CHAPTER I

INTRODUCTION

A quadratic field $\mathbb{Q}(\sqrt{d})$ is a subfield of the complex numbersof the form $\left\{b+c\sqrt{d} \,\middle|\, b,c \in \mathbb{Q}\right\}$, where d is a square-free integer. For each $a = b+c\sqrt{d}$ of $\mathbb{Q}(\sqrt{d})$, $N(a) = a^2-c^2d$ and $T(a) = 2b$ are called the norm and trace of a. If both N(a) and T(a) are in $\mathbb{Z}$ we say that a is an integer in $\mathbb{Q}(\sqrt{d})$. The integers in $\mathbb{Z}$ will be referred to as rational integers, We denote the set of integers of $\mathbb{Q}(\sqrt{d})$ by I_d. It is well known that I_d is an integral domain. For certain values of d, I_d is a unique factorization domain. However, there are many values of d for which I_d is not a unique factorization domain. The purpose of our study is to find a method to obtain all possible factorizations of rational integers into productsof irreducible integers.

Chapter II providesnecessary background in algebra and number theory. Chapter III provides results which are used as tools of the method of factorization. Examples are also provided to illustrate our method.