver j. Hence, transmitter i encodes message m_{ii} and m_{ji} into a block codeword $\{x_i[n]\}_{n=1}^{N}$ with an average transmit power constraint

$$\frac{1}{N}\sum_{n=1}^{N} |x_i[n]|^2 \leq 1, \quad i = 1, 2,$$

for arbitrary block length N. Note that the outcome of each encoder depends only on its own messages. Signal-to-noise ratio (SNR) and interference-to-noise ratio (INR) can be defined to capture the channel gains as follows:

$$\mathsf{SNR}_i = |h_{ii}|^2, \text{ and } \mathsf{INR}_i = |h_{ij}|^2, \ i, j = 1, 2, \ i \neq j.$$

Receiver-Cooperative Links: These links are noiseless with capacity $\mathsf{C}^{\mathsf{B}}_{ij}$ from receiver i to j, for $(i, j) = (1, 2), (2, 1)$, where $0 \leq \mathsf{C}^{\mathsf{B}}_{ij} \leq \mathsf{C}^{\mathsf{B}*}_{ij}$ and $\mathsf{C}^{\mathsf{B}*}_{ij}$ denotes the maximum value of the capacity of receiver-cooperative link from receiver i to j. Encoding at each receiver is causal in the sense that the cooperation signal from receiver i to j, $u_{ij}[n]$, is only a function of the received signal at receiver i, $\{y_i[1], \ldots, y_i[n-1]\}$ and the cooperation signal from receiver *j* to *i*, $\{u_{ji}[1], \ldots, u_{ji}[n-1]\}$, for any time index $n = 1, 2, \ldots, N$.

3.2 Strategies, Achievable Rates, and Capacity Region

We give the definitions for the coding strategies, achievable rate of the strategy, and the capacity region of the channel.

Definition 3.1 (Strategy and Average Probability of error) *An* $(M_{11}, M_{12}, M_{21}, M_{22}, N)-$ *strategy for the X channel consists of the following: for* i, j *= 1, 2,* $i \neq j$*,*

- *Message sets* $\mathcal{M}_{ii} := \{1, 2, \ldots, M_{ii}\}$ *and* $\mathcal{M}_{ji} := \{1, 2, \ldots, M_{ji}\}$ *for transmitter* i*;*

- *Message sets* $\mathcal{M}_{ii} := \{1, 2, \ldots, M_{ii}\}$ *and* $\mathcal{M}_{ij} := \{1, 2, \ldots, M_{ij}\}$ *for receiver* i*;*

- *Encoding function* $e_i^{(N)} : \mathcal{M}_{ii} \times \mathcal{M}_{ji} \to \mathbb{C}^N,\ m_{ii} \times m_{ji} \mapsto x_i^N$ *at transmitter* i*;*

- *Set of relay function* $\{r_i^{(n)}\}_{n=1}^N$ *such that* $u_{ij}[n] = r_i^{(n)}(y_i^{n-1}, u_{ji}^{n-1}) \in \{1, 2, \ldots, 2^{\mathsf{C}_{ij}^{\mathsf{B}}}\}$, $\forall n = 1, 2, \ldots, N$ *at receiver* i*;*

- *Decoding function* $d_i^{(N)} : \mathbb{C}^N \times \{1, 2, \ldots, 2^{N\mathsf{C}_{ji}^{\mathsf{B}}}\} \to \mathcal{M}_{ii} \times \mathcal{M}_{ij},\ (y_i^N, u_{ji}^N) \mapsto \hat{m}_{ii} \times \hat{m}_{ij}$ *at receiver* i*.*

The average probability of error

$$P_e^{(N)} := \frac{1}{M_{11}M_{12}M_{21}M_{22}} \times \sum_{\substack{m_{11}\in\mathcal{M}_{11}\\ m_{12}\in\mathcal{M}_{12}\\ m_{21}\in\mathcal{M}_{21}\\ m_{22}\in\mathcal{M}_{22}}} \Pr\left\{\begin{array}{ll} d_1^{(N)}(y_1^N, u_{21}^N) \neq m_{11} \times m_{12} \text{ or} & \mid m_{11}, m_{12}, m_{21}, \\ d_2^{(N)}(y_2^N, u_{12}^N) \neq m_{21} \times m_{22} & \mid m_{22} \text{ are sent} \end{array}\right\}$$

Definition 3.2 (Achievable Rates for the general case) *A rate hexatruple* $(R_{1c}, R_{2c}, R_{11p}, R_{12p}, R_{21p}, R_{22p})$ *is said to be achievable if for any small* $\epsilon > 0$ *and for all sufficiently large N, there exists a* $(M_{1c}, M_{2c}, M_{11p}, M_{12p}, M_{21p}, M_{22p}, N)$ *strategy with* $M_{ic} \geq 2^{NR_{ic}}$ *and* $M_{ijp} \geq 2^{NR_{ijp}}$*, for i, j = 1,2, such that* $P_e^{(N)} < \epsilon$*. The achievable rate region is the closure of the set of achievable rates. This definition is used in Theorem 6.3.*

Definition 3.3 (Achievable Rates) *A rate quadruple* $(R_{11}, R_{12}, R_{21}, R_{22})$ *is said to be achievable if for any small* $\epsilon > 0$ *and for all sufficiently large N, there exists a* $(M_{11}, M_{12}, M_{21}, M_{22}, N)$ *strategy with* $M_{ij} \geq 2^{NR_{ij}}$*, for i, j = 1,2, such that* $P_e^{(N)} < \epsilon$*. The achievable rate region is the closure of the set of achievable rates. This definition is used in Theorem 6.8.*

Definition 3.4 (Capacity Region) *The capacity region* $\mathscr{C}$ *of the X channel is the closure of the set of the achievable rate* $(R_{11}, R_{12}, R_{21}, R_{22})$*.*

3.3 Classification of the Two-User Gaussian X Channel

In the work [12], Prasad and Chockalingam classify the two-user Gaussian X channel into the two broad classes which each class is also divided into two subclasses as follows:

- *Strong Gaussian X channel*

1. Type I: the direct channels are stronger than the cross channels, i.e., $\mathsf{SNR}_1 \geq \mathsf{INR}_2$ and $\mathsf{SNR}_2 \geq \mathsf{INR}_1$.
2. Type II: the cross channels are stronger than the direct channels, i.e., $\mathsf{SNR}_1 \leq \mathsf{INR}_2$ and $\mathsf{SNR}_2 \leq \mathsf{INR}_1$.

- *Mixed Gaussian X channel*

1. Type I: one of the direct channels is stronger than the corresponding cross channel and the other cross channel is stronger than the corresponding direct channel, i.e., $\mathsf{SNR}_1 \geq \mathsf{INR}_2$ and $\mathsf{SNR}_2 \leq \mathsf{INR}_1$.
2. Type II: one of the cross channels is stronger than the corresponding direct channel and the other direct channel is stronger than the corresponding cross channel, i.e., $\mathsf{SNR}_1 \leq \mathsf{INR}_2$ and $\mathsf{SNR}_2 \geq \mathsf{INR}_1$.

In addition to the general case, we also consider the strong Gaussian X channel type I case in this dissertation. However, we modify the constraints in the strong Gaussian X channel type I case by replacing the symbol "$\geq$" with "$>$", that is, $\mathsf{SNR}_1 > \mathsf{INR}_2$ and $\mathsf{SNR}_2 > \mathsf{INR}_1$.

3.4 Notations

We use the following notations throughout in the rest of this dissertation.

- $h(\cdot)$ and $I(\cdot)$ denote the differential entropy of a continuous random variable or vector, and mutual information, respectively.
- For a real number a, $(a)^+ := \max(0, a)$ denotes its positive part.
- $\mathbb{C}$ denotes the set of all complex numbers.
- $\mathcal{CN}(0, 1)$ denotes complex Gaussian random variable with zero mean and unit variance.
- For set $A \subseteq \mathbb{R}^k$ in an k-dimensional space, conv$\{A\}$ denotes the convex hull of the set A.
- Let x^N denote the sequence $\{x[1], \cdots, x[N]\}$ where $[\cdot]$ denote time indices.
- Unless otherwise stated, all logarithms $\log(\cdot)$ are of the base 2.

CHAPTER IV

AN OUTER BOUND ON CAPACITY REGION FOR THE TWO-USER GAUSSIAN X CHANNEL WITH LIMITED RECEIVER COOPERATION

In this chapter, we provide an outer bound on the capacity region containing upper bounds for the two-user Gaussian X channel with limited receiver cooperation in Lemma 4.1. Ideas for proving upper bounds are outlined in Section 4.1 and all details are given in Section 4.3. This outer bound is used to evaluate the performance of our proposed strategy in the case of strong Gaussian X channel type I which details are shown completely in Chapter VI.

4.1 An Outer Bound

For finding an outer bound on the capacity region for the two-user Gaussian X channel with limited receiver cooperation, we use Fano's inequality, data processing inequality and genie-aided techniques, etc. The result is given in the following theorem.

Lemma 4.1 *$\mathscr{C} \subseteq \overline{\mathscr{C}}$, where an outer bound on the capacity region of the two-user Gaussian X channel with limited receiver cooperation $\overline{\mathscr{C}}$ consists of nonnegative rate quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ satisfying the following inequalities.*

$$R_{11} \leq \log(1 + \mathsf{SNR}_1) + \min\Big\{\mathsf{C}_{21}^{\mathsf{B}}, \log\Big(1 + \frac{\mathsf{INR}_2}{1 + \mathsf{SNR}_1}\Big)\Big\} \tag{4.1}$$

$$R_{12} \leq \log(1 + \mathsf{INR}_1) + \min\Big\{\mathsf{C}_{21}^{\mathsf{B}}, \log\Big(1 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_1}\Big)\Big\} \tag{4.2}$$

$$R_{21} \leq \log(1 + \mathsf{INR}_2) + \min\Big\{\mathsf{C}_{12}^{\mathsf{B}}, \log\Big(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\Big)\Big\} \tag{4.3}$$

$$R_{22} \leq \log(1 + \mathsf{SNR}_2) + \min\Big\{\mathsf{C}_{12}^{\mathsf{B}}, \log\Big(1 + \frac{\mathsf{INR}_1}{1 + \mathsf{SNR}_2}\Big)\Big\} \tag{4.4}$$

$$R_{11} + R_{12} \leq \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + \mathsf{C}_{21}^{\mathsf{B}} \tag{4.5}$$

$$R_{21} + R_{22} \leq \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_2) + \mathsf{C}_{12}^{\mathsf{B}} \tag{4.6}$$

$$R_{11}+R_{12} \leq \log\Big(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) - \log\Big(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big) \tag{4.7}$$

$$R_{21}+R_{22} \leq \log\Big(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) - \log\Big(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big) \tag{4.8}$$

$$R_{11}+R_{12}+R_{21} \leq \log(1+\mathsf{INR}_2)+\log(1+\mathsf{INR}_1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2})+\mathsf{C}^{\mathsf{B}}_{21}+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.9}$$

$$R_{11}+R_{12}+R_{22} \leq \log(1+\mathsf{SNR}_2)+\log(1+\mathsf{SNR}_1+\frac{\mathsf{INR}_1}{1+\mathsf{SNR}_2})+\mathsf{C}^{\mathsf{B}}_{21}+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.10}$$

$$R_{11}+R_{21}+R_{22} \leq \log(1+\mathsf{SNR}_1)+\log(1+\mathsf{SNR}_2+\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1})+\mathsf{C}^{\mathsf{B}}_{21}+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.11}$$

$$R_{12}+R_{21}+R_{22} \leq \log(1+\mathsf{INR}_1)+\log(1+\mathsf{INR}_2+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1})+\mathsf{C}^{\mathsf{B}}_{21}+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.12}$$

$$R_{11}+R_{12}+R_{21} \leq \log(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\log\Big(1+\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big)+\mathsf{C}^{\mathsf{B}}_{21} \tag{4.13}$$

$$R_{11}+R_{12}+R_{22} \leq \log(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\log\Big(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big)+\mathsf{C}^{\mathsf{B}}_{21} \tag{4.14}$$

$$R_{11}+R_{21}+R_{22} \leq \log(1+\mathsf{SNR}_2+\mathsf{INR}_2)+\log\Big(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big)+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.15}$$

$$R_{12}+R_{21}+R_{22} \leq \log(1+\mathsf{SNR}_2+\mathsf{INR}_2)+\log\Big(1+\frac{\mathsf{INR}_1}{1+\mathsf{SNR}_2}\Big)+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.16}$$

$$R_{11}+R_{12}+R_{21} \leq \log\Big(1+\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}+\mathsf{INR}_1+\mathsf{SNR}_2+\tfrac{\mathsf{INR}_2}{1+\mathsf{INR}_2}+\tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}_2}\Big) + \log(1+\mathsf{INR}_2)-\log\Big(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big)+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.17}$$

$$R_{11}+R_{12}+R_{22} \leq \log\Big(1+\mathsf{SNR}_1+\tfrac{\mathsf{INR}_1}{1+\mathsf{SNR}_2}+\tfrac{\mathsf{SNR}_2}{1+\mathsf{SNR}_2}+\mathsf{INR}_2+\tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{SNR}_2}\Big) + \log(1+\mathsf{SNR}_2)-\log\Big(1+\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big)+\mathsf{C}^{\mathsf{B}}_{12} \tag{4.18}$$

$$R_{11}+R_{21}+R_{22} \leq \log\Big(1+\tfrac{\mathsf{SNR}_1}{1+\mathsf{SNR}_1}+\mathsf{INR}_1+\mathsf{SNR}_2+\tfrac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}+\tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{SNR}_1}\Big) + \log(1+\mathsf{SNR}_1)-\log\Big(1+\frac{\mathsf{INR}_1}{1+\mathsf{SNR}_2}\Big)+\mathsf{C}^{\mathsf{B}}_{21} \tag{4.19}$$

$$R_{12}+R_{21}+R_{22} \leq \log\Big(1+\mathsf{SNR}_1+\tfrac{\mathsf{INR}_1}{1+\mathsf{INR}_1}+\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}+\mathsf{INR}_2+\tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}_1}\Big) + \log(1+\mathsf{INR}_1)-\log\Big(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big)+\mathsf{C}^{\mathsf{B}}_{21} \tag{4.20}$$

$$R_{11}+R_{12}+R_{21} \leq \log\Big(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) - \log\Big(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big) \tag{4.21}$$

$$R_{11}+R_{12}+R_{22} \leq \log\Big(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) - \log\Big(1+\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big) \tag{4.22}$$

$$R_{11} + R_{21} + R_{22} \leq \log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\right) - \log\left(1 + \frac{\mathsf{INR}_1}{1 + \mathsf{SNR}_2}\right) \tag{4.23}$$

$$R_{12} + R_{21} + R_{22} \leq \log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\right) - \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) \tag{4.24}$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\left(1 + \mathsf{SNR}_1 + \frac{\mathsf{INR}_1}{1 + \mathsf{SNR}_2}\right) + \log\left(1 + \mathsf{SNR}_2 + \frac{\mathsf{INR}_2}{1 + \mathsf{SNR}_1}\right) + \mathsf{C}^{\mathsf{B}}_{12} + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.25}$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\left(1 + \mathsf{INR}_1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) + \log\left(1 + \mathsf{INR}_2 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_1}\right) + \mathsf{C}^{\mathsf{B}}_{12} + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.26}$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\left(1 + \tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2} + \mathsf{INR}_1 + \mathsf{SNR}_2 + \tfrac{\mathsf{INR}_2}{1+\mathsf{INR}_2} + \tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}_2}\right) + \log\left(1 + \mathsf{INR}_2 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_1}\right) - \log\left(1 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_1}\right) + \mathsf{C}^{\mathsf{B}}_{12} \tag{4.27}$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\left(1 + \mathsf{SNR}_1 + \tfrac{\mathsf{INR}_1}{1+\mathsf{INR}_1} + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1} + \mathsf{INR}_2 + \tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}_1}\right) + \log\left(1 + \mathsf{INR}_1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) - \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.28}$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\begin{pmatrix} 1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 \\ +|h_{11}h_{22} - h_{12}h_{21}|^2 \end{pmatrix} \tag{4.29}$$

$$2R_{11} + R_{12} + R_{21} + R_{22} \leq \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) + \log\left(1 + \mathsf{INR}_2 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_1}\right) + \mathsf{C}^{\mathsf{B}}_{12} + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.30}$$

$$R_{11} + 2R_{12} + R_{21} + R_{22} \leq \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + \log\left(1 + \frac{\mathsf{INR}_1}{1 + \mathsf{SNR}_2}\right) + \log\left(1 + \mathsf{SNR}_2 + \frac{\mathsf{INR}_2}{1 + \mathsf{SNR}_1}\right) + \mathsf{C}^{\mathsf{B}}_{12} + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.31}$$

$$R_{11} + R_{12} + 2R_{21} + R_{22} \leq \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_2) + \log\left(1 + \frac{\mathsf{INR}_2}{1 + \mathsf{SNR}_1}\right) + \log\left(1 + \mathsf{SNR}_1 + \frac{\mathsf{INR}_1}{1 + \mathsf{SNR}_2}\right) + \mathsf{C}^{\mathsf{B}}_{12} + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.32}$$

$$R_{11} + R_{12} + R_{21} + 2R_{22} \leq \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_2) + \log\left(1 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_1}\right) + \log\left(1 + \mathsf{INR}_1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) + \mathsf{C}^{\mathsf{B}}_{12} + \mathsf{C}^{\mathsf{B}}_{21} \tag{4.33}$$

$$2R_{11} + R_{12} + R_{21} + R_{22} \leq \log\Big(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\Big) + \log\Big(1 + \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big) + \log\Big(1 + \mathsf{INR}_2 + \frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big) - \log\Big(1 + \frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big) + \mathsf{C}_{12}^{\mathsf{B}} \tag{4.34}$$

$$R_{11} + 2R_{12} + R_{21} + R_{22} \leq \log\Big(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\Big) + \log\Big(1 + \frac{\mathsf{INR}_1}{1+\mathsf{SNR}_2}\Big) + \log\Big(1 + \mathsf{SNR}_2 + \frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big) - \log\Big(1 + \frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big) + \mathsf{C}_{12}^{\mathsf{B}} \tag{4.35}$$

$$R_{11} + R_{12} + 2R_{21} + R_{22} \leq \log\Big(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\Big) + \log\Big(1 + \frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big) + \log\Big(1 + \mathsf{SNR}_1 + \frac{\mathsf{INR}_1}{1+\mathsf{SNR}_2}\Big) - \log\Big(1 + \frac{\mathsf{INR}_1}{1+\mathsf{SNR}_2}\Big) + \mathsf{C}_{21}^{\mathsf{B}} \tag{4.36}$$

$$R_{11} + R_{12} + R_{21} + 2R_{22} \leq \log\Big(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\Big) + \log\Big(1 + \frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big) + \log\Big(1 + \mathsf{INR}_1 + \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big) - \log\Big(1 + \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big) + \mathsf{C}_{21}^{\mathsf{B}} \tag{4.37}$$

$$2R_{11} + R_{12} + R_{21} + R_{22} \leq \log\Big(1 + \mathsf{SNR}_1 + \tfrac{\mathsf{INR}_1}{1+\mathsf{INR}_1} + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1} + \mathsf{INR}_2 + \tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}_1}\Big) + \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + \mathsf{C}_{21}^{\mathsf{B}} \tag{4.38}$$

$$R_{11} + 2R_{12} + R_{21} + R_{22} \leq \log\Big(1 + \tfrac{\mathsf{SNR}_1}{1+\mathsf{SNR}_1} + \mathsf{INR}_1 + \mathsf{SNR}_2 + \tfrac{\mathsf{INR}_2}{1+\mathsf{SNR}_1} + \tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{SNR}_1}\Big) + \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + \mathsf{C}_{21}^{\mathsf{B}} \tag{4.39}$$

$$R_{11} + R_{12} + 2R_{21} + R_{22} \leq \log\Big(1 + \mathsf{SNR}_1 + \tfrac{\mathsf{INR}_1}{1+\mathsf{SNR}_2} + \tfrac{\mathsf{SNR}_2}{1+\mathsf{SNR}_2} + \mathsf{INR}_2 + \tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{SNR}_2}\Big) + \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_2) + \mathsf{C}_{12}^{\mathsf{B}} \tag{4.40}$$

$$R_{11} + R_{12} + R_{21} + 2R_{22} \leq \log\Big(1 + \tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2} + \mathsf{INR}_1 + \mathsf{SNR}_2 + \tfrac{\mathsf{INR}_2}{1+\mathsf{INR}_2} + \tfrac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}_2}\Big) + \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_2) + \mathsf{C}_{12}^{\mathsf{B}} \tag{4.41}$$

Proof: All above bounds can be upper bounded by mutual information via Fano's inequality and data processing inequality. Next, they are decomposed into two parts: 1) terms which are similar to ones in Gaussian X channels without cooperation and 2) terms which indicate the enhancement from cooperation. To upper bound the first part, we use the genie-aided techniques such as [2, 9, 14] where genies provide side information signals that are carefully chosen to the receivers. In the second part, we consider that both cooperation signals u_{21}^N and u_{12}^N are a function of both received signals (y_1^N, y_2^N) and use the straightforward bounding techniques for others. Details are given thoroughly in Section 4.3.

Next, for $i, j = 1, 2$ and $i \neq j$, we give a brief outline for our proposed outer bound as follows:

First of all, bounds (4.1)−(4.4) are the upper bounds of individual rates. In the genie-aided channel, a genie gives side information x_j^N to receiver i for upper bounds R_{ii} and x_i^N to receiver i for upper bounds R_{ij}. Therefore, there is no interference at receiver i. The gain of receiver cooperation is the minimum value between $\mathsf{C}_{ji}^{\mathsf{B}}$ and $\log\left(1+\frac{\mathsf{INR}_j}{1+\mathsf{SNR}_i}\right)$ for (4.1) and (4.4) and $\mathsf{C}_{ij}^{\mathsf{B}}$ and $\log\left(1+\frac{\mathsf{SNR}_j}{1+\mathsf{INR}_i}\right)$ for (4.2) and (4.3).

Bounds (4.5)−(4.6) on $R_{ii} + R_{ij}$ are straightforward cut-set upper bounds of the sum of 2 rates by setting $m_{ji} = m_{jj} = \phi$. The gain of cooperation from receiver *i* to *j* is upper bounded by $\mathsf{C}_{ij}^{\mathsf{B}}$.

Bounds (4.7)−(4.8) are derived by providing side information y_i^N and m_{ji} to receiver i. In these cases, the gain of cooperation from receiver *j* to *i* is absorbed into a power gain[1].

Bounds (4.9)−(4.12) correspond to the Z-channel bounds. A genie gives interfering information s_{ji}^N and m_{ji} to receiver i and x_j^N and m_{ii} to receiver j for bounds (4.9) and (4.12) and s_{jj}^N and m_{jj} to receiver i and x_i^N and m_{ij} to receiver j for bounds (4.10) and (4.11). The gain of receiver cooperation is the sum of $\mathsf{C}_{ij}^{\mathsf{B}}$ and $\mathsf{C}_{ji}^{\mathsf{B}}$.

Bounds (4.13)−(4.16) also correspond to the Z-channel bounds and the Huang-Cadambe-Jafar (HCJ) upper bounds for the Gaussian X channel without cooperation [5]. For these bounds, a genie gives y_j^N to receiver i for $i, j = 1, 2$ and $i \neq j$. Therefore, the gain from receiver j to i is absorbed into the power gain and the other gain is upper bounded by $\mathsf{C}_{ij}^{\mathsf{B}}$.

Bounds (4.17) and (4.20) are derived by providing side information y_j^N, s_{ji}^N and m_{ji} to receiver i and x_j^N and m_{ii} to receiver j. Bounds (4.18) and (4.19) are derived by providing side information y_j^N, s_{jj}^N and m_{jj} to receiver i and x_i^N and m_{ij} to receiver j. Since a genie gives x_j^N and m_{ii} to receiver j for (4.17) and (4.20) and x_i^N and m_{ij} to receiver j for (4.18) and (4.19), i.e., it means that there is no interference at receiver j, therefore, the cooperation from receiver *j* to *i* provides the power gain. In addition, the gain of cooperation from receiver *i* to *j* is upper bounded by $\mathsf{C}_{ij}^{\mathsf{B}}$.

[1]The power gain which is identified in [2] occurs in the saturation region where receiver cooperation is inefficient. This gain is also bounded, regardless of the cooperation rate.

Bounds (4.21) and (4.22) are derived by giving side information y_j^N to receiver i and y_i^N, m_{ii} and m_{ij} to receiver j. In addition, bounds (4.23) and (4.24) are derived by giving side information y_j^N, m_{ji} and m_{jj} to receiver i and y_i^N to receiver j. The gain from the receiver cooperation in both sides is absorbed into a power gain.

Bounds (4.25)$-$(4.26) correspond to the Etkin-Tse-Wang (ETW) upper bounds for the interference channel without cooperation [9] which is extended to the Gaussian X channel without cooperation [5]. In the genie-aided channel, a genie give side information $\tilde{s}_{jj}^N$ and m_{jj} to receiver i for (4.25) and $\tilde{s}_{ji}^N$ and m_{ji} to receiver i for (4.26). The gain of receiver cooperation is upper bounded by $\mathsf{C}_{12}^{\mathsf{B}} + \mathsf{C}_{21}^{\mathsf{B}}$.

Bounds (4.27)$-$(4.28) on $R_{11} + R_{12} + R_{21} + R_{22}$ are derived by giving side information y_j^N and $\tilde{s}_{ji}^N$ to receiver i and $\tilde{s}_{ij}^N$ to receiver j. Since a genie gives y_j^N to receiver i, therefore, the gain from receiver j to i is absorbed into the power gain and the other gain is upper bounded by $\mathsf{C}_{ij}^{\mathsf{B}}$.

Bound (4.29) is straightforward cut-set upper bound of the sum of 4 rates.

Bounds (4.30)$-$(4.33) correspond to the Niesen-Maddah-Ali (NMA) upper bounds for the Gaussian X channel without cooperation [12]. In the genie-aided channel, the gain of receiver cooperation is upper bounded by $\mathsf{C}_{12}^{\mathsf{B}} + \mathsf{C}_{21}^{\mathsf{B}}$.

Bounds (4.34) and (4.37) on $2R_{ii}+R_{ij}+R_{ji}+R_{jj}$ are derived by giving side information y_j^N to the first receiver i, y_j^N and x_j^N to the second receiver i and $\tilde{s}_{ij}^N$ to receiver j. Similarly, bounds (4.35) and (4.36) on $R_{ii} + 2R_{ij} + R_{ji} + R_{jj}$ are derived by giving side information y_j^N to the first receiver i, y_j^N and x_i^N to the second receiver i and $\tilde{s}_{ii}^N$ to receiver j. In the genie-aided channel, the structure based on Z-channel is created and thus the gain from one direction of receiver cooperation is absorbed into the power gain. The other gain is upper bounded by $\mathsf{C}_{ij}^{\mathsf{B}}$.

Bounds (4.38) and (4.41) on $2R_{ii}+R_{ij}+R_{ji}+R_{jj}$ are derived by giving side information y_j^N to the first receiver i and y_i^N and $\tilde{s}_{ij}^N$ to receiver j. Similarly, bounds (4.39) and (4.40) on $R_{ii} + 2R_{ij} + R_{ji} + R_{jj}$ are derived by giving side information y_j^N to the first receiver i and y_i^N and $\tilde{s}_{ii}^N$ to receiver j. In the genie-aided channel, the point-to-point MIMO channel is created and thus the gain from both directions of receiver cooperation is absorbed into the multiple-antenna systems. The other gain is upper bounded by $\mathsf{C}_{ij}^{\mathsf{B}}$.

Note that the derivation of all bounds works for all SNR's and INR's. ■

Remark 4.2 (Dependence on Phases) *The sum-rate upper bounds on (4.7)–(4.8), (4.17)–(4.24), (4.27)– (4.29) and (4.34)–(4.41) not only depend on* SNR*'s and* INR*'s but also on the phases of channel coefficients, i.e.,* $|h_{11}h_{22} - h_{12}h_{21}|^2$*. When these upper bounds are active, an outer bound depends on phases [2].*

Next, the effectiveness of our proposed outer bound is given in the following section.

4.2 Effectiveness of the proposed outer bound

To test the effectiveness of an outer bound proposed in Lemma 4.1, we compare this result with the existing results [2–4, 14, 15, 31] in the two ways as follows:

1. Comparison of some upper bounds in Lemma 4.1 with the known results in the following two cases:

   - Non-cooperation case:

     (a) Substituting $\mathsf{C}_{12}^{\mathsf{B}} = \mathsf{C}_{21}^{\mathsf{B}} = 0$ in the upper bounds (4.13)–(4.16) and (4.25)–(4.26), it is easily seen that these upper bounds are identical to the results of Lemma 5.2 and Theorem 5.3, respectively, in [14]. In addition, the upper bounds (4.30)–(4.33) with setting $\mathsf{C}_{12}^{\mathsf{B}} = \mathsf{C}_{21}^{\mathsf{B}} = 0$ are also the same as the result of Lemma 10 in [15].

     (b) Considering the case where each transmitter has only one message by letting $m_{12} = m_{21} = \phi$, i.e., $R_{12} = R_{21} = 0$, and substituting $\mathsf{C}_{12}^{\mathsf{B}} = \mathsf{C}_{21}^{\mathsf{B}} = 0$ in the upper bounds (4.1), (4.4), (4.14)–(4.15), (4.26), (4.30) and (4.33), it is easily seen that these bounds are the same as all upper bounds in an outer bound on the capacity region for the two-user deterministic interference channel in [3, 4].

   - Receiver cooperation case:

     (a) Considering the case where each transmitter has only one message by letting $m_{12} = m_{21} = \phi$, i.e., $R_{12} = R_{21} = 0$, it is easily seen that the set of the upper bounds consisting of (4.1), (4.4), (4.14)–(4.15), (4.26), (4.29)–(4.30), (4.33), (4.38) and (4.41) is identical to an outer bound on the capacity region for the two-user Gaussian interference channel with limited receiver cooperation, i.e, Lemma 5.1 in [2].

(b) Considering the case where each transmitter has only one message by letting $m_{12} = m_{21} = \phi$, i.e., $R_{12} = R_{21} = 0$, and letting that receiver 1 suffers from interference and noise but receiver 2 suffers only from noise, it is easily seen that the set of the upper bounds consisting of (4.1) with $\mathsf{C}^{\mathsf{B}}_{21} = 0$, (4.4), (4.14) and (4.29) with some modifications and disappearance of h_{12} is the same as an outer bound on the capacity region for the two-user asymmetric interference channel with limited receiver cooperation, i.e., Theorem 2 in [31].

(c) Comparing (4.29) with (9) in Lemma 5.1 [2] without setting any parameters in our channel model as (i) and (ii) above, we see obviously that both upper bounds from the different channel model are the same.

Conclusion: With considerations above, **it can be easily seen that** some of the proposed upper bounds in Lemma 4.1 are the same as the existing results [2–4, 14, 15, 31] by setting a certain set of parameters. *This means that* these bounds are more generalized than those in several communication scenarios.

2. Comparison of the region of the proposed outer bound with the region of an outer bound of the two-user Gaussian interference channel with limited receiver cooperation [2] and that of the two-user Gaussian interference without receiver cooperation [3, 4] through some numerical examples.

   In this comparison, it consists of the following 3 steps:

   (a) *Defining parameters:* In order to compare our result with the existing results [2–4] properly, we define the following parameters. Let

   - $R_1 = R_{11} + R_{12}$;
   - $R_2 = R_{21} + R_{22}$;
   - $R_1 + R_2 = R_{11} + R_{12} + R_{21} + R_{22}$;
   - $2R_1 + R_2 = 2R_{11} + 2R_{12} + R_{21} + R_{22}$;
   - $R_1 + 2R_2 = R_{11} + R_{12} + 2R_{21} + 2R_{22}$.

Remind that $\mathsf{SNR}_i = |h_{ii}|^2$ and $\mathsf{INR}_i = |h_{ij}|^2$ for $i, j = 1, 2$ and $i \neq j$.

(b) *Calculating parameters:* Based on the assigned parameters in the first step, we can calculate each parameter from our results as follows:

- $R_1 = \min\{(4.1) + (4.2), (4.5), (4.7)\}$;
- $R_2 = \min\{(4.3) + (4.4), (4.6), (4.8)\}$;
- $R_1 + R_2 = \min\{(4.25), (4.26), (4.27), (4.28), (4.29)\}$;
- $2R_1 + R_2 = \min\{(4.25) + (4.5), (4.26) + (4.5), (4.27) + (4.5), (4.28)+(4.5), (4.29)+(4.5), (4.25)+(4.7), (4.26)+(4.7), (4.27)+(4.7), (4.28)+(4.7), (4.29)+(4.7), (4.30)+(4.2), (4.31)+(4.1), (4.34)+(4.2), (4.35)+(4.1), (4.38)+(4.2), (4.39)+(4.1), (4.9)+(4.10), (4.13)+(4.14), (4.17) + (4.18), (4.21) + (4.22)\}$;
- $R_1 + 2R_2 = \min\{(4.25) + (4.6), (4.26) + (4.6), (4.27) + (4.6), (4.28)+(4.6), (4.29)+(4.6), (4.25)+(4.8), (4.26)+(4.8), (4.27)+(4.8), (4.28)+(4.8), (4.29)+(4.8), (4.32)+(4.4), (4.33)+(4.3), (4.36)+(4.4), (4.37)+(4.3), (4.40)+(4.4), (4.41)+(4.3), (4.11)+(4.12), (4.15)+(4.16), (4.19) + (4.20), (4.23) + (4.24)\}$.

(c) *Plotting the region*: Following the same method in [20], we can plot the region of the proposed outer bound from the received results in the second step.

Next, using the three steps above, we compare the region of our obtained result with the region of the outer bounds in the two-user Gaussian interference channel with limited receiver cooperation [2] and without cooperation [3,4] in the following three cases.

- **Case 1:** We consider the symmetric case where $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 20$ dB, $\mathsf{INR}_1 = \mathsf{INR}_2 = 15$ dB and $\mathsf{C}^{\mathsf{B}}_{12} = \mathsf{C}^{\mathsf{B}}_{21} = 2$. The received result (see Figure 4.1) shows that our proposed outer bound is *a superset* of the outer bounds [2] and [3, 4]. Furthermore, we observe that the maximum gap between our proposed outer bound and the outer bounds [2] and [3,4] are 2 and 2.393 bits/s/Hz, respectively.
- **Case 2:** We consider the symmetric case where $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 20$ dB, $\mathsf{INR}_1 = \mathsf{INR}_2 =7$ dB and $\mathsf{C}^{\mathsf{B}}_{12} = \mathsf{C}^{\mathsf{B}}_{21} = 2$. In this case, the values of INR_1

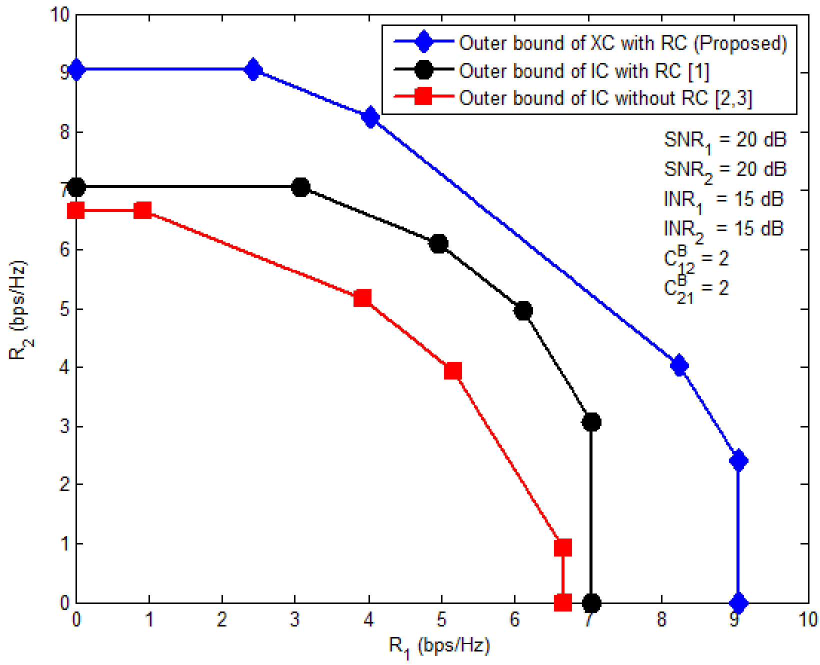


Figure 4.1: Comparison of our proposed outer bound with outer bounds [2] and [3,4] where $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 20$ dB, $\mathsf{INR}_1 = \mathsf{INR}_2 = 15$ dB and $\mathsf{C}^{\mathsf{B}}_{12} = \mathsf{C}^{\mathsf{B}}_{21} = 2$

and INR_2 are reduced from 15 dB in the first case to 7 dB. The obtained re-

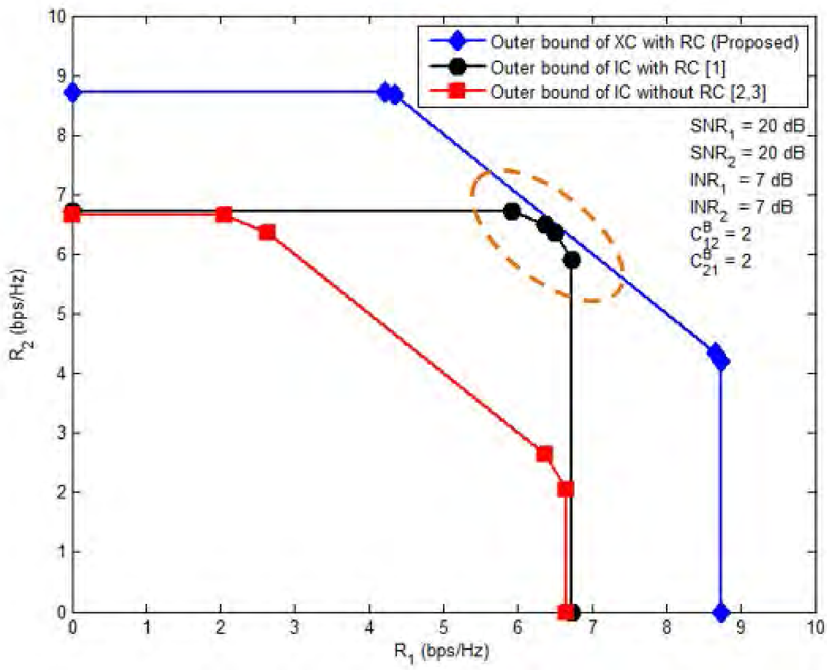


Figure 4.2: Comparison of our proposed outer bound with outer bounds [2] and [3,4] where $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 20$ dB, $\mathsf{INR}_1 = \mathsf{INR}_2 = 7$ dB and $\mathsf{C}^{\mathsf{B}}_{12} = \mathsf{C}^{\mathsf{B}}_{21} = 2$

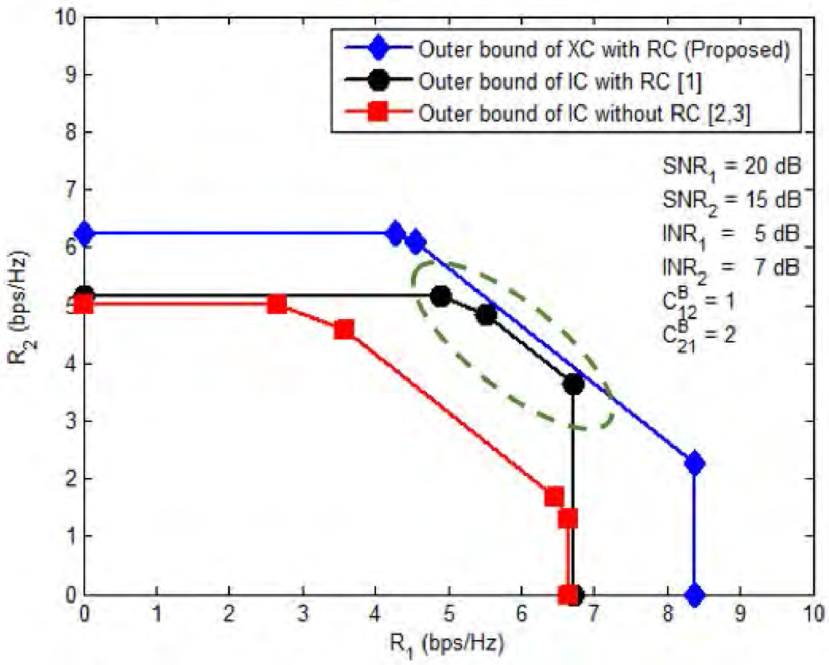


Figure 4.3: Comparison our proposed outer bound with outer bounds [2] and [3, 4] where $\mathsf{SNR}_1 = 20$ dB, $\mathsf{SNR}_2 = 15$ dB, $\mathsf{INR}_1 = 5$ dB, $\mathsf{INR}_2 = 7$ dB, $\mathsf{C}^{\mathsf{B}}_{12} = 1$ and $\mathsf{C}^{\mathsf{B}}_{21} = 2$

sult (see Figure 4.2) shows that our proposed outer bound is *a superset* of the outer bounds [2] and [3, 4]. Furthermore, we observe that the maximum gap between our proposed outer bound and the outer bounds [2] and [3, 4] are 2 and 2.04 bits/s/Hz, respectively. However, there is a region (indicated by the dashed ellipse) where the gap between our outer bound and the outer bound [2] approaches to zero.

- **Case 3:** We consider the asymmetric case where $\mathsf{SNR}_1 = 20$ dB, $\mathsf{SNR}_2 = 15$ dB, $\mathsf{INR}_1 = 7$ dB, $\mathsf{INR}_2 = 5$ dB, $\mathsf{C}^{\mathsf{B}}_{12} = 1$, $\mathsf{C}^{\mathsf{B}}_{21} = 1$. The received result (see Figure 4.3) shows that our proposed outer bound is *a superset* of the outer bounds [2] and [3,4]. Furthermore, we observe that the maximum gap between our proposed outer bound and the outer bounds [2] and [3, 4] are 1.073 and 1.206 bits/s/Hz, respectively. The result in this case is similar to the result in the second case, i.e., there is a region (indicated by the dashed ellipse) where the gap between our outer bound and the outer bound [2] closes to zero.

Conclusion: From three comparison cases above, based on defining parameters in the first stage for reducing our results from 4 dimensions to 2 dimensions, **it is obvi-**

ously seen that the outer bounds in the two-user Gaussian interference channel with limited receiver cooperation [2] and without cooperation [3,4] are obviously **subsets** of our proposed outer bound.

Next, in addition to compare between our proposed outer bound and the outer bound [2], we compare above two outer bound with an achievable rate region from our result in Chapter VI and an achievable rate region [2].

Comparisons of outer bounds and achievable rate regions

For better understanding comparisons, we first define $Outer_{proposed}$, $Outer_{IC-RC}$, $Achievable_{proposed}$ and $Achievable_{IC-RC}$ as our proposed outer bound, the outer bound [2], an achievable rate region from our result in Chapter VI and an achievable rate region in [2], respectively. Comparisons of outer bound and achievable rate region of our work with those of the work [2] have details as follows:

- When comparing our proposed outer bound with the outer bound [2] as shown in the three cases above (see Figure 4.1– Figure 4.3), the obtained results can be concluded that our proposed outer bound is *a superset* of the outer bound [2], i.e., $Outer_{proposed} \supseteq Outer_{IC-RC}$. Especially, the first symmetric case, the maximum gap between our proposed outer bound and the outer bound [2] is 2 bps/Hz.

- When comparing our proposed outer bound and an achievable rate region of our proposed strategy in the strong Gaussian X channel type I case (see more details in Chapter VI) with the same defining parameters in the the first step, we obtain that the maximum gap of this comparison is 3 bps/Hz. This gap value is larger than the maximum gap of comparing between our proposed outer bound and the outer bound [2] in the first symmetric case. Therefore, we can state that our proposed outer bound and the outer bound [2] are *a superset* of an achievable rate region of our proposed strategy in the strong Gaussian X channel type I case, i.e., $Outer_{proposed} \supseteq Outer_{IC-RC} \supseteq Achievable_{proposed}$.

- When comparing the outer bound and an achievable rate region in [2], Wang and Tse [2] show that the gap between them is equal to 2 bits/s/Hz/user at the most, independent of channel parameters. Hence, we obtain that the outer bound in [2] is *a superset* of an achievable rate region in [2], i.e., $Outer_{IC-RC} \supseteq Achievable_{IC-RC}$.

The results from three comparisons above can be concluded that

$$Outer_{proposed} \supseteq Outer_{IC-RC} \supseteq Achievable_{proposed} \supseteq Achievable_{IC-RC}.$$

Remark 4.3 *All upper bounds in Lemma 4.1 have the following relationship:*

$$\mathrm{A} = \mathrm{B} \cup \mathrm{C} = \mathrm{C}^c \cup \mathrm{C}$$

where A, B and C denote the set of all upper bounds (4.1)–(4.41), the set of upper bounds (4.5)–(4.12), (4.27)–(4.28), and (4.34)–(4.41) which is the novel result and the set of upper bounds (4.1)–(4.4), (4.13)– (4.26) and (4.30)–(4.37) which is also the novel result and can cover the previous results in [2,4,14], respectively, by specifying some parameters suitably in these bounds.

Remark 4.4 (Benefit of Our Result) *The obtained outer bound tells us about communication limits of the two-user Gaussian X channel with limited receiver cooperation. In addition, it can be used to assess the performance of any proposed methods for transmitting messages in this channel.*

4.3 Proof of Lemma 4.1

In this section, we give the details for proving Theorem 4.1 which are based on the genie-aided techniques [2, 9, 14]. For this proof, we define auxiliary information s_{ij} and side information $\tilde{s}_{ij}$ as follows:

$$s_{ij} := h_{ij}x_j + z_i, \quad \tilde{s}_{ij} := h_{ij}x_j + \tilde{z}_i$$

where, for $i, j = 1, 2$, z_i and $\tilde{z}_i$ are i.i.d $\mathcal{CN}(0, 1)$ and independent of everything else. Both s_{ij} and $\tilde{s}_{ij}$ have the same marginal distribution.

Bounds (4.1) on R_{11}, (4.2) on R_{12}, (4.3) on R_{21} and (4.4) on R_{22}:

Proof: We compute the upper bound on (4.1) by using Fano's inequality, data processing inequality, and chain rule: if R_{11} is achievable,

$$\begin{aligned}
N(R_{11}-\epsilon_N) &\overset{(a)}{\le} I(m_{11}; y_1^N, u_{21}^N)\\
&\overset{(b)}{\le} I(m_{11}; y_1^N, u_{21}^N, x_2^N)\\
&\overset{(c)}{\le} I(m_{11}; y_1^N, u_{21}^N | x_2^N)\\
&\overset{(d)}{=} I(m_{11}; y_1^N | x_2^N) + I(m_{11}; u_{21}^N | y_1^N, x_2^N)\\
&= h(y_1^N|x_2^N) - h(y_1^N|x_2^N, m_{11}) + I(m_{11}; u_{21}^N | y_1^N, x_2^N)\\
&\le h(y_1^N|x_2^N) - h(y_1^N|x_1^N, x_2^N) + I(m_{11}; u_{21}^N | y_1^N, x_2^N)\\
&\overset{(e)}{\le} N\log\big(1+\mathsf{SNR}_1\big) + I(m_{11}; u_{21}^N | y_1^N, x_2^N)
\end{aligned}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to a genie providing side information x_2^N to receiver 1. (c) is due to the fact that m_{11} and x_2^N are independent. (d) is due to chain rule. (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Next, we upper bound term $I(m_{11}; u_{21}^N | y_1^N, x_2^N)$ which is the augmentation from cooperation by using the relationship u_{21}^N is a function of (y_1^N, y_2^N) as follows:

$$\begin{aligned}
I(m_{11}; u_{21}^N | y_1^N, x_2^N) &= h(m_{11}|y_1^N, x_2^N) - h(m_{11}|x_2^N, y_1^N, u_{21}^N)\\
&\overset{(a)}{\le} h(m_{11}|y_1^N, x_2^N) - h(m_{11}|x_2^N, y_1^N, u_{21}^N, y_2^N)\\
&\overset{(b)}{=} h(m_{11}|y_1^N, x_2^N) - h(m_{11}|x_2^N, y_1^N, y_2^N)\\
&= I(m_{11}; y_2^N | y_1^N, x_2^N)\\
&= h(y_2^N|y_1^N, x_2^N) - h(y_2^N|y_1^N, x_2^N, m_{11}, m_{21})\\
&\le h(y_2^N|y_1^N, x_2^N) - h(y_2^N|y_1^N, x_2^N, x_1^N)\\
&= h(s_{21}^N|s_{11}^N) - h(z_2^N|z_1^N)\\
&\le N\log\Big(1+\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_1}\Big)
\end{aligned}$$

where (a) is due to the fact that conditioning reduces entropy. (b) is due to the fact that u_{21}^N is a function of (y_1^N, y_2^N). In addition, we can see that term $I(m_{11}; u_{21}^N | y_1^N, x_2^N) \le H(u_{21}^N) \le N\mathsf{C}_{21}^{\mathsf{B}}$.

Hence, similarly if a genie provides side information x_1^N to receiver 1 for R_{12}, x_2^N to receiver 2 for R_{21} and x_2^N to receiver 2 for R_{22}, we have bounds (4.1)−(4.4). ∎

Bounds (4.5) on $R_{11} + R_{12}$ and (4.6) on $R_{21} + R_{22}$:

Proof: In this proof, we show only bound (4.5) and the other bound can be shown similarly. To upper bound (4.5), we set $m_{21} = m_{22} = \phi$. If (R_{11}, R_{12}) is achievable, we can write

$$\begin{aligned} N(R_{11} + R_{12} - \epsilon_N) &\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) \\ &\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N | y_1^N) \\ &\overset{(c)}{\leq} h(y_1^N) - h(y_1^N | m_{11}, m_{12}) + \mathrm{H}(u_{21}^N) \\ &\overset{(d)}{\leq} N \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + N \mathsf{C}_{21}^{\mathsf{B}} \end{aligned}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule. (c) is due to the fact that $I(m_{11}; u_{21}^N | y_1^N, x_2^N) \leq H(u_{21}^N)$. (d) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints and $H(u_{21}^N) \leq N\mathsf{C}_{21}^{\mathsf{B}}$.

Hence, similarly for $R_{21} + R_{22}$, we have bounds (4.5)–(4.6). ■

Bounds (4.7) on $R_{11} + R_{12}$ and (4.8) on $R_{21} + R_{22}$:

Proof: In this proof, we show only bound (4.7) and the other bound can be shown similarly. To upper bound (4.7), a genie provides side information y_2^N and m_{21} to receiver 1. If (R_{11}, R_{12}) is achievable, we can write

$$\begin{aligned} N(R_{11} + R_{12} - \epsilon_N) &\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) \\ &\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N, m_{21}) \\ &\overset{(c)}{=} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N | m_{21}) \\ &\overset{(d)}{=} I(m_{11}, m_{12}; y_1^N, y_2^N | m_{21}) \\ &= h(y_1^N, y_2^N | m_{21}) - h(y_1^N, y_2^N | m_{11}, m_{12}, m_{21}) \\ &= h(y_1^N, y_2^N) - h(s_{12}^N, s_{22}^N | m_{12}) \\ &= h(y_1^N, y_2^N) - h(s_{12}^N | m_{12}) - h(s_{22}^N | s_{12}^N, m_{12}) \\ &\leq h(y_1^N, y_2^N) - h(z_1^N) - h(s_{22}^N | s_{12}^N) \\ &\overset{(e)}{\leq} N\{\text{RHS of (4.7)}\} \end{aligned}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to a genie providing side information y_2^N and m_{21} to receiver 1. (c) is due to

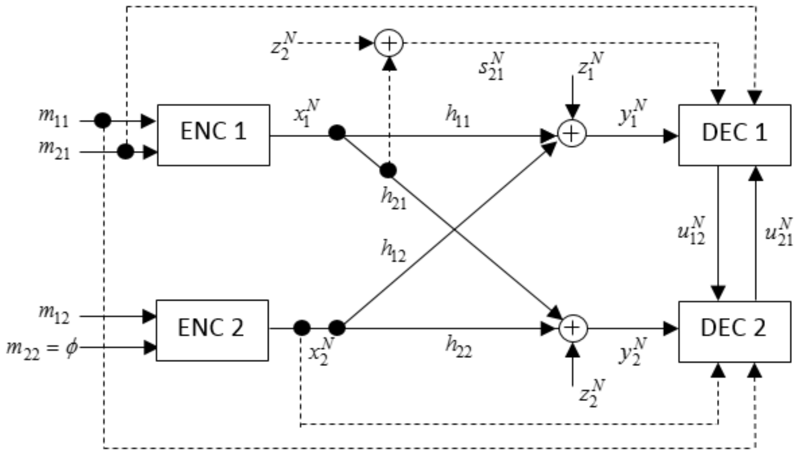


Figure 4.4 Side information structure for bound (4.9)

the fact that m_{11}, m_{12} and m_{21} are independent. (d) is due to the fact that u_{21}^N is a function of (y_1^N, y_2^N). (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, similarly if a genie provides y_1^N and m_{12} to receiver 2 for $R_{21} + R_{22}$, we have bounds (4.7)−(4.8). ■

Bounds (4.9) on $R_{11} + R_{12} + R_{21}$, (4.10) on $R_{11} + R_{12} + R_{22}$, (4.11) on $R_{11} + R_{21} + R_{22}$ and (4.12) on $R_{12} + R_{21} + R_{22}$:

Proof: In this proof, we show only (4.9) and other bounds can be shown similarly. To upper bound $R_{11} + R_{12} + R_{21}$, we set message $m_{22} = \phi$. A genie gives side information s_{21}^N and m_{21} to receiver 1 and x_2^N and m_{11} to receiver 2 (refer to Figure 4.4). If (R_{11}, R_{12}, R_{21}) is achievable, we can write

$$
\begin{aligned}
&N(R_{11} + R_{12} + R_{21} - \epsilon_N) \\
&\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N) \\
&\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N | y_1^N) + I(m_{21}; y_2^N) + I(m_{21}; u_{12}^N | y_2^N) \\
&\overset{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + H(u_{21}^N) + I(m_{21}; y_2^N) + H(u_{12}^N) \\
&\overset{(d)}{\leq} I(m_{11}, m_{12}; y_1^N, s_{21}^N, m_{21}) + I(m_{21}; y_2^N, x_2^N, m_{11}) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= I(m_{11}, m_{12}; y_1^N, s_{21}^N | m_{21}) + I(m_{21}; y_2^N | x_2^N, m_{11}) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= I(m_{11}, m_{12}; s_{21}^N | m_{21}) + I(m_{11}, m_{12}; y_1^N | s_{21}^N, m_{21}) + I(m_{21}; y_2^N | x_2^N, m_{11}) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}}
\end{aligned}
$$

$$= h(s_{21}^N|m_{21}) - h(s_{21}^N|m_{21}, m_{11}, m_{12}) + h(y_1^N|s_{21}^N, m_{21}) - h(y_1^N|s_{21}^N, m_{21}, m_{11}, m_{12})$$
$$+ h(y_2^N|x_2^N, m_{11}) - h(y_2^N|x_2^N, m_{11}, m_{21}) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}}$$
$$\leq h(s_{21}^N|m_{21}) - h(z_2^N) + h(y_1^N|s_{21}^N) - h(z_1^N) + h(s_{21}^N|m_{11}) - h(z_2^N) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}}$$
$$\stackrel{(e)}{\leq} N\{\text{RHS of (4.9)}\}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule. (c) is due to the fact that $I(m_{11}, m_{12}; u_{21}^N|y_1^N) \leq H(u_{21}^N)$ and $I(m_{21}; u_{12}^N|y_2^N) \leq H(u_{12}^N)$. (d) is due to the fact that a genie provides side information s_{21}^N and m_{21} to receiver 1 and x_2^N and m_{11} to receiver 2 and $H(u_{ij}^N) \leq N\mathsf{C}_{ij}^N$. (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, similarly if a genie provides side information s_{22}^N and m_{22} to receiver 1 and x_1^N and m_{12} to receiver 2 for $R_{11}+R_{12}+R_{22}$ (with setting $m_{21} = \phi$), x_2^N and m_{21} to receiver 1 and s_{11}^N and m_{11} to receiver 2 for $R_{11}+R_{21}+R_{22}$ (with setting $m_{12} = \phi$), and x_1^N and m_{22} to receiver 1 and s_{12}^N and m_{12} to receiver 2 for $R_{12}+R_{21}+R_{22}$ (with setting $m_{11} = \phi$), we have shown bounds (4.9)–(4.12). ■

Bounds (4.13) on $R_{11}+R_{12}+R_{21}$, (4.14) on $R_{11}+R_{12}+R_{22}$, (4.15) on $R_{11}+R_{21}+R_{22}$ and (4.16) on $R_{12}+R_{21}+R_{22}$:

Proof: In this proof, we show only (4.13) and other bounds can be shown similarly. To upper bound $R_{11}+R_{12}+R_{21}$, we set message $m_{22} = \phi$. Let a genie give side information y_1^N, m_{11} and m_{12} to receiver 2 (refer to Figure 4.5). If (R_{11}, R_{12}, R_{21}) is achievable, we obtain

$$N(R_{11} + R_{12} + R_{21} - \epsilon_N)$$
$$\stackrel{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N)$$
$$\stackrel{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N|y_1^N) + I(m_{21}; y_2^N, u_{12}^N, y_1^N, m_{11}, m_{12})$$
$$\stackrel{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + H(u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N, y_1^N|m_{11}, m_{12})$$
$$\stackrel{(d)}{\leq} I(m_{11}, m_{12}; y_1^N) + N\mathsf{C}_{21}^{\mathsf{B}} + I(m_{21}; y_1^N, y_2^N|m_{11}, m_{12})$$
$$= h(y_1^N) - h(y_1^N|m_{11}, m_{12}) + h(y_1^N, y_2^N|m_{11}, m_{12}) - h(y_1^N, y_2^N|m_{11}, m_{12}, m_{21}) + N\mathsf{C}_{21}^{\mathsf{B}}$$

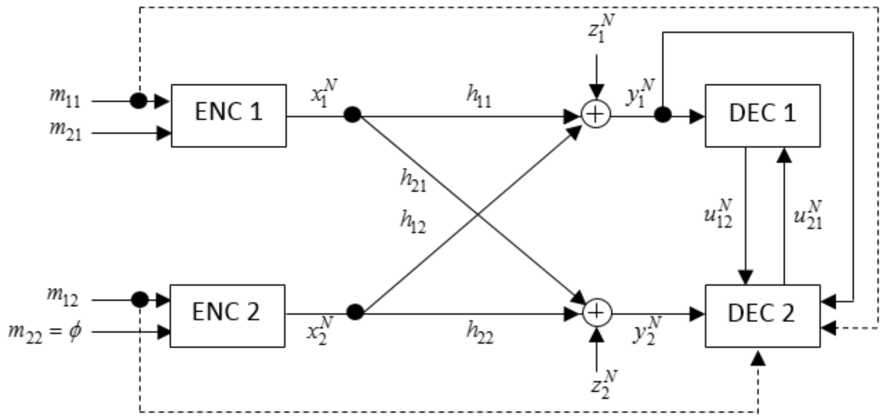


Figure 4.5 Side information structure for bound (4.13)

$$= h(y_1^N) - h(y_1^N|m_{11}, m_{12}) + h(y_1^N|m_{11}, m_{12}) + h(y_2^N|y_1^N, m_{11}, m_{12})$$
$$- h(y_1^N, y_2^N|m_{11}, m_{12}, m_{21}) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$\leq h(y_1^N) + h(y_2^N|y_1^N, m_{11}, m_{12}, x_2^N) - h(y_1^N, y_2^N|m_{11}, m_{12}, m_{21}, x_1^N, x_2^N) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$\leq h(y_1^N) + h(s_{21}^N|s_{11}^N) - h(z_1^N, z_2^N) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$\overset{(e)}{\leq} N\{\text{RHS of (4.13)}\}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule and a genie providing side information y_1^N, m_{11} and m_{12} to receiver 2. (c) is due to the fact that $I(m_{11}, m_{12}; u_{21}^N|y_1^N) \leq H(u_{21}^N)$ and messages m_{11}, m_{12} and m_{21} are independent. (d) is due to the fact that u_{21}^N is a function of (y_1^N, y_2^N) and $H(u_{21}^N) \leq N\mathsf{C}_{21}^N$. (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, similarly if a genie provides side information y_1^N, m_{11} and m_{12} to receiver 2 for $R_{11}+R_{12}+R_{22}$ (with setting $m_{21} = \phi$), y_2^N, m_{21} and m_{22} to receiver 1 for $R_{11}+R_{21}+R_{22}$ (with setting $m_{12} = \phi$), and y_2^N, m_{21} and m_{22} to receiver 1 for $R_{12}+R_{21}+R_{22}$ (with setting $m_{11} = \phi$), we have shown bounds (4.13)–(4.16). ■

Bounds (4.17) on $R_{11}+R_{12}+R_{21}$, (4.18) on $R_{11}+R_{12}+R_{22}$, (4.19) on $R_{11}+R_{21}+R_{22}$ and (4.20) on $R_{12}+R_{21}+R_{22}$:

Proof: In this proof, we show only (4.17) and other bounds can be shown similarly. Let a genie give side information y_2^N, s_{21}^N and m_{21} to receiver 1 and x_2^N and m_{11} to receiver

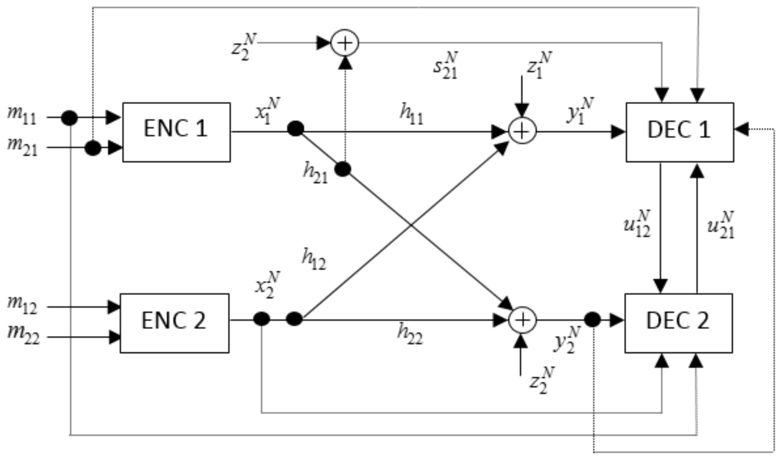


Figure 4.6 Side information structure for bound (4.17)

2 (refer to Figure 4.6). If (R_{11}, R_{12}, R_{21}) is achievable, we obtain

$$
\begin{aligned}
&N(R_{11} + R_{12} + R_{21} - \epsilon_N) \\
&\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N) \\
&\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N) + I(m_{21}; u_{12}^N|y_2^N) \\
&\overset{(c)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N, s_{21}^N, m_{21}) + I(m_{21}; y_2^N, x_2^N, m_{11}) + \mathrm{H}(u_{12}^N) \\
&\overset{(d)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N, s_{21}^N|m_{21}) + I(m_{21}; y_2^N|x_2^N, m_{11}) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(e)}{\leq} I(m_{11}, m_{12}; s_{21}^N|m_{21}) + I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N|s_{21}^N, m_{21}) + I(m_{21}; y_2^N|x_2^N, m_{11}) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(f)}{\leq} I(m_{11}, m_{12}; s_{21}^N|m_{21}) + I(m_{11}, m_{12}; y_1^N, y_2^N|s_{21}^N, m_{21}) + I(m_{21}; y_2^N|x_2^N, m_{11}) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= h(s_{21}^N|m_{21}) - h(s_{21}^N|m_{21}, m_{11}, m_{12}) + h(y_1^N, y_2^N|s_{21}^N, m_{21}) - h(y_1^N, y_2^N|s_{21}^N, m_{21}, m_{11}, m_{12}) \\
&+ h(y_2^N|x_2^N, m_{11}) - h(y_2^N|x_2^N, m_{11}, m_{21}) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= h(s_{21}^N|m_{21}) - h(z_2^N) + h(y_1^N, y_2^N|s_{21}^N) - h(s_{12}^N, s_{22}^N|m_{12}) + h(s_{21}^N|m_{11}) - h(z_2^N) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= h(y_1^N, y_2^N|s_{21}^N) - h(s_{12}^N|m_{12}) - h(s_{22}^N|s_{12}^N, m_{12}) + h(s_{21}^N|m_{21}) - h(z_2^N) + h(s_{21}^N|m_{11}) \\
&- h(z_2^N) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\leq h(y_1^N, y_2^N|s_{21}^N) - h(z_2^N) - h(s_{22}^N|s_{12}^N) + h(s_{21}^N) - h(z_2^N) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(g)}{\leq} N\{\text{RHS of } (4.17)\}
\end{aligned}
$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule. (c) is due to the fact that a genie provides side information y_2^N, s_{21}^N

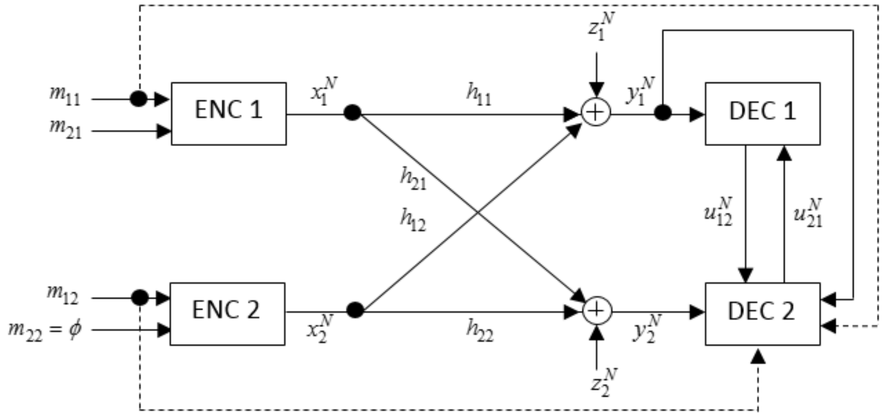


Figure 4.7 Side information structure for bound (4.21)

and m_{21} to receiver 1 and x_2^N and m_{11} to receiver 2 and $I(m_{21}; u_{12}^N|y_2^N) \leq H(u_{12}^N)$. (d) is due to the fact that messages m_{11}, m_{12} and m_{21} are independent and m_{11}, m_{21} and x_2^N are also independent and $H(u_{12}^N) \leq N\mathsf{C}_{21}^{\mathsf{B}}$. (e) is due to chain rule. (f) is due to the fact that u_{21}^N is a function of (y_1^N, y_2^N). (g) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, similarly if a genie provides side information y_2^N, s_{22}^N and m_{22} to receiver 1 and x_1^N and m_{12} to receiver 2 for $R_{11} + R_{12} + R_{22}$, x_2^N and m_{21} to receiver 1 and y_1^N, s_{11}^N and m_{11} to receiver 2 for $R_{11} + R_{21} + R_{22}$, x_1^N and m_{22} to receiver 1 and y_1^N, s_{12}^N and m_{12} to receiver 2 for $R_{12} + R_{21} + R_{22}$, we have shown bounds (4.17)−(4.20). ■

Bounds (4.21) on $R_{11}+R_{12}+R_{21}$*, (4.22) on* $R_{11}+R_{12}+R_{22}$*, (4.23) on* $R_{11}+R_{21}+R_{22}$ *and (4.24) on* $R_{12} + R_{21} + R_{22}$*:*

Proof: In this proof, we show only (4.21) and other bounds can be shown similarly. Let a genie gives side information y_2^N to receiver 1 and y_1^N, m_{11} and m_{12} to receiver 2 (refer to Figure 4.7). If (R_{11}, R_{12}, R_{21}) is achievable, we obtain

$$\begin{aligned}
N(R_{11} + R_{12} + R_{21} - \epsilon_N) &\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N) \\
&\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N) + I(m_{21}; y_2^N, u_{12}^N, y_1^N, m_{11}, m_{12}) \\
&\overset{(c)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N) + I(m_{21}; y_2^N, u_{12}^N, y_1^N|m_{11}, m_{12}) \\
&\overset{(d)}{\leq} I(m_{11}, m_{12}; y_1^N, y_2^N) + I(m_{21}; y_2^N, y_1^N|m_{11}, m_{12}) \\
&= I(m_{11}, m_{12}, m_{21}; y_1^N, y_2^N)
\end{aligned}$$

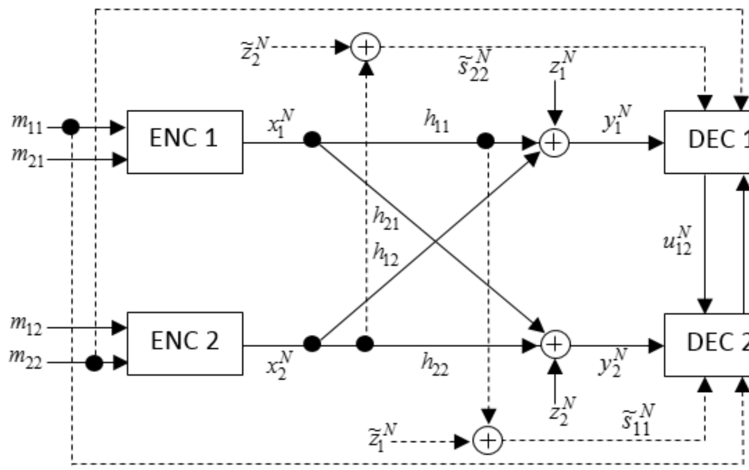


Figure 4.8 Side information structure for bound (4.25)

$$
\begin{aligned}
&= h(y_1^N, y_2^N) - h(y_1^N, y_2^N|m_{11}, m_{12}, m_{21}) \\
&= h(y_1^N, y_2^N) - h(s_{12}^N, s_{22}^N|m_{12}) \\
&= h(y_1^N, y_2^N) - h(s_{12}^N|m_{12}) - h(s_{22}^N|s_{12}^N) \\
&= h(y_1^N, y_2^N) - h(z_1^N) - h(s_{22}^N) \\
&\overset{\text{(e)}}{\leq} N\{\text{RHS of (4.21)}\}
\end{aligned}
$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to a genie providing side information y_2^N to receiver 1 and y_1^N, m_{11} and m_{12} to receiver 2. (c) is due to the fact that m_{11}, m_{12} and m_{21} are independent. (d) is due to the fact that u_{ij}^N is a function of (y_1^N, y_2^N). (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, similarly if a genie provides side information y_2^N to receiver 1 and y_1^N, m_{11} and m_{12} to receiver 2 for $R_{11} + R_{12} + R_{22}$, y_2^N, m_{21} and m_{22} to receiver 1 and y_1^N to receiver 2 for $R_{11} + R_{21} + R_{22}$ and $R_{12} + R_{21} + R_{22}$, we have shown bounds (4.21)–(4.24). ■

Bounds (4.25)–(4.26) on $R_{11} + R_{12} + R_{21} + R_{22}$:

Proof: To upper bound (4.25), a genie gives side information $\tilde{s}_{22}^N$ and m_{22} to receiver 1 and $\tilde{s}_{11}^N$ and m_{11} to receiver 2 (refer to Figure 4.8). If $(R_{11}, R_{12}, R_{22}, R_{22})$ is achievable, we have

$$N(R_{11}+R_{12}+R_{21}+R_{22}-\epsilon_N)$$

$$\overset{(a)}{\leq} I(m_{11},m_{12};y_1^N,u_{21}^N)+I(m_{21},m_{22};y_2^N,u_{12}^N)$$

$$\overset{(b)}{\leq} I(m_{11},m_{12};y_1^N)+I(m_{11},m_{12};u_{21}^N|y_1^N)+I(m_{21},m_{22};y_2^N)+I(m_{21},m_{22};u_{12}^N|y_2^N)$$

$$\overset{(c)}{\leq} I(m_{11},m_{12};y_1^N)+I(m_{21},m_{22};y_2^N)+\mathrm{H}(u_{21}^N)+\mathrm{H}(u_{12}^N)$$

$$\overset{(d)}{\leq} I(m_{11},m_{12};y_1^N,\tilde{s}_{22}^N,m_{22})+I(m_{21},m_{22};y_2^N,\tilde{s}_{11}^N,m_{11})+N\mathsf{C}_{21}^{\mathsf{B}}+N\mathsf{C}_{12}^{\mathsf{B}}$$

$$\overset{(e)}{=} I(m_{11},m_{12};y_1^N,\tilde{s}_{22}^N|m_{22})+I(m_{21},m_{22};y_2^N,\tilde{s}_{11}^N|m_{11})+N\mathsf{C}_{21}^{\mathsf{B}}+N\mathsf{C}_{12}^{\mathsf{B}}$$

$$\overset{(f)}{=} I(m_{11},m_{12};\tilde{s}_{22}^N|m_{22})+I(m_{11},m_{12};y_1^N|m_{22},\tilde{s}_{22}^N)+I(m_{21},m_{22};\tilde{s}_{11}^N|m_{11})$$

$$+I(m_{21},m_{22};y_2^N|m_{11},\tilde{s}_{11}^N)+N\mathsf{C}_{21}^{\mathsf{B}}+N\mathsf{C}_{12}^{\mathsf{B}}$$

$$\overset{(g)}{\leq} h(\tilde{s}_{22}^N|m_{22})-h(\tilde{s}_{22}^N|x_2^N,m_{11})+h(y_1^N|\tilde{s}_{22}^N,m_{22})-h(y_1^N|\tilde{s}_{22}^N,x_2^N,m_{11})+h(\tilde{s}_{11}^N|m_{11})$$

$$-h(\tilde{s}_{11}^N|x_1^N,m_{22})+h(y_2^N|\tilde{s}_{11}^N,m_{11})-h(y_2^N|\tilde{s}_{11}^N,x_1^N,m_{22})+N\mathsf{C}_{21}^{\mathsf{B}}+N\mathsf{C}_{12}^{\mathsf{B}}$$

$$\leq h(\tilde{s}_{22}^N|m_{22})-h(\tilde{z}_2^N)+h(y_1^N|\tilde{s}_{22}^N)-h(s_{11}^N|m_{11})+h(\tilde{s}_{11}^N|m_{11})-h(\tilde{z}_1^N)+h(y_2^N|\tilde{s}_{11}^N)$$

$$-h(s_{22}^N|m_{22})+N\mathsf{C}_{21}^{\mathsf{B}}+N\mathsf{C}_{12}^{\mathsf{B}}$$

$$=h(y_1^N|\tilde{s}_{22}^N)-h(\tilde{z}_2^N)+h(y_2^N|\tilde{s}_{11}^N)-h(\tilde{z}_1^N)+N\mathsf{C}_{21}^{\mathsf{B}}+N\mathsf{C}_{12}^{\mathsf{B}}$$

$$\overset{(h)}{\leq} N\{\text{RHS of (4.25)}\}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule. (c) is due to the fact that $I(m_{ii},m_{ij};u_{ji}^N|y_i^N) \leq H(u_{ji}^N)$. (d) is due to the fact that a genie provides side information $\tilde{s}_{ii}^N$ and m_{ii} to receiver j, for $i,j=1,2$ and $H(u_{ji}^N) \leq N\mathsf{C}_{ji}^{\mathsf{B}}$. (e) is due to the fact that m_{11},m_{12},m_{21} and m_{22} are independent. (f) is due to chain rule. (g) is due to the fact that x_i^N is a function of messages (m_{ii},m_{ji}). (h) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, similarly if a genie gives side information $\tilde{s}_{21}^N$ and m_{21} to receiver 1 and $\tilde{s}_{12}^N$ and m_{12} to receiver 2 for the other bound, we have shown bounds (4.25)–(4.26). ■

Bounds (4.27)–(4.28) on $R_{11}+R_{12}+R_{21}+R_{22}$:

Proof: To upper bound (4.27), a genie gives side information y_2^N and $\tilde{s}_{21}^N$ to receiver 1 and $\tilde{s}_{12}^N$ to receiver 2 (refer to Figure 4.9). If $(R_{11},R_{12},R_{22},R_{22})$ is achievable, we obtain

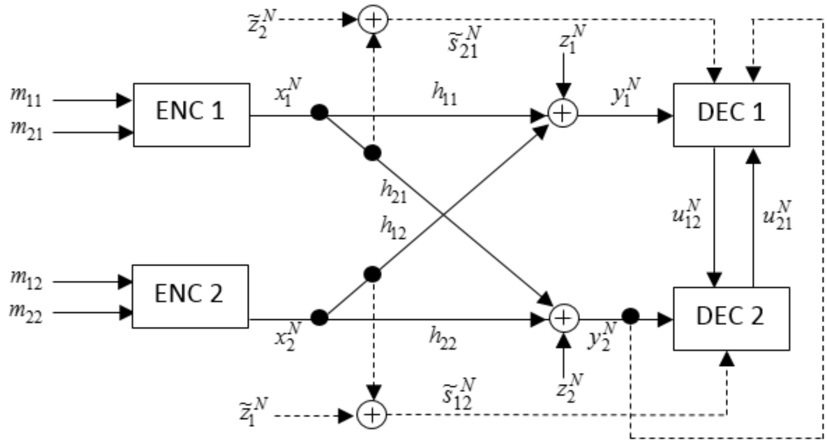


Figure 4.9 Side information structure for bound (4.27)

$$
\begin{aligned}
&N(R_{11}+R_{12}+R_{21}+R_{22}-\epsilon_N)\\
&\overset{(a)}{\leq} I(m_{11},m_{12};y_1^N,u_{21}^N)+I(m_{21},m_{22};y_2^N,u_{12}^N)\\
&\overset{(b)}{\leq} I(m_{11},m_{12};y_1^N,u_{21}^N,y_2^N,\tilde{s}_{21}^N)+I(m_{21},m_{22};y_2^N,\tilde{s}_{12}^N)+I(m_{21},m_{22};u_{12}^N|y_2^N)\\
&\overset{(c)}{\leq} I(m_{11},m_{12};y_1^N,u_{21}^N,y_2^N|\tilde{s}_{21}^N)+I(m_{11},m_{12};\tilde{s}_{21}^N)+I(m_{21},m_{22};y_2^N,\tilde{s}_{12}^N)+\mathrm{H}(u_{12}^N)\\
&\overset{(d)}{\leq} I(m_{11},m_{12};y_1^N,y_2^N|\tilde{s}_{21}^N)+h(\tilde{s}_{21}^N)-h(\tilde{s}_{21}^N|m_{11},m_{12})+h(y_2^N,\tilde{s}_{12}^N)\\
&-h(y_2^N,\tilde{s}_{12}^N|m_{21},m_{22})+N\mathsf{C}_{12}^{\mathsf{B}}\\
&\leq h(y_1^N,y_2^N|\tilde{s}_{21}^N)-h(y_1^N,y_2^N|\tilde{s}_{21}^N,m_{11},m_{12})+h(\tilde{s}_{21}^N)-h(\tilde{z}_2^N)+h(\tilde{s}_{12}^N)+h(y_2^N|\tilde{s}_{12}^N)\\
&-h(s_{21}^N,\tilde{z}_1^N)+N\mathsf{C}_{12}^{\mathsf{B}}\\
&\leq h(y_1^N,y_2^N|\tilde{s}_{21}^N)-h(s_{12}^N,s_{22}^N)+h(\tilde{s}_{21}^N)-h(\tilde{z}_2^N)+h(\tilde{s}_{12}^N)+h(y_2^N|\tilde{s}_{12}^N)-h(s_{21}^N)\\
&-h(\tilde{z}_1^N)+N\mathsf{C}_{12}^{\mathsf{B}}\\
&=h(y_1^N,y_2^N|\tilde{s}_{21}^N)+h(y_2^N|\tilde{s}_{12}^N)-h(s_{22}^N|s_{12}^N)-h(\tilde{z}_2^N)-h(\tilde{z}_1^N)+N\mathsf{C}_{12}^{\mathsf{B}}\\
&\overset{(e)}{\leq} N\{\text{RHS of (4.27)}\}
\end{aligned}
$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule and a genie providing side information y_2^N and $\tilde{s}_{21}^N$ to receiver 1 and $\tilde{s}_{12}^N$ to receiver 2. (c) is due to chain rule and the fact that $I(m_{21},m_{22};u_{12}^N|y_2^N) \leq \mathrm{H}(u_{12}^N)$. (d) is due to the fact that u_{21}^N is a function of (y_1,y_2) and $\mathrm{H}(u_{12}^N) \leq N\mathsf{C}_{12}^{\mathsf{B}}$. (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

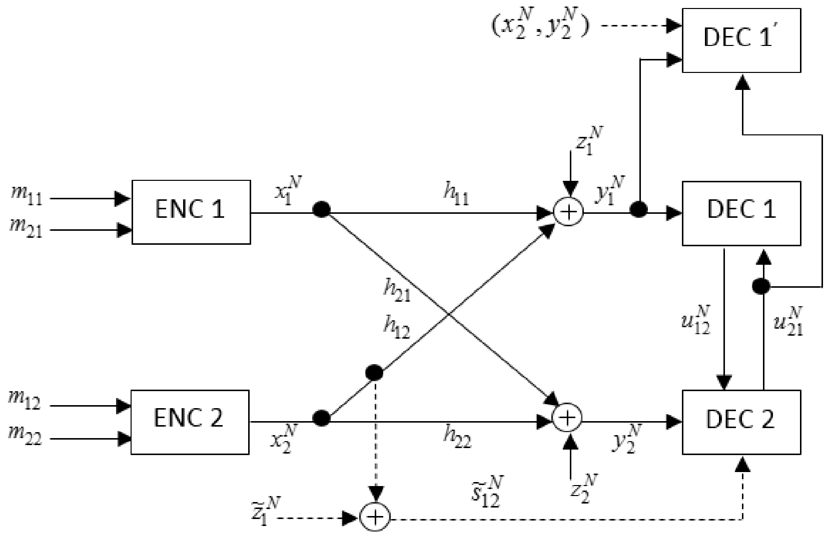


Figure 4.10 Side information structure for bound (4.30)

Hence, and similarly if a genie gives side information y_1^N and $\tilde{s}_{12}^N$ to receiver 2 and $\tilde{s}_{21}^N$ to receiver 1 for the other bound, we have shown bounds (4.27)−(4.28). ■

Bound (4.29) on $R_{11} + R_{12} + R_{21} + R_{22}$:

Proof: This is the straightforward cut-set upper bound: if $(R_{11}, R_{12}, R_{22}, R_{22})$ is achievable, we obtain

$$
\begin{aligned}
N(R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) &\le I(m_{11}, m_{12}, m_{21}, m_{22}; y_1^N, y_2^N) \\
&= h(y_1^N, y_2^N) - h(z_1^N, z_2^N) \\
&\le N\{\text{RHS of } (4.29)\}
\end{aligned}
$$

where $\epsilon_N \to 0$ as $N \to \infty$.

Hence, we have shown bounds (4.29). ■

Bounds (4.30) on $2R_{11} + R_{12} + R_{21} + R_{22}$*, (4.31) on* $R_{11} + 2R_{12} + R_{21} + R_{22}$*, (4.32) on* $R_{11} + R_{12} + 2R_{21} + R_{22}$ *and (4.33) on* $R_{11} + R_{12} + R_{21} + 2R_{22}$:

Proof: In this proof, we show only (4.30) and other bounds can be shown similarly. Now, let a genie gives side information y_2^N and x_2^N to the second receiver 1 and $\tilde{s}_{12}^N$ to receiver 2 (refer to Figure 4.10). If $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable, we get

$$
\begin{aligned}
&N(2R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) \\
&\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{11}; y_1^N, u_{21}^N) + I(m_{21}, m_{22}; y_2^N, u_{12}^N) \\
&\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N|y_1^N) + I(m_{11}; y_1^N, u_{21}^N, y_2^N, x_2^N) + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) \\
&+ I(m_{21}, m_{22}; u_{12}^N|y_2^N) \\
&\overset{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}; y_1^N, u_{21}^N, y_2^N|x_2^N) + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) + \mathrm{H}(u_{21}^N) + \mathrm{H}(u_{12}^N) \\
&\overset{(d)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}; y_1^N, y_2^N|x_2^N) + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= h(y_1^N) - h(y_1^N|m_{11}, m_{12}) + h(y_1^N, y_2^N|x_2^N) - h(y_1^N, y_2^N|x_2^N, m_{11}) + h(y_2^N, \tilde{s}_{12}^N) \\
&- h(y_2^N, \tilde{s}_{12}^N|m_{21}, m_{22}) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\leq h(y_1^N) - h(s_{12}^N) + h(s_{11}^N, s_{21}^N) - h(z_1^N, z_2^N) + h(y_2^N, \tilde{s}_{12}^N) - h(s_{21}^N, \tilde{z}_1^N) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= h(y_1^N) + h(s_{11}^N|s_{21}^N) + h(y_2^N|\tilde{s}_{12}^N) - h(z_1^N) - h(z_2^N) - h(\tilde{z}_1^N) + N\mathsf{C}_{21}^{\mathsf{B}} + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(e)}{\leq} N\{\text{RHS of (4.30)}\}
\end{aligned}
$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule and the fact that a genie gives side information y_2^N and x_2^N to the second receiver 1 and $\tilde{s}_{12}^N$ to receiver 2. (c) is due to the fact that m_{11} and x_2^N are independent and $I(m_{ii}, m_{ij}; u_{ji}^N|y_{\mathrm{i}}^N) \leq \mathrm{H}(u_{ji}^N)$. (d) is due to the fact that u_{21}^N is a function of (y_1, y_2) and $\mathrm{H}(u_{ij}^N) \leq N\mathsf{C}_{ij}^{\mathsf{B}}$. (e) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, and similarly if a genie provides side information y_2^N and x_1^N to the second receiver 1 and $\tilde{s}_{11}^N$ to receiver 2 for $R_{11} + 2R_{12} + R_{21} + R_{22}$, y_1^N and x_2^N to the second receiver 2 and $\tilde{s}_{22}^N$ to receiver 1 for $R_{11} + R_{12} + 2R_{21} + R_{22}$ and y_1^N and x_1^N to the second receiver 2 and $\tilde{s}_{21}^N$ to receiver 1 for $R_{11} + R_{12} + R_{21} + 2R_{22}$, we have shown bounds (4.30)–(4.33). ■

Bounds (4.34) on $2R_{11} + R_{12} + R_{21} + R_{22}$, (4.35) on $R_{11} + 2R_{12} + R_{21} + R_{22}$, (4.36) on $R_{11} + R_{12} + 2R_{21} + R_{22}$ and (4.37) on $R_{11} + R_{12} + R_{21} + 2R_{22}$:

Proof: In this proof, we show only (4.34) and other bounds can be shown similarly. Now, let a genies give side information y_2^N to the first receiver 1, y_2^N and x_2^N to the second receiver 1 and $\tilde{s}_{12}^N$ to receiver 2 (refer to Figure 4.11). If $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable,

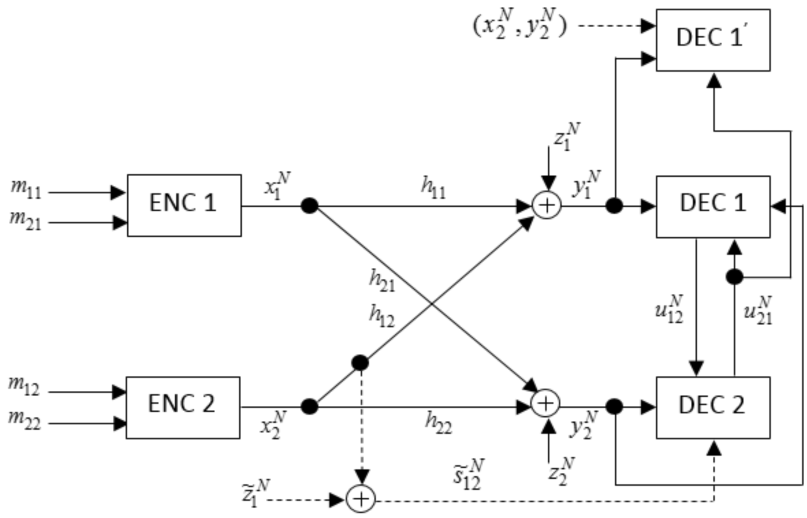


Figure 4.11 Side information structure for bound (4.34)

we obtain

$$
\begin{aligned}
&N(2R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) \\
&\overset{(a)}{\le} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{11}; y_1^N, u_{21}^N) + I(m_{21}, m_{22}; y_2^N, u_{12}^N) \\
&\overset{(b)}{\le} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N) + I(m_{11}; y_1^N, u_{21}^N, y_2^N, x_2^N) + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) \\
&+ I(m_{21}, m_{22}; u_{12}^N | y_2^N) \\
&\overset{(c)}{\le} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N) + I(m_{11}; y_1^N, u_{21}^N, y_2^N | x_2^N) + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) + \mathrm{H}(u_{12}^N) \\
&\overset{(d)}{\le} I(m_{11}, m_{12}; y_1^N, y_2^N) + I(m_{11}; y_1^N, y_2^N | x_2^N) + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&= h(y_1^N, y_2^N) - h(y_1^N, y_2^N | m_{11}, m_{12}) + h(y_1^N, y_2^N | x_2^N) - h(y_1^N, y_2^N | x_2^N, m_{11}) + h(y_2^N, \tilde{s}_{12}^N) \\
&- h(y_2^N, \tilde{s}_{12}^N | m_{21}, m_{22}) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\le h(y_1^N, y_2^N) - h(s_{12}^N, s_{22}^N) + h(s_{11}^N, s_{21}^N) - h(z_1^N, z_2^N) + h(\tilde{s}_{12}^N) + h(y_2^N | \tilde{s}_{12}^N) - h(s_{21}^N) \\
&- h(\tilde{z}_1^N) + N\mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(e)}{\le} N\{\text{RHS of } (4.34)\}
\end{aligned}
$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule and the fact that a genie gives side information y_2^N to the first receiver 1, y_2^N and x_2^N to the second receiver 2 and $\tilde{s}_{12}^N$ to receiver 2. (c) is due to the fact that m_{11} and x_2^N are independent and $I(m_{21}, m_{22}; u_{12}^N | y_2^N) \le \mathrm{H}(u_{12}^N)$. (d) is due to the fact that u_{21}^N is a function of (y_1, y_2) and $\mathrm{H}(u_{12}^N) \le N\mathsf{C}_{12}^{\mathsf{B}}$. (e) is due to the fact that i.i.d.

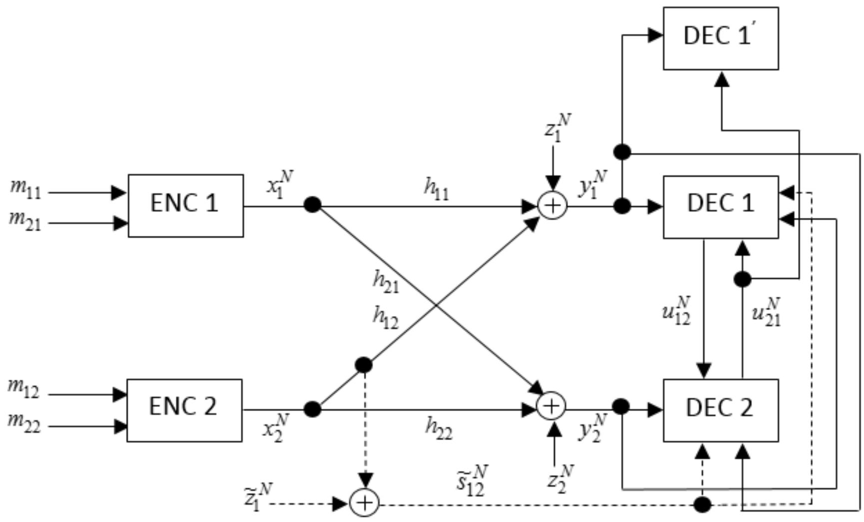


Figure 4.12 Side information structure for bound (4.38)

Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, and similarly if a genie provides side information y_2^N to the first receiver 1, y_2^N and x_1^N to the second receiver 2 and $\tilde{s}_{11}^N$ to receiver 2 for $R_{11} + 2R_{12} + R_{21} + R_{22}$, y_1^N to the first receiver 2, y_1^N and x_2^N to the second receiver 2 and $\tilde{s}_{22}^N$ to receiver 1 for $R_{11} + R_{12} + 2R_{21} + R_{22}$ and y_1^N to the first receiver 2, y_1^N and x_1^N to the first receiver 2 and $\tilde{s}_{21}^N$ to receiver 1 for $R_{11} + R_{12} + R_{21} + 2R_{22}$, we have shown bounds (4.34)−(4.37). ■

Bounds (4.38) on $2R_{11} + R_{12} + R_{21} + R_{22}$, (4.39) on $R_{11} + 2R_{12} + R_{21} + R_{22}$, (4.40) on $R_{11} + R_{12} + 2R_{21} + R_{22}$ and (4.41) on $R_{11} + R_{12} + R_{21} + 2R_{22}$:

Proof: In this proof, we show only (4.38) and other bounds can be shown similarly. Now, let a genie gives side information y_2^N and $\tilde{s}_{12}^N$ to the first receiver 1 and y_1^N and $\tilde{s}_{12}^N$ to receiver 2 (refer to Figure. 4.12). If $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable, we can write

$$
\begin{aligned}
&N(2R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) \\
&\overset{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{11}; y_1^N, u_{21}^N) + I(m_{21}, m_{22}; y_2^N, u_{12}^N) \\
&\overset{(b)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N, \tilde{s}_{12}^N) + I(m_{11}; y_1^N) + I(m_{11}; u_{21}^N | y_1^N) \\
&+ I(m_{21}, m_{22}; y_2^N, u_{12}^N, y_1^N, \tilde{s}_{12}^N) \\
&\overset{(c)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N | \tilde{s}_{12}^N) + I(m_{11}, m_{12}; \tilde{s}_{12}^N) + I(m_{21}, m_{22}; y_2^N, u_{12}^N, y_1^N | \tilde{s}_{12}^N) \\
&+ I(m_{21}, m_{22}; \tilde{s}_{12}^N) + I(m_{11}; y_1^N) + \mathrm{H}(u_{21}^N)
\end{aligned}
$$

$$\stackrel{\text{(d)}}{\leq} I(m_{11}, m_{12}; y_1^N, y_2^N | \tilde{s}_{12}^N) + I(m_{11}, m_{12}; \tilde{s}_{12}^N) + I(m_{21}, m_{22}; y_2^N, y_1^N | \tilde{s}_{12}^N) + I(m_{21}, m_{22}; \tilde{s}_{12}^N)$$
$$+ I(m_{11}; y_1^N) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$\stackrel{\text{(e)}}{=} I(m_{11}, m_{12}; y_1^N, y_2^N | \tilde{s}_{12}^N) + I(m_{11}, m_{12}; \tilde{s}_{12}^N) + I(m_{21}, m_{22}; y_2^N, y_1^N | \tilde{s}_{12}^N, m_{11}, m_{12})$$
$$+ I(m_{21}, m_{22}; \tilde{s}_{12}^N | m_{11}, m_{12}) + I(m_{11}; y_1^N) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$= I(m_{11}, m_{12}, m_{21}, m_{22}; y_1^N, y_2^N | \tilde{s}_{12}^N) + I(m_{11}, m_{12}, m_{21}, m_{22}; \tilde{s}_{12}^N) + I(m_{11}; y_1^N) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$= h(y_1^N, y_2^N | \tilde{s}_{12}^N) - h(y_1^N, y_2^N | \tilde{s}_{12}^N, m_{11}, m_{12}, m_{21}, m_{22}) + h(\tilde{s}_{12}^N) - h(\tilde{s}_{12}^N | m_{11}, m_{12}, m_{21}, m_{22})$$
$$+ h(y_1^N) - h(y_1^N | m_{11}) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$\leq h(y_1^N, y_2^N | \tilde{s}_{12}^N) - h(z_1^N, z_2^N) + h(\tilde{s}_{12}^N) - h(\tilde{z}_1^N) + h(y_1^N) - h(s_{12}^N) + N\mathsf{C}_{21}^{\mathsf{B}}$$
$$\stackrel{\text{(f)}}{\leq} N\{\text{RHS of (4.38)}\}$$

where $\epsilon_N \to 0$ as $N \to \infty$. (a) is due to Fano's inequality and data processing inequality. (b) is due to chain rule and a genie giving side information y_2^N and $\tilde{s}_{12}^N$ to the first receiver 1 and y_1^N and $\tilde{s}_{12}^N$ to receiver 2. (c) is due to chain rule and the fact that $I(m_{11}; u_{21}^N | y_1^N) \leq \mathrm{H}(u_{21}^N)$. (d) is due to the fact that u_{ij}^N is a function of (y_1, y_2) and $\mathrm{H}(u_{21}^N) \leq N\mathsf{C}_{21}^{\mathsf{B}}$. (e) is due to the fact that conditioning reduces entropy and (m_{11}, m_{12}) and (m_{21}, m_{22}) are independent. (f) is due to the fact that i.i.d. Gaussian distribution maximizes differential entropy under covariance constraints.

Hence, and similarly if a genie provides side information y_2^N and $\tilde{s}_{11}^N$ to the first receiver 1 and y_1^N and $\tilde{s}_{11}^N$ to receiver 2 for $R_{11} + 2R_{12} + R_{21} + R_{22}$, y_1^N and $\tilde{s}_{22}^N$ to the first receiver 2 and y_2^N and $\tilde{s}_{22}^N$ to receiver 1 for $R_{11} + R_{12} + 2R_{21} + R_{22}$ and y_1^N and $\tilde{s}_{21}^N$ to the first receiver 2 and y_2^N and $\tilde{s}_{21}^N$ to receiver 1 for $R_{11} + R_{12} + R_{21} + 2R_{22}$, we have shown bounds (4.38)–(4.41). ■

CHAPTER V

GENERALIZED DEGREES OF FREEDOM CHARACTERIZATION

In this chapter, we can earn further comprehensions with the effect of receiver cooperation on the two-user Gaussian X channel by characterizing the *generalized degrees of freedom* (GDoF), a natural generalization of the notion of degrees of freedom (DoF) in point-to-point communication to multiuser scenarios, of the sum capacity from Lemma 4.1 in the symmetric channel setting where $\mathsf{SNR}_1 = \mathsf{SNR}_2 = \mathsf{SNR}$, $\mathsf{INR}_1 = \mathsf{INR}_2 = \mathsf{INR}$ and $\mathsf{C}^{\mathsf{B}}_{21} = \mathsf{C}^{\mathsf{B}}_{12} = \mathsf{C}^{\mathsf{B}}$. Furthermore, we also show the behavior of the gain from receiver cooperation under the symmetric channel setting.

5.1 Generalized Degrees of Freedom

We use the notation of the GDoF that is initially proposed in [9] to characterize the asymptotic behavior of the capacity region with respect to growth of SNR as fixing α and κ in symmetric channel setting case. The GDoF of the sum capacity [2, 9] is defined as

$$d(\alpha,\kappa) := \lim_{\substack{\text{fix } \alpha,\kappa \\ \mathsf{SNR}\to\infty}} \frac{C_{\Sigma}(\mathsf{SNR},\mathsf{INR},\mathsf{C}^{\mathsf{B}})}{\log \mathsf{SNR}} \tag{5.1}$$

where

$$\lim_{\mathsf{SNR}\to\infty} \frac{\log \mathsf{INR}}{\log \mathsf{SNR}} = \alpha, \quad \lim_{\mathsf{SNR}\to\infty} \frac{\mathsf{C}^{\mathsf{B}}}{\log \mathsf{SNR}} = \kappa.$$

and $C_{\Sigma}(\mathsf{SNR},\mathsf{INR},\mathsf{C}^{\mathsf{B}})$ is the sum capacity of the two-user Gaussian X channel with limited receiver cooperation, i.e., $C_{\Sigma}(\mathsf{SNR},\mathsf{INR},\mathsf{C}^{\mathsf{B}}) = R_{11} + R_{12} + R_{21} + R_{22}$.

Note that α and κ are called the normalized interference level and the normalized capacity of the receiver-cooperative link, respectively.

We also use approximations [9] such as

$$\log(1+\mathsf{SNR}+\mathsf{INR}) \approx \max(\log(\mathsf{SNR}), \log(\mathsf{INR})) \tag{5.2}$$

$$\log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) \approx \left(\log\frac{\mathsf{SNR}}{\mathsf{INR}}\right)^{+} \tag{5.3}$$

to give an expansion of the capacity region of the Gaussian X channel with receiver cooperation which is accurate to the first order approximation. Both (5.2)−(5.3) are very useful in the derivation of the generalized degrees of freedom region.

Remark 5.1 *The limit of (5.1) does not exist since it has different values due to certain channel realizations. This event occurs when* $\alpha = 1$*, where the phase of the channel gains has the effect on both inner and outer bounds [9].*

Next, we provide sum capacity under the symmetric channel setting from the results in Chapter IV.

5.2 Approximate Symmetric Sum Capacity

This section gives an approximate symmetric sum capacity as follows:

Corollary 5.2 *(Approximate Symmetric Sum Capacity):*

$$\begin{aligned} C_\Sigma \leq\ & 2\log\big(1+\mathsf{SNR}\big)+2\log\big(1+\mathsf{INR}\big) \\ & +\min\Big\{4\mathsf{C}^{\mathsf{B}}, 2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+2\log\Big(1+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big)\Big\} \end{aligned} \tag{5.4}$$

$$C_\Sigma \leq 2\log\big(1+\mathsf{INR}+\mathsf{SNR}\big)+2\mathsf{C}^{\mathsf{B}} \tag{5.5}$$

$$C_\Sigma \leq 2\log\Big(1+2\mathsf{SNR}+2\mathsf{INR}+|h_{11}h_{22}-h_{12}h_{21}|^2\Big)-2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big) \tag{5.6}$$

$$C_\Sigma \leq \frac{1}{3}\begin{bmatrix} 2\log(1+\mathsf{SNR})+2\log\Big(1+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big) \\ +2\log(1+\mathsf{INR})+2\log\Big(1+\mathsf{INR}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+8\mathsf{C}^{\mathsf{B}} \end{bmatrix} \tag{5.7}$$

$$C_\Sigma \leq \frac{1}{3}\Big[4\log(1+\mathsf{SNR}+\mathsf{INR})+2\log\Big(1+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big)+2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+4\mathsf{C}^{\mathsf{B}}\Big] \tag{5.8}$$

$$C_\Sigma \leq \frac{1}{3}\begin{bmatrix} 2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}+\mathsf{INR}+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{INR}}+\frac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}}\Big) \\ +2\log\Big(1+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}+\frac{\mathsf{SNR}}{1+\mathsf{SNR}}+\mathsf{INR}+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) \\ +2\log(1+\mathsf{SNR})+2\log(1+\mathsf{INR})-2\log\Big(1+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big) \\ -2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big) \end{bmatrix} \tag{5.9}$$

$$C_\Sigma \le \frac{1}{3}\begin{bmatrix} 4\log\Big(1+2\mathsf{SNR}+2\mathsf{INR}+|h_{11}h_{22}-h_{12}h_{21}|^2\Big)-2\log\Big(1+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big) \\ -2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big) \end{bmatrix} \tag{5.10}$$

$$C_\Sigma \le 2\log\Big(1+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big)+2\mathsf{C}^{\mathsf{B}} \tag{5.11}$$

$$C_\Sigma \le 2\log\Big(1+\mathsf{INR}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+2\mathsf{C}^{\mathsf{B}} \tag{5.12}$$

$$C_\Sigma \le \log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}+\mathsf{INR}+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{INR}}+\frac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}}\Big) \\ +\log\Big(1+\mathsf{INR}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)-\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+\mathsf{C}^{\mathsf{B}} \tag{5.13}$$

$$C_\Sigma \le \log\Big(1+2\mathsf{SNR}+2\mathsf{INR}+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) \tag{5.14}$$

$$C_\Sigma \le \frac{1}{5}\begin{bmatrix} 2\log\Big(1+\mathsf{INR}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+2\log\Big(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big) \\ +2\log\Big(1+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big)+2\log\Big(1+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big) \\ +4\log(1+\mathsf{SNR}+\mathsf{INR})+8\mathsf{C}^{\mathsf{B}} \end{bmatrix} \tag{5.15}$$

$$C_\Sigma \le \frac{1}{5}\begin{bmatrix} 4\log\Big(1+2\mathsf{SNR}+2\mathsf{INR}+|h_{11}h_{22}-h_{12}h_{21}|^2\Big) \\ +2\log\Big(1+\mathsf{INR}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\Big)+2\log\Big(1+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}\Big)+4\mathsf{C}^{\mathsf{B}} \end{bmatrix} \tag{5.16}$$

$$C_\Sigma \le \frac{1}{5}\begin{bmatrix} 2\log\Big(1+\mathsf{SNR}+\mathsf{INR}+\frac{\mathsf{INR}}{1+\mathsf{INR}}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}+\frac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}}\Big) \\ +2\log\Big(1+\mathsf{SNR}+\mathsf{INR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}+\frac{\mathsf{SNR}}{1+\mathsf{SNR}}+\frac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{SNR}}\Big) \\ +4\log(1+\mathsf{SNR}+\mathsf{INR})+4\mathsf{C}^{\mathsf{B}} \end{bmatrix} \tag{5.17}$$

Note that the symmetric sum capacity (5.4)–(5.17) can be obtained by calculating the sum rate C_Σ from our proposed outer bound in Lemma 4.1 as follows:

- Sum capacity C_Σ (5.4) is obtained by adding (4.1)−(4.4).
- Sum capacity C_Σ (5.5) is obtained by adding (4.5)−(4.6).
- Sum capacity C_Σ (5.6) is obtained by adding (4.7)−(4.8).
- Sum capacity C_Σ (5.7) is obtained by adding (4.9)−(4.12).
- Sum capacity C_Σ (5.8) is obtained by adding (4.13)−(4.16).
- Sum capacity C_Σ (5.9) is obtained by adding (4.17)−(4.20)
- Sum capacity C_Σ (5.10) is obtained by adding (4.21)−(4.24)
- Sum capacity C_Σ (5.11)−(5.12) are the symmetric case of (4.25)−(4.26), respectively.

- Sum capacity C_Σ (5.13) is the symmetric case of (4.27) or (4.28).
- Sum capacity C_Σ (5.14) is the symmetric case of (4.29).
- Sum capacity C_Σ (5.15) is obtained by adding (4.30)−(4.33).
- Sum capacity C_Σ (5.16) is obtained by adding (4.34)−(4.37).
- Sum capacity C_Σ (5.17) is obtained by adding (4.38)−(4.41).

where $C_\Sigma = R_{11} + R_{12} + R_{21} + R_{22}$.

Remark 5.3 *Our results in Lemma 4.1 and Corollary 5.2 show that the sum rates rely on the phases of the channel gains considerably when a capacity of the receiver-cooperative link* C^{B} *is so large that MIMO sum-rate cut-set or MIMO sum-rate based on genie-aided bounds are active.*

Next, we explore the GDoF from the results in Colloraly 5.2.

5.3 Generalized Degrees of Freedom of the Symmetric Sum Capacity

Before computing the GDoF of the sum capacity, we first consider the important point, i.e., phases in MIMO situations, and then propose the method to solve this problem as follows:

In general, the characterization of the GDoF in several communication scenarios, i.e., interference channel [9], or X channel [14], etc., cannot consider the impact of phases in MIMO situations. For solving this problem, Wang and Tse [2] propose the following lemma which uses an i.i.d. uniform distribution on the phases of the channel gains instead of claiming that the limit of (5.1) exists for all channel realizations.

Lemma 5.4 ([2]) *Let*

$$|h_{ij}| = g_{ij},\ \angle h_{ij} = \Theta_{ij}, \forall i, j \in \{1, 2\}$$

where g_{ij}'s are deterministic and Θ_{ij}'s are i.i.d. uniformly distributed over [0, 2π].

Proof: See Appendix E in [2]. ■

In this dissertation, we use Lemma 5.4 to consider the term $A_1 := \log(1 + 2\mathsf{SNR} + 2\mathsf{INR} + |h_{11}h_{22} - h_{12}h_{21}|^2)$ that appears in (5.6), (5.10), (5.14) and (5.16). Then, we obtain the limit $\mathcal{L}_1(\alpha,\kappa) := \lim_{\substack{\text{fix } \alpha,\kappa \\ \mathsf{SNR}\to\infty}} \frac{A_1}{\log \mathsf{SNR}}$. Furthermore, we extend the concept of Lemma 5.4 to manage the terms $A_2 := \log(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}+\mathsf{INR}+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{INR}}+\frac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{INR}})$ and $A_3 := \log(1+\mathsf{SNR}+\frac{\mathsf{INR}}{1+\mathsf{SNR}}+\frac{\mathsf{SNR}}{1+\mathsf{INR}}+\mathsf{INR}+\frac{|h_{11}h_{22}-h_{12}h_{21}|^2}{1+\mathsf{SNR}})$ in (5.9), (5.13), (5.17). Then, we also get the limit for both $\mathcal{L}_2(\alpha,\kappa) := \lim_{\substack{\text{fix } \alpha,\kappa \\ \mathsf{SNR}\to\infty}} \frac{A_2}{\log \mathsf{SNR}}$ and $\mathcal{L}_3(\alpha,\kappa) := \lim_{\substack{\text{fix } \alpha,\kappa \\ \mathsf{SNR}\to\infty}} \frac{A_3}{\log \mathsf{SNR}}$. Hence, with considerations above, it is seen that the limit of (5.1) exists *almost surely*.

After solving the phases issue above, we give the GDoF of the sum capacity with symmetric channel setting in the following theorem.

Theorem 5.5 (Generalized Degrees of Freedom of the Symmetric Sum Capacity) *The GDoF of the sum capacity for the two-user Gaussian X channel with limited receiver cooperation in symmetric channel setting is given as follows:*

For $0 \le \alpha < 1$,

$$\begin{aligned} d(\alpha,\kappa) = \min\Big\{ & 2+2\alpha+4\kappa, 4, 2+2\kappa, 2-\frac{2\alpha}{3}+\frac{4\kappa}{3}, 2\max(\alpha,(1-\alpha)^+)+2\kappa, \\ & \frac{1}{3}\Big[4+\max(4\alpha, 2\alpha+2(1-\alpha)^+)+8\kappa\Big], \frac{1}{3}\Big[2(2-\alpha)^+ + 2 + 4\alpha\Big], \\ & \frac{1}{3}\Big[6+2\alpha\Big], (2-\alpha)^+ + \max(\alpha,(1-\alpha)^+) - (1-\alpha)^+ + \kappa, 2, \\ & \frac{1}{5}\Big[8-2\alpha+2\max(\alpha,(1-\alpha)^+)+8\kappa\Big], \frac{1}{5}\Big[10+2\max(\alpha,(1-\alpha)^+)+4\kappa\Big], \\ & \frac{1}{5}\Big[2(2-\alpha)^+ + 6 + 4\kappa\Big]\Big\} \end{aligned} \tag{5.18}$$

For $\alpha \ge 1$,

$$\begin{aligned} d(\alpha,\kappa) = \min\Big\{ & 2+2\alpha+4\kappa, 4\alpha, 2\alpha+2\kappa, 2\alpha-\frac{2}{3}+\frac{4\kappa}{3}, 2\max(1,(\alpha-1)^+)+2\kappa, \\ & \frac{1}{3}\Big[4\alpha+\max(4, 2+2(\alpha-1)^+)+8\kappa\Big], \frac{1}{3}\Big[2\alpha+2(2\alpha-1)^+ + 4\Big], \\ & \frac{1}{3}\Big[6\alpha+2\Big], 2\alpha-(1-\alpha)^+ + \kappa, 2\alpha, \frac{1}{5}\Big[8\alpha-2+2\max(1,(\alpha-1)^+)+8\kappa\Big], \\ & \frac{1}{5}\Big[10\alpha+2\max(1,(\alpha-1)^+)+4\kappa\Big], \frac{1}{5}\Big[2(2\alpha-1)^+ + 6\alpha + 4\kappa\Big]\Big\} \end{aligned} \tag{5.19}$$

Note that the GDoFs in (5.18)–(5.19) are calculated directly from Corollary 5.2 together with solving the issue of phases in MIMO situations above as follows:

1. Using the approximations (5.2)$-$(5.3) with (5.4)$-$(5.17) to obtain the approximate version of them.

2. Substituting the approximate version of (5.4)−(5.5), (5.7)−(5.8), (5.11)−(5.12) and (5.15) obtained from the first step in (5.1)

3. Substituting the approximate version of (5.6), (5.9)−(5.10), (5.13)−(5.14) and (5.16)−(5.17) in (5.1) where terms involving $|h_{11}h_{22} - h_{12}h_{21}|^2$ in these inequalities are solved using the concept of Lemma 5.4.

In the next section, we show the characterization of the GDoF of the symmetric sum capacity with respect to the normalized interference level (α) and the normalized capacity of the receiver-cooperative link (κ).

5.4 Results and Discussion

First, the GDoFs ($d(\alpha, \kappa)$) of the sum capacity for the symmetric two-user Gaussian X channel with and without limited receiver cooperation from (5.18)−(5.19) versus the normalized interference level (α) are plotted in Figure 5.1.

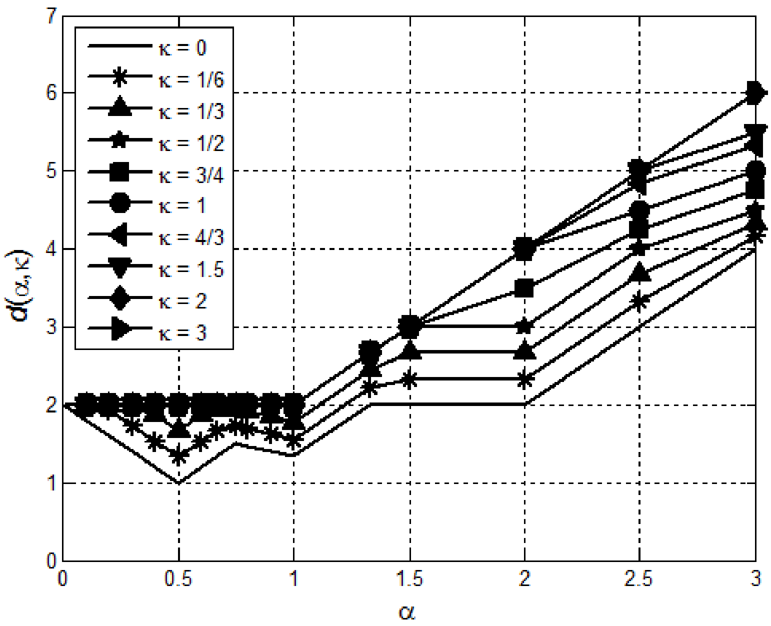


Figure 5.1: The GDoF for the symmetric two-user Gaussian X channel with/without receiver cooperation

From Figure 5.1, the result shows that

1. Our system obtains the gain from the receiver cooperation for $\alpha \in [0, 3]$ when we compare the GDoF between the receiver cooperation case ($\kappa > 0$) and non-

cooperation case ($\kappa = 0$)[1]. This implies that the performance of our system can be improved when amount of exchanged information between both receivers increases. However, the obtained gains from the receiver cooperation at the different values of α do not equal.

2. Our system achieves the full receiver cooperation. To consider this issue, we divide the considered range of α into 2 parts as follows:

   (a) For $0 \leq \alpha < 1$: As shown in Figure 5.2, the GDoF increases with varying gain value when κ increases and equals 2 for all α's $\in [0, 1)$ at $\kappa = \frac{1}{2}$. For the GDoF curve, we see that it changes from a sawtooth curve to a linear line with a slope of 0 when κ increases from 0 to $\frac{1}{2}$ (see in Figure 5.2 (b)). However, the GDoF still equals 2 with a slope of the GDoF curve = 0 even though $\kappa > \frac{1}{2}$ (see in Figure 5.2 (a)). This implies that there is no more gain in the GDoF when $\kappa > \frac{1}{2}$ since full receiver cooperation performance is obtained. Note that in Figure 5.2(b), we observe that

   - $d(\alpha, \kappa = \frac{1}{3}) = 2$ when $0 \leq \alpha \leq 0.3$ and $\alpha = \frac{2}{3}$ and
   - $d(\alpha, \kappa = \frac{1}{6}) = 2$ when $0 \leq \alpha \leq 0.1$

   (b) For $1 \leq \alpha \leq 3$: As shown in Figure 5.3, the GDoF increases with varying gain value when κ increases. The GDoF curve changes from a step to a linear line with a slope of 2 when κ increases from 0 to 2. Furthermore, we observe that the GDoF saturates since full receiver cooperation performance is achieved when $\kappa \geq \kappa^*$ in the following subrange of α.

      i. For $1 < \alpha \leq \frac{3}{2}$, we obtain $\kappa^* = \frac{1}{2}$

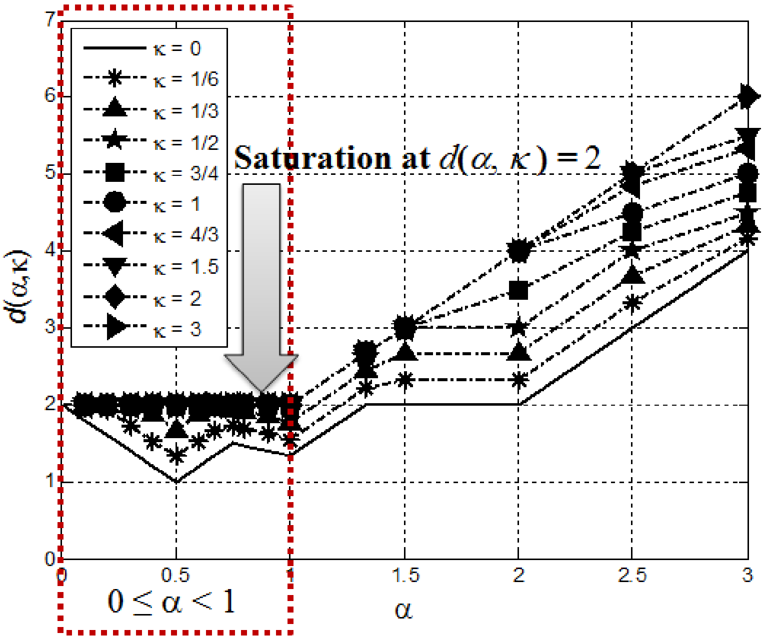
      ii. For $\frac{3}{2} < \alpha \leq 2$, we obtain $\kappa^* = 1$

      iii. For $2 < \alpha \leq \frac{5}{2}$, we obtain $\kappa^* = \frac{3}{2}$

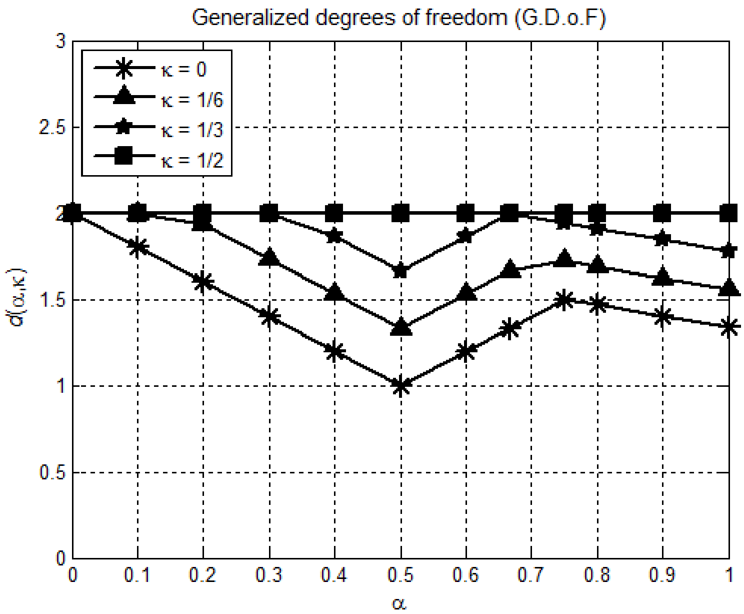
      iv. For $\frac{5}{2} < \alpha \leq 3$, we obtain $\kappa^* = 2$

In addition, from Figure 5.4, when we use $\alpha = \frac{4}{3}$ as the line for dividing the GDoF curves into 2 areas, we easily see that, at each κ, the GDoF in the range $\frac{4}{3} \leq \alpha \leq 3$ is larger than or equal to the GDoF in the range $0 \leq \alpha \leq \frac{4}{3}$. This result shows that the

[1]The GDoF of the symmetric Gaussian X channel defined in Section 3.1 is the same as the result in [14], i.e., the solid line in Figure 5.1.



(a)



(b)

Figure 5.2: Characteristic of the GDoF for the symmetric two-user Gaussian X channel with receiver cooperation (a) Considering $0 \leq \alpha < 1$ and $\max(\kappa) = 3$ (b) Zooming Figure 5.2(a) by focusing on $0 \leq \alpha < 1$ and $\max(\kappa) = \frac{1}{3}$

performance of this system is improved considerably when it is at medium to high interference environments.

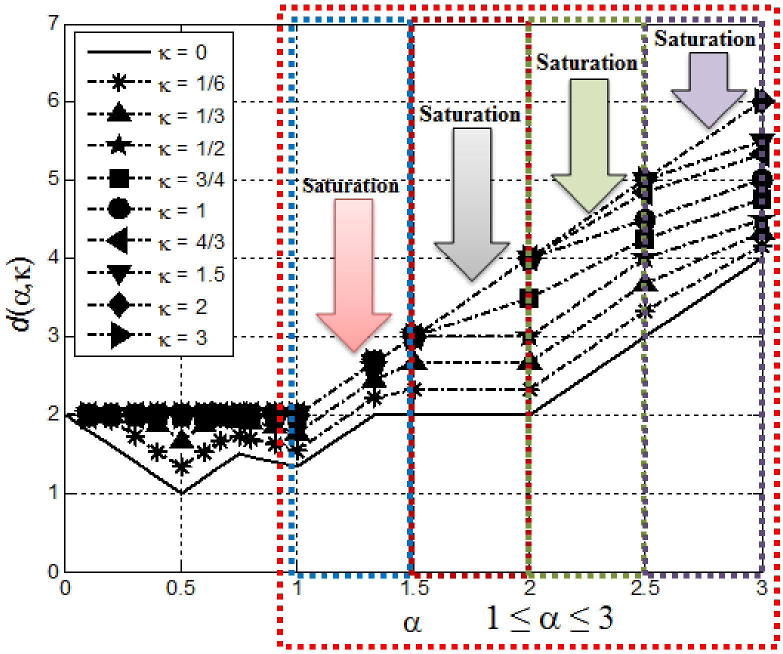


Figure 5.3: Characteristic of the GDoF for the symmetric two-user Gaussian X channel with receiver cooperation when $1 \leq \alpha \leq 3$

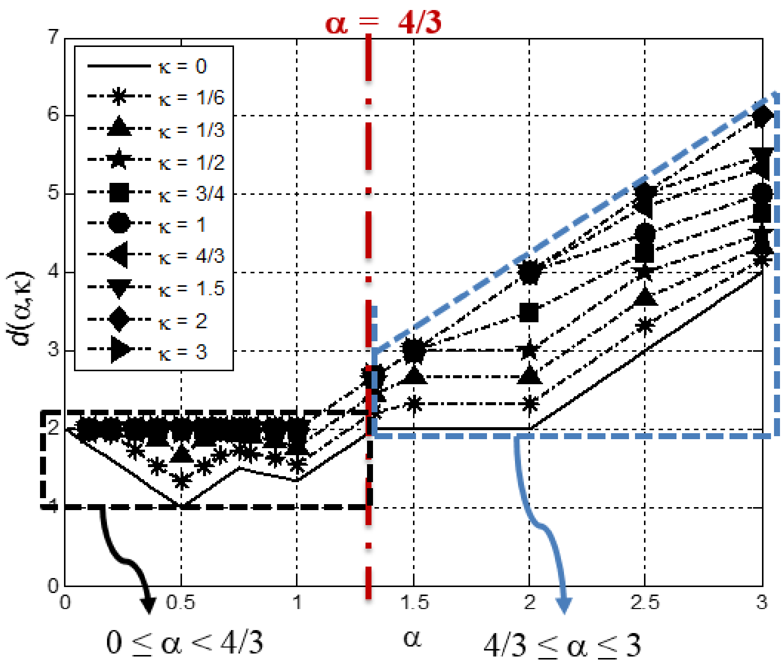


Figure 5.4: Comparison between the GDoF for $0 \leq \alpha \leq \frac{4}{3}$ and the GDoF for $\frac{4}{3} \leq \alpha \leq 3$ when $0 \leq \kappa < 3$

Remark 5.6 *The interesting notice of the obtained results in Figure 5.1 is a growth of the GDoF ($d(\alpha, \kappa)$) without bound when*

- $\alpha \geq 2$ *at* $\kappa = 0, \frac{1}{6}, \frac{1}{2}$.
- $\alpha \geq 1$ *at* $\kappa = \frac{3}{4}, 1, \frac{4}{3}, \frac{3}{2}, 2, 3$.

In the case $\kappa = 0$ corresponding to the two-user Gaussian X channel without receiver cooperation, our received result is identical to $d(\alpha, \kappa = 0) = 2\alpha - 2$ for $\alpha \geq 2$ in Theorem 3.2 [14]. For $\kappa \geq 0$, tendency of $d(\alpha, \kappa)$ is similar to the case $\kappa = 0$, i.e., $d(\alpha, \kappa)$ grows as α increases.

Second, we present the behavior of the obtained gain from limited receiver cooperation by plotting the GDoF ($d(\alpha, \kappa)$) at α = $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{2}$, 2, $\frac{5}{2}$ and 3 versus the normalized capacity of the receiver-cooperative link κ. The result is shown in Figure 5.5.

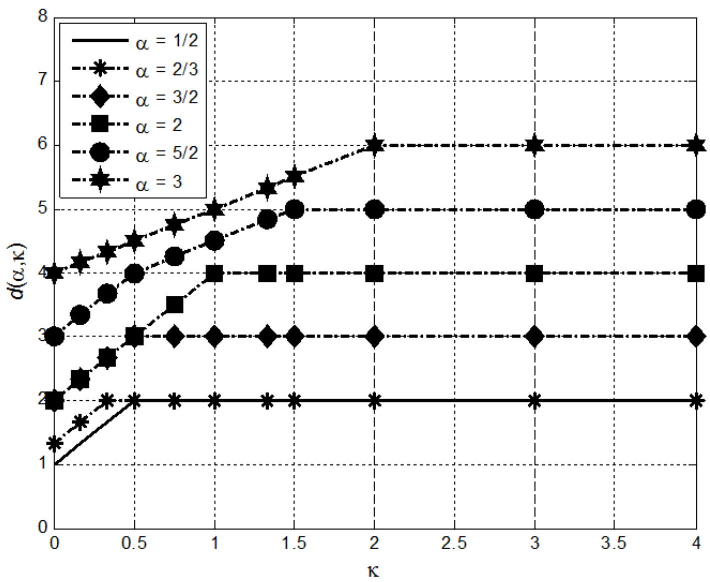


Figure 5.5 Gain from receiver cooperation when considering at α = $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{2}$, 2, $\frac{5}{2}$ and 3

From Figure 5.5, we see that

- At $\alpha = \frac{1}{2}$: The GDoF curve increases linearly and its slope $= 2$ when κ increases from 0 to $\frac{1}{2}$ and we obtain the GDoF value $d(\alpha = \frac{1}{2}, \kappa = \frac{1}{2}) = 2$. However, when $\kappa \geq \frac{1}{2}$, the GDoF curve's the slope changes to 0 and the GDoF value does not change, i.e., $d(\frac{1}{2}, \kappa) = 2$ for $\frac{1}{2} \leq \kappa \leq 4$.

- At $\alpha = \frac{2}{3}$: The GDoF curve increases linearly and its slope $= 2$ when κ increases from 0 to $\frac{1}{3}$ and we obtain the GDoF value $d(\alpha = \frac{2}{3}, \kappa = \frac{1}{3}) = 2$. However, when $\kappa \geq \frac{1}{3}$, the GDoF curve's the slope changes to 0 and the GDoF value does not change, i.e., $d(\frac{2}{3}, \kappa) = 2$ for $\frac{1}{3} \leq \kappa \leq 4$.

- At $\alpha = \frac{3}{2}$: The GDoF curve increases linearly and its slope $= 2$ when κ increases from 0 to $\frac{1}{2}$ and we obtain the GDoF value $d(\alpha = \frac{3}{2}, \kappa = \frac{1}{2}) = 3$. However, when $\kappa \geq \frac{1}{2}$, the GDoF curve's the slope changes to 0 and the GDoF value does not change, i.e., $d(\frac{3}{2}, \kappa) = 3$ for $\frac{1}{2} \leq \kappa \leq 4$.

- At $\alpha = 2$: The GDoF curve increases linearly and its slope $= 2$ when κ increases from 0 to 1 and we obtain the GDoF value $d(\alpha = 2, \kappa = \frac{1}{)} = 4$. However, when $\kappa \geq 1$, the GDoF curve's the slope changes to 0 and the GDoF value does not change, i.e., $d(2, \kappa) = 4$ for $1 \leq \kappa \leq 4$.

- At $\alpha = \frac{5}{2}$: The GDoF curve increases linearly and its slope $= 2$ and 1 when κ increases from 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to $\frac{3}{2}$, respectively, and we obtain the GDoF value $d(\alpha = \frac{5}{2}, \kappa = \frac{3}{2}) = 5$. However, when $\kappa \geq \frac{3}{2}$, the GDoF curve's the slope changes to 0 and the GDoF value does not change, i.e., $d(\frac{5}{2}, \kappa) = 5$ for $\frac{3}{2} \leq \kappa \leq 4$.

- At $\alpha = 3$: The GDoF curve increases linearly and its slope $= \frac{1}{2}$ when κ increases from 0 to 2 and we obtain the GDoF value $d(\alpha = 3, \kappa = 2) = 6$. However, when $\kappa \geq 2$, the GDoF curve's the slope changes to 0 and the GDoF value does not change, i.e., $d(3, \kappa) = 6$ for $2 \leq \kappa \leq 4$.

From the results above, it is clear that the receiver cooperation is obviously efficient in "*the linear region*" as defined in [2], i.e, the region which the GDoF is proportional to the normalized capacity of the receiver-cooperative link (κ) with a positive slope until it gets full receiver cooperation performance at a specific point $\kappa = \kappa^*$ as shown in Figure 5.5, where

1. $\kappa^* = \frac{1}{3}$ for $\alpha = \frac{2}{3}$

2. $\kappa^* = \frac{1}{2}$ for $\alpha = \frac{1}{2}, \frac{3}{2}$

3. $\kappa^* = 1$ for $\alpha = 2$.

4. $\kappa^* = \frac{3}{2}$ for $\alpha = \frac{5}{2}$

5. $\kappa^* = 2$ for $\alpha = 3$

Note that this result corresponds to the obtained result in Figure 5.2 and Figure 5.3.

CHAPTER VI

ACHIEVABLE RATE REGIONS FOR THE TWO-USER GAUSSIAN X CHANNEL WITH LIMITED RECEIVER COOPERATION

In this chapter, we propose the strategies which consist of transmission schemes based on HK strategy and cooperative protocol based on QMF scheme and then derive the achievable rate regions for the two-user Gaussian X channel with limited receiver cooperation in both the general case and the strong Gaussian X channel type I case. We show that our strategy in the strong Gaussian X channel type I case achieving the capacity region universally to within 2 bit/s/Hz per message, regardless of channel parameters, for the two-user Gaussian X channel with limited receiver cooperation.

6.1 Motivation of Strategy

Before providing our strategy for the two-user Gaussian X channel with limited receiver cooperation, we first give the idea for transmission scheme and then reveal the reason why we choose the two-round strategy based on QMF scheme as our cooperative protocol in this section.

6.1.1 Idea for Transmission Scheme

The idea for our transmission scheme comes from the objective of sending and receiving messages from each transmitter to the corresponding receivers. Therefore, we divide all transmitted messages $\{m_{ij}\}$ into two groups as follows:

1. In the first group, message m_{ii} is sent from transmitter i to receiver i, for $i = 1, 2$.

2. In the second group, message m_{ij} is sent from transmitter i to receiver j, for $(i, j) = (1, 2)$ or $(2, 1)$.

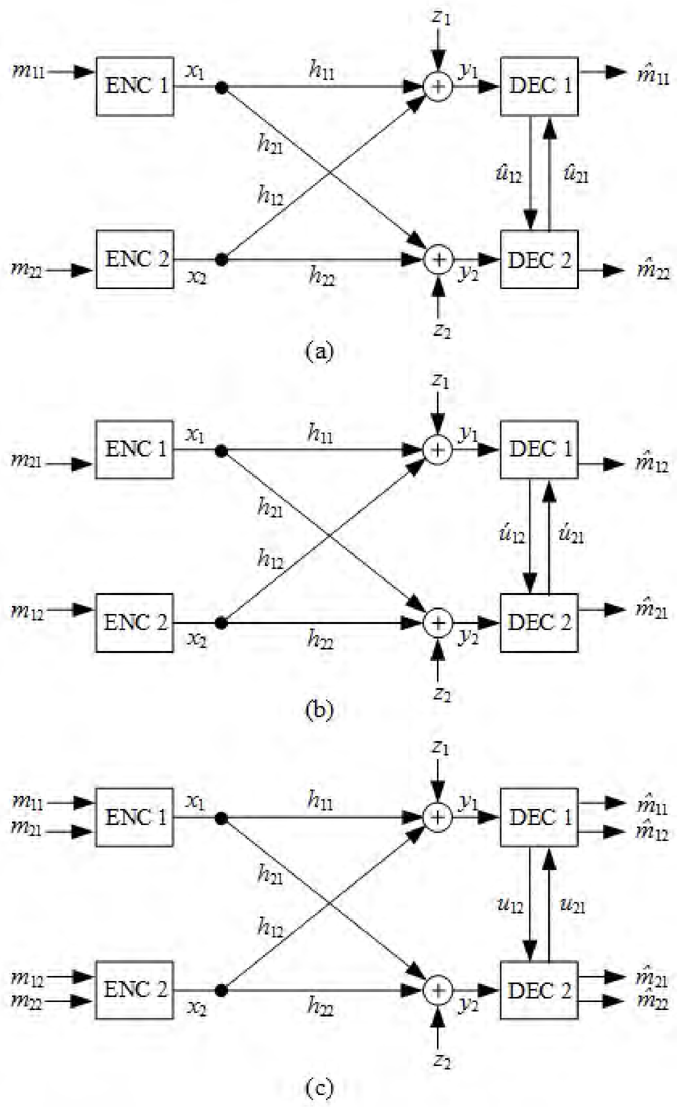


Figure 6.1: Idea for transmission scheme in the two-user Gaussian X channel with receiver cooperation, (a) Transmitting messages in the first two-user Gaussian interference channel with receiver cooperation, (b) Transmitting messages in the second two-user Gaussian interference channel with receiver cooperation and (c) Transmitting messages in the two-user Gaussian X channel with receiver cooperation viewed as the superposition of (a) and (b).

From dividing into groups above, we imagine the two-user Gaussian X channel with limited receiver cooperation as superposing of two Gaussian interference channels with limited receiver cooperation as shown in Figure 6.1. For $i, j = 1, 2$ and $i \neq j$, we have

- In Figure 6.1 (a), message m_{ii} in the first group is sent from transmitter *i* and then it is decoded correctly with limited receiver cooperation at receiver *i*. This communication scenario is similar to the work [2],

- In Figure 6.1 (b), message m_{ji} in the second group is sent from transmitter *i* and then it are decoded with limited receiver cooperation at receiver *j*. This communication scenario is different from the work [2] due to the different purpose of decoding at each receiver,

- In Figure 6.1 (c), messages m_{ii} and m_{ji} are sent from transmitter *i*. At receiver *i*, messages m_{ii} and m_{ij} are decoded corretly with limited receiver cooperation. Figure 6.1 (c) shows that the two-user Gaussian X channel with limited receiver cooperation can be viewed as superposing of two Gaussian interference channels from Figure 6.1 (a) and Figure 6.1 (b).

In this dissertation, we aim at the two-user Gaussian X channel with limited receiver cooperation where *each transmitter sends simultaneously two different messages to two receivers* and both receivers are allowed to exchange *a certain amount of information* between them. From the requirement above and the viewpoint of Figure 6.1, therefore, all messages in our system can be sent by using HK strategy [20] in our transmission scheme.

6.1.2 Cooperative Protocol

In [2], Wang and Tse reveal that their proposed cooperative protocol which is based on the QMF scheme achieves the optimal number of GDoF for all value of the normalized interference (α) and the normalized capacity of the receiver-cooperative link (κ). In addition, they also show that strategies based on conventional compress-forward or decode-forward scheme which are used in [29, 30] are not proper for receiver cooperation to mitigate interference in certain regimes because both schemes do not achieve the optimal GDoF universally for all α's and κ. Therefore, from the key advantage above, we use the cooperative protocol of the work [2] in the two-user Gaussian X channel with limited receiver cooperation.

6.2 Proposed Strategies

In this section, we describe the proposed strategies consisting of two parts, i.e., the transmission scheme and the cooperative protocol, and derive the achievable rate regions for the general case and the strong Gaussian X channel type I case.

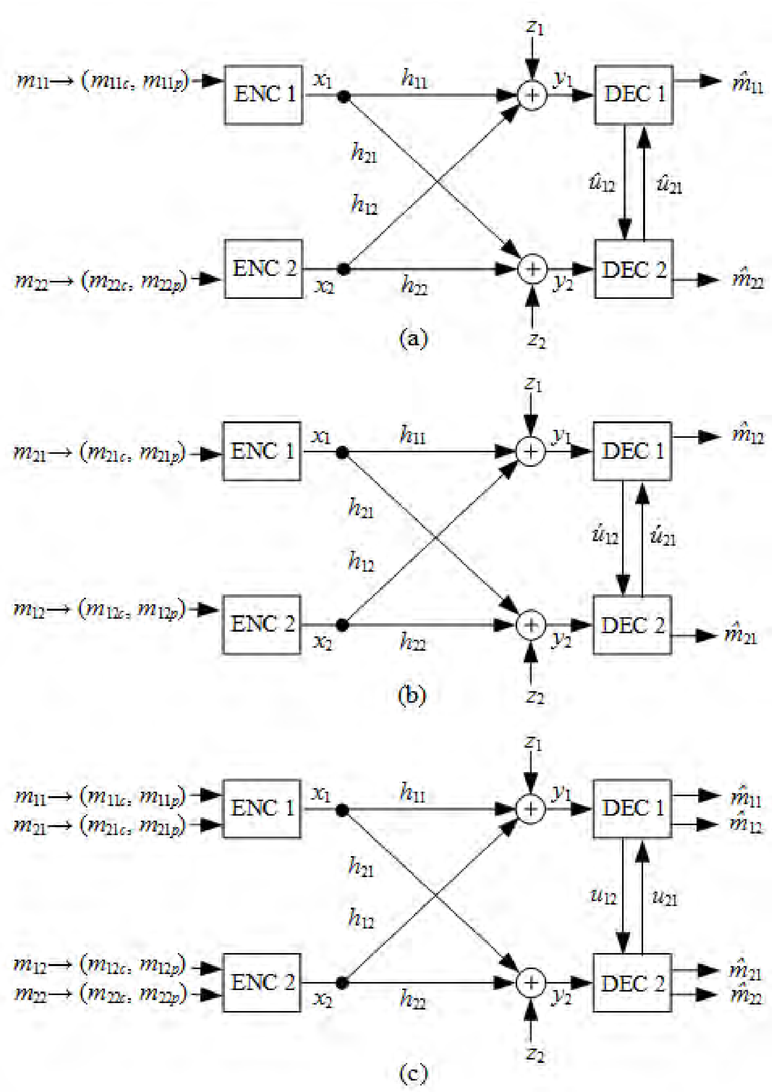


Figure 6.2: Transmission scheme in the general case, (a) Transmitting messages in the first two-user Gaussian interference channel with receiver cooperation, (b) Transmitting messages in the second two-user Gaussian interference channel with receiver cooperation and (c) Transmitting messages in the two-user Gaussian X channel with receiver cooperation viewed as the superposition of (a) and (b).

6.2.1 General Case

For $i, j = 1, 2$ and $i \neq j$, idea for transmitting messages in the general case based on the perspective of our motivation as shown in Figure 6.1 and using the HK strategy is depicted in Figure 6.2 and can be described as follows:

- In Figure 6.2 (a), message m_{ii} at transmitter *i* is split into common and private messages, i.e., (m_{iic}, m_{iip}). At receiver *i*, messages (m_{iic}, m_{iip}) are decoded correctly with limited receiver cooperation.

- In Figure 6.2 (b), message m_{ji} at transmitter *i* is split into common and private messages, i.e., (m_{jic}, m_{jip}). At receiver *i*, messages (m_{ijc}, m_{ijp}) are decoded correctly with limited receiver cooperation.

- In Figure 6.2 (c), message m_{ii} and m_{ji} at transmitter *i* are split into messages (m_{iic}, m_{iip}) and (m_{jic}, m_{jip}). At receiver *i*, messages (m_{iic}, m_{iip}) and (m_{ijc}, m_{ijp}) are decoded correctly with limited receiver cooperation. Figure 6.2 (c) shows that the two-user Gaussian X channel with limited receiver cooperation can be viewed as superposing of two Gaussian interference channels from Figure 6.2 (a) and Figure 6.2 (b).

With the consideration above, our transmission scheme in this case has the details as follows:

6.2.1.1 Transmission Scheme

Each transmitter consists of two independent messages m_{ii} and m_{ji} that are sent to receiver i and j, respectively, for $i, j = 1, 2$ and $i \neq j$. Each transmitter splits each own message into common and private messages, that is, $m_{ii} \rightarrow (m_{iic}, m_{iip})$ and $m_{ji} \rightarrow (m_{jic}, m_{jip})$. Hence, we have four independent submessages in each transmitter. Each common message can be decoded by both receivers, while each private message is decoded only at own receiver. In each transmitter, two common messages (m_{iic}, m_{jic}) are used for generating the common codeword and then these two common messages are rearranged as a new common message $m_{ic} \in \{1, ..., 2^{N(R_{iic}+R_{jic})}\}$. Next, each transmitter generates each private codeword by serving common codeword as the cloud center. Finally, each transmitter generates the codeword which is superposed over triple of common codeword and two private codewords. From the transmission scheme above, each message is encoded into a codebook drawn from a Gaussian random codebook with a certain power. For transmitter i, the power for its two private codewords and one common codeword are Q_{iip}, Q_{jip}, and $Q_{ic} = 1 - (Q_{iip} + Q_{jip})$, respectively.

Note that our transmission scheme above which is based on the HK strategy can be called *two-layer HK scheme* and is similar to the work [11, 13] but it does not use the Marton's binning technique as the work [11].

Remark 6.1 (Managing the power-splitting of private signals in the general case) *For the issue of managing the power-splitting of all private signals in the general case, we do not propose or adopt any method to solve this problem. We only intend to provide the achievable rates based on the proposed strategy consisting the transmission scheme and the cooperative protocol in the general terms for this case as shown in Section 6.2.1.3.*

6.2.1.2 Cooperative Protocol

We use the two-round strategy ($\mathrm{STG}_{j\to i\to j}$) from [2] with some modifications in the decoding processes. This protocol is based on the quantize-map-and-forward (QMF) scheme and its processing order is: receiver j quantize-and-bins, receiver i decoded-and-bins and receiver j decodes. Its achievable rate region is denoted by $\mathscr{R}_{j\to i\to j}$. By time sharing, the achievable rate region is obtained by $\mathscr{R} := \mathrm{conv}\{\mathscr{R}_{2\to 1\to 2} \cup \mathscr{R}_{1\to 2\to 1}\}$, i.e., the convex hull of the union of two rate regions. Next, we describe the two-round strategy $\mathrm{STG}_{2\to 1\to 2}$.

- *Quantize-Binning:* Receiver 2 (serving as relay) quantizes its received signal by a pregenerated Gaussian quantization codebook with proper distortion and sends out a bin index determined by a pregenerated binning function ($l_{21} = b_2(\hat{y}_2^N)$) to receiver 1. Based on our transmission scheme, the private signals x_{21p}, x_{22p} and the noise that it meets are not the required information for receiver 1. Therefore, the natural configuration is to set the distortion level equal to the sum of the power level of the noise, the total power level of the private signals x_{21p}, x_{22p} and the term $|h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}$ relating to the channel gain and phase of channel results from sending m_{21p} and m_{22p} that are contained in the codewords via wireless channel from transmitter 1 and 2, respectively.

- *Decode-Binning:* Receiver 1 retrieves the receiver-cooperative side information, and then decodes two common messages and its two own private messages by searching in transmitters' codebooks for a codeword quadruple (indexed by two common messages (m_{1c}, m_{2c}) and receiver 1's two private messages (m_{11p}, m_{12p})) that is jointly typical with its received signal and some quantization codeword in the given bin. If there is such unique codeword quadruple, i.e., there exists an unique indices, it can easily obtain two desired common messages m_{11c}, m_{12c} from m_{1c}, m_{2c}, respectively. Otherwise, declare an error. After receiver 1 decodes already, it uses two pregenerated binning function to bin the two common messages m_{1c}, m_{2c} ($l_{12}^{(ic)} = b_1^{(ic)}(m_{ic}),\ \text{for } i = 1, 2$) and sends out these two bin indices to receiver 2.

- *Decoding:* After receiving two bin indices from the receiver-cooperative side information, receiver 2 decodes two common messages m_{1c}, m_{2c} and its two own private messages m_{21p}, m_{22p} by searching in the corresponding bins (containing common messages) and receiver 2's two private codebooks for a codeword quadruple that is jointly typical with its received signal. Finally, if there is such unique codeword quadruple, i.e., there exists an unique indices, it can easily obtain two desired common messages m_{21c}, m_{22c} from m_{1c}, m_{2c}, respectively. Otherwise, declare an error.

Remark 6.2 *Although the cooperative protocol is similar to that in the work [2], but there are two modification points based on codebook generation of transmitted messages as follows:*

1. *In Quantize-Binning stage, the quantization distortion equals the total power of the undesired signals, i.e., the noise power and the total power of private signals* x_{21p}, x_{22p} *and the term* $|h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}$ *involving with the channel gains and the phases of channel for transmitting private messages* m_{21p} *and* m_{22p} *that are contained in the codewords via wireless channel from transmitter 1 and 2, respectively.*

2. *In Decode-Binning and Decoding stages, the set of messages (*$m_{1c}, m_{2c}, m_{11p}, m_{12p}$*) and (*$m_{1c}, m_{2c}, m_{21p}, m_{22p}$*) are decoded correctly, respectively.*

6.2.1.3 Achievable Rates

In the following theorem, we establish the achievable rates using the two-round strategy ($\text{STG}_{2\to1\to2}$) for the two-user Gaussian X channel with limited receiver cooperation in the general case. Let R_{ic} R_{iip}, and R_{ijp} denote the rates for transmitter i's common message, private message m_{iip}, and private message m_{ijp}, respectively, for $i, j = 1, 2$ and $i \neq j$.

Theorem 6.3 *(Achievable Rate Region for $STG_{2\to1\to2}$): The rate sextuple ($R_{1c}, R_{2c}, R_{11p}, R_{12p}, R_{21p}, R_{22p}$) in the general case satisfying the following constraints is achievable:*

Constraint at transmitter 1:

$$R_{1c} = R_{11c} + R_{21c} \tag{6.1}$$

Constraints at receiver 1:

$$R_{11p} \le \min\Big\{ I(x_{11p}; y_1|x_{1c}, x_{2c}, x_{12p}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{11p}; y_1, \hat{y}_2|x_{1c}, x_{2c}, x_{12p}) \Big\} \tag{6.2}$$

$$R_{12p} \le \min\Big\{ I(x_{12p}; y_1|x_{1c}, x_{2c}, x_{11p}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{12p}; y_1, \hat{y}_2|x_{1c}, x_{2c}, x_{11p}) \Big\} \tag{6.3}$$

$$R_{1c} + R_{11p} \le \min\Big\{ I(x_{1c}, x_{11p}; y_1|x_{2c}, x_{12p}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{1c}, x_{11p}; y_1, \hat{y}_2|x_{2c}, x_{12p}) \Big\} \tag{6.4}$$

$$R_{2c} + R_{12p} \le \min\Big\{ I(x_{2c}, x_{12p}; y_1|x_{1c}, x_{11p}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{2c}, x_{12p}; y_1, \hat{y}_2|x_{1c}, x_{11p}) \Big\} \tag{6.5}$$

$$R_{11p} + R_{12p} \le \min\Big\{ I(x_{11p}, x_{12p}; y_1|x_{1c}, x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{11p}, x_{12p}; y_1, \hat{y}_2|x_{1c}, x_{2c}) \Big\} \tag{6.6}$$

$$R_{1c} + R_{11p} + R_{12p} \le \min\Big\{ I(x_{1c}, x_{11p}, x_{12p}; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{1c}, x_{11p}, x_{12p}; y_1, \hat{y}_2|x_{2c}) \Big\} \tag{6.7}$$

$$R_{2c} + R_{12p} + R_{11p} \le \min\Big\{ I(x_{2c}, x_{12p}, x_{11p}; y_1|x_{1c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{2c}, x_{12p}, x_{11p}; y_1, \hat{y}_2|x_{1c}) \Big\} \tag{6.8}$$

$$R_{1c} + R_{11p} + R_{2c} + R_{12p} \le \min\Big\{ I(x_{1c}, x_{11p}, x_{2c}, x_{12p}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+, I(x_{1c}, x_{11p}, x_{2c}, x_{12p}; y_1, \hat{y}_2) \Big\} \tag{6.9}$$

where

$$\xi_1 = I(\hat{y}_2, y_2|x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1)$$

Constraint at transmitter 2:

$$R_{2c} = R_{12c} + R_{22c} \tag{6.10}$$

Constraints at receiver 2:

$$R_{21p} \le I(x_{21p}; y_2|x_{1c}, x_{2c}, x_{22p}) \quad (6.11)$$

$$R_{22p} \le I(x_{22p}; y_2|x_{1c}, x_{2c}, x_{21p}) \quad (6.12)$$

$$R_{1c} + R_{21p} \le I(x_{1c}, x_{21p}; y_2|x_{2c}, x_{22p}) + \mathsf{C}_{12}^{\mathsf{B}} \quad (6.13)$$

$$R_{2c} + R_{22p} \le I(x_{2c}, x_{22p}; y_2|x_{1c}, x_{21p}) + \mathsf{C}_{12}^{\mathsf{B}} \quad (6.14)$$

$$R_{21p} + R_{22p} \le I(x_{21p}, x_{22p}; y_2|x_{1c}, x_{2c}) \quad (6.15)$$

$$R_{1c} + R_{21p} + R_{22p} \le I(x_{1c}, x_{21p}, x_{22p}; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \quad (6.16)$$

$$R_{2c} + R_{21p} + R_{22p} \le I(x_{2c}, x_{21p}, x_{22p}; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} \quad (6.17)$$

$$R_{1c} + R_{21p} + R_{2c} + R_{22p} \le I(x_{1c}, x_{21p}, x_{2c}, x_{22p}; y_2) + \mathsf{C}_{12}^{\mathsf{B}} \quad (6.18)$$

over all joint distribution

$$p(x_{1c})p(x_{11p}|x_{1c})p(x_{21p}|x_{1c})p(x_1|x_{1c}, x_{11p}, x_{21p})$$
$$\times\, p(x_{2c})p(x_{12p}|x_{2c})p(x_{22p}|x_{2c})p(x_2|x_{2c}, x_{12p}, x_{22p}).$$

For $i, j = 1, 2$ and $i \neq j$, the superposition codebook generating random variable $x_i = x_{ic} + x_{iip} + x_{jip}$, where both private codebook generating random variables $x_{iip} \sim \mathcal{CN}(0, Q_{iip})$ and $x_{jip} \sim \mathcal{CN}(0, Q_{jip})$ are independent of the common codebook generating random variable $x_{ic} \sim \mathcal{CN}(0, Q_{ic})$. The quantization codebook generating random variable $\hat{y}_2 = y_2 + \hat{z}_2$, where $\hat{z}_2 \sim \mathcal{CN}(0, \Delta_2)$ is independent of everything else and Δ_2 denotes the quantization distortion at receiver 2.

Proof: See Section 6.4. ■

From Theorem 6.3, we provide some comments on these rate constraints as follows:

- First, m_{1c} and m_{2c} must be decoded correctly at receiver 1 because they are used to help receiver 2. Since the rate constraints (6.4) and (6.5) which are the same as the rate constraint R_{1c} and R_{2c}, respectively, as seen in Section 6.4.2 for deriving the probability of $E_S^{(1)}$ involve with m_{1c} and m_{2c}. Therefore, both (6.4) and (6.5) are obviously required.

- Second, in the set of the rate constraints at receiver 1, on the right-hand side each inequality is minimum of two terms. The first term corresponds to the case when receiver 1 can only determine a set of candidates of quantized $\hat{y}_2^N$. The second term corresponds to the case when the cooperative link is strong enough to carry the quantized $\hat{y}_2^N$ accurately.

- Finally, there is no gain in R_{21p} and R_{22p} in the set of the rate constraints at receiver 2 because receiver 1 only help receiver 2 decode m_{1c} and m_{2c}.

Remark 6.4 *(Effectiveness of an Achievable Rate Region in Theorem 6.3) Considering the special case when the capacity of the receiver-cooperative links is zero, i.e.,* $\mathsf{C}_{12}^{\mathsf{B}} = \mathsf{C}_{21}^{\mathsf{B}} = 0$, *in Theorem 6.3, and compare it with [11] when setting* $R_{ijp} = R_{ijp^*}$ *for* $i, j = 1, 2$ *and assigning the time-sharing random variable* g *to be a constant (no time sharing) in the result of [11]. With the above comparison, we obtain that our proposed rate region reduces to the best known achievable rate region for the two-user Gaussian X channel without receiver cooperation in [11]. This means that our proposed achievable rate region is larger than or equal to the one of the two-user Gaussian X channel without receiver cooperation case.*

Remark 6.5 *(Achievable Rate Region with the Perfect Cooperation) This dissertation studies the case of limited rate receiver cooperation where noiseless receiver-cooperative links have finite capacity* $0 \leq \mathsf{C}_{ij}^{\mathsf{B}} \leq \mathsf{C}_{ij}^{\mathsf{B}*}$, *for* $i, j = 1, 2$ *and* $i \neq j$, *as given in Section 3.1. When we consider an achievable rate region in Theorem 6.3 for the perfect cooperation case where both receivers can share* y_1^N *and* y_2^N *perfectly, it can see that this rate region contains achievable rates with the maximum value of the finite capacity of links, i.e.,* $\mathsf{C}_{ij}^{\mathsf{B}*}$. *Furthermore, comparing this rate region with the region for receiver 2 in Theorem 6.3 when* $\mathsf{C}_{12}^{\mathsf{B}}$ *is sufficiently large, it obviously obtain that the region for receiver 2 does not exceed an achievable rate region with the perfect cooperation since* $\mathsf{C}_{12}^{\mathsf{B}} \leq \mathsf{C}_{12}^{\mathsf{B}*}$.

We define the following notations which are used over the rest of this chapter: for $i, j = 1, 2$ and $i \neq j$,

$$\mathsf{SNR}_i^{iip} := |h_{ii}|^2 Q_{iip} = \mathsf{SNR}_i Q_{iip}, \mathsf{SNR}_i^{ijp} := |h_{ii}|^2 Q_{ijp} = \mathsf{SNR}_i Q_{ijp},$$
$$\mathsf{INR}_i^{ijp} := |h_{ij}|^2 Q_{ijp} = \mathsf{INR}_i Q_{ijp}, \ \ \mathsf{INR}_i^{jjp} := |h_{ij}|^2 Q_{jjp} = \mathsf{INR}_i Q_{jjp}.$$

Next, we calculate the rate loss term ξ_1 which is in the set of the rate constraints at receiver 1 as follows:

$$\begin{aligned}
\xi_1 &= I(\hat{y}_2, y_2|x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1) \\
&= h(\hat{y}_2|x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1) - h(\hat{y}_2|x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1, y_2) \\
&= h(h_{21}x_{21p} + h_{22}x_{22p} + z_2 + \tilde{z}_2|h_{11}x_{21p} + h_{12}x_{22p} + z_1) - h(\hat{z}_2) \\
&= \log\left(\frac{1+\Delta_2}{\Delta_2} + \frac{\mathsf{INR}_2^{21p} + \mathsf{SNR}_2^{22p} + |h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}}{\Delta_2(1+\mathsf{SNR}_1^{21p} + \mathsf{INR}_1^{22p})}\right) \\
&\le \log\left(\frac{1+\Delta_2 + \mathsf{INR}_2^{21p} + \mathsf{SNR}_2^{22p} + |h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}}{\Delta_2}\right). \qquad (6.19)
\end{aligned}$$

From (6.19), it is easily seen that the rate loss term ξ_1 can be upper bounded by 1 bit by selecting $\Delta_2 = 1 + \mathsf{INR}_2^{21p} + \mathsf{SNR}_2^{22p} + |h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}$.

Remark 6.6 *(Reason for Choosing Δ_2 in the General Case) Based on our transmission scheme in Sect. 6.2.1.1, the undesirable signals from y_2^N at receiver 1 are the private signals involving m_{21p} and m_{22p} and noise which it meets. Choosing $\Delta_2 = 1 + \mathsf{INR}_2^{21p} + \mathsf{SNR}_2^{22p} + |h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}$ which is equal to the aggregate power of the undesirable signals, where the first three terms of the aggregate power of the undesirable signals correspond to the power level of noise and the private signals involving m_{21p} and m_{22p}, respectively, in y_2 for receiver 1 and the last term $|h_{11}h_{22} - h_{12}h_{21}|^2 Q_{21p}Q_{22p}$ relating to the channel gain and phase of channel results from sending m_{21p} and m_{22p} that are contained in the codewords via wireless channel from transmitter 1 and 2, respectively. Hence, ξ_1 in (6.19) can be upper bounded by 1 bit with choosing Δ_2 as above and the undesirable signals are managed by treating them as noise at receiver 1.*

6.2.2 Strong Gaussian X Channel Type I Case

In Section 3.3, we show the classification of the two-user Gaussian X channel based on the work [12]. This dissertation considers only on the strong Gaussian X channel type I case, i.e., $\mathsf{SNR}_1 > \mathsf{INR}_2$ and $\mathsf{SNR}_2 > \mathsf{INR}_1$. With the point of view in Figure 6.1, idea for transmitting messages in this case is shown in Figure 6.3, where the bold arrow indicates the strong channel gain, and has the following details:

1. In Figure 6.3 (a), the first interference channel corresponds to the weak interference channel with receiver cooperation. Therefore, both messages m_{11} and m_{22} can be split into common and private messages, i.e., (m_{11c},m_{11p}) and (m_{22c},m_{22p}), respectively, and then messages (m_{11c},m_{11p}) and (m_{22c},m_{22p}) are decoded with cooperation at receiver 1 and 2, respectively.

2. In Figure 6.3 (b), the second interference channel corresponds to the strong interference channel with receiver cooperation. Therefore, both messages m_{12} and m_{21} are whole common messages, i.e., m_{12c} and m_{21c}, respectively, and messages m_{12c} and m_{21c} are decoded with cooperation at receiver 1 and 2, respectively.

3. Finally, we obtain the strong Gaussian X channel type I with receiver cooperation as Figure 6.3 (c) by superimposing two interference channels with receiver cooperation from Figure 6.3 (a) and Figure 6.3 (b) and messages m_{iic},m_{iip},m_{jic} are sent from transmitter i and then messages m_{iic},m_{iip},m_{ijc} are decoded at receiver i, for $i, j = 1, 2$ and $i \neq j$.

With the point of view in Figure 6.1 and the concept of a simple power split construction in [9], our proposed transmission scheme in this case is showed in Figure 6.3 and can be described as follows:

6.2.2.1 Transmission Scheme

Each transmitter consists of two independent messages m_{ii} and m_{ji} that are sent to receiver i and j, respectively, for $i, j = 1, 2$ and $i \neq j$. In transmitter i, message m_{ii} is split into common and private messages, i.e., $m_{ii} \rightarrow (m_{iic}, m_{iip})$, whereas message m_{ji} is whole common message. Hence, we have three independent submessages in each transmitter. Each common message can be decoded by both receivers, while a private message is decoded only at own receiver. In each transmitter, two common messages (m_{iic}, m_{jic}) are used for generating a common codeword and then these two common messages are rearranged as a new common message $m_{ic} \in \{1, ..., 2^{N(R_{iic}+R_{jic})}\}$. Finally, each transmitter generates a private codeword by serving common codeword as the cloud center. From the transmission scheme above, each message is encoded into a codebook drawn from a Gaussian random codebook with a certain power. For transmitter i, the power for its private and common codewords are Q_{iip} and $Q_{ic} = 1 - Q_{iip}$, respectively.

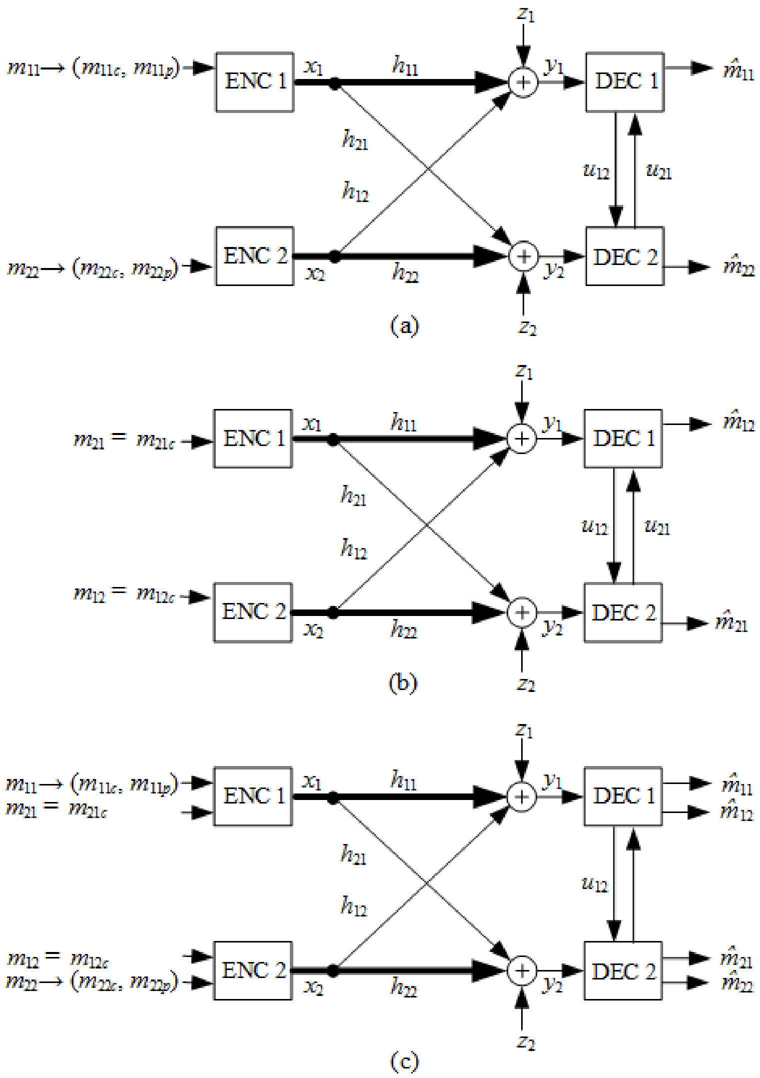


Figure 6.3: Transmission scheme in the the strong Gaussian X channel type I case, (a) Transmitting messages in the weak interference channel with receiver cooperation, (b) Transmitting messages in the strong interference channel with receiver cooperation and (c) Transmission scheme for the strong Gaussian X channel type I with receiver cooperation viewed as the superposition of two different interference channels ((a) and (b)).

The power split configuration is such that $Q_{iip} + Q_{ic} = 1$ and $\mathsf{INR}_i^{jjp} := |h_{ij}|^2 Q_{ijp} \leq 1$ if $\mathsf{SNR}_i > \mathsf{INR}_j$. Using a simple power-splitting configuration from [9], we set the power of each private message as follows:

$$Q_{iip} = \min\left\{\frac{1}{\mathsf{INR}_j}, 1\right\} \tag{6.20}$$

Remark 6.7 (Comparison our transmission scheme with the previous works) *From the proposed transmission scheme for the strong Gaussian X channel type I above, we see that*

1. *In each transmitter, there is only one private message for encoding which corresponds to the result in [15].*
2. *A common codeword in our work is generated from two different common messages but a common codeword in [2] is generated from a common message.*
3. *A transmitted codeword from each transmitter in our work which is the sum of a common codeword and a private codeword (see details in Section 6.5) is similar to [2].*

6.2.2.2 Cooperative Protocol

Using the proposed transmission scheme in Section 6.2.2.1, i.e., message m_{ii} is split into m_{iic} and m_{iip} while making m_{ij} as the whole common message m_{ijc} for $i, j = 1, 2$ and $i \neq j$. Therefore, the cooperative protocol with $\mathrm{STG}_{2\to1\to2}$ in Section 6.2.1.2 is reduced to the original version [2] by decoding messages $(m_{1c}, m_{2c}, m_{11p})$ and $(m_{1c}, m_{2c}, m_{22p})$ in the decode-binning and decode stages, respectively.

Note that the details of the cooperative protocol with $\mathrm{STG}_{2\to1\to2}$ in this subsection are similar to the ones in Section 6.2.1.2. Hence, we omit them.

6.2.2.3 Achievable Rates

In the following theorem, we establish the achievable rates using the two-round strategy ($\mathrm{STG}_{2\to1\to2}$) for the two-user Gaussian X channel with limited receiver cooperation in the strong Gaussian X channel type I case. Let R_{ic} and R_{iip} denote the rates for ith common message and private message m_{iip}, respectively, for $i = 1, 2$.

Theorem 6.8 *(Achievable Rate Region for $STG_{2\to 1\to 2}$): The rate tuple $(R_{1c}, R_{2c}, R_{11p}, R_{22p})$ in the strong Gaussian X channel type I case satisfying the following constraints is achievable:*

Constraint at transmitter 1:

$$R_{1c} = R_{11c} + R_{21c} \tag{6.21}$$

Constraints at receiver 1:

$$R_{11p} \le \min\Big\{I(x_1; y_1|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+, I(x_1; y_1, \hat{y}_2|x_{1c}, x_{2c})\Big\} \tag{6.22}$$

$$R_{2c} \le \min\Big\{I(x_{2c}; y_1|x_1) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+, I(x_{2c}; y_1, \hat{y}_2|x_1)\Big\} \tag{6.23}$$

$$R_{1c} + R_{11p} \le \min\Big\{I(x_1; y_1|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+, I(x_1; y_1, \hat{y}_2|x_{2c})\Big\} \tag{6.24}$$

$$R_{2c} + R_{11p} \le \min\Big\{I(x_{2c}, x_1; y_1|x_{1c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+, I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c})\Big\} \tag{6.25}$$

$$R_{1c} + R_{11p} + R_{2c} \le \min\Big\{I(x_1, x_{2c}; y_1) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+, I(x_1, x_{2c}; y_1, \hat{y}_2)\Big\} \tag{6.26}$$

where

$$\xi_1 = I(\hat{y}_2; y_2|x_{1c}, x_{2c}, x_1, y_1)$$

Constraint at transmitter 2:

$$R_{2c} = R_{12c} + R_{22c} \tag{6.27}$$

Constraints at receiver 2:

$$R_{22p} \le I(x_2; y_2|x_{1c}, x_{2c}) \tag{6.28}$$

$$R_{1c} \le I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} \tag{6.29}$$

$$R_{2c} + R_{22p} \le I(x_2; y_2|x_{1c}) + \mathsf{C}^{\mathsf{B}}_{12} \tag{6.30}$$

$$R_{1c} + R_{22p} \le I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}^{\mathsf{B}}_{12} \tag{6.31}$$

$$R_{1c} + R_{2c} + R_{22p} \le I(x_{1c}, x_2; y_2) + \mathsf{C}^{\mathsf{B}}_{12} \tag{6.32}$$

where $x_i = x_{ic} + x_{iip}$ is the superposition codebook generating random variable with $x_{ic} \sim \mathcal{CN}(0, Q_{ic})$ and $x_{iip} \sim \mathcal{CN}(0, Q_{iip})$ that is independent of x_{ic}. $\hat{y}_2 = y_2 + \hat{z}_2$ is the quantization codebook generating random variable and $\hat{z}_2 \sim \mathcal{CN}(0, \Delta_2)$ where Δ_2 is the quantization distortion at receiver 2.

Proof: See all details in Section 6.5. ■

We provide some comments on these rate constraints as follows:

1. The achievable rate region in Theorem 6.8 is similar to Theorem 4.3 in [2] for the two-user interference channels with limited receiver cooperation, but there are two key different points as follows:

   (a) In our work, the rate constraints at both transmitters in (6.21) and (6.27) are required in our work but not in [2].

   (b) The rate constraint for R_{1c} where receiver 2 is required to decode m_{1c} correctly.

Next, the rate loss ξ_1 in the set of the rate constraints at receiver 1 can be calculated as the following:

$$\begin{aligned}
\xi_1 &= I(\hat{y}_2, y_2 | x_{1c}, x_{2c}, x_1, y_1) \\
&= h(\hat{y}_2 | x_{1c}, x_{2c}, x_1, y_1) - h(\hat{y}_2 | x_{1c}, x_{2c}, x_1, y_1, y_2) \\
&= h(h_{22}x_{22p} + z_2 + \hat{z}_2 | h_{12}x_{22p} + z_1) - h(\hat{z}_2) \\
&= \log\Big(\frac{1+\Delta_2}{\Delta_2} + \frac{\mathsf{SNR}_2^{22p}}{(1+\mathsf{INR}_1^{22p})\Delta_2}\Big) \\
&\leq \log\Big(\frac{1+\Delta_2+\mathsf{SNR}_2^{22p}}{\Delta_2}\Big).
\end{aligned} \tag{6.33}$$

By choosing $\Delta_2 = 1 + \mathsf{SNR}_2^{22p}$, the rate loss ξ_1 is upper bounded by 1 bit.

Remark 6.9 *The above chosen distortion ($\Delta_2 = 1 + \mathsf{SNR}_2^{22p}$) may not be optimal. The achievable rates can be further improved if we optimize over all possible distortions [2, 32]. For instance, if the cooperative link capacity is large, the distortion level could be lowered to obtain a finer description of the received signal. With the above selection for the distortion, however, this achievable rate region can be within a constant gap to the capacity region, regardless of channel parameters.*

6.3 Characterization of the Capacity Region to Within 2 Bits

In this section, we show the performance of the proposed strategy in the case of strong Gaussian X channel type I. First of all, we present the main result of this chapter in Section 6.3.1. Second, we provide the achievable rate region in the terms of $(R_{11}, R_{12}, R_{21}, R_{22})$ by using the result of Theorem 6.8 in Section 6.3.2. Finally, we show that our proposed strategy in the strong Gaussian X channel type I case is within 2 bits/s/Hz per message compared with an outer bound on the capacity region of the two-user Gaussian X channel with limited receiver cooperation from Lemma 4.1.

6.3.1 Capacity Region to Within 2 Bits

In this dissertation, we investigate only in the Gaussian strong X channel type I case, i.e., $\mathsf{SNR}_1 > \mathsf{INR}_2$ and $\mathsf{SNR}_2 > \mathsf{INR}_1$. Our main result states in the following theorem (Recall that $\overline{\mathscr{C}}$ is an outer bound on the capacity region shown in Lemma 4.1):

Theorem 6.10 (*Within 2-bit Gap to Capacity Region):*

$$\mathscr{R} \subseteq \mathscr{C} \subseteq \overline{\mathscr{C}} \subseteq \mathscr{R} \oplus ([0,2] \times [0,2] \times [0,2] \times [0,2]),$$

Proof: This theorem is proved by Lemma 6.12. ■

6.3.2 Achievable Rate Region for the Strong Gaussian X Channel Type I Case

First, we consider $\mathrm{STG}_{2\to1\to2}$ and get the set of achievable rates $(R_{1c}, R_{2c}, R_{11p}, R_{22p})$ from Theorem 6.8. Remind that the rate loss $\xi_1 \leq 1$ when we choose $\Delta_2 = 1 + \mathsf{SNR}_2^{22p}$.

To simplify computations, we follow the same line in [2] by replacing (6.22) and (6.24) with

$$R_{11p} \leq I(x_1; y_1 | x_{1c}, x_{2c}),$$
$$R_{1c} + R_{11p} \leq I(x_1; y_1 | x_{2c})$$

in the following computations.

For i, j = 1, 2 and $i \neq j$, the achievable rate region in terms of $(R_{11}, R_{12}, R_{21}, R_{22})$ for $\mathscr{R}_{1\to2\to1}$ can be computed from the following three constraints:

1. $R'_{ii} = R_{iip} + R_{ic}$
2. $R'_i = R_{iip} + R_{ic} + R_{jc}$
3. $R''_i = R_{iip} + 2R_{ic}$ and $R''_j = R_{jjp} + R_{jc}$

Next, rewriting the above three constraints as follows:

1. $R_{iip} = R'_{ii} - R_{ic}$

2. $R_{iip} = R'_{i} - R_{ic} - R_{jc}$

3. $R_{iip} = R''_{i} - 2R_{ic}$ and $R_{jjp} = R''_{j} - R_{jc}$

Then applying the Fourier-Motzkin algorithm to each rewritten constraint and removing redundant terms (details omitted). Finally, after collecting and reordering the above obtained results, we get the achievable rate region $\mathscr{R}_{2\to1\to2}$ which consists of nonnegative $(R_{11}, R_{12}, R_{21}, R_{22})$ satisfying

$$R_{11} \leq I(x_1; y_1|x_{2c}) - R_{21c} \tag{6.34}$$

$$R_{11} \leq I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \tag{6.35}$$

$$R_{12} \leq I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c} \tag{6.36}$$

$$R_{12} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{22c} \tag{6.37}$$

$$R_{21} \leq I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \tag{6.38}$$

$$R_{22} \leq I(x_2; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \tag{6.39}$$

$$R_{22} \leq I(x_{2c}; y_1|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} \tag{6.40}$$

$$R_{22} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c} \tag{6.41}$$

$$\begin{aligned}
R_{11} + R_{12} &\leq I(x_1, x_{2c}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c} \\
R_{11} + R_{12} &\leq I(x_1, x_{2c}; y_1, \hat{y}_2) - R_{21c} - R_{22c} \\
R_{11} + R_{12} &\leq I(x_{2c}; y_1|x_1) + I(x_1; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c} \\
R_{11} + R_{12} &\leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_1; y_1|x_{2c}) - R_{21c} - R_{22c}
\end{aligned} \tag{6.42}$$

$$R_{11} + R_{12} \leq \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}; y_2|x_2) \\ +(\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} - R_{22c} \end{Bmatrix} \tag{6.43}$$

$$R_{11} + R_{12} \leq \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{21c} - R_{22c} \end{Bmatrix} \tag{6.44}$$

$$R_{11} + R_{12} \leq \begin{Bmatrix} I(x_{2c}, x_{1p}; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{21c} - R_{22c} \end{Bmatrix} \tag{6.45}$$

$$R_{11} + R_{12} \leq I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} - R_{22c} \tag{6.46}$$

$$\begin{aligned}
R_{21} + R_{22} &\leq I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \\
R_{21} + R_{22} &\leq I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}
\end{aligned} \tag{6.47}$$

$$R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c};y_1|x_1)+I(x_2;y_2|x_{1c},x_{2c})+I(x_{1c};y_2|x_2) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \end{Bmatrix} \tag{6.48}$$

$$R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_2;y_2|x_{1c},x_{2c})+I(x_{1c};y_2|x_2) \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \end{Bmatrix}$$

$$R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{11c} - R_{12c} \end{Bmatrix} \tag{6.49}$$

$$R_{21}+R_{22} \le I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$$

$$R_{11}+R_{12}+R_{21} \le I(x_1,x_{2c};y_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ - R_{22c} \tag{6.50}$$

$$R_{11}+R_{12}+R_{21} \le I(x_1,x_{2c};y_1,\hat{y}_2) - R_{22c} \tag{6.51}$$

$$R_{11}+R_{12}+R_{21} \le I(x_{2c};y_1|x_1)+I(x_1;y_1|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ - R_{22c} \tag{6.52}$$

$$R_{11}+R_{12}+R_{21} \le I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_1;y_1|x_{2c}) - R_{22c} \tag{6.53}$$

$$R_{11}+R_{12}+R_{21} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+I(x_{1c};y_2|x_2) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \end{Bmatrix} \tag{6.54}$$

$$R_{11}+R_{12}+R_{21} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c};y_2|x_2) \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \end{Bmatrix} \tag{6.55}$$

$$R_{11}+R_{12}+R_{21} \le \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c};y_2|x_2)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{22c} \end{Bmatrix} \tag{6.56}$$

$$R_{11}+R_{12}+R_{21} \le I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c};y_2|x_2)+\mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \tag{6.57}$$

$$R_{11}+R_{12}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \end{Bmatrix} \tag{6.58}$$

$$R_{11}+R_{12}+R_{22} \le I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \tag{6.59}$$

$$R_{11}+R_{12}+R_{22} \le I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2)+\mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \tag{6.60}$$

$$R_{11}+R_{12}+R_{22} \le I(x_1,x_{2c};y_1)+I(x_2;y_2|x_{1c},x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ - R_{21c}$$

$$R_{11}+R_{12}+R_{22} \le I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_2;y_2|x_{1c},x_{2c}) - R_{21c} \tag{6.61}$$

$$R_{11}+R_{12}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c},x_{2c}) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \end{Bmatrix} \tag{6.62}$$

$$R_{11}+R_{12}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c},x_{2c}) \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \end{Bmatrix} \tag{6.63}$$

$$R_{11}+R_{12}+R_{22} \leq \left\{\begin{aligned} & I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c}) \\ & +(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}-R_{21c} \end{aligned}\right\} \tag{6.64}$$

$$R_{11}+R_{12}+R_{22} \leq \left\{\begin{aligned} & I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1) \\ & +I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{21c} \end{aligned}\right\} \tag{6.65}$$

$$R_{11}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ \\ & +\mathsf{C}^{\mathsf{B}}_{12}-R_{12c} \end{aligned}\right\} \tag{6.66}$$

$$R_{11}+R_{21}+R_{22} \leq I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{12c} \tag{6.67}$$

$$R_{11}+R_{21}+R_{22} \leq I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{12c}$$

$$R_{11}+R_{21}+R_{22} \leq I(x_1,x_{2c};y_1)+I(x_2;y_2|x_{1c},x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+-R_{12c} \tag{6.68}$$

$$R_{11}+R_{21}+R_{22} \leq I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_2;y_2|x_{1c},x_{2c})-R_{12c}$$

$$R_{11}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c},x_{2c}) \\ & +(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}-R_{21c} \end{aligned}\right\} \tag{6.69}$$

$$R_{11}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c};y_2|x_2) \\ & +I(x_2;y_2|x_{1c},x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{12c} \end{aligned}\right\} \tag{6.70}$$

$$R_{11}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c}) \\ & +(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}-R_{12c} \end{aligned}\right\} \tag{6.71}$$

$$R_{11}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1) \\ & +I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{12c} \end{aligned}\right\}$$

$$R_{12}+R_{21}+R_{22} \leq I(x_{1c},x_2;y_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{11c} \tag{6.72}$$

$$R_{12}+R_{21}+R_{22} \leq I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c})+2\mathsf{C}^{\mathsf{B}}_{12}-R_{11c} \tag{6.73}$$

$$R_{12}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_{2c};y_1|x_1)+I(x_2;y_2|x_{1c},x_{2c})+I(x_{1c};y_2|x_2) \\ & +(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}-R_{11c} \end{aligned}\right\} \tag{6.74}$$

$$R_{12}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_2;y_2|x_{1c},x_{2c})+I(x_{1c};y_2|x_2) \\ & +\mathsf{C}^{\mathsf{B}}_{12}-R_{11c} \end{aligned}\right\} \tag{6.75}$$

$$R_{12}+R_{21}+R_{22} \leq \left\{\begin{aligned} & I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ \\ & +\mathsf{C}^{\mathsf{B}}_{12}-R_{11c} \end{aligned}\right\} \tag{6.76}$$

$$R_{12}+R_{21}+R_{22} \leq I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{11c} \tag{6.77}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c}) \\ +(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \tag{6.78}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12} \tag{6.79}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2)+\mathsf{C}^{\mathsf{B}}_{12} \tag{6.80}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le I(x_1,x_{2c};y_1)+I(x_2;y_2|x_{1c},x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ \tag{6.81}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_2;y_2|x_{1c},x_{2c}) \tag{6.82}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c};y_2|x_2) \\ +I(x_2;y_2|x_{1c},x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \tag{6.83}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c};y_2|x_2) \\ +I(x_2;y_2|x_{1c},x_{2c})+\mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \tag{6.84}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \tag{6.85}$$

$$R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \tag{6.86}$$

$$2R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c}) \\ +I(x_1,x_{2c};y_1)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}-R_{21c} \end{Bmatrix}$$

$$2R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c}) \\ +I(x_1,x_{2c};y_1,\hat{y}_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{21c} \end{Bmatrix}$$

$$2R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c},x_1;y_1|x_{1c}) \\ +I(x_{1c};y_2|x_2)+I(x_{1c},x_2;y_2|x_{2c}) \\ +(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + 2\mathsf{C}^{\mathsf{B}}_{12}-R_{21c} \end{Bmatrix} \tag{6.87}$$

$$2R_{11}+R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c}) \\ +I(x_{1c};y_2|x_2)+I(x_{1c},x_2;y_2|x_{2c})+2\mathsf{C}^{\mathsf{B}}_{12} \\ -R_{21c} \end{Bmatrix} \tag{6.88}$$

$$R_{11}+2R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12}-R_{22c} \end{Bmatrix} \tag{6.89}$$

$$R_{11}+2R_{12}+R_{21}+R_{22} \le I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c},x_2;y_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{22c} \tag{6.90}$$

$$R_{11}+2R_{12}+R_{21}+R_{22} \le \begin{Bmatrix} I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c}) \\ +2I(x_{2c};y_1|x_1)+2(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}-R_{22c} \end{Bmatrix} \tag{6.91}$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c})\\+2I(x_{2c};y_1,\hat{y}_2|x_1)+\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.92)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c};y_1|x_1)+I(x_{2c},x_1;y_1|x_{1c})\\+I(x_{1c},x_2;y_2|x_{2c})+2(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+\\+\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.93)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{2c},x_1;y_1|x_{1c})\\+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^++\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.94)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c};y_1|x_1)+I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})\\+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^++\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.95)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})\\+I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.96)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c};y_1|x_1)+I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2)\\+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^++\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.97)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_1;y_1|x_{1c},x_{2c})\\+I(x_{1c},x_2;y_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.98)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_1,x_{2c};y_1)+I(x_2;y_2|x_{1c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^++\mathsf{C}^{\mathsf{B}}_{12}\\-R_{22c}\end{matrix}\right\}$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_2;y_2|x_{1c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\quad(6.99)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c})+I(x_{1c};y_2|x_2)\\+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^++2\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.100)$$

$$R_{11}+2R_{12}+R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_2;y_2|x_{1c})\\+I(x_{1c};y_2|x_2)+2\mathsf{C}^{\mathsf{B}}_{12}-R_{22c}\end{matrix}\right\}\quad(6.101)$$

$$R_{11}+R_{12}+2R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_1,x_{2c};y_1)+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^+\\+\mathsf{C}^{\mathsf{B}}_{12}-R_{11c}\end{matrix}\right\}\quad(6.102)$$

$$R_{11}+R_{12}+2R_{21}+R_{22}\leq I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}-R_{11c}\quad(6.103)$$

$$R_{11}+R_{12}+2R_{21}+R_{22}\leq I(x_1;y_1|x_{2c})+I(x_{1c},x_2;y_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{11c}$$

$$R_{11}+R_{12}+2R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_1;y_1|x_{2c})+I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c})\\+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^++\mathsf{C}^{\mathsf{B}}_{12}-R_{11c}\end{matrix}\right\}\quad(6.104)$$

$$R_{11}+R_{12}+2R_{21}+R_{22}\leq\left\{\begin{matrix}I(x_1;y_1|x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})\\+\mathsf{C}^{\mathsf{B}}_{12}-R_{11c}\end{matrix}\right\}$$

$$R_{11}+R_{12}+2R_{21}+R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +2I(x_{1c};y_2|x_2)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + 2\mathsf{C}_{12}^{\mathsf{B}}-R_{11c} \end{array}\right\} \quad (6.105)$$

$$R_{11}+R_{12}+2R_{21}+R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +2I(x_{1c};y_2|x_2)+2\mathsf{C}_{12}^{\mathsf{B}}-R_{11c} \end{array}\right\} \quad (6.106)$$

$$R_{11}+R_{12}+2R_{21}+R_{22} \leq \left\{\begin{array}{l} I(x_1,x_{2c};y_1)+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{1c};y_2|x_2)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}-R_{11c} \end{array}\right\} \quad (6.107)$$

$$R_{11}+R_{12}+2R_{21}+R_{22} \leq \left\{\begin{array}{l} I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{1c};y_2|x_2)+\mathsf{C}_{12}^{\mathsf{B}}-R_{11c} \end{array}\right\}$$

$$R_{11}+R_{12}+2R_{21}+R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c};y_2|x_2) \\ +I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +2\mathsf{C}_{12}^{\mathsf{B}}-R_{11c} \end{array}\right\} \quad (6.108)$$

$$R_{11}+R_{12}+2R_{21}+R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_{1c};y_2|x_2) \\ +I(x_{1c},x_2;y_2|x_{2c})+2\mathsf{C}_{12}^{\mathsf{B}}-R_{11c} \end{array}\right\} \quad (6.109)$$

$$R_{11}+R_{12}+R_{21}+2R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{1c},x_2;y_2)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)+\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\}$$

$$R_{11}+R_{12}+R_{21}+2R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{1c},x_2;y_2)+\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\}$$

$$R_{11}+R_{12}+R_{21}+2R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c}) \\ +2(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)+\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\} \quad (6.110)$$

$$R_{11}+R_{12}+R_{21}+2R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{2c};y_1|x_1)+I(x_{1c},x_2;y_2|x_{2c}) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)+\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\}$$

$$R_{11}+R_{12}+R_{21}+2R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c}) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)+\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\}$$

$$R_{11}+R_{12}+R_{21}+2R_{22} \leq \left\{\begin{array}{l} I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_2;y_2|x_{1c},x_{2c}) \\ +I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c}) \\ +\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\}.$$

Remark 6.11 (Maximum value of each proposed bound) *All of the above bounds except the bounds of the sum of 4 rates, i.e., $R_{11}+R_{12}+R_{21}+R_{22}$, involve with parameters β and γ in the common rate constraints (6.1) and (6.10) at both transmitters which are rewritten the following forms for convenience in the later consideration:*

$$R_{1c} = R_{11c} + R_{21c} = \beta R_{1c} + (1-\beta)R_{1c} \tag{6.111}$$

$$R_{2c} = R_{12c} + R_{22c} = (1-\gamma)R_{2c} + \gamma R_{2c} \tag{6.112}$$

where $\beta, \gamma \in [0,1]$. After calculating, it is easily seen that

- R_{11} *bound has the maximum value when* $\beta = 1$*;*
- R_{12} *bound has the maximum value when* $\gamma = 0$*;*
- R_{21} *bound has the maximum value when* $\beta = 0$*;*
- R_{22} *bound has the maximum value when* $\gamma = 1$*;*
- $R_{11} + R_{12}$ *bound has the maximum value when* $\beta = 1$ *and* $\gamma = 0$*;*
- $R_{21} + R_{22}$ *bound has the maximum value when* $\beta = 0$ *and* $\gamma = 1$*;*
- $R_{11} + R_{12} + R_{21}$ *bound has the maximum value when* $\gamma = 0$*;*
- $R_{11} + R_{12} + R_{22}$ *bound has the maximum value when* $\beta = 1$*;*
- $R_{11} + R_{21} + R_{22}$ *bound has the maximum value when* $\gamma = 1$*;*
- $R_{12} + R_{21} + R_{22}$ *bound has the maximum value when* $\beta = 0$*;*
- $2R_{11} + R_{12} + R_{21} + R_{22}$ *bound has the maximum value when* $\beta = 1$*;*
- $R_{11} + 2R_{12} + R_{21} + R_{22}$ *bound has the maximum value when* $\gamma = 0$*;*
- $R_{11} + R_{12} + 2R_{21} + R_{22}$ *bound has the maximum value when* $\beta = 0$*;*
- $R_{11} + R_{12} + R_{21} + 2R_{22}$ *bound has the maximum value when* $\gamma = 1$.

Next, for the achievable rate region $\mathscr{R}_{2\to1\to2}$, we will show that all bounds at their maximum values except (6.37), (6.43)−(6.49), (6.58)−(6.60), (6.62)−(6.71), (6.73), (6.87)−(6.110) are within a constant gap from the corresponding upper bounds in Lemma 4.1.

By symmetry, however, we can obtain strategy $\mathscr{R}_{1\to2\to1}$. Similarly to the perspective of the work [2], rate points in $\mathscr{R}_{1\to2\to1}$ can recompense the problematic bounds (6.37), (6.43)–(6.49), (6.58)–(6.60), (6.62)–(6.71), (6.73), (6.87)–(6.110) in strategy $\mathscr{R}_{2\to1\to2}$ by time-sharing. Hence, the resulting achievable rate region $\mathscr{R} := \text{conv}\{\mathscr{R}_{2\to1\to2} \cup \mathscr{R}_{1\to2\to1}\}$ is within a bounded gap from the upper bounds in Lemma 4.1. We give the following lemma.

Lemma 6.12 (*Rate Region in the Two-User Gaussian Strong X Channel Type I)*

$$\mathscr{R} \subseteq \mathscr{C} \subseteq \overline{\mathscr{C}} \subseteq \mathscr{R} \oplus ([0,2] \times [0,2] \times [0,2] \times [0,2]),$$

in the two-user Gaussian strong X channel type I.

Proof: We need the following claims:

Claim 6.13 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the R_{12} bound (6.37) is active and $R_{11}+R_{21}+R_{22}$ bounds are also active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and $R_{11}+R_{21}+R_{22}$ bound intersect can be obtained. This condition is depicted in Figure. 6.4.*

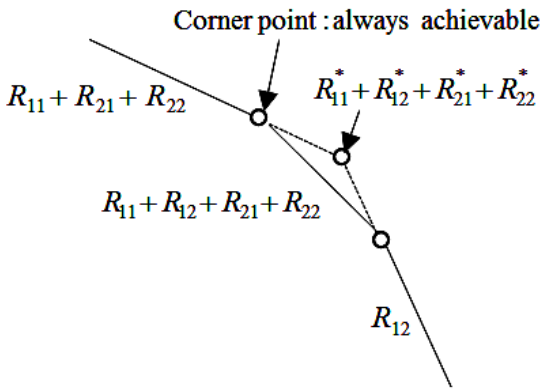


Figure 6.4 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.13

Proof: In this condition, the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the required corner point. Details is given in Appendix 6.6. ■

Claim 6.14 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{11}+R_{12}$ bound (6.43), ..., (6.45) or (6.46) is active and $R_{21} + R_{22}$ bounds are also active, the corner point where $R_{11} + R_{12} + R_{21} + R_{22}$ bound and $R_{21} + R_{22}$ bound intersect can be obtained. This condition is depicted in Figure. 6.5.*

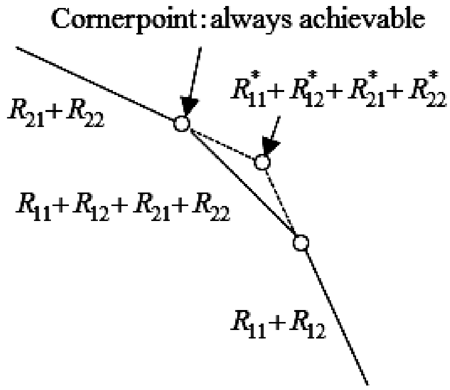


Figure 6.5 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.14

Proof: In this condition, the value of $R_{11} + R_{12} + R_{21} + R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11} + R_{12} + R_{21} + R_{22}$ at the required corner point. Details is given in Appendix 6.6. ■

Claim 6.15 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{21} + R_{22}$ bound (6.47), (6.48) or (6.49) is active and $R_{11} + R_{12}$ bounds are also active, the corner point where $R_{11} + R_{12} + R_{21} + R_{22}$ bound and $R_{11} + R_{12}$ bound intersect can be obtained. This condition is depicted in Figure. 6.6.*

Proof: In this condition, the value of $R_{11} + R_{12} + R_{21} + R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11} + R_{12} + R_{21} + R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■

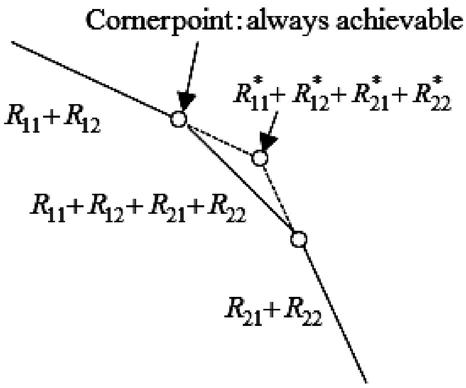
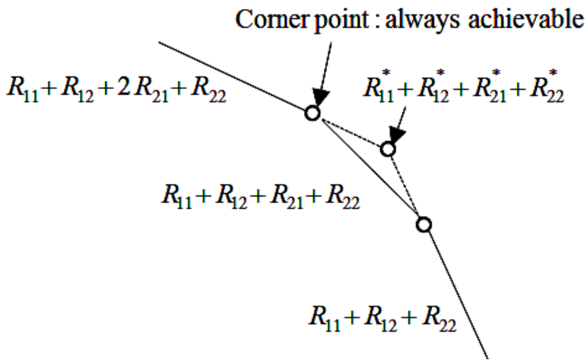
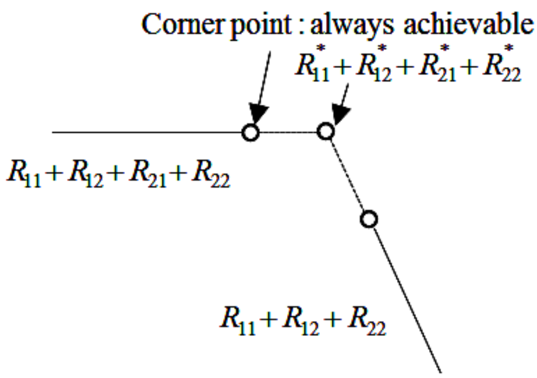


Figure 6.6 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.15



(a)



(b)

Figure 6.7: Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.16. (a) $R_{11}+R_{12}+2R_{21}+R_{22}$ bound is active. (b) $R_{11}+R_{12}+2R_{21}+R_{22}$ bound is not active.

Claim 6.16 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{11}+R_{12}+R_{22}$ bound (6.58), ..., (6.64) or (6.65) is active,*

1. *if $R_{11}+R_{12}+2R_{21}+R_{22}$ bounds are active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and $R_{11}+R_{12}+2R_{21}+R_{22}$ bound intersect can be obtained;*

2. *if $R_{11}+R_{12}+2R_{21}+R_{22}$ bounds are not active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and R_{21} bound intersect can be obtained.*

   These two conditions are depicted in Figure. 6.7.

Proof: In both conditions, the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■

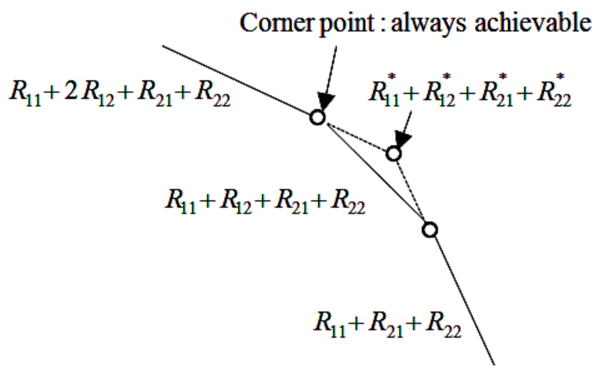
Claim 6.17 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{11}+R_{21}+R_{22}$ bound (6.66), ..., (6.70) or (6.71) is active,*

1. *if $R_{11}+2R_{12}+R_{21}+R_{22}$ bounds are active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and $R_{11}+2R_{12}+R_{21}+R_{22}$ bound intersect can be obtained;*

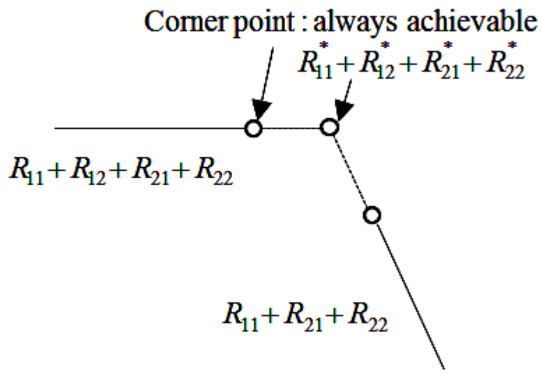
2. *if $R_{11}+2R_{12}+R_{21}+R_{22}$ bounds are not active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and R_{12} bound intersect can be obtained.*

   These two conditions are depicted in Figure. 6.8.

Proof: In both conditions, the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■



(a)



(b)

Figure 6.8: Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.17. (a) $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound is active. (b) $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound is not active.

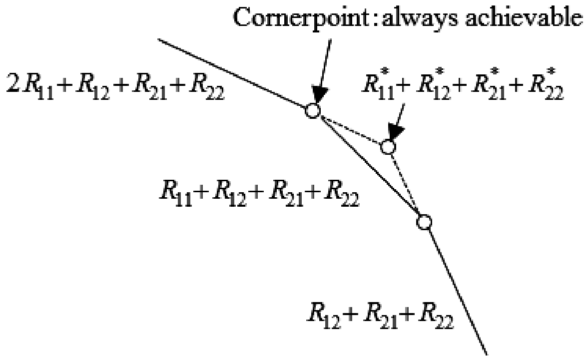
Claim 6.18 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{12} + R_{21} + R_{22}$ bound (6.73) is active,*

1. *if $2R_{11} + R_{12} + R_{21} + R_{22}$ bounds are active, the corner point where $R_{11} + R_{12} + R_{21} + R_{22}$ bound and $2R_{11} + R_{12} + R_{21} + R_{22}$ bound intersect can be obtained;*

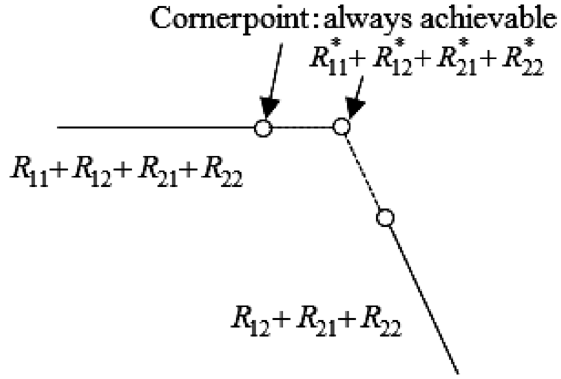
2. *if $2R_{11} + R_{12} + R_{21} + R_{22}$ bounds are not active, the corner point where $R_{11} + R_{12} + R_{21} + R_{22}$ bound and R_{11} bound intersect can be obtained.*

   These two conditions are depicted in Figure. 6.9.

Proof: In both conditions, the value of $R_{11} + R_{12} + R_{21}+ R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11} + R_{12} + R_{21} + R_{22}$ at the required corner point. Details are given in Appendix 6.6. ∎



(a)



(b)

Figure 6.9: Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.18. (a) $2R_{11} + R_{12} + R_{21} + R_{22}$ bound is active. (b) $2R_{11} + R_{12} + R_{21} + R_{22}$ bound is not active.

Claim 6.19 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound (6.87) or (6.88) is active and $R_{12} + R_{21} + R_{22}$ bounds are also active, the corner point where $R_{11} + R_{12} + R_{21} + R_{22}$ bound and $R_{12} + R_{21} + R_{22}$ bound intersect can be obtained. This condition is depicted in Figure. 6.10.*

Proof: In this condition, the value of $R_{11} + R_{12} + R_{21}+ R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11} + R_{12} + R_{21} + R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■

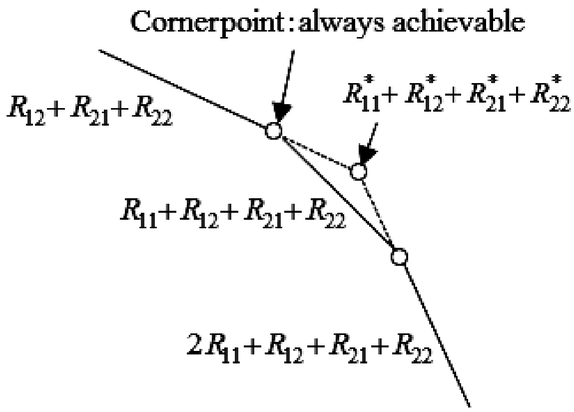


Figure 6.10 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.19

Claim 6.20 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{11}+2R_{12}+R_{21}+R_{22}$ bound (6.89), . . . , (6.100) or (6.101) is active and $R_{11}+R_{21}+R_{22}$ bounds are also active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and $R_{11}+R_{21}+R_{22}$ bound intersect can be obtained. This condition is depicted in Figure. 6.11.*

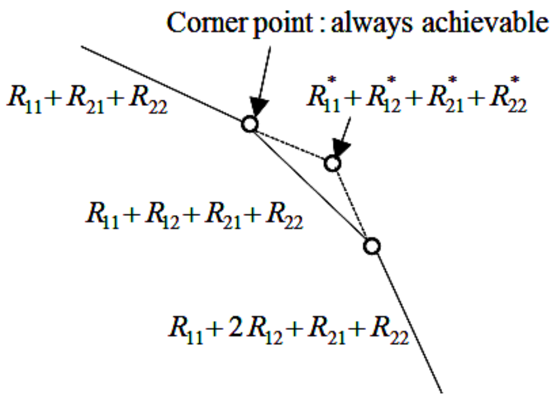


Figure 6.11 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.20

Proof: In this condition, the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■

Claim 6.21 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{11}+R_{12}+2R_{21}+R_{22}$ bound (6.102), . . . , (6.108) or (6.109) is active and $R_{11}+R_{12}+R_{22}$ bounds are also active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and $R_{11}+R_{12}+R_{22}$ bound intersect can be obtained. This condition is depicted in Figure. 6.12.*

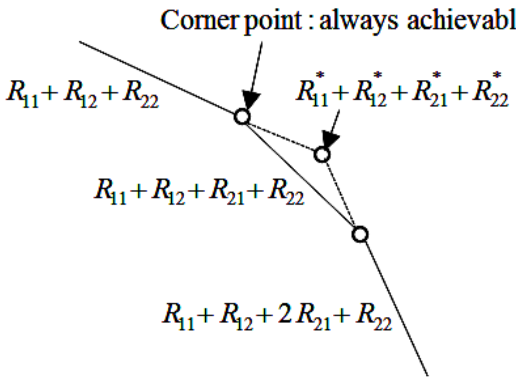


Figure 6.12 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.21

Proof: In this condition, the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■

Claim 6.22 *In the achievable rate region $\mathscr{R}_{2\to1\to2}$, whenever the $R_{11}+R_{12}+R_{21}+2R_{22}$ bound (6.110) is active and $R_{11}+R_{12}+R_{21}$ bounds are also active, the corner point where $R_{11}+R_{12}+R_{21}+R_{22}$ bound and $R_{11}+R_{12}+R_{21}$ bound intersect can be obtained. This condition is depicted in Figure. 6.13.*

Proof: In this condition, the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the intersection of the dashed lines is always greater than the value of $R_{11}+R_{12}+R_{21}+R_{22}$ at the required corner point. Details are given in Appendix 6.6. ■

Therefore, the R_{12} bound (6.37), the $R_{11}+R_{12}$ bounds (6.43)– (6.46), the $R_{21}+R_{22}$ bounds (6.47)–(6.49), the $R_{11}+R_{12}+R_{22}$ bounds (6.58)–(6.65), the $R_{11}+R_{21}+R_{22}$ bounds (6.66)–(6.71), the $R_{12}+R_{21}+R_{22}$ bound (6.73), the $2R_{11}+R_{12}+R_{21}+R_{22}$ bounds (6.87)–(6.88), the $R_{11}+2R_{12}+R_{21}+R_{22}$ bounds (6.89)–(6.101), the $R_{11}+R_{12}+2R_{21}+R_{22}$ bounds (6.102)–(6.109) and the $R_{11}+R_{12}+R_{21}+2R_{22}$ bounds (6.110) and, by symmetry,

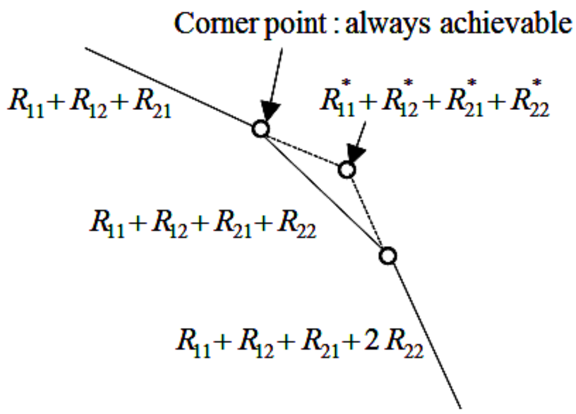


Figure 6.13 Condition on $\mathscr{R}_{2\to1\to2}$ for Claim 6.22

theirs corresponding R_{21}, $R_{11}+R_{12}$, $R_{21}+R_{22}$, $R_{11}+R_{12}+R_{21}$, $R_{11}+R_{12}+R_{22}$, $R_{11}+R_{21}+R_{22}$, $2R_{11}+R_{12}+R_{21}+R_{22}$, $R_{11}+2R_{12}+R_{21}+R_{22}$, $R_{11}+R_{12}+2R_{21}+R_{22}$ and $R_{11}+R_{12}+R_{21}+2R_{22}$ bounds in the achievable rate region $\mathscr{R}_{1\to2\to1}$ do not appear in $\mathscr{R}=\text{conv}\{\mathscr{R}_{2\to1\to2}\cup\mathscr{R}_{1\to2\to1}\}$ and $\mathscr{R}$ is within 2 bits per message to the approximate capacity region in Theorem 4.1. Next, we first consider the bounds in the achievable rate region $\mathscr{R}_{2\to1\to2}$ except the bounds (6.37), (6.43)−(6.49), (6.58)−(6.60), (6.62)−(6.71), (6.73), (6.87)−(6.110). We claim that

Claim 6.23 *The bounds in the achievable rate region $\mathscr{R}_{2\to1\to2}$ except (6.37), (6.43)−(6.49), (6.58)−(6.60), (6.62)−(6.71), (6.73), (6.87)−(6.110) satisfy:*

- R_{11} *bound is within 2 bits to upper bounds when* $\beta=1$*;*
- R_{12} *bound is within 2 bits to upper bounds when* $\gamma=0$*;*
- R_{21} *bound is within 2 bits to upper bounds when* $\beta=0$*;*
- R_{22} *bound is within 2 bits to upper bounds when* $\gamma=1$*;*
- $R_{11}+R_{12}$ *bound is within 3 bits to upper bounds when* $\beta=1$ *and* $\gamma=0$*;*
- $R_{21}+R_{22}$ *bound is within 3 bits to upper bounds when* $\beta=0$ *and* $\gamma=1$*;*
- $R_{11}+R_{12}+R_{21}$ *bound is within 4 bits to upper bounds when* $\gamma=0$*;*
- $R_{11}+R_{12}+R_{22}$ *bound is within 4 bits to upper bounds when* $\beta=1$*;*

- *$R_{11} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds when $\gamma = 1$;*

- *$R_{12} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds when $\beta = 0$;*

- *$R_{11} + R_{12} + R_{21} + R_{22}$ bound is within 5 bits to upper bounds;*

- *$2R_{11} + R_{12} + R_{21} + R_{22}$ bound is within 6 bits to upper bounds when $\beta = 1$;*

- *$R_{11} + 2R_{12} + R_{21} + R_{22}$ bound is within 6 bits to upper bounds when $\gamma = 0$;*

- *$R_{11} + R_{12} + 2R_{21} + R_{22}$ bound is within 6 bits to upper bounds when $\beta = 0$;*

- *$R_{11} + R_{12} + R_{21} + 2R_{22}$ bound is within 6 bits to upper bounds when $\gamma = 1$.*

Proof: See Appendix 6.6. ■

6.4 Proof of Theorem 6.3

We will first describe the detail of our proposed strategy and analyze the error probability.

6.4.1 Description of the Strategy

Codebook Generation: Transmitter i splits its own messages $m_{ii} \to (m_{iic}, m_{iip})$ and $m_{ji} \to (m_{jic}, m_{jip})$ for $i, j \in \{1, 2\}$ and $i \neq j$. We consider block length-N encoding and generate codebooks as follows:

Fix a joint distribution

$$\begin{aligned} p(x_{1c}, x_{11p}, x_{21p}, x_{2c}, x_{12p}, x_{22p}, x_1, x_2) = p(x_{1c})p(x_{11p}|x_{1c})p(x_{21p}|x_{1c})p(x_1|x_{1c}, x_{11p}, x_{21p}) \\ \times\, p(x_{2c})p(x_{12p}|x_{2c})p(x_{22p}|x_{2c})p(x_2|x_{2c}, x_{12p}, x_{22p}) \end{aligned}$$

- First we generate $2^{N(R_{iic}+R_{jic})}$ independent common codewords $x_{ic}^N(m_{iic}, m_{jic})$, $m_{iic} \in \{1, \cdots, 2^{NR_{iic}}\}$ and $m_{jic} \in \{1, \cdots, 2^{NR_{jic}}\}$, according to distribution $p(x_{ic}^N) = \prod_{n=1}^{N} p(x_{ic}[n])$ with $x_{ic}[n] \sim \mathcal{CN}(0, Q_{ic})$ for all n.

- For convenience, we combine and rearrange two codeword indices (m_{iic}, m_{jic}) into $m_{ic} \in \{1, \ldots, 2^{NR_{ic}}\}$, where $R_{ic} = R_{iic} + R_{jic}$. Therefore, we also denote these independent common codewords with $x_{ic}^N(m_{ic})$.

- Then for each common codeword $x_{ic}^N(m_{ic})$ serving as a cloud center, we generate $2^{NR_{iip}}$ independent codewords $x_{iip}^N(m_{ic}, m_{iip})$, $m_{iip} \in \{1, \ldots, 2^{NR_{iip}}\}$, according to conditional distribution $p(x_{iip}^N|x_{ic}^N) = \prod_{n=1}^N p(x_{iip}[n]|x_{ic}[n])$ with $x_{iip}[n] \sim \mathcal{CN}(0, Q_{iip})$ for all n.

- Similarly, for each common codeword $x_{ic}^N(m_{ic})$, we generate $2^{NR_{jip}}$ independent codewords $x_{jip}^N(m_{ic}, m_{jip})$, $m_{jip} \in \{1, \ldots, 2^{NR_{jip}}\}$, according to conditional distribution $p(x_{jip}^N|x_{ic}^N) = \prod_{n=1}^N p(x_{jip}[n]|x_{ic}[n])$ with $x_{jip}[n] \sim \mathcal{CN}(0, Q_{jip})$ for all n.

- Finally, at transmitter i, we generate a codeword x_i^N according to conditional distribution $p(x_i^N|x_{ic}^N, x_{iip}^N, x_{jip}^N) = \prod_{n=1}^N p(x_i[n]|x_{ic}[n], x_{iip}[n], x_{jip}[n])$.

Note that the configuration of power is $Q_{ic} + Q_{iip} + Q_{jip} = 1$.

For receiver 2 serving as relay, a quantization codebook $\hat{\mathscr{Y}}_2$ of size$|\hat{\mathscr{Y}}_2| = 2^{N\hat{R}_2}$ is generated randomly according to marginal distribution $p(\hat{y}_2^N)$, marginalized over joint distribution $p(y_2^N, x_{1c}^N, x_{11p}^N, x_{2c}^N, x_{12p}^N)p(\hat{y}_2^N|y_2^N, x_{1c}^N, x_{11p}^N, x_{2c}^N, x_{12p}^N)$, where

$$p(\hat{y}_2^N|y_2^N, x_{1c}^N, x_{11p}^N, x_{2c}^N, x_{12p}^N) = \prod_{n=1}^N p(\hat{y}_2[n]|y_2[n], x_{1c}[n], x_{11p}[n], x_{2c}[n], x_{12p}[n])$$

The conditional distribution is such that for all n, $\hat{y}_2[n] = y_2[n] + \hat{z}_2[n]$, where $\hat{z}_2 \sim \mathcal{CN}(0, \Delta_2)$, independent of everything else. Each element in the codebook $\hat{\mathscr{Y}}_2$ is mapped into $\{1, \ldots, 2^{N\mathsf{C}_{21}^{\mathsf{B}}}\}$ with a uniformly generated random mapping

$$b_2 : \hat{\mathscr{Y}}_2 \to \{1, \ldots, 2^{N\mathsf{C}_{21}^{\mathsf{B}}}\}, \hat{y} \mapsto l_{21} \; (binning)$$

For receiver 1 serving as relay, it generates two binning functions $b_1^{(1c)}$ and $b_1^{(2c)}$ independently according to uniform distributions, such that the message set $\{1 \le m_{ic} \le 2^{NR_{ic}}\}$ is divided into $2^{\phi_1^{(ic)} N\mathsf{C}_{12}^{\mathsf{B}}}$, for $i = 1, 2$, where $0 \le \phi_1^{(ic)} \le 1, \phi_1^{(1c)} + \phi_1^{(2c)} = 1$ and

$$b_1^{(ic)} : \{1, \ldots, 2^{NR_{ic}}\} \to \{1, \ldots, 2^{\phi_1^{(ic)} N\mathsf{C}_{12}^{\mathsf{B}}}\},$$
$$m_{ic} \mapsto l_{12}^{ic} \in \{1, \ldots, 2^{\phi_1^{(ic)} N\mathsf{C}_{12}^{\mathsf{B}}}\}.$$

Encoding: Transmitter i sends out messages (m_{iic}, m_{iip}) and (m_{jic}, m_{jip}) to receiver i and j, respectively, according to its codebooks. For receiver 2 serving as relay, it selects the quantization codeword $\hat{y}_2^N$ which is jointly typical with y_2^N (if there is more than one, it selects the one with smallest index) and then sends out the bin index l_{21} stand for

$\hat{y}_2^N$. After decoding $(m_{1c}, m_{11p}, m_{2c}, m_{12p})$, receiver 1 sends out bin indices $(l_{12}^{(1c)}, l_{12}^{(2c)})$ corresponding to binning functions $(b_1^{(1c)}, b_1^{(2c)})$.

Decoding at Receiver 1: First of all, upon receiving signal y_1^N and receiver-cooperative side information l_{21}, it constructs a list of message quadruples (two common messages and its two own private messages), each element of this list indices a codeword quadruple that is jointly typical with its received signal from the transmitter-receiver link. Let $L(y_1^N)$ denote a list of candidates as follows:

$$L(y_1^N) := \Big\{ \underline{m} := (m_{1c}, m_{11p}, m_{2c}, m_{12p}) | \\ \big(x_{1c}^N(m_{1c}), x_{11p}^N(m_{1c}, m_{11p}), x_{2c}^N(m_{2c}), x_{12p}^N(m_{2c}, m_{12p}), y_1^N\big) \in \mathcal{A}_\epsilon^{(N)} \Big\}$$

where $A_\epsilon^{(N)}$ denotes the set of joint ϵ-typical *N*-sequences [1].

After that, for each element $\underline{m} \in L(y_1^N)$, it constructs an ambiguity set of quantization codewords $B(\underline{m})$ where each element of this set is jointly typical with the quadruple and the received signal. $B(\underline{m})$ is defined as follows:

$$B(\underline{m}) := \Big\{ \hat{y}_2^N \in \hat{\mathscr{Y}}_2 | \\ \big(\hat{y}_2^N, x_{1c}^N(m_{1c}), x_{11p}^N(m_{1c}, m_{11p}), x_{2c}^N(m_{2c}), x_{12p}^N(m_{2c}, m_{12p}), y_1^N\big) \in \mathcal{A}_\epsilon^{(N)} \Big\}$$

Finally, it searches through all ambiguity sets and find one that contains a quantization codeword with the same bin index it received. Declare the transmitted message is $\hat{\underline{m}}$ if there exists an unique $\hat{\underline{m}}$ such that $\exists \hat{y}_2^N \in B(\underline{m})$ with $b_2(\hat{y}_2^N) = l_{21}$. Otherwise, it declares an error.

Decoding at receiver 2: After obtaining two bin indices $(l_{12}^{(1c)}, l_{12}^{(2c)})$, receiver 2 searches for an unique message quadruple $(m_{1c}, m_{21p}, m_{2c}, m_{22p})$ such that

$$\big(x_{1c}^N(m_{1c}), x_{21p}^N(m_{1c}, m_{21p}), x_{2c}^N(m_{2c}), x_{22p}^N(m_{2c}, m_{22p}), y_1^N\big) \in \mathcal{A}_\epsilon^{(N)}$$

and $b_1^{(ic)}(m_{ic}) = l_{12}^{(ic)}$, for *i* = 1, 2. If there is no such unique quadruple, an error is declared.

6.4.2 Error Probability Analysis

Error probability analysis at receiver 1: Without loss of generality, we assume that all transmitted messages are 1's. Following the same analysis in [2], we consider the case where receiver 2 serves as a relay to help receiver 1 decode its own messages.

At receiver 1, by law of large numbers, the the probability that the actually transmitted messages $\underline{1} := (m_{1c} = 1, m_{11p} = 1, m_{2c} = 1, m_{12p} = 1) \notin L(y_1^N)$ approaches to zero as $N \to \infty$. Furthermore, the probability that $B(\underline{1})$ does not contain the actually selected $\hat{y}_2^N$ is also negligible when N is sufficiently large. Next, we consider the following error events:

1. First, no quantization codeword is jointly typical with the received signal. From the known result of source coding, this probability approaches to zero as $N \to \infty$ if $\hat{R}_2 \geq I(\hat{y}_2; y_2)$.

2. Second, there exists $\underline{m} \neq \underline{1}$ which is in both the candidate list $L(y_1^N)$ and the ambiguity set $B(\underline{m})$ contains some quantization codeword $\hat{y}_2^N$ with bin index $b_2(\hat{y}_2^N) = l_{21}$. This event can divide into two cases:

   (a) $\hat{y}_2^N \in B(\underline{m})$ is not the truly selected quantization codeword.

   (b) $\hat{y}_2^N \in B(\underline{m})$ is indeed the selected quantization codeword.

Next, we analyze the error probability of these two typical error events above. For any nonempty $S \subseteq \{1c, 11p, 2c, 12p\}$, we define error events as follows:

- Let $E_S^{(1)}$ denote the event that there exists some $\underline{m} \neq \underline{1}$, (where $m_s \neq 1, \forall s \in S$ and $m_s = 1, \forall s \notin S$), such that $\underline{m} \in L(y_1^N)$ and $B(\underline{m})$ contains some $\hat{y}_2^N(k), k \in \{1, 2, \cdots, 2^{N\hat{R}_2}\}$ with $b_2(\hat{y}_2^N(k)) = l_{21}$. Note that $\hat{y}_2^N(k)$ is not the actually selected quantization codeword $\hat{y}_2^N(1)$.

- Let $E_S^{(2)}$ denote the event that there exists some $\underline{m} \neq \underline{1}$, (where $m_s \neq 1, \forall s \in S$ and $m_s = 1, \forall s \notin S$), such that $\underline{m} \in L(y_1^N)$ and $B(\underline{m})$ contains $\hat{y}_2^N(1)$.

Probability of $E_S^{(1)}$: For convenience, let $\underline{x}^N(\underline{m})$ denote the vector of codewords corresponding to message $\underline{m}$, i.e., $(x_{1c}^N(m_{1c}), x_{11p}^N(m_{1c}, m_{11p}), x_{2c}^N(m_{2c}), x_{12p}^N(m_{2c}, m_{12p}))$. The probability of the error event $E_S^{(1)}$ can upper bound as (6.114).

$$
\begin{aligned}
&\Pr\{E_S^{(1)}\} \\
&\quad \leq \sum_{\substack{\underline{m}: m_s \neq 1, \\ \forall s \in S}} \sum_{k \neq 1} \Pr\{\underline{m} \in L(y_1^N), \hat{y}_2^N(k) \in B(\underline{m}), b(\hat{y}_2^N(k)) = l_{21}\} \\
&\quad \leq \sum_{\substack{\underline{m}: m_s \neq 1, \\ \forall s \in S}} \sum_{k \neq 1} \Pr\{\left(\hat{y}_2^N(k), x^N(\underline{m}), y_1^N\right) \in A_\epsilon^{(N)}, b(\hat{y}_2^N(k)) = l_{21}\}
\end{aligned}
$$

$$\overset{(a)}{=} 2^{-N\mathsf{C}_{21}^{\mathsf{B}}} \sum_{\substack{\underline{m}:m_s\neq 1,\\ \forall s\in S}} \sum_{k\neq 1} \Pr\{\big(\hat{y}_2^N(k), x^N(\underline{m}), y_1^N\big) \in A_\epsilon^{(N)}\}$$

$$\leq 2^{N\left(\sum_{s\in S} R_s\right)} 2^{-N\mathsf{C}_{21}^{\mathsf{B}}} \sum_{k\neq 1} \Pr\{\big(\hat{y}_2^N(k), x^N(\underline{m}), y_1^N\big) \in A_\epsilon^{(N)}\} \tag{6.113}$$

$$\leq 2^{N\left(\sum_{s\in S} R_s\right)} 2^{-N\mathsf{C}_{21}^{\mathsf{B}}} \Pr\{\underline{m} \in L(y_1^N)\} \cdot 2^{N\hat{R}_2} \cdot 2^{-N(I(\hat{y}_2;x_{1c},x_{11p},x_{2c},x_{12p},y_1)-3\epsilon)} \tag{6.114}$$

where (a) is due to the independent uniform binning. In (6.113), for $k \neq 1$, $\hat{y}_2^N(k)$ is independent of $\big(x^N(\underline{m}), y_1^N\big)$. We can upper bound $\sum_{k\neq 1} \Pr\{\big(\hat{y}_2^N(k), x^N(\underline{m}), y_1^N\big) \in A_\epsilon^{(N)}\}$ by using Theorem 15.2.2 in [1] and derive with the same line of Appendix A in [2]. Therefore, we obtain (6.114).

From the results in the works [11] and [13], we obtain

$$\Pr\{\underline{m} \in L(y_1^N)\} \leq \begin{cases} 2^{-N(I(x_{11p};y_1|x_{1c},x_{2c},x_{12p})-\epsilon')} & S=\{11p\} \\ 2^{-N(I(x_{12p};y_1|x_{1c},x_{2c},x_{11p})-\epsilon')} & S=\{12p\} \\ 2^{-N(I(x_{1c},x_{11p};y_1|x_{2c},x_{12p})-\epsilon')} & S=\{1c\} \\ 2^{-N(I(x_{2c},x_{12p};y_1|x_{1c},x_{11p})-\epsilon')} & S=\{2c\} \\ 2^{-N(I(x_{1c},x_{11p};y_1|x_{2c},x_{12p})-\epsilon')} & S=\{1c,11p\} \\ 2^{-N(I(x_{2c},x_{12p};y_1|x_{1c},x_{11p})-\epsilon')} & S=\{2c,12p\} \\ 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1)-\epsilon')} & S=\{1c,2c\} \\ 2^{-N(I(x_{11p},x_{12p};y_1|x_{1c},x_{2c})-\epsilon')} & S=\{11p,12p\} \\ 2^{-N(I(x_{11p},x_{2c},x_{12p};y_1|x_{1c})-\epsilon')} & S=\{2c,11p\} \\ 2^{-N(I(x_{1c},x_{11p},x_{12p};y_1|x_{2c})-\epsilon')} & S=\{1c,12p\} \\ 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1)-\epsilon')} & S=\{1c,11p,2c\} \\ 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1)-\epsilon')} & S=\{1c,2c,12p\} \\ 2^{-N(I(x_{11p},x_{2c},x_{12p};y_1|x_{1c})-\epsilon')} & S=\{11p,2c,12p\} \\ 2^{-N(I(x_{1c},x_{11p},x_{12p};y_1|x_{2c})-\epsilon')} & S=\{1c,11p,12p\} \\ 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1)-\epsilon')} & S=\{1c,11p,2c,12p\} \end{cases}$$

where $\epsilon' = 4\epsilon$. Note that it is alike in the X channel without cooperation as in [11] and [13], that is, receiver 1 can decode both m_{1c} and m_{2c} correctly.

Therefore, after eliminating the redundant terms, the probability of the first kind of error event vanishes as $N \to \infty$ if the rates satisfy the following inequalities:

$$R_{11p} \leq I(x_{11p}; y_1 | x_{1c}, x_{2c}, x_{12p}) + \psi$$

$$R_{12p} \leq I(x_{12p}; y_1 | x_{1c}, x_{2c}, x_{11p}) + \psi$$

$$R_{1c} + R_{11p} \leq I(x_{1c}, x_{11p}; y_1 | x_{2c}, x_{12p}) + \psi$$

$$R_{2c} + R_{12p} \leq I(x_{2c}, x_{12p}; y_1 | x_{1c}, x_{11p}) + \psi$$

$$R_{11p} + R_{12p} \leq I(x_{11p}, x_{12p}; y_1 | x_{1c}, x_{2c}) + \psi$$

$$R_{1c} + R_{11p} + R_{12p} \leq I(x_{1c}, x_{11p}, x_{12p}; y_1 | x_{2c}) + \psi$$

$$R_{2c} + R_{11p} + R_{12p} \leq I(x_{11p}, x_{2c}, x_{12p}; y_1 | x_{1c}) + \psi$$

$$R_{1c} + R_{11p} + R_{2c} + R_{12p} \leq I(x_{1c}, x_{11p}, x_{2c}, x_{12p}; y_1) + \psi$$

where $\psi = \mathsf{C}_{21}^{\mathsf{B}} - \hat{R}_2 + I(\hat{y}_2; x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1)$.

Furthermore, $\Pr\{E_S^{(1)}\}$ can be alternately upper bounded as follows:

$$\begin{aligned} &\Pr\{E_S^{(1)}\} \\ &\quad \leq \sum_{\substack{\underline{m}: m_s \neq 1, \\ \forall s \in S}} \Pr\{\underline{m} \in L(y_1^N)\} \cdot \Pr\{\exists k \neq 1, \hat{y}_2^N(k) \in B(\underline{m}), b(\hat{y}_2^N(k)) = l_{21} | \underline{m} \in L(y_1^N)\} \\ &\quad \leq 2^{N(\sum_{s \in S} R_s)} \Pr\{\underline{m} \in L(y_1^N)\} \end{aligned} \tag{6.115}$$

Hence, the probability of the first kind of error event disappears as $N \to 0$ if the rates satisfy the following inequalities:

$$R_{11p} \leq I(x_{11p}; y_1 | x_{1c}, x_{2c}, x_{12p}) + \psi^+$$

$$R_{12p} \leq I(x_{12p}; y_1 | x_{1c}, x_{2c}, x_{11p}) + \psi^+$$

$$R_{1c} + R_{11p} \leq I(x_{1c}, x_{11p}; y_1 | x_{2c}, x_{12p}) + \psi^+$$

$$R_{2c} + R_{12p} \leq I(x_{2c}, x_{12p}; y_1 | x_{1c}, x_{11p}) + \psi^+$$

$$R_{11p} + R_{12p} \leq I(x_{11p}, x_{12p}; y_1 | x_{1c}, x_{2c}) + \psi^+$$

$$R_{1c} + R_{11p} + R_{12p} \leq I(x_{1c}, x_{11p}, x_{12p}; y_1 | x_{2c}) + \psi^+$$

$$R_{2c} + R_{11p} + R_{12p} \leq I(x_{11p}, x_{2c}, x_{12p}; y_1 | x_{1c}) + \psi^+$$

$$R_{1c} + R_{11p} + R_{2c} + R_{12p} \leq I(x_{1c}, x_{11p}, x_{2c}, x_{12p}; y_1) + \psi^+$$

Finally, substituting $\hat{R}_2 = I(\hat{y}_2; y_2)$ and using the Markov relation $(x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1) - y_2 - \hat{y}_2$, we have the rate loss term as follows:

$$\begin{aligned}\xi_1 &:= \hat{R}_2 - I(\hat{y}_2; x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1)\\ &= I(\hat{y}_2; y_2) - I(\hat{y}_2; x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1)\\ &= I(\hat{y}_2; y_2 | x_{1c}, x_{11p}, x_{2c}, x_{12p}, y_1)\end{aligned}$$

Probability of $E_S^{(2)}$: We can upper bound the probability of $E_S^{(2)}$ as follows:

$$\begin{aligned}\Pr\{E_S^{(2)}\} &\le \sum_{\underline{m}:m_s\neq 1, \forall s\in S} \Pr\{\hat{y}_2^N(1) \in B(\underline{m}), \underline{m} \in L(y_1^N)\}\\ &= \sum_{\underline{m}:m_s\neq 1, \forall s\in S} \Pr\{(\hat{y}_2^N(1), \underline{x}^N(\underline{m}), y_1^N) \in A_\epsilon^{(N)}\}\\ &\le \begin{cases} \Theta \cdot 2^{-N(I(x_{11p};y_1,\hat{y}_2|x_{1c},x_{2c},x_{12p})-\epsilon')} & S=\{11p\}\\ \Theta \cdot 2^{-N(I(x_{12p};y_1,\hat{y}_2|x_{1c},x_{2c},x_{11p})-\epsilon')} & S=\{12p\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p};y_1,\hat{y}_2|x_{2c},x_{12p})-\epsilon')} & S=\{1c\}\\ \Theta \cdot 2^{-N(I(x_{2c},x_{12p};y_1,\hat{y}_2|x_{1c},x_{11p})-\epsilon')} & S=\{2c\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p};y_1,\hat{y}_2|x_{2c},x_{12p})-\epsilon')} & S=\{1c,11p\}\\ \Theta \cdot 2^{-N(I(x_{2c},x_{12p};y_1,\hat{y}_2|x_{1c},x_{11p})-\epsilon')} & S=\{2c,12p\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1,\hat{y}_2)-\epsilon')} & S=\{1c,2c\}\\ \Theta \cdot 2^{-N(I(x_{11p},x_{12p};y_1,\hat{y}_2|x_{1c},x_{2c})-\epsilon')} & S=\{11p,12p\}\\ \Theta \cdot 2^{-N(I(x_{11p},x_{2c},x_{12p};y_1,\hat{y}_2|x_{1c})-\epsilon')} & S=\{2c,11p\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p},x_{12p};y_1,\hat{y}_2|x_{2c})-\epsilon')} & S=\{1c,12p\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1,\hat{y}_2)-\epsilon')} & S=\{1c,11p,2c\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1,\hat{y}_2)-\epsilon')} & S=\{1c,2c,12p\}\\ \Theta \cdot 2^{-N(I(x_{11p},x_{2c},x_{12p};y_1,\hat{y}_2|x_{1c})-\epsilon')} & S=\{11p,2c,12p\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p},x_{12p};y_1,\hat{y}_2|x_{2c})-\epsilon')} & S=\{1c,11p,12p\}\\ \Theta \cdot 2^{-N(I(x_{1c},x_{11p},x_{2c},x_{12p};y_1,\hat{y}_2)-\epsilon')} & S=\{1c,11p,2c,12p\} \end{cases}\end{aligned}$$

where $\Theta = 2^{N(\sum_{s\in S} Rs)}$ and $\epsilon' = 4\epsilon$.

After eliminating the redundant terms, therefore, the probability of the second kind of error event vanishes for sufficiently large N if the rates satisfy the following inequalities:

$$\begin{aligned}
R_{11p} &\leq I(x_{11p}; y_1, \hat{y}_2 | x_{1c}, x_{2c}, x_{12p}) \\
R_{12p} &\leq I(x_{12p}; y_1, \hat{y}_2 | x_{1c}, x_{2c}, x_{11p}) \\
R_{1c} + R_{11p} &\leq I(x_{1c}, x_{11p}; y_1, \hat{y}_2 | x_{2c}, x_{12p}) \\
R_{2c} + R_{12p} &\leq I(x_{2c}, x_{12p}; y_1, \hat{y}_2 | x_{1c}, x_{11p}) \\
R_{11p} + R_{12p} &\leq I(x_{11p}, x_{12p}; y_1, \hat{y}_2 | x_{1c}, x_{2c}) \\
R_{1c} + R_{11p} + R_{12p} &\leq I(x_{1c}, x_{11p}, x_{12p}; y_1, \hat{y}_2 | x_{2c}) \\
R_{2c} + R_{11p} + R_{12p} &\leq I(x_{11p}, x_{2c}, x_{12p}; y_1, \hat{y}_2 | x_{1c}) \\
R_{1c} + R_{11p} + R_{2c} + R_{12p} &\leq I(x_{1c}, x_{11p}, x_{2c}, x_{12p}; y_1, \hat{y}_2)
\end{aligned}$$

Error Probability Analysis of Receiver 2: After receiving the two bin indices, receiver 2 can decode $(m_{1c}, m_{21p}, m_{2c}, m_{22p})$ with the smaller candidate message sets, i.e., m_{1_c} and m_{2_c}. Following the same line as [2], we obtain the rate region that is achievable for receiver 2 to decode successfully as follows:

$$\begin{aligned}
R_{21p} &\leq I(x_{21p}; y_2 | x_{1c}, x_{2c}, x_{22p}) \\
R_{22p} &\leq I(x_{22p}; y_2 | x_{1c}, x_{2c}, x_{21p}) \\
R_{1c} + R_{21p} &\leq I(x_{1c}, x_{21p}; y_2 | x_{2c}, x_{22p}) + \mathsf{C}_{12}^{\mathsf{B}} \\
R_{2c} + R_{22p} &\leq I(x_{2c}, x_{22p}; y_2 | x_{2c}, x_{21p}) + \mathsf{C}_{12}^{\mathsf{B}} \\
R_{21p} + R_{22p} &\leq I(x_{21p}, x_{22p}; y_2 | x_{1c}, x_{2c}) \\
R_{1c} + R_{21p} + R_{22p} &\leq I(x_{1c}, x_{21p}, x_{22p}; y_2 | x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \\
R_{2c} + R_{21p} + R_{22p} &\leq I(x_{2c}, x_{21p}, x_{22p}; y_2 | x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} \\
R_{1c} + R_{21p} + R_{2c} + R_{22p} &\leq I(x_{1c}, x_{21p}, x_{2c}, x_{22p}; y_2) + \mathsf{C}_{12}^{\mathsf{B}}.
\end{aligned}$$

Note that the upper bounds for R_{21p}, R_{22p} and $R_{21p} + R_{22p}$ in the rate region above do not gain from receiver cooperation since receiver 1 does not decode the private messages m_{21p}, m_{22p}.

6.5 Proof of Theorem 6.8

Codebook Generation: In transmitter *i*, a message m_{ii} is split into m_{iic} and m_{iip} and a message m_{ji} is considered as a common message m_{jic}. We consider block length-N encoding and generate codebooks as follows:

- First we generate $2^{N(R_{iic}+R_{jic})}$ independent common codewords $x_{ic}^N(m_{iic}, m_{jic})$, $m_{iic} \in \{1, \cdots, 2^{NR_{iic}}\}$ and $m_{jic} \in \{1, \cdots, 2^{NR_{jic}}\}$, according to distribution $p(x_{ic}^N) = \prod_{n=1}^N p(x_{ic}[n])$ with $x_{ic}[n] \sim \mathcal{CN}(0, Q_{ic})$ for all n.

- For convenience, we combine and rearrange two codeword indices (m_{iic}, m_{jic}) into $m_{ic} \in \{1, \ldots, 2^{NR_{ic}}\}$, where $R_{ic} = R_{iic} + R_{jic}$. Therefore, we also denote these independent common codewords with $x_{ic}^N(m_{ic})$.

- Finally, for each common codeword $x_{ic}^N(m_{ic})$ serving as a cloud center, we generate $2^{NR_{iip}}$ independent codewords $x_i^N(m_{ic}, m_{iip})$, $m_{iip} \in \{1, \ldots, 2^{NR_{iip}}\}$, according to conditional distribution $p(x_i^N|x_{ic}^N) = \prod_{n=1}^N p(x_i[n]|x_{ic}[n])$ such that for all n, $x_i[n] = x_{ic}[n] + x_{iip}[n]$, with $x_{iip}[n] \sim \mathcal{CN}(0, Q_{iip})$ and independent of everything else.

The power split configuration is such that $Q_{ic} + Q_{iip} = 1$ and $\mathsf{INR}_i^{jjp} := Q_{ijp}|h_{ij}|^2 \le 1$ if $\mathsf{SNR}_i > \mathsf{INR}_j$. Using a simple power-splitting configuration from [9], we set the power of each private codeword by $Q_{iip} = \min\left\{\frac{1}{\mathsf{INR}_j}, 1\right\}$.

The details for generating codebook when receiver 1 and 2 serve as relay, encoding and decoding at receiver 1 and 2 and analyzing the error probability at receiver 1 and 2 are the same as the proof of Lemma 5.1 in [2], they are thus omitted here.

6.6 Proof of Claim 6.13, Claim 6.14, Claim 6.15, Claim 6.16, Claim 6.17, Claim 6.18, Claim 6.19, Claim 6.20, Claim 6.21, Claim 6.22 and Claim 6.23

6.6.1 Proof of Claim 6.13

Proof: There are three possible $R_{11} + R_{21} + R_{22}$ bounds. Hence, we consider the following three cases:

1) If the bound

$$R_{11} + R_{21} + R_{22} \le I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{21} + R_{22}$ bound and the R_{12} bound (6.37) intersect satisfies

$$\begin{aligned} R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22} &= \Big\{I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c}\Big\} \\ &\quad + \Big\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{22c}\Big\} \\ &= \Big\{I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}^{\mathsf{B}}_{12}\Big\} \\ &\quad + \Big\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{12c} - R_{22c}\Big\} \\ &= (6.80) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{12c} - R_{22c}\} \ge 0$ refers to (6.37).

2) If the bound

$$R_{11} + R_{21} + R_{22} \le I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{21} + R_{22}$ bound and the R_{12} bound (6.37) intersect satisfies

$$\begin{aligned} R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22} &= \Big\{I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c}\Big\} \\ &\quad + \Big\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{22c}\Big\} \\ &= \Big\{I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c})\Big\} \\ &\quad + \Big\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{12c} - R_{22c}\Big\} \\ &= (6.82) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{12c} - R_{22c}\} \geq 0$ refers to (6.37).

3) If the bound

$$R_{11} + R_{21} + R_{22} \leq I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{21} + R_{22}$ bound and the R_{12} bound (6.37) intersect satisfies

$$\begin{aligned} R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* = &\left\{ \begin{matrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) \\ + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \end{matrix} \right\} \\ &+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{22c} \right\} \\ &= \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \right\} \\ &+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{12c} - R_{22c} \right\} \\ &= (6.86) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1, \hat{y}_2|x_1) - R_{12c} - R_{22c}\} \geq 0$ refers to (6.37).

Therefore, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + R_{21} + R_{22}$ bound intersect can be acquired.

6.6.2 Proof of Claim 6.14

Proof: In this proof, we consider only (6.43) and other bounds can be shown similarly. There are three possible $R_{21} + R_{22}$ bounds. Hence, we consider the following three cases:

1) If the bound

$$R_{21} + R_{22} \leq I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{21} + R_{22}$ bound and the $R_{11} + R_{12}$ bound (6.43) intersect satisfies

$$\begin{aligned} R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* = &\left\{ \begin{matrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}; y_2|x_2) \\ + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} - R_{22c} \end{matrix} \right\} \\ &+ \left\{ I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \right\} \end{aligned}$$

$$= \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} \right\}$$
$$+ \left\{ \begin{array}{l} \{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \\ +\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \end{array} \right\}$$
$$= (6.80)$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \geq 0$ and $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refer to (6.36) and (6.38), respectively.

2) If the bound

$$R_{21} + R_{22} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{21} + R_{22}$ bound and the $R_{11} + R_{12}$ bound (6.43) intersect satisfies

$$R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* = \left\{ \begin{array}{l} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}; y_2|x_2) \\ +(\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} - R_{22c} \end{array} \right\}$$
$$+ \left\{ \begin{array}{l} I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{11c} - R_{12c} \end{array} \right\}$$
$$= \left\{ \begin{array}{l} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) \\ +I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} \end{array} \right\}$$
$$+ \left\{ \begin{array}{l} \{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \\ +\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \end{array} \right\}$$
$$\geq (6.86)$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \geq 0$ and $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refer to (6.36) and (6.38), respectively.

3) If the bound

$$R_{21} + R_{22} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{21} + R_{22}$ bound and the $R_{11} + R_{12}$

bound (6.43) intersect satisfies

$$R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* = \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}; y_2|x_2) \\ +(\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} - R_{22c} \end{Bmatrix}$$
$$+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \right\}$$
$$= \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \right\}$$
$$+ \begin{Bmatrix} \{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \\ +\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \end{Bmatrix}$$
$$= (6.86)$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \geq 0$ and $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refer to (6.36) and (6.38), respectively.

Therefore, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{21} + R_{22}$ bound intersect can be acquired.

6.6.3 Proof of Claim 6.15

Proof: In this proof, we consider only (6.47) and other bounds can be shown similarly. There are four possible $R_{11} + R_{12}$ bounds. Hence, we consider the following four cases:

1) If the bound

$$R_{11} + R_{12} \leq I(x_1, x_{2c}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{12}$ bound and the $R_{21} + R_{22}$ bound (6.47) intersect satisfies

$$R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* = \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \right\}$$
$$+ \left\{ I(x_1, x_{2c}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c} \right\}$$
$$= \left\{ I(x_1, x_{2c}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c} \right\}$$
$$+ \left\{ I(x_2; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \right\}$$
$$+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \right\}$$
$$= (6.50) + (6.39)$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refers to (6.38).

2) If the bound

$$R_{11} + R_{12} \leq I(x_1, x_{2c}; y_1, \hat{y}_2) - R_{21c} - R_{22c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{12}$ bound and the $R_{21} + R_{22}$ bound (6.47) intersect satisfies

$$\begin{aligned} R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* &= \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \right\} \\ &+ \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) - R_{21c} - R_{22c} \right\} \\ &= \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) - R_{22c} \right\} + \left\{ I(x_2; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \right\} \\ &+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \right\} \\ &= (6.51) + (6.39) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refers to (6.38).

3) If the bound

$$R_{11} + R_{12} \leq I(x_{2c}; y_1|x_1) + I(x_1; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{12}$ bound and the $R_{21} + R_{22}$ bound (6.47) intersect satisfies

$$\begin{aligned} R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* &= \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \right\} \\ &+ \left\{ I(x_{2c}; y_1|x_1) + I(x_1; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c} \right\} \\ &= \left\{ I(x_{2c}; y_1|x_1) + I(x_1; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c} \right\} \\ &+ \left\{ I(x_2; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \right\} \\ &+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \right\} \\ &= (6.52) + (6.39) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refers to (6.38).

4) If the bound

$$R_{11} + R_{12} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_1; y_1|x_{2c}) - R_{21c} - R_{22c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{12}$ bound and the $R_{21} + R_{22}$ bound (6.47) intersect satisfies

$$\begin{aligned} R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* &= \Big\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c} \Big\} \\ &\quad + \Big\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_1; y_1|x_{2c}) - R_{21c} - R_{22c} \Big\} \\ &= \Big\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_1; y_1|x_{2c}) - R_{22c} \Big\} \\ &\quad + \Big\{ I(x_2; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \Big\} \\ &\quad + \Big\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \Big\} \\ &= (6.53) + (6.39) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refers to (6.38).

Therefore, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + R_{12}$ bound intersect can be acquired.

6.6.4 Proof of Claim 6.16

Proof: In this proof, we consider only (6.58) and other bounds can be shown similarly. For the first condition, there are three possible $R_{11} + R_{12} + 2R_{21} + R_{22}$ bounds. Hence, we consider the following three cases:

1) If the bound

$$R_{11} + R_{12} + 2R_{21} + R_{22} \leq I(x_1; y_1|x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound and the $R_{11} + R_{12} + R_{22}$ bound (6.58) intersect satisfies

$$\begin{aligned} 2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \Big\{ I(x_1; y_1|x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \Big\} \\ &\quad + \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \end{array} \right\} \end{aligned}$$

$$= \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} \end{array} \right\}$$

$$+ \left\{ I(x_1; y_1|x_{2c}) - R_{21c} \right\} + \left\{ I(x_{1c}, x_2; y_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \right\}$$

$$= (6.78) + (6.34) + (6.72)$$

which is greater than twice the active sum rate bound.

2) If the bound

$$R_{11} + R_{12} + 2R_{21} + R_{22} \leq \left\{ \begin{array}{l} I(x_1; y_1|x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} \end{array} \right\}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound and the $R_{11} + R_{12} + R_{22}$ bound (6.58) intersect satisfies

$$2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} = \left\{ \begin{array}{l} I(x_1; y_1|x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{array} \right\}$$

$$+ \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} \end{array} \right\}$$

$$+ \left\{ I(x_1; y_1|x_{2c}) - R_{21c} \right\}$$

$$+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \right\}$$

$$= (6.78) + (6.34) + (6.77)$$

which is greater than twice the active sum rate bound.

3) If the bound

$$R_{11} + R_{12} + 2R_{21} + R_{22} \leq \left\{ \begin{array}{l} I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} \end{array} \right\}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound and the $R_{11} + R_{12} + R_{22}$ bound (6.58) intersect satisfies

$$2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} = \left\{ \begin{array}{l} I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{array} \right\}$$

$$+ \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{array} \right\}$$

$$
\begin{aligned}
&= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\
&+ \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{21c} \right\} \\
&+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \right\} \\
&= (6.78) + (6.61) + (6.38)
\end{aligned}
$$

which is greater than twice the active sum rate bound.

Therefore, in the first condition, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound intersect can be achieved.

For the second condition, there is one possible R_{21} bound. Hence, we consider the following case:

If the bound

$$
R_{21} \leq I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}
$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the R_{21} bound and the $R_{11} + R_{12} + R_{22}$ bound (6.58) intersect satisfies

$$
\begin{aligned}
R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{21c} \end{Bmatrix} \\
&+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \right\} \\
&= \left\{ I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \right\} \\
&+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \right\} \\
&= (6.78)
\end{aligned}
$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\} \geq 0$ refers to (6.38).

Therefore, in the second condition, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the R_{21} bound intersect can be achieved.

6.6.5 Proof of Claim 6.17

Proof: In this proof, we consider only (6.66) and other bounds can be shown similarly. For the first condition, there is one possible $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound. Hence, we consider the following case:

If the bound

$$R_{11} + 2R_{12} + R_{21} + R_{22} \leq I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + R_{21} + R_{22}$ bound (6.66) intersect satisfies

$$\begin{aligned}
2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \begin{Bmatrix} I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{22c} \end{Bmatrix} \\
&+ \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \end{Bmatrix} \\
&= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\
&+ \left\{ I(x_1, x_{2c}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c} \right\} \\
&+ \left\{ I(x_2; y_2|x_{1c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \right\} \\
&= (6.78) + (6.50) + (6.39)
\end{aligned}$$

which is greater than twice the active sum rate bound.

Therefore, in the first condition, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound intersect can be achieved.

For the second condition, there is one possible R_{12} bound. Hence, we consider the following case:

If the bound

$$R_{12} \leq I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the R_{12} bound and the $R_{11} + R_{21} + R_{22}$ bound (6.66) intersect satisfies

$$\begin{aligned}
R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \\ -R_{12c} \end{Bmatrix} \\
&+ \left\{ I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c} \right\} \\
&= \left\{ I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \right\} \\
&+ \left\{ I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c} \right\} \\
&= (6.78)
\end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{2c}; y_1|x_1) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - R_{12c} - R_{22c}\} \geq 0$ refers to (6.36).

Therefore, in the second condition, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the R_{12} bound intersect can be achieved.

6.6.6 Proof of Claim 6.18

Proof: For the first condition, there are two possible $2R_{11} + R_{12} + R_{21} + R_{22}$ bounds. Hence, we consider the following two cases:

1) If the bound

$$2R_{11} + R_{12} + R_{21} + R_{22} \leq \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2|x_{2c}) + I(x_1, x_{2c}; y_1) \\ +(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{Bmatrix}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{12} + R_{21} + R_{22}$ bound (6.73) intersect satisfies

$$\begin{aligned} 2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \right\} \\ &\quad + \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2|x_{2c}) + I(x_1, x_{2c}; y_1) \\ +(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{Bmatrix} \\ &= \begin{Bmatrix} I(x_1, x_{2c}; y_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} - R_{21c} \end{Bmatrix} \\ &\quad + \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + \mathsf{C}^{\mathsf{B}}_{12} \right\} + \mathsf{C}^{\mathsf{B}}_{12} \\ &\geq (6.78) + (6.80) \end{aligned}$$

which is greater than twice the active sum rate bound.

2) If the bound

$$2R_{11} + R_{12} + R_{21} + R_{22} \leq \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2|x_{2c}) + I(x_1, x_{2c}; y_1, \hat{y}_2) \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{Bmatrix}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{12}+R_{21}+R_{22}$ bound (6.73) intersect satisfies

$$\begin{aligned}2R_{11}^*+2R_{12}^*+2R_{21}^*+2R_{22}^* &= \left\{I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c})+2\mathsf{C}_{12}^{\mathsf{B}}-R_{11c}\right\}\\ &+\left\{\begin{array}{l}I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c})\\ +I(x_1,x_{2c};y_1,\hat{y}_2)+\mathsf{C}_{12}^{\mathsf{B}}-R_{21c}\end{array}\right\}\\ &=\left\{I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c})+\mathsf{C}_{12}^{\mathsf{B}}\right\}\\ &+\left\{I(x_{1c},x_2;y_2|x_{2c})+I(x_1,x_{2c};y_1,\hat{y}_2)-R_{11c}-R_{21c}\right\}\\ &+2\mathsf{C}_{12}^{\mathsf{B}}\\ &>(6.80)+(6.82)\end{aligned}$$

which is greater than twice the active sum rate bound.

Therefore, in the first condition, we conclude that the corner point where the $R_{11}+R_{12}+R_{21}+R_{22}$ bound and the $2R_{11}+R_{12}+R_{21}+R_{22}$ bound intersect can be achieved.

For the second condition, there are two possible R_{11} bounds. Hence, we consider the following two cases:

1) If the bound

$$R_{11}\leq I(x_1;y_1|x_{2c})-R_{21c}$$

is active. The point $R_{11}^*+R_{12}^*+R_{21}^*+R_{22}^*$ where the R_{11} bound and the $R_{12}+R_{21}+R_{22}$ bound (6.73) intersect satisfies

$$\begin{aligned}R_{11}^*+R_{12}^*+R_{21}^*+R_{22}^* &= \left\{I(x_{1c};y_2|x_2)+I(x_2;y_2|x_{1c})+2\mathsf{C}_{12}^{\mathsf{B}}-R_{11c}\right\}\\ &+\left\{I(x_1;y_1|x_{2c})-R_{21c}\right\}\\ &=\{I(x_2;y_2|x_{1c})+I(x_1;y_1|x_{2c})+\mathsf{C}_{12}^{\mathsf{B}}\}\\ &+\left\{I(x_{1c};y_2|x_2)+\mathsf{C}_{12}^{\mathsf{B}}-R_{11c}-R_{21c}\right\}\\ &=(6.80)\end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c};y_2|x_2)+\mathsf{C}_{12}^{\mathsf{B}}-R_{11c}-R_{21c}\}$ refers to (6.38).

2) If the bound

$$R_{11}\leq I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c};y_2|x_2)+\mathsf{C}_{12}^{\mathsf{B}}-R_{21c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the R_{11} bound and the $R_{12} + R_{21} + R_{22}$ bound (6.73) intersect satisfies

$$\begin{aligned} R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^* &= \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \right\} \\ &+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \right\} \\ &= \left\{ I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}) + I(x_1; y_1|x_{1c}, x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \right\} \\ &+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \right\} + \mathsf{C}_{12}^{\mathsf{B}} \\ &\geq (6.80) \end{aligned}$$

which is greater than the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c}\}$ refers to (6.38).

Therefore, in the second condition, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the R_{11} bound intersect can be achieved.

6.6.7 Proof of Claim 6.19

Proof: This proof considers only (6.87) and the other can be shown similarly. Since there are five possible $R_{12} + R_{21} + R_{22}$ bounds. Hence, we consider the following five cases:

1) If the bound

$$R_{12} + R_{21} + R_{22} \leq I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{12} + R_{21} + R_{22}$ bound and the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound (6.87) intersect satisfies

$$\begin{aligned} 2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \left\{ \begin{array}{l} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \end{array} \right\} \\ &+ \left\{ I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \right\} \\ &= \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{array} \right\} \\ &+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} \right\} \\ &+ \left\{ I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{21c} \right\} \\ &= (6.78) + (6.80) \end{aligned}$$

which is greater than twice the active sum rate bound. Note that $\{I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c} - R_{21c}\} \geq 0$ refers to (6.38).

2) If the bound

$$R_{12} + R_{21} + R_{22} \leq \begin{Bmatrix} I(x_{2c}; y_1|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{Bmatrix}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{12} + R_{21} + R_{22}$ bound and the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound (6.87) intersect satisfies

$$\begin{aligned} 2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + 2\mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{Bmatrix} \\ &+ \begin{Bmatrix} I(x_{2c}; y_1|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) \\ +(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{Bmatrix} \\ &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \\ &+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \right\} \\ &+ \begin{Bmatrix} I(x_{2c}; y_1|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) \\ +(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{Bmatrix} \\ &= (6.78) + (6.35) + (6.74) \end{aligned}$$

which is greater than twice the active sum rate bound.

3) If the bound

$$R_{12} + R_{21} + R_{22} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{12} + R_{21} + R_{22}$ bound and the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound (6.87) intersect satisfies

$$\begin{aligned} 2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + 2\mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{Bmatrix} \\ &+ \begin{Bmatrix} I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{Bmatrix} \end{aligned}$$

$$
\begin{aligned}
&= \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} \end{array} \right\} \\
&+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \right\} \\
&+ \left\{ \begin{array}{l} I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{11c} \end{array} \right\} \\
&= (6.78) + (6.35) + (6.75)
\end{aligned}
$$

which is greater than twice the active sum rate bound.

4) If the bound

$$
R_{12} + R_{21} + R_{22} \leq I(x_{2c}; y_1|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c}
$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{12} + R_{21} + R_{22}$ bound and the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound (6.87) intersect satisfies

$$
\begin{aligned}
2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \left\{ \begin{array}{l} I(x_1; y_1, |x_{1c}, x_{2c}) + I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + 2\mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \end{array} \right\} \\
&+ \left\{ \begin{array}{l} I(x_{2c}; y_1|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} \end{array} \right\} \\
&= \left\{ I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \right\} \\
&+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{21c} \right\} \\
&+ \left\{ \begin{array}{l} I(x_{2c}; y_1|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} \end{array} \right\} \\
&= (6.78) + (6.35) + (6.76)
\end{aligned}
$$

which is greater than twice the active sum rate bound.

5) If the bound

$$
R_{12} + R_{21} + R_{22} \leq I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}^{\mathsf{B}}_{12} - R_{11c}
$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{12} + R_{21} + R_{22}$ bound and the $2R_{11} + R_{12} + R_{21} + R_{22}$ bound (6.87) intersect satisfies

$$
\begin{aligned}
2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \left\{ \begin{array}{l} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + 2\mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \end{array} \right\} \\
&+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \right\} \\
&= \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{array} \right\} \\
&+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c} \right\} \\
&+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} \right\} \\
&= (6.78) + (6.35) + (6.77)
\end{aligned}
$$

which is greater than twice the active sum rate bound.

Therefore, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{12} + R_{21} + R_{22}$ bound intersect can be acquired.

6.6.8 Proof of Claim 6.20

Proof: In this proof, we consider only (6.89) and the other bounds can be shown similarly. Since there are three possible $R_{11} + R_{21} + R_{22}$ bounds. Hence, we consider the following three cases:

1) If the bound

$$
R_{11} + R_{21} + R_{22} \leq I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c}
$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{21} + R_{22}$ bound and the $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound (6.89) intersect satisfies

$$
\begin{aligned}
2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \end{array} \right\} \\
&+ \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \right\} \\
&= \left\{ I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} \right\} \\
&+ \left\{ \begin{array}{l} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{12c} - R_{22c} \end{array} \right\} \\
&= (6.80) + (6.78)
\end{aligned}
$$

which is greater than twice the active sum rate bound.

2) If the bound

$$R_{11} + R_{21} + R_{22} \leq I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{21} + R_{22}$ bound and the $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound (6.89) intersect satisfies

$$\begin{aligned} 2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \end{Bmatrix} \\ &+ \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c} \right\} \\ &= \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) \right\} \\ &+ \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{12c} - R_{22c} \end{Bmatrix} \\ &= (6.82) + (6.78) \end{aligned}$$

which is greater than twice the active sum rate bound.

3) If the bound

$$R_{11} + R_{21} + R_{22} \leq I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c}$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{21} + R_{22}$ bound and the $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound (6.89) intersect satisfies

$$\begin{aligned} 2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \end{Bmatrix} \\ &+ \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \end{Bmatrix} \\ &= \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\ &+ \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} - R_{12c} - R_{22c} \end{Bmatrix} \\ &= (6.86) + (6.78) \end{aligned}$$

which is greater than twice the active sum rate bound.

Therefore, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + R_{21} + R_{22}$ bound intersect can be acquired.

6.6.9 Proof of Claim 6.21

Proof: In this proof, we consider only (6.102) and the other bounds can be shown similarly. Since there are two possible $R_{11} + R_{12} + R_{22}$ bounds. Hence, we consider the following two cases:

1) If the bound

$$R_{11} + R_{12} + R_{22} \le I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - R_{21c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{12} + R_{22}$ bound and the $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound (6.102) intersect satisfies

$$\begin{aligned}
2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \begin{Bmatrix} I(x_1, x_{2c}; y_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} \end{Bmatrix} \\
&\quad + \left\{ I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - R_{21c} \right\} \\
&= \left\{ I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \right\} \\
&\quad + \begin{Bmatrix} I(x_1, x_{2c}; y_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} - R_{21c} \end{Bmatrix} \\
&= (6.81) + (6.78)
\end{aligned}$$

which is greater than twice the active sum rate bound.

2) If the bound

$$R_{11} + R_{12} + R_{22} \le I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{21c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{12} + R_{22}$ bound and the $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound (6.102) intersect satisfies

$$\begin{aligned}
2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \begin{Bmatrix} I(x_1, x_{2c}; y_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} \end{Bmatrix} \\
&\quad + \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{21c} \right\} \\
&= \left\{ I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) \right\} \\
&\quad + \begin{Bmatrix} I(x_1, x_{2c}; y_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{11c} - R_{21c} \end{Bmatrix} \\
&= (6.82) + (6.78)
\end{aligned}$$

which is greater than twice the active sum rate bound.

Therefore, we conclude that the corner point where the $R_{11}+R_{12}+R_{21}+R_{22}$ bound and the $R_{11}+R_{12}+R_{22}$ bound intersect can be acquired.

6.6.10 Proof of Claim 6.22

Proof: Since there are eight possible $R_{11}+R_{12}+R_{21}$ bounds. Hence, we consider the following eight cases:

1) If the bound

$$R_{11}+R_{12}+R_{21} \leq I(x_1,x_{2c};y_1)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}-R_{22c}$$

is active. The point $R^*_{11}+R^*_{12}+R^*_{21}+R^*_{22}$ where the $R_{11}+R_{12}+R_{21}$ bound and the $R_{11}+R_{12}+R_{21}+2R_{22}$ bound (6.110) intersect satisfies

$$\begin{aligned}
2R^*_{11}+2R^*_{12}+2R^*_{21}+2R^*_{22} &= \left\{\begin{matrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+2(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}+\mathsf{C}^{\mathsf{B}}_{12}-R_{12c} \end{matrix}\right\} \\
&+\left\{I(x_1,x_{2c};y_1)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}-R_{22c}\right\} \\
&= \left\{\begin{matrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+} \\ +\mathsf{C}^{\mathsf{B}}_{12} \end{matrix}\right\} \\
&+\left\{I(x_1,x_{2c};y_1)+I(x_2;y_2|x_{1c},x_{2c})+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}\right\} \\
&+\left\{I(x_{2c};y_1|x_1)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}-R_{12c}-R_{22c}\right\} \\
&= (6.78)+(6.81)
\end{aligned}$$

which is greater than twice the active sum rate bound. Note that $\{I(x_{2c};y_1|x_1)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}-R_{12c}-R_{22c}\}\geq 0$ refers to (6.36).

2) If the bound

$$R_{11}+R_{12}+R_{21} \leq I(x_1,x_{2c};y_1,\hat{y}_2)-R_{22c}$$

is active. The point $R^*_{11}+R^*_{12}+R^*_{21}+R^*_{22}$ where the $R_{11}+R_{12}+R_{21}$ bound and the

$R_{11}+R_{12}+R_{21}+2R_{22}$ bound (6.110) intersect satisfies

$$2R_{11}^*+2R_{12}^*+2R_{21}^*+2R_{22}^* = \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+2(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \end{Bmatrix}$$
$$+\left\{I(x_1,x_{2c};y_1,\hat{y}_2)-R_{22c}\right\}$$
$$= \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix}$$
$$+\left\{I(x_1,x_{2c};y_1,\hat{y}_2)+I(x_2;y_2|x_{1c},x_{2c})\right\}$$
$$+\left\{I(x_{2c};y_1|x_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+-R_{12c}-R_{22c}\right\}$$
$$=(6.78)+(6.82)$$

which is greater than twice the active sum rate bound. Note that $\{I(x_{2c};y_1|x_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+-R_{12c}-R_{22c}\}\geq 0$ refers to (6.36).

3) If the bound

$$R_{11}+R_{12}+R_{21}\leq I(x_{2c};y_1|x_1)+I(x_1;y_1|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+-R_{22c}$$

is active. The point $R_{11}^*+R_{12}^*+R_{21}^*+R_{22}^*$ where the $R_{11}+R_{12}+R_{21}$ bound and the $R_{11}+R_{12}+R_{21}+2R_{22}$ bound (6.110) intersect satisfies

$$2R_{11}^*+2R_{12}^*+2R_{21}^*+2R_{22}^* = \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+2(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \end{Bmatrix}$$
$$+\left\{I(x_{2c};y_1|x_1)+I(x_1;y_1|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+-R_{22c}\right\}$$
$$= \begin{Bmatrix} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix}$$
$$+\begin{Bmatrix} I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+I(x_1;y_1|x_{2c}) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \end{Bmatrix}$$
$$+\left\{I(x_{2c};y_1|x_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+-R_{12c}-R_{22c}\right\}$$
$$=(6.78)+(6.81)$$

which is greater than twice the active sum rate bound. which is greater than twice the active sum rate bound. Note that $\{I(x_{2c};y_1|x_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+-R_{12c}-R_{22c}\}\geq 0$ refers to (6.36).

4) If the bound

$$R_{11}+R_{12}+R_{21} \leq I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_1;y_1|x_{2c})-R_{22c}$$

is active. The point $R_{11}^*+R_{12}^*+R_{21}^*+R_{22}^*$ where the $R_{11}+R_{12}+R_{21}$ bound and the $R_{11}+R_{12}+R_{21}+2R_{22}$ bound (6.110) intersect satisfies

$$\begin{aligned}
2R_{11}^*+2R_{12}^*+2R_{21}^*+2R_{22}^* &= \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+2(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ +\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\} \\
&+ \left\{I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_1;y_1|x_{2c})-R_{22c}\right\} \\
&= \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{array}\right\} \\
&+ \left\{I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_1;y_1|x_{2c})-R_{22c}\right\} \\
&+ \left\{I(x_{2c};y_1|x_1)+I(x_2;y_2|x_{1c},x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ -R_{12c}\right\} \\
&= (6.78)+(6.53)+(6.40)
\end{aligned}$$

which is greater than twice the active sum rate bound.

5) If the bound

$$R_{11}+R_{12}+R_{21} \leq \left\{\begin{array}{l} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+I(x_{1c};y_2|x_2)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}}-R_{22c} \end{array}\right\}$$

is active. The point $R_{11}^*+R_{12}^*+R_{21}^*+R_{22}^*$ where the $R_{11}+R_{12}+R_{21}$ bound and the $R_{11}+R_{12}+R_{21}+2R_{22}$ bound (6.110) intersect satisfies

$$\begin{aligned}
2R_{11}^*+2R_{12}^*+2R_{21}^*+2R_{22}^* &= \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1) \\ +I(x_{1c},x_2;y_2|x_{2c})+2(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ +\mathsf{C}_{12}^{\mathsf{B}}-R_{12c} \end{array}\right\} \\
&+ \left\{\begin{array}{l} I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+I(x_{1c};y_2|x_2) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ +\mathsf{C}_{12}^{\mathsf{B}}-R_{22c} \end{array}\right\} \\
&= \left\{\begin{array}{l} I(x_{2c},x_1;y_1|x_{1c})+I(x_{1c},x_2;y_2|x_{2c})+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{array}\right\} \\
&+ \left\{\begin{array}{l} I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c};y_2|x_2)+I(x_{2c};y_1|x_1) \\ +(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ +\mathsf{C}_{12}^{\mathsf{B}}-R_{22c} \end{array}\right\} \\
&+ \left\{I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1|x_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^+ -R_{12c}\right\} \\
&= (6.78)+(6.54)+(6.40)
\end{aligned}$$

which is greater than twice the active sum rate bound.

6) If the bound

$$R_{11} + R_{12} + R_{21} \leq I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}; y_2|x_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{22c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{12} + R_{21}$ bound and the $R_{11} + R_{12} + R_{21} + 2R_{22}$ bound (6.110) intersect satisfies

$$\begin{aligned}
2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + 2(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c} \end{Bmatrix} \\
&+ \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}; y_2|x_2) \\ +\mathsf{C}^{\mathsf{B}}_{12} - R_{22c} \end{Bmatrix} \\
&= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ \\ +\mathsf{C}^{\mathsf{B}}_{12} \end{Bmatrix} \\
&+ \begin{Bmatrix} I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + I(x_{2c}; y_1|x_1) \\ +(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{22c} \end{Bmatrix} \\
&+ \left\{ I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c} \right\} \\
&= (6.78) + (6.54) + (6.41)
\end{aligned}$$

which is greater than twice the active sum rate bound.

7) If the bound

$$R_{11} + R_{12} + R_{21} \leq I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{22c}$$

is active. The point $R^*_{11} + R^*_{12} + R^*_{21} + R^*_{22}$ where the $R_{11} + R_{12} + R_{21}$ bound and the $R_{11} + R_{12} + R_{21} + 2R_{22}$ bound (6.110) intersect satisfies

$$\begin{aligned}
2R^*_{11} + 2R^*_{12} + 2R^*_{21} + 2R^*_{22} &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + 2(\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c} \end{Bmatrix} \\
&+ \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} \\ -R_{22c} \end{Bmatrix}
\end{aligned}$$

$$
\begin{aligned}
&= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\
&+ \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}, x_{2c}) \\ +(\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\
&+ \left\{ I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c} \right\} \\
&= (6.78) + (6.83)
\end{aligned}
$$

which is greater than twice the active sum rate bound. Note that $\{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \geq 0$ refers to (6.36).

8) If the bound

$$
R_{11} + R_{12} + R_{21} \leq I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}
$$

is active. The point $R_{11}^* + R_{12}^* + R_{21}^* + R_{22}^*$ where the $R_{11} + R_{12} + R_{21}$ bound and the $R_{11} + R_{12} + R_{21} + 2R_{22}$ bound (6.110) intersect satisfies

$$
\begin{aligned}
2R_{11}^* + 2R_{12}^* + 2R_{21}^* + 2R_{22}^* &= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) \\ +I(x_{1c}, x_2; y_2|x_{2c}) + 2(\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c} \end{Bmatrix} \\
&+ \left\{ I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c} \right\} \\
&= \begin{Bmatrix} I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\
&+ \begin{Bmatrix} I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}, x_{2c}) \\ +\mathsf{C}_{12}^{\mathsf{B}} \end{Bmatrix} \\
&+ \left\{ I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c} \right\} \\
&= (6.78) + (6.84)
\end{aligned}
$$

which is greater than twice the active sum rate bound. Note that $\{I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{12c} - R_{22c}\} \geq 0$ refers to (6.36).

Therefore, we conclude that the corner point where the $R_{11} + R_{12} + R_{21} + R_{22}$ bound and the $R_{11} + R_{12} + R_{21}$ bound intersect can be acquired.

In addition, we use the following lemma which is proposed in [22] to help our proof.

Lemma 6.24

$$\log\big(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\big)$$
$$\geq \log\Big(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\Big)+\log(1+\mathsf{SNR}_2+\mathsf{INR}_2) \qquad (6.116)$$

Proof: See all details at Lemma B.6 in [22]. ■

Furthermore, using the same line as Lemma B.6 in [22], we also obtain

$$\log\big(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\big)$$
$$\geq \log\Big(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\Big)+\log(1+\mathsf{SNR}_1+\mathsf{INR}_1) \qquad (6.117)$$

Next, for proving in the following claim, we assign the following terms which are considerably useful for our proof:

1. Quantization distortion at receiver 2 $(\Delta_2)=1+\mathsf{SNR}_2^{22p}$. This value of Δ_2 is chosen for upper bounding the rate loss term ξ_1 in (6.33) by 1 bit, where $\mathsf{SNR}_i^{iip}=\mathsf{SNR}_i Q_{iip}=\mathsf{SNR}_i\min\Big\{1,\frac{1}{\mathsf{INR}_j}\Big\}\geq\frac{\mathsf{SNR}_i}{1+\mathsf{INR}_j}$ and $0\leq\mathsf{INR}_i^{jjp}\leq 1$ for $i,j=1,2$ and $i\neq j$.

2. For $\beta,\gamma\in[0,1]$, we set $R_{11c}=\beta R_{1c}$, $R_{21c}=R_{1c}-\beta R_{1c}$, $R_{12c}=R_{2c}-\gamma R_{2c}$ and $R_{22c}=\gamma R_{2c}$, where R_{1c} and R_{2c} are the rate constraints without cooperation for the common codeword at transmitter 1 and 2, respectively, that are calculated as follows:

$$R_{1c}=I(x_{1c};y_2|x_2)=\log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right),$$
$$R_{2c}=I(x_{2c};y_1|x_1)=\log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right).$$

Remark 6.25 (Constraint common rates (6.111) and (6.112) at the asymptotic values of β and γ): *Remind that the constraint common rates (6.111) and (6.112) are*

$$R_{1c}=R_{11c}+R_{21c}=\beta R_{1c}+(1-\beta)R_{1c}$$
$$R_{2c}=R_{12c}+R_{22c}=(1-\gamma)R_{2c}+\gamma R_{2c}$$

where $\beta,\gamma\in[0,1]$. *When considering at the asymptotic values of* β *and* γ*, these two constraint common rates can be considered as the following 4 cases:*

1. *Case* $\beta = 0$*: We obtain* $R_{1c} = R_{21c}$.

2. *Case* $\beta = 1$*: We obtain* $R_{1c} = R_{11c}$.

3. *Case* $\gamma = 0$*: We obtain* $R_{2c} = R_{12c}$.

4. *Case* $\gamma = 1$*: We obtain* $R_{2c} = R_{22c}$.

There result are useful to understand the individual rate and the sum-rate in the next remark.

Remark 6.26 (Individual rate at the asymptotic values of β and γ): *Remind that, based on the proposed strategy as shown in Appendix 6.5 for the two-user Gaussian X channel with limited receiver cooperation in the case of strong Gaussian X channel type I, the transmitted codeword can be written as follows:* $x_i[n] = x_{ic}[n] + x_{iip}[n]$ *for all* n *and* $i = 1, 2$. *The rate for each message that corresponds to the proposed strategy is shown in the following relationships:* $R_{ii} = R_{iip} + R_{iic}$ *and* $R_{ij} = R_{ijc}$, *for* $i, j = 1, 2$ *and* $i \neq j$, *where* R_{iic} *and* R_{ijc} *are defined in (6.111) and (6.112).*

1. *The individual rate at the asymptotic values of* β *and* γ *can be considered in the following 4 cases:*

   (a) *Case* $\beta = 0$*: Since* $R_{1c} = R_{21c}$*; therefore, we obtain*

      i. $R_{11} = R_{11p}$

      ii. $R_{21} = R_{21c} = R_{1c}$

      This result shows that R_{11} *and* R_{21} *are the rate of a private codeword* x_{11p}^N *and the rate of a common codeword* x_{1c}^N*, respectively.*

   (b) *Case* $\beta = 1$*: Since* $R_{1c} = R_{11c}$*; therefore, we obtain*

      i. $R_{11} = R_{11p} + R_{11c} = R_{11p} + R_{1c}$

      ii. $R_{21} = R_{21c} = 0$

      This result shows that R_{11} *is the sum rate of a private codeword* x_{11p}^N *and a common codeword* x_{1c}^N. $R_{21} = 0$.

   (c) *Case* $\gamma = 0$*: Since* $R_{2c} = R_{12c}$*; therefore, we obtain*

      i. $R_{12} = R_{12c} = R_{2c}$

ii. $R_{22} = R_{22p}$

This result shows that R_{12} *and* R_{22} *are equal to the rate of a common codeword* x_{2c}^N *and the rate of a private codeword* x_{22p}^N*, respectively.*

(d) *Case* $\gamma = 1$*: Since* $R_{2c} = R_{22c}$*; therefore, we obtain*

i. $R_{12} = R_{12c} = 0$

ii. $R_{22} = R_{22p} + R_{22c} = R_{22p} + R_{2c}$

This result shows that R_{22} *is the sum rate of a private codeword* x_{22p}^N *and a common codeword* x_{2c}^N*.* $R_{12} = 0$*.*

2. *The sum of 2 rates at the asymptotic values of* β *and* γ *can be considered in the following 4 cases:*

(a) *Case* $\beta = 0$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{21c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain*

i. $R_{11} + R_{12} = R_{11p} + R_{12c} = R_{11p} + R_{2c}$

ii. $R_{21} + R_{22} = R_{21c} + R_{22p} = R_{22p} + R_{1c}$

This result shows that $R_{ii} + R_{ij}$ *is the sum rate of a private codeword* x_{iip}^N *and a common codeword* x_{jc}^N*.*

(b) *Case* $\beta = 0$ *and* $\gamma = 1$*: Since* $R_{1c} = R_{21c}$ *and* $R_{2c} = R_{22c}$*; therefore, we obtain* $R_{12} = 0$ *and*

i. $R_{11} + R_{12} = R_{11p}$

ii. $R_{21} + R_{22} = R_{21c} + R_{22p} + R_{22c} = R_{22p} + R_{1c} + R_{2c}$

This result shows that $R_{11} + R_{12}$ *equals the rate of a private codeword* x_{11p}^N *and* $R_{21} + R_{22}$ *is the sum rate of a private codeword* x_{22p}^N *and two common codewords* x_{1c}^N *and* x_{2c}^N*.*

(c) *Case* $\beta = 1$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain* $R_{21} = 0$ *and*

i. $R_{11} + R_{12} = R_{11p} + R_{11c} + R_{12c} = R_{11p} + R_{1c} + R_{2c}$

ii. $R_{21} + R_{22} = R_{22p}$

This result shows that $R_{11}+R_{12}$ *is the sum rate of a private codeword* x_{11p}^N *and two common codewords* x_{1c}^N *and* x_{2c}^N *and* $R_{21}+R_{22}=R_{22}$ *is equal to the rate of a private codeword* x_{22p}^N.

(d) *Case* $\beta=1$ *and* $\gamma=1$*: Since* $R_{1c}=R_{11c}$ *and* $R_{2c}=R_{22c}$*; therefore, we obtain* $R_{12}=R_{21}=0$ *and*

i. $R_{11}+R_{12}=R_{11p}+R_{11c}=R_{11p}+R_{1c}$

ii. $R_{21}+R_{22}=R_{22p}+R_{22c}=R_{22p}+R_{2c}$

This result shows that $R_{ii}+R_{ij}$ *is the sum rate of a private codeword* x_{iip}^N *and a common codeword* x_{ic}^N.

3. *The sum of 3 rates at the asymptotic values of* β *and* γ *can be considered in the following 4 cases:*

(a) *Case* $\beta=0$ *and* $\gamma=0$*: Since* $R_{1c}=R_{21c}$ *and* $R_{2c}=R_{12c}$*; therefore, we obtain*

i. $R_{11}+R_{12}+R_{21}=R_{11p}+R_{12c}+R_{21c}=R_{11p}+R_{2c}+R_{1c}$

ii. $R_{11}+R_{12}+R_{22}=R_{11p}+R_{12c}+R_{22p}=R_{11p}+R_{2c}+R_{22p}$

iii. $R_{11}+R_{21}+R_{22}=R_{11p}+R_{21c}+R_{22p}=R_{11p}+R_{1c}+R_{22p}$

iv. $R_{12}+R_{21}+R_{22}=R_{12c}+R_{21c}+R_{22p}=R_{2c}+R_{1c}+R_{22p}$.

This result shows that $R_{ii}+R_{ij}+R_{ji}$ *is the sum rate of a private codeword* x_{iip}^N *and two common codewords* x_{ic}^N *and* x_{jc}^N *and* $R_{ii}+R_{ij}+R_{jj}$ *is the sum rate of two private codewords* x_{iip}^N *and* x_{jjp}^N *and a common codeword* x_{2c}^N.

(b) *Case* $\beta=0$ *and* $\gamma=1$*: Since* $R_{1c}=R_{21c}$ *and* $R_{2c}=R_{22c}$*; therefore, we obtain* $R_{12}=0$ *and*

i. $R_{11}+R_{12}+R_{21}=R_{11p}+R_{21c}=R_{11p}+R_{1c}$*, i.e., the sum rate of a private codeword* x_{11p}^N *and a common codeword* x_{1c}^N.

ii. $R_{11}+R_{12}+R_{22}=R_{11p}+R_{22p}+R_{22c}=R_{11p}+R_{22p}+R_{2c}$*, i.e., the sum rate of two private codewords* x_{11p}^N *and* x_{22p}^N *and a common codewords* x_{2c}^N.

iii. $R_{11}+R_{21}+R_{22}=R_{11p}+R_{21c}+R_{22p}+R_{22c}=R_{11p}+R_{1c}+R_{22p}+R_{2c}$*, i.e., the sum rate of two private codewords* x_{11p}^N *and* x_{22p}^N *and two common codewords* x_{1c}^N *and* x_{2c}^N.

iv. $R_{12} + R_{21} + R_{22} = R_{21c} + R_{22p} + R_{22c} = R_{1c} + R_{22p} + R_{2c}$, *i.e., the sum rate of a private codeword* x_{22p}^N *and two common codewords* x_{1c}^N *and* x_{2c}^N.

(c) *Case* $\beta = 1$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain* $R_{21} = 0$ *and*

i. $R_{11} + R_{12} + R_{21} = R_{11p} + R_{11c} + R_{12c} = R_{11p} + R_{1c} + R_{2c}$, *i.e., the sum rate of a private codeword* x_{11p}^N *and two common codewords* x_{1c}^N *and* x_{2c}^N.

ii. $R_{11} + R_{12} + R_{22} = R_{11p} + R_{11c} + R_{12c} + R_{22p} = R_{11p} + R_{1c} + R_{2c} + R_{22p}$, *i.e., the sum rate of two private codewords* x_{11p}^N *and* x_{22p}^N *and two common codewords* x_{1c}^N *and* x_{2c}^N.

iii. $R_{11} + R_{21} + R_{22} = R_{11p} + R_{11c} + R_{22p} = R_{11p} + R_{1c} + R_{22p}$, *i.e., the sum rate of two private codewords* x_{11p}^N *and* x_{22p}^N *and a common codeword* x_{1c}^N.

iv. $R_{12} + R_{21} + R_{22} = R_{12c} + R_{22p} = R_{2c} + R_{22p}$, *i.e., the sum rate of a private codeword* x_{22p}^N *and a common codeword* x_{2c}^N.

(d) *Case* $\beta = 1$ *and* $\gamma = 1$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{22c}$*; therefore, we obtain* $R_{12} = R_{21} = 0$ *and*

i. $R_{11} + R_{12} + R_{21} = R_{11p} + R_{11c} = R_{11p} + R_{1c}$

ii. $R_{11} + R_{12} + R_{22} = R_{11p} + R_{11c} + R_{22p} + R_{22c} = R_{11p} + R_{1c} + R_{22p} + R_{2c}$

iii. $R_{11} + R_{21} + R_{22} = R_{11p} + R_{11c} + R_{22p} + R_{22c} = R_{11p} + R_{1c} + R_{22p} + R_{2c}$

iv. $R_{12} + R_{21} + R_{22} = R_{22p} + R_{22c} = R_{22p} + R_{2c}$

This result shows that $R_{ii} + R_{ij} + R_{ji}$ *is the sum rate of a private codeword* x_{iip}^N *and a common codeword* x_{ic}^N *and* $R_{ii} + R_{ij} + R_{jj}$ *is the sum rate of two private codewords* x_{iip}^N *and* x_{jjp}^N *and two common codewords* x_{ic}^N *and* x_{jc}^N.

4. *The sum of 4 rates at the asymptotic values of* β *and* γ *can be considered in the following 4 cases:*

(a) *Case* $\beta = 0$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{21c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain*

$R_{11} + R_{12} + R_{21} + R_{22} = R_{11p} + R_{12c} + R_{21c} + R_{22p} = R_{11p} + R_{2c} + R_{1c} + R_{22p}$

(b) *Case* $\beta = 0$ *and* $\gamma = 1$*: Since* $R_{1c} = R_{21c}$ *and* $R_{2c} = R_{22c}$*; therefore, we obtain* $R_{12} = 0$ *and* $R_{11} + R_{12} + R_{21} + R_{22} = R_{11p} + R_{21c} + R_{22p} + R_{22c} = R_{11p} + R_{1c} + R_{22p} + R_{2c}$

(c) *Case* $\beta = 1$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain* $R_{21} = 0$ *and* $R_{11} + R_{12} + R_{21} + R_{22} = R_{11p} + R_{11c} + R_{12c} + R_{22p} = R_{11p} + R_{1c} + R_{2c} + R_{22p}$

(d) *Case* $\beta = 1$ *and* $\gamma = 1$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{22c}$*; therefore, we obtain* $R_{12} = R_{21} = 0$ *and* $R_{11} + R_{12} + R_{21} + R_{22} = R_{11p} + R_{11c} + R_{22c} + R_{22p} = R_{11p} + R_{1c} + R_{2c} + R_{22p}$

From the above result, it is obviously seen that the sum of 4 rates for 4 cases has the same value, i.e., it is the sum of the rate of two private codewords x_{11p}^N *and* x_{22p}^N *and the rates of two common codewords* x_{1c}^N *and* x_{2c}^N*. Therefore, we can conclude that the sum of four rates does not depend on the asymptotic values of* β *and* γ*.*

5. *The sum of 5 rates at the asymptotic values of* β *and* γ *can be considered in the following 4 cases:*

(a) *Case* $\beta = 0$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{21c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain*

i. $2R_{11} + R_{12} + R_{21} + R_{22} = 2R_{11p} + R_{12c} + R_{21c} + R_{22p} = 2R_{11p} + R_{2c} + R_{1c} + R_{22p}$

ii. $R_{11} + 2R_{12} + R_{21} + R_{22} = R_{11p} + 2R_{12c} + R_{21c} + R_{22p} = R_{11p} + 2R_{2c} + R_{1c} + R_{22p}$

iii. $R_{11} + R_{12} + 2R_{21} + R_{22} = R_{11p} + R_{12c} + 2R_{21c} + R_{22p} = R_{11p} + R_{2c} + 2R_{1c} + R_{22p}$

iv. $R_{11} + R_{12} + R_{21} + 2R_{22} = R_{11p} + R_{12c} + R_{21c} + 2R_{22p} = R_{11p} + R_{2c} + R_{1c} + 2R_{22p}$

This result shows that $2R_{ii} + R_{ij} + R_{ji} + R_{jj}$ *is the sum of twice rate of a private codeword* x_{iip}^N*, rates of two common codewords* x_{ic}^N *and* x_{jc}^N *and a rate of a private codeword* x_{jjp}^N *and* $R_{ii} + 2R_{ij} + R_{ji} + R_{jj}$ *is the sum of rates of two private codewords* x_{iip}^N *and* x_{jjp}^N*, twice rate of a common codeword* x_{jc}^N *and a rate of a common codeword* x_{ic}^N*.*

(b) *Case* $\beta = 0$ *and* $\gamma = 1$*: Since* $R_{1c} = R_{21c}$ *and* $R_{2c} = R_{22c}$*; therefore, we obtain* $R_{12} = 0$ *and* *i.* $2R_{11} + R_{12} + R_{21} + R_{22} = 2R_{11p} + R_{21c} + R_{22p} + R_{22c} = 2R_{11p} + R_{1c} + R_{22p} + R_{2c}$, *i.e., the sum of twice rate of a private codeword* x_{11p}^N, *the rates of two common codewords* x_{1c}^N *and* x_{2c}^N *and the rate of a private codeword* x_{22p}^N.

ii. $R_{11} + 2R_{12} + R_{21} + R_{22} = R_{11p} + R_{21c} + R_{22p} + R_{22c} = R_{11p} + R_{1c} + R_{22p} + R_{2c}$, *i.e., the sum of the rates of two private codewords* x_{11p}^N *and* x_{22p}^N *and the rates of two common codewords* x_{1c}^N *and* x_{2c}^N.

iii. $R_{11} + R_{12} + 2R_{21} + R_{22} = R_{11p} + 2R_{21c} + R_{22p} + R_{22c} = R_{11p} + 2R_{1c} + R_{22p} + R_{2c}$, *i.e., the sum of the rates of two private codewords* x_{11p}^N *and* x_{22p}^N, *twice rate of a common codeword* x_{1c}^N *and the rate of a common codeword* x_{2c}^N.

iv. $R_{11} + R_{12} + R_{21} + 2R_{22} = R_{11p} + R_{21c} + 2R_{22p} + 2R_{22c} = R_{11p} + R_{1c} + 2R_{22p} + 2R_{2c}$, *i.e., the sum of the rate of a private codeword* x_{11p}^N, *the rate of a common codeword* x_{1c}^N, *twice rate of a private codeword* x_{22p}^N *and twice rate of a common codeword* x_{2c}^N.

(c) Case $\beta = 1$ *and* $\gamma = 0$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{12c}$*; therefore, we obtain* $R_{21} = 0$ *and*

i. $2R_{11} + R_{12} + R_{21} + R_{22} = 2R_{11p} + 2R_{11c} + R_{12c} + R_{22p} = 2R_{11p} + 2R_{1c} + R_{2c} + R_{22p}$, *i.e., the sum of twice rate of a private codeword* x_{11p}^N, *twice rate of a common codeword* x_{1c}^N, *the rate of a private codeword* x_{22p}^N *and the rate of a common codeword* x_{2c}^N.

ii. $R_{11} + 2R_{12} + R_{21} + R_{22} = R_{11p} + R_{11c} + 2R_{12c} + R_{22p} = R_{11p} + R_{1c} + 2R_{2c} + R_{22p}$, *i.e., the sum of the rate of a private codeword* x_{11p}^N, *the rate of a common codeword* x_{1c}^N, *twice rates of a common codeword* x_{2c}^N *and the rate of a private codeword* x_{22p}^N.

iii. $R_{11} + R_{12} + 2R_{21} + R_{22} = R_{11p} + R_{11c} + R_{12c} + R_{22p} = R_{11p} + R_{1c} + R_{2c} + R_{22p}$, *i.e., the sum of the rates of two private codewords* x_{11p}^N *and* x_{22p}^N *and the rates of two common codewords* x_{1c}^N *and* x_{2c}^N.

iv. $R_{11} + R_{12} + R_{21} + 2R_{22} = R_{11p} + R_{11c} + R_{12c} + 2R_{22p} = R_{11p} + R_{1c} + R_{2c} + 2R_{22p}$, *i.e., the sum of the rate of the private codeword* x_{11p}^N, *the rates of two common codewords* x_{1c}^N *and* x_{2c}^N *and twice rate of a private codeword*

x_{22p}^N.

(d) *Case* $\beta = 1$ *and* $\gamma = 1$*: Since* $R_{1c} = R_{11c}$ *and* $R_{2c} = R_{22c}$*; therefore, we obtain* $R_{12} = R_{21} = 0$ *and*

i. $2R_{11} + R_{12} + R_{21} + R_{22} = 2R_{11p} + 2R_{11c} + R_{22c} + R_{22p} = 2R_{11p} + 2R_{1c} + R_{2c} + R_{22p}$.

ii. $R_{11} + 2R_{12} + R_{21} + R_{22} = R_{11p} + R_{11c} + R_{22c} + R_{22p} = R_{11p} + R_{1c} + R_{2c} + R_{22p}$.

iii. $R_{11} + R_{12} + 2R_{21} + R_{22} = R_{11p} + R_{11c} + R_{22c} + R_{22p} = R_{11p} + R_{1c} + R_{2c} + R_{22p}$.

iv. $R_{11} + R_{12} + R_{21} + 2R_{22} = R_{11p} + R_{11c} + 2R_{22c} + 2R_{22p} = R_{11p} + R_{1c} + 2R_{2c} + 2R_{22p}$.

This result shows that $2R_{ii} + R_{ij} + R_{ji} + R_{jj}$ *is the sum of twice rate of a private codeword* x_{iip}^N*, twice rate of a common codeword* x_{ic}^N*, the rate of a common codeword* x_{jc}^N *and the rate of a private codeword* x_{jjp}^N *and* $R_{ii} + 2R_{ij} + R_{ji} + R_{jj}$ *is the sum of the rates of two private codewords* x_{iip}^N *and* x_{jjp}^N*, and the rates of two common codewords* x_{ic}^N *and* x_{jc}^N.

Note that the all above relationships are useful for better understanding the proof of Claim 6.23 in the next section.

6.6.11 Proof of Claim 6.23

Proof: In this proof, we consider the bounds with the different values of β and γ as follows:

1. R_{11} bound: We have two bounds as follows:

- Case $\beta = 1$:

  - First,

$I(x_1; y_1|x_{2c}) - R_{21c}$
$= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) - (R_{1c} - \beta R_{1c})$
$\geq \log\left(\frac{1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right)$
which is within 2 bits to the upper bound $\log(1 + \mathsf{SNR}_1 + \mathsf{INR}_2)$ in (4.1).

• Second,

$I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{21c}$
$= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - (R_{1c} - \beta R_{1c})$
$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
which is within 2 bits to the upper bound $\log(1 + \mathsf{SNR}_1 + \mathsf{INR}_2)$ in (4.1).

- Case $\beta = 0$: In this case, it is easily seen that above two bounds of R_{11} are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\beta = 1$. Therefore, we omit it.

2. R_{12} bound: The bound

- Case $\gamma = 0$:

• $I(x_{2c}; y_1|x_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c}$
$= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - \gamma R_{2c}$
$= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+$
which is within 2 bits to the upper bound $\log(1 + \mathsf{INR}_1) + \mathsf{C}_{21}^{\mathsf{B}}$ in (4.2).

- Case $\gamma = 1$: In this case, it is seen that above bound of R_{12} is less than the corresponding upper bound. However, it is not within a constant gap. The proof is similar to case $\gamma = 0$. Therefore, we omit it.

3. R_{21} bound: The bound

- Case $\beta = 0$:

• $I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
$= \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$
$= \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
which is within 1 bit to the upper bound $\log(1 + \mathsf{INR}_2) + \mathsf{C}_{12}^{\mathsf{B}}$ in (4.3).

- Case $\beta = 1$: In this case, it is seen that above bound of R_{21} is less than the corresponding upper bound. However, it is not within a constant gap. The proof is similar to case $\beta = 0$. Therefore, we omit it.

4. R_{22} bound: We have three bounds as follows:

- Case $\gamma = 1$:

  • First,

  $I(x_{2c}, x_2; y_2|x_{1c}) + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c}$
  $= \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}^{\mathsf{B}}_{12} - (R_{2c} - \gamma R_{2c})$
  $\geq \log\left(\frac{1+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}^{\mathsf{B}}_{12}$
  which is within 1 bits to the upper bound $\log(1 + \mathsf{SNR}_2) + \mathsf{C}^{\mathsf{B}}_{12}$ in (4.4).

  • Second,

  $I(x_{2c}; y_1|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - R_{12c}$
  $= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - (R_{2c} - \gamma R_{2c})$
  $\geq \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+$
  $\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{(1+\mathsf{INR}_1^{22p})(1+\mathsf{INR}_1^{22p})}\right)$
  which is within 2 bits to the upper bound $\log(1 + \mathsf{SNR}_2 + \mathsf{INR}_1)$ in (4.4).

  • Third,

  $I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c}$
  $= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) - (R_{2c} - \gamma R_{2c})$
  $\overset{(a)}{\geq} \log\left(\frac{2(1+\mathsf{INR}_1+\mathsf{SNR}_2)}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$
  $\geq \log\left(\frac{1+\mathsf{SNR}_2+\mathsf{INR}_1}{1+\mathsf{INR}_2^{11p}}\right) - 1$
  where $\Delta_2 = 1+\mathsf{SNR}_2^{22p}$, $\mathsf{INR}_1^{22p} \leq 1$ and (a) is due to $(1 + \Delta_2)(1 + \mathsf{INR}_1^{22p}) + \mathsf{SNR}_2^{22p}$ $\leq 2(1 + \Delta_2) + \mathsf{SNR}_2^{22p} = 4 + 3\mathsf{SNR}_2^{22p} \leq 4\Delta_2$. This bound is within 2 bits to the upper bound $\log(1 + \mathsf{SNR}_2 + \mathsf{INR}_1)$ in (4.4).

- Case $\gamma = 0$: In this case, it is easily seen that above two bounds of R_{22} are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\gamma = 1$. Therefore, we omit it.

5. $R_{11} + R_{12}$ bound: We have four bounds as follows:

- Case $\beta = 1$ and $\gamma = 0$:

  • First,

  $I(x_1, x_{2c}; y_1) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - R_{21c} - R_{22c}$
  $= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ - (R_{1c} - \beta R_{1c}) - \gamma R_{2c}$
  $= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+$
  which is within $1 + 1 = 2$ bits to the upper bound (4.5).

• Second,

$I(x_1, x_{2c}; y_1, \hat{y}_2) - R_{21c} - R_{22c}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) - (R_{1c} - \beta R_{1c}) - \gamma R_{2c}$
$\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{\Delta_2}\right) - 2$

Using (6.117), therefore, this bound is within 2 bits to the upper bound (4.7).

• Third,

$I(x_{2c}; y_1|x_1) + I(x_1; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c} - R_{22c}$
$= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - (R_{1c} - \beta R_{1c}) - \gamma R_{2c}$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{(1+\mathsf{INR}_1^{22p})(1+\mathsf{INR}_1^{22p})}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+$

which is within 2+1 = 3 bits to the upper bound (4.5).

• Fourth,

$I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_1; y_1|x_{2c}) - R_{21c} - R_{22c}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) - (R_{1c} - \beta R_{1c}) - \gamma R_{2c}$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right) + \log\left(\frac{1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right)$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) - 1$

which is within 2 bits to the upper bound (4.7).

- Case $\beta = 0$ and $\gamma = 0$, case $\beta = 0$ and $\gamma = 1$ and case $\beta = 1$ and $\gamma = 1$: In these cases, it is seen that above four bounds of $R_{11} + R_{12}$ are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\beta = 1$ and $\gamma = 0$. Therefore, we omit it.

6. $R_{21} + R_{22}$ bound: We have three bounds as follows:

- Case $\beta = 0$ and $\gamma = 1$:

• First,

$I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$
$= \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c} - (R_{2c} - \gamma R_{2c})$
$= \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$

which is within 1 bits to the upper bound (4.6).

• Second,

$I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$

$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
$- \beta R_{1c} - (R_{2c} - \gamma R_{2c})$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
$\geq \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{(1+\mathsf{INR}_2^{11p})(1+\mathsf{INR}_2^{11p})}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
which is within 3 bits to the upper bound (4.6).

• Third,

$I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c} - R_{12c}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c} - (R_{2c} - \gamma R_{2c})$
$\geq \log\left(\frac{2(1+\mathsf{INR}_1+\mathsf{SNR}_2)}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
$\geq \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
which is within 2 bits to the upper bound (4.6).

- Case $\beta = 0$ and $\gamma = 0$, case $\beta = 1$ and $\gamma = 0$ and case $\beta = 1$ and $\gamma = 1$: In these cases, above three bounds of $R_{21} + R_{22}$ are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\beta = 0$ and $\gamma = 1$. Therefore, we omit it.

7. $R_{11} + R_{12} + R_{21}$ bound: We have eight bounds as follows:

- Case $\gamma = 0$:

• First,

$I(x_1, x_{2c}; y_1) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c}$
$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - \gamma R_{2c}$
$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+$
which is within 3 bits to the upper bound (4.13).

• Second,

$I(x_1, x_{2c}; y_1, \hat{y}_2) - R_{22c}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) - \gamma R_{2c}$
$\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{\Delta_2}\right) - 2$
which is within 2 bits to the upper bound (4.21).

• Third,

$I(x_{2c}; y_1|x_1) + I(x_1; y_1|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{22c}$

$= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - \gamma R_{2c}$

$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{(1+\mathsf{INR}_1^{22p})(1+\mathsf{INR}_1^{22p})}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+$

which is within $2+1=3$ bits to the upper bound (4.13).

- Fourth,

$I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_1; y_1|x_{2c}) - R_{22c}$

$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) - \gamma R_{2c}$

$\geq \log\left(\frac{2(1+\mathsf{INR}_1+\mathsf{SNR}_2)}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right)$

$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) - 1$

Using (6.117), therefore, this bound is within 2 bits to the upper bound (4.21).

- Fifth,

$I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}; y_2|x_{2c}, x_{22p}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}$

$= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - \gamma R_{2c}$

$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{(1+\mathsf{INR}_1^{22p})(1+\mathsf{INR}_1^{22p})}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$

which is within 3+1 = 4 bits to the upper bound (4.9).

- Sixth,

$I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}; y_2|x_{2c}, x_{22p}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}$

$= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \gamma R_{2c}$

$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{\Delta_2}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$

$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$

Using (6.117), therefore, this bound is within 3 bits to the upper bound (4.17).

- Seventh,

$I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}$

$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - \gamma R_{2c}$

$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$

which is within 2+1 = 3 bits to the upper bound (4.9).

- Eighth,

$I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}$

$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
$- \gamma R_{2c}$

$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 2$

which is within 3 bits to the upper bound (4.17).

- Case $\gamma = 1$: In this case, above eight bounds of $R_{11} + R_{12} + R_{21}$ are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\gamma = 0$. Therefore, we omit it.

8. $R_{11} + R_{12} + R_{22}$ bound: We have two bounds as follows:

- Case $\beta = 1$

  • First,

$$I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - R_{21c}$$
$$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(1 + \frac{\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ - (R_{1c} - \beta R_{1c})$$
$$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(1 + \frac{\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+$$

which is within 2+1 = 3 bits to the upper bound (4.14).

  • Second,

$$I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{21c}$$
$$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(1 + \frac{\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$
$$- (R_{1c} - \beta R_{1c})$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{4\Delta_2}\right) + \log\left(1 + \mathsf{SNR}_2^{22p}\right) - 1$$
$$\geq \log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\right) - 3$$

which is within 3 bits to the upper bound (4.22).

- Case $\beta = 0$: In this case, above two bounds of $R_{11} + R_{12} + R_{22}$ are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\beta = 1$. Therefore, we omit it.

9. $R_{11} + R_{21} + R_{22}$ bound: We have three bounds as follows:

- Case $\gamma = 1$

  • First,

$$I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c}$$
$$= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - (R_{2c} - \gamma R_{2c})$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$$

which is within 2 bits to the upper bound (4.15).

• Second,

$$I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c}) - R_{12c}$$
$$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$
$$- (R_{2c} - \gamma R_{2c})$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{\Delta_2}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) - 2$$

which is within 3 bits to the upper bound (4.23).

• Third,

$$I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{12c}$$
$$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{11p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$
$$- (R_{2c} - \gamma R_{2c}) + \mathsf{C}_{12}^{\mathsf{B}}$$
$$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$$
$$\geq \log(1 + \mathsf{INR}_1) + \log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$$

which is within 3 bits to the upper bound (4.27).

- Case $\gamma = 0$: In this case, above three bounds of $R_{11} + R_{21} + R_{22}$ are less than the corresponding upper bounds. However, they are not within a constant gap. The proof is similar to case $\beta = 1$. Therefore, we omit it.

10. $R_{12} + R_{21} + R_{22}$ bound: We have five bounds as follows:

- Case $\beta = 0$:

• First,

$$I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$$
$$= \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$$
$$= \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$$

which is within 2 bits to the upper bound (4.16).

• Second,

$$I(x_{2c}; y_1|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_{2c}, x_{22p}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$$
$$= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$$
$$\geq \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{(1+\mathsf{INR}_2^{11p})(1+\mathsf{INR}_2^{11p})}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$$

which is within 3+1 = 4 bits to the upper bound (4.12).

• Third,

$I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
$\geq \log(1+\mathsf{INR}_1) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
Applying Lemma 6.24, this bound is within 3 bits to the upper bound (4.16).

• Fourth,

$I(x_{2c}; y_1|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
$= \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$
$\geq \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$
which is within 2+1 = 3 bits to the upper bound (4.12).

• Fifth,

$I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$
$\geq \log\left(\frac{(1+\mathsf{INR}_1+\mathsf{SNR}_2)}{2\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
$\geq \log(1+\mathsf{INR}_1) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
$\geq \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
which is within 3 bits to the upper bound (4.16).

- Case $\beta = 1$: In this case, although above eight bounds of $R_{12} + R_{21} + R_{22}$ are less than the corresponding upper bounds but they are not within a constant gap. The proof is similar to case $\beta = 0$. Therefore, we omit it.

11. $R_{11} + R_{12} + R_{21} + R_{22}$ bound: We have nine bounds as follows:

- First,

$I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$
$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$
which is within $2 + 1 = 3$ bits to the upper bound (4.26).

- Second,

$I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$

$+ \mathsf{C}_{12}^{\mathsf{B}}$
$\geq \log\Big(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{4\Delta_2}\Big) + \log\Big(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\Big) + \mathsf{C}_{12}^{\mathsf{B}}$
$\geq \log\Big(1 + \mathsf{SNR}_1^{11p} + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2^{11p} + |h_{11}h_{22} - h_{12}h_{21}|^2 Q_{11p}\Big)$
$+ \log\Big(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\Big) - \log(1 + \mathsf{SNR}_2^{22p}) + \mathsf{C}_{12}^{\mathsf{B}} - 2$
which is within 3 bits to the upper bound (4.27).

Note that we can lower bound for this bound in the alternative form which is useful for considering the third bound as follows:

$$\begin{aligned}
&I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \\
&= I(x_{2c}; y_1, \hat{y}_2|x_{1c}) + I(x_1; y_1, \hat{y}_2|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \\
&= I(x_{2c}; \hat{y}_2|x_{1c}) + I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(a)}{\geq} I(x_{2c}; y_2|x_{1c}) - 1 + I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} \\
&\overset{(b)}{\geq} I(x_{1c}, x_2; y_2) + I(x_1; y_1|x_{1c}, x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - 1
\end{aligned}$$

where (a) is due to

$$\begin{aligned}
I(x_{2c}; \hat{y}_2|x_{1c}) &= \log\Big(\frac{1 + \Delta_2 + \mathsf{SNR}_2 + \mathsf{INR}_2^{11p}}{1 + \Delta_2 + \mathsf{SNR}_2^{22p} + \mathsf{INR}_2^{11p}}\Big) \\
&\geq \log\Big(\frac{1 + \mathsf{SNR}_2 + \mathsf{INR}_2^{11p}}{1 + (1 + \mathsf{SNR}_2^{22p}) + \mathsf{SNR}_2^{22p} + \mathsf{INR}_2^{11p}}\Big) \\
&\geq \log\Big(\frac{1 + \mathsf{SNR}_2 + \mathsf{INR}_2^{11p}}{1 + \mathsf{SNR}_2^{22p} + \mathsf{INR}_2^{11p}}\Big) - 1 \\
&= I(x_{2c}; y_2|x_{1c}) - 1
\end{aligned}$$

and (b) is due to

$$\begin{aligned}
I(x_{2c}; y_2|x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) &= I(x_{2c}; y_2, x_{1c}) + I(x_{1c}, x_2; y_2|x_{2c}) \\
&\geq I(x_{2c}; y_2) + I(x_{1c}, x_2; y_2|x_{2c}) \\
&= I(x_{1c}, x_2; y_2)
\end{aligned}$$

- Third,
  $I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}}$
  $= \log\Big(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\Big) + \log\Big(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\Big) + \mathsf{C}_{12}^{\mathsf{B}}$
  $\geq \log\Big(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\Big) + \log\Big(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\Big) + \mathsf{C}_{12}^{\mathsf{B}}$
  Using the lower bound of the alternative form for the above second bound, therefore, this bound is within 2 bits to the upper bound (4.27).

- Fourth,

$I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+$
$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+$
Using the same consideration as the third bound, this bound is within 2+1 = 3 bits to the upper bound (4.28).

- Fifth,

$I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_2; y_2|x_{1c}, x_{2c})$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(1 + \frac{\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$
$\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{4\Delta_2}\right) + \log\left(1 + \mathsf{SNR}_2^{22p}\right) - 1$
$= \log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\right) - 3$
which is within 3 bits to the upper bound (4.29).

- Sixth,

$I(x_{2c}, x_1; y_1|x_{1c}) + I(x_{1c}; y_2|x_2) + I(x_{22p}; y_2|x_{1c}, x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$
$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{(1+\mathsf{INR}_2^{11p})(1+\mathsf{INR}_2^{11p})}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$
which is within $3 + 1 = 4$ bits to the upper bound (4.26).

- Seventh,

$I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_{1c}; y_2|x_2) + I(x_2; y_2|x_{1c}, x_{2c}) + \mathsf{C}^{\mathsf{B}}_{12}$
$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right)$
$+ \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}^{\mathsf{B}}_{12}$
$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{(1+\mathsf{INR}_2^{11p})(1+\mathsf{INR}_2^{11p})}\right) +$
$\mathsf{C}^{\mathsf{B}}_{12}$
$\geq \log\left(1 + \mathsf{SNR}_1^{11p} + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2^{11p} + |h_{11}h_{22} - h_{12}h_{21}|^2 Q_{11p}\right)$
$+ \log(1 + \mathsf{INR}_2 + \mathsf{SNR}_2^{22p}) - \log(1 + \mathsf{SNR}_2^{22p}) + \mathsf{C}^{\mathsf{B}}_{12} - 4$
which is within 4 bits to the upper bound (4.27).

- Eighth,

$I(x_1; y_1|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$
$= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$
$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{(1+\mathsf{INR}_1^{22p})(1+\mathsf{INR}_1^{22p})}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$

which is within 3+1 = 4 bits to the upper bound (4.26).

- Ninth,

$I(x_1;y_1|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}^{\mathsf{B}}_{12}$
$=\log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{11p})+\mathsf{SNR}_2^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\mathsf{C}^{\mathsf{B}}_{12}$
$\geq\log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\mathsf{C}^{\mathsf{B}}_{12}$
$\geq\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\mathsf{C}^{\mathsf{B}}_{12}-1$
Using the concept of (6.117), therefore, it easily see that this bound is within 3 bits to the upper bound (4.27).

12. $2R_{11}+R_{12}+R_{21}+R_{22}$ bound: We have two bounds as follows:

- Case $\beta=1$:

  - First,

$I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c})+I(x_1,x_{2c};y_1)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}+\mathsf{C}^{\mathsf{B}}_{12}-R_{21c}$
$=\log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}$
$+\mathsf{C}^{\mathsf{B}}_{12}-(R_{1c}-\beta R_{1c})$
$\geq\log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right)+(\mathsf{C}^{\mathsf{B}}_{21}-\xi_1)^{+}+\mathsf{C}^{\mathsf{B}}_{12}$
Using the concept of (6.117), therefore, this bound is within 3+1 = 4 bits to the upper bound (4.30).

  - Second,

$I(x_1;y_1|x_{1c},x_{2c})+I(x_{1c},x_2;y_2|x_{2c})+I(x_1,x_{2c};y_1,\hat{y}_2)+\mathsf{C}^{\mathsf{B}}_{12}-R_{21c}$
$=\log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$
$+\log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right)+\mathsf{C}^{\mathsf{B}}_{12}-(R_{1c}-\beta R_{1c})$
$\geq\log\left(\frac{1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{4\Delta_2}\right)$
$+\mathsf{C}^{\mathsf{B}}_{12}$
$\geq\log\left(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2\right)$
$+\log(1+\mathsf{INR}_2+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1})-\log(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1})+\log(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2})+\mathsf{C}^{\mathsf{B}}_{12}-4$
which is within 4 bits to the upper bound (4.34).

- Case $\beta=0$: In this case, although above two bounds of $2R_{11}+R_{12}+R_{21}+R_{22}$ are less than the upper bounds but they are not within a constant gap. The proof is similar to case $\beta=1$. Therefore, we omit it.

13. $R_{11}+2R_{12}+R_{21}+R_{22}$ bound: We have a bound as follows:

- Case $\gamma = 0$:

  • $I(x_1, x_{2c}; y_1) + I(x_2; y_2|x_{1c}) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - R_{22c}$
  $= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}} - \gamma R_{2c}$
  $\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}_{21}^{\mathsf{B}} - \xi_1)^+ + \mathsf{C}_{12}^{\mathsf{B}}$
  which is within 5 bits to the upper bound (4.31).

- Case $\gamma = 1$: In this case, although above bound of $R_{11} + 2R_{12} + R_{21} + R_{22}$ is less than the corresponding upper bound but it is not within a constant gap. The proof is similar to case $\gamma \to 0$. Therefore, we omit it.

14. $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound: We have three bounds as follows:

- Case $\beta = 0$:

  • First,
  $I(x_1; y_1|x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
  $= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$
  $\geq \log\left(\frac{1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
  Using the concept of (6.117) with (4.40), therefore, this bound which is within 4 bits to the upper bound (4.40).

  • Second,
  $I(x_1; y_1|x_{2c}) + I(x_{2c}; y_1, \hat{y}_2|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
  $= \log\left(\frac{1+\mathsf{INR}_1^{22p}+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{11p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} -$
  βR_{1c}
  $\geq \log\left(\frac{1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}}$
  $\geq \log\left(\frac{1+\mathsf{SNR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - 1$
  which is within 5 bits to the upper bound (4.40).

  • Third,
  $I(x_1, x_{2c}; y_1, \hat{y}_2) + I(x_{2c}; y_2|x_{1c}, x_{2c}) + I(x_{1c}; y_2|x_2) + \mathsf{C}_{12}^{\mathsf{B}} - R_{11c}$
  $= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$
  $+ \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}_{12}^{\mathsf{B}} - \beta R_{1c}$
  $\geq \log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2+|h_{11}h_{22}-h_{12}h_{21}|^2}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2}{1+\mathsf{INR}_2^{11p}}\right)$
  $+ \mathsf{C}_{12}^{\mathsf{B}}$

$$= \log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2\right) + \log(1 + \mathsf{INR}_2) + \mathsf{C}^{\mathsf{B}}_{12} - 4$$

Using (6.117), therefore, this bound is within 4+1 = 5 bits to the upper bound (4.40).

- Case $\beta = 1$: In this case, although above three bounds of $R_{11} + R_{12} + 2R_{21} + R_{22}$ are less than the corresponding upper bounds but they are not within a constant gap. The proof is similar to case $\beta = 0$. Therefore, we omit it.

15. $R_{11} + R_{12} + R_{21} + 2R_{22}$ bound: We have five bounds as follows:

- Case $\gamma = 1$:

• First,

$$I(x_{2c}, x_1; y_1|x_{1c}) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c}$$
$$= \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - (R_{2c} - \gamma R_{2c})$$
$$\geq \log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12}$$

which is within $3+1 = 4$ bits to the upper bound (4.33).

• Second,

$$I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{1c}, x_2; y_2) + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c}$$
$$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}^{\mathsf{B}}_{12} - (R_{2c} - \gamma R_{2c})$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}^{\mathsf{B}}_{12}$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{\Delta_2}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2}{1+\mathsf{INR}_2^{11p}}\right) + \mathsf{C}^{\mathsf{B}}_{12} - 2$$

which is within 4 bits to the upper bound (4.41)

• Third,

$$I(x_{2c}, x_1; y_1, \hat{y}_2|x_{1c}) + I(x_2; y_2|x_{1c}, x_{2c}) + I(x_{2c}; y_1|x_1) + I(x_{1c}, x_2; y_2|x_{2c}) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - R_{12c}$$
$$= \log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + \log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right) + \log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right) + (\mathsf{C}^{\mathsf{B}}_{21} - \xi_1)^+ + \mathsf{C}^{\mathsf{B}}_{12} - (R_{2c} - \gamma R_{2c})$$
$$\geq \log\left(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2 Q_{11p}}{4\Delta_2}\right) + \log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$

$$+\log\left(\frac{1+\mathsf{INR}_1}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^{+}+\mathsf{C}_{12}^{\mathsf{B}}$$
$$=\log\left(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right)+\log(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\log(1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p})$$
$$+\log(1+\mathsf{INR}_1)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^{+}+\mathsf{C}_{12}^{\mathsf{B}}-5$$

Using the concept of (6.117), this bound is within 5+1 = 6 bits to the upper bound (4.33)

• Fourth,

$$I(x_{2c},x_1;y_1|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})$$
$$+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^{+}+\mathsf{C}_{12}^{\mathsf{B}}-R_{12c}$$
$$=\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right)$$
$$+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^{+}+\mathsf{C}_{12}^{\mathsf{B}}-(R_{2c}-\gamma R_{2c})$$
$$\geq\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_1^{11p}}{1+\mathsf{INR}_1^{22p}}\right)+\log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$
$$+(\mathsf{C}_{21}^{\mathsf{B}}-\xi_1)^{+}+\mathsf{C}_{12}^{\mathsf{B}}$$

which is within 4+1 = 5 bits to the upper bound (4.33)

• Fifth,

$$I(x_{2c},x_1;y_1,\hat{y}_2|x_{1c})+I(x_2;y_2|x_{1c},x_{2c})+I(x_{2c};y_1,\hat{y}_2|x_1)+I(x_{1c},x_2;y_2|x_{2c})+\mathsf{C}_{12}^{\mathsf{B}}$$
$$-R_{12c}$$
$$=\log\left(\frac{(1+\Delta_2)(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1)+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2Q_{11p}}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right)$$
$$+\log\left(\frac{(1+\Delta_2)(1+\mathsf{INR}_1)+\mathsf{SNR}_2}{(1+\Delta_2)(1+\mathsf{INR}_1^{22p})+\mathsf{SNR}_2^{22p}}\right)+\log\left(\frac{1+\mathsf{INR}_2^{11p}+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)$$
$$+\mathsf{C}_{12}^{\mathsf{B}}-(R_{2c}-\gamma R_{2c})$$
$$\geq\log\left(\frac{1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2Q_{11p}}{4\Delta_2}\right)+\log\left(\frac{1+\mathsf{INR}_1+\mathsf{SNR}_2}{2\Delta_2}\right)$$
$$+\log\left(\frac{1+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\log\left(\frac{1+\mathsf{INR}_2+\mathsf{SNR}_2^{22p}}{1+\mathsf{INR}_2^{11p}}\right)+\mathsf{C}_{12}^{\mathsf{B}}-2$$
$$\geq\log\left(1+\mathsf{SNR}_1^{11p}+\mathsf{INR}_1+\mathsf{SNR}_2+\mathsf{INR}_2^{11p}+|h_{11}h_{22}-h_{12}h_{21}|^2Q_{11p}\right)$$
$$+\log(1+\mathsf{SNR}_2+\mathsf{INR}_2)+\mathsf{C}_{12}^{\mathsf{B}}-5$$

which is within 5 bits to the upper bound (4.41)

- Case $\gamma=0$: In this case, although above two bounds of $R_{11}+R_{12}+R_{21}+2R_{22}$ are less than the uppers bounds but they are not within a constant gap. The proof is similar to case $\gamma=1$. Therefore, we omit it.

Therefore, we obtain that the bounds in the achievable rate region $\mathscr{R}_{2\to1\to2}$ satisfy:

- R_{11} bound is within 2 bits to upper bounds when $\beta=1$;
- R_{12} bound is within 2 bits to upper bounds when $\gamma=0$;

- R_{21} bound is within 1 bits to upper bounds when $\beta = 0$;
- R_{22} bound is within 2 bits to upper bounds when $\gamma = 1$;
- $R_{11} + R_{12}$ bound is within 3 bits to upper bounds when $\beta = 1$ and $\gamma = 0$;
- $R_{21} + R_{22}$ bound is within 3 bits to upper bounds when $\beta = 0$ and $\gamma = 1$;
- $R_{11} + R_{12} + R_{21}$ bound is within 4 bits to upper bounds when $\gamma = 0$;
- $R_{11} + R_{12} + R_{22}$ bound is within 3 bits to upper bounds when $\beta = 1$;
- $R_{11} + R_{21} + R_{22}$ bound is within 3 bits to upper bounds when $\gamma = 1$;
- $R_{12} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds when $\beta = 0$;
- $R_{11} + R_{12} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds;
- $2R_{11} + R_{12} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds when $\beta = 1$;
- $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds when $\gamma = 0$;
- $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound is within 5 bits to upper bounds when $\beta = 0$;
- $R_{11} + R_{12} + R_{21} + 2R_{22}$ bound is within 6 bits to upper bounds when $\gamma = 1$.

Finally, by symmetry, we can relax the above obtained results without loss of generality for R_{21}, $R_{11}+R_{12}+R_{22}$, $R_{11}+R_{21}+R_{22}$, $R_{11}+R_{12}+R_{21}+R_{22}$, $2R_{11}+R_{12}+R_{21}+R_{22}$, $R_{11} + 2R_{12} + R_{21} + R_{22}$, $R_{11} + R_{12} + 2R_{21} + R_{22}$ bounds as follows:

- R_{21} bound is within 2 bits to upper bounds when $\beta = 0$;
- $R_{11} + R_{12} + R_{22}$ bound is within 4 bits to upper bounds when $\beta = 1$;
- $R_{11} + R_{21} + R_{22}$ bound is within 4 bits to upper bounds when $\gamma = 1$;
- $R_{11} + R_{12} + R_{21} + R_{22}$ bound is within 5 bits to upper bounds;
- $2R_{11} + R_{12} + R_{21} + R_{22}$ bound is within 6 bits to upper bounds when $\beta = 1$;
- $R_{11} + 2R_{12} + R_{21} + R_{22}$ bound is within 6 bits to upper bounds when $\gamma = 0$;
- $R_{11} + R_{12} + 2R_{21} + R_{22}$ bound is within 6 bits to upper bounds when $\beta = 0$;

CHAPTER VII

CONCLUSION

In this dissertation, we give an attempt to understand the fundamental limits of the two-user Gaussian X channel with limited receiver cooperation. The better comprehension of these limits leads us to use cooperation in practice efficiently for managing interference in the two-user Gaussian X channel channel. Furthermore, we provide the strategies for communications in this channel for the general case and the strong Gaussian X channel type I case. Three main results of this dissertation can be concluded as follows:

First, as shown in Chapter IV, we give an outer bound based on the Fano's inequality, the data processing inequality and the genie-aided techniques for the two-user Gaussian X channel with limited receiver cooperation. The obtained results present that some of upper bounds contained in our proposed outer bound are identical to the known results in several communication scenarios such as the two-user Gaussian interference channel with/without receiver cooperation [2–4], the two-user Gaussian X channel without receiver cooperation [14, 15] and the two-user Gaussian Z-interference channel with receiver cooperation [31] by setting a certain set of parameters. Furthermore, we show that the region of our proposed outer bound is larger than the region of an outer bound on capacity region of the two-user interference channel with limited receiver cooperation [2] and without cooperation [3, 4]

Second, as shown in Chapter V, the proposed outer bound in Chapter IV is then used to find the GDoF under the symmetric channel setting. The received results show that the GDoF can be improved obviously by increasing a certain amount of information which is exchanged between both receivers. However, it is seen that the system reaches the saturation of the receiver cooperation, i.e., there is no more gain when the normalized capacity of the receiver-cooperative link κ is larger than or equal κ^* where

1. $\kappa^* = \frac{1}{3}$ for $\alpha = \frac{2}{3}$
2. $\kappa^* = \frac{1}{2}$ for $\alpha = \frac{1}{2}, \frac{3}{2}$

3. $\kappa^* = 1$ for $\alpha = 2$

4. $\kappa^* = \frac{3}{2}$ for $\alpha = \frac{5}{2}$

5. $\kappa^* = 2$ for $\alpha = 3$

Finally, as shown in Chapter VI, we propose the strategies for the two-user Gaussian X channel with rate-limited receiver cooperation and give achievable rate regions in both the general case and the strong Gaussian X channel type I case. Our results show that the proposed strategy achieves the capacity region to within 2 bits/s/Hz per message, regardless of channel parameters, for the case of the strong Gaussian X channel type I when parameters β and γ in the common rate constraints at each transmitter are set such that each bound from our proposed strategy except (6.37), (6.43)$-$(6.49), (6.58)$-$(6.60), (6.62)$-$(6.71), (6.73), (6.87)$-$(6.110) reaches its maximum value.

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Appendix

List of Publications

Tan-a-ram, S. and Benjapolakul, W. "Upper Bounds for the Two-User Gaussian X Channel with Limited Receiver Cooperation", submitted to Frequenz (Journal of RF-Engineering and Telecommunications), under review. Content taken from Chapter IV and Chapter V.

Tan-a-ram, S. and Benjapolakul, W. "Achievable Rate Regions for the Two-User Gaussian X Channel with Limited Receiver Cooperation: General Case", submitted to IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, under review. Content taken from Chapter VI.

Biography

Surapol Tan-a-ram received the B.Eng. degree in telecommunication engineering from King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, in 2000 and the M.Eng degree in electrical engineering from Chulalongkorn University, Bangkok, Thailand, in 2002. Since 2003, he has been with National Electronics and Computer Technology Center (NECTEC), National Science and Technology Development Agency (NSTDA), Pathum Thani, Thailand. He has been pursuing the Ph.D. degree in Electrical Engineering at Chulalongkorn University, Bangkok, Thailand, since 2012.

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