CHAPTER IV
RESULTS AND DISCUSSION

4.1 Determination the Flow Regimes

In Figure 4.1, the superficial water velocities are plotted against the superficial air velocities for only bubble and slug flow because the research was directed especially to the slug flow regime. An Increase in the superficial air velocity, while the superficial water velocity was being held constant changed the flow pattern if the air velocity was adequate. A bubble to slug transition occurred over a small range of values, but not at a single point as well as a transition point predicted by Nicklin's model, Eqn. (4) with a void fraction equaled 0.1. Experiments of Nicklin indicate that if a void fraction equals 0.1 is somewhat arbitrarily taken for the bubble to slug transition, little significant error will arise in pressure-gradient and void fraction calculations. However the transition covered the point predicted by Nicklin's model.

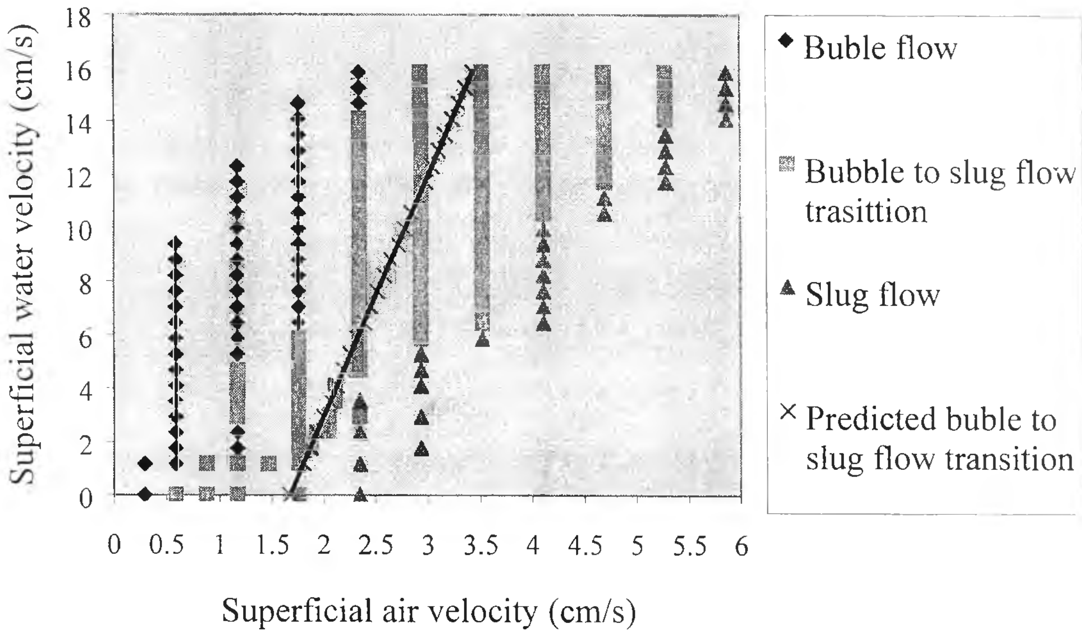

Figure 4.1 Flow pattern map generated from a 19 mm inside diameter tube.

4.2 Determination the Rise Velocities of a Single Slug (u_b) and Their Slug Length

Three different size single slugs of thirty one slugs that I studied in the range of 3.00 to 42.14 cm slug length, which were tracked by a camcorder are shown in Figure 4.2. The length of slug was read from each picture by Adobe Photoshop program and slug velocities were measured.

The results are shown in Figure 4.3 in which the rising velocities are plotted as a function of the slug length for water at room temperature. It was evident that the rise velocities of all slugs were very close to those predicted by Eqn. (6), ($u_b=c(gD)^{1/2}$, c=0.35). They were independent of slug length.

In Figure 4.4, values of c in Eqn. (6) from an experiment are plotted as a function of slug length. The average value of c was 0.33, which was very close to the theoretical value (c=0.35).

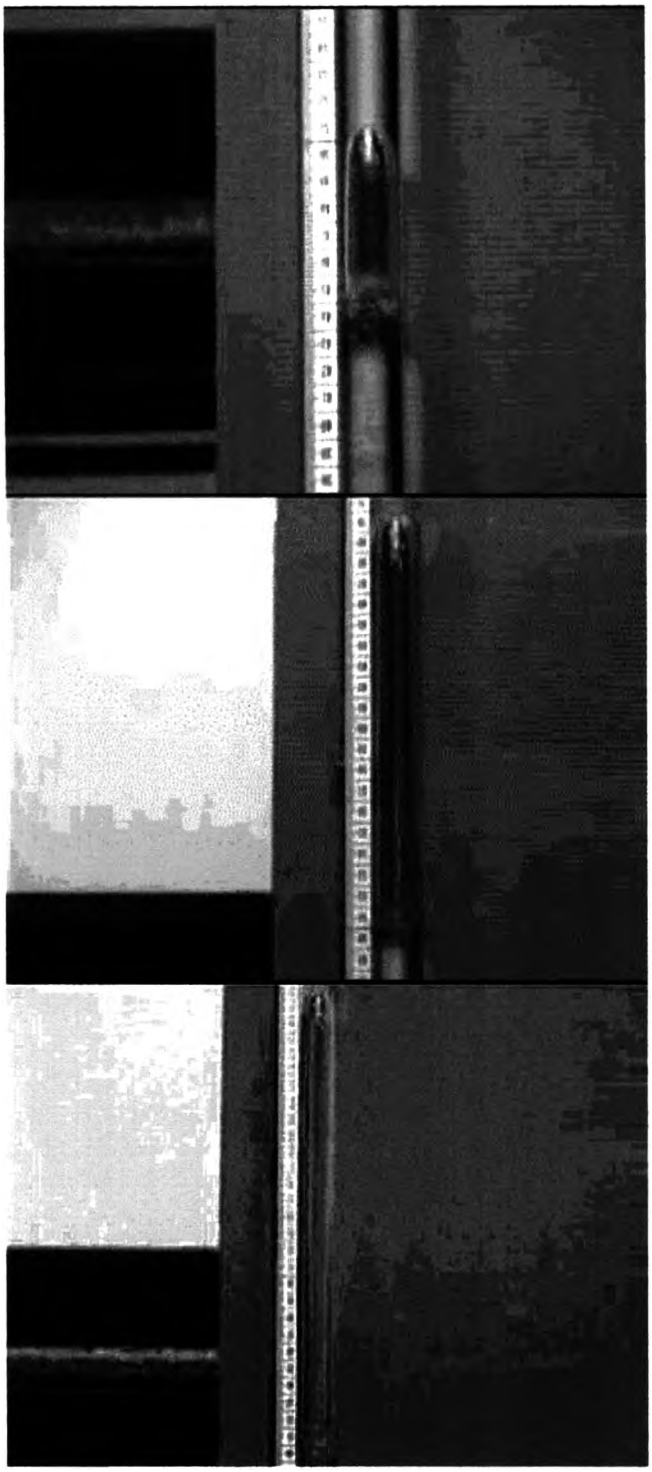

Figure 4.2 Single Slugs with 6.25, 19.73 and 26.88 cm slug length, respectively.

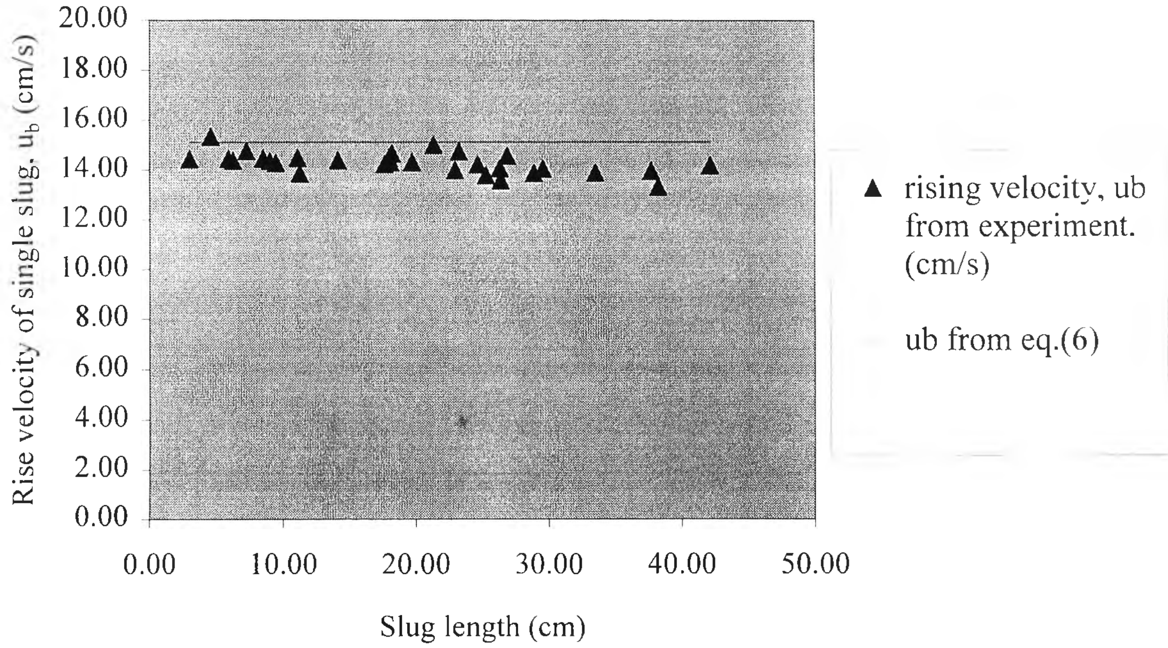

Figure 4.3 Relationship between the rise velocity of single slugs and slug length.

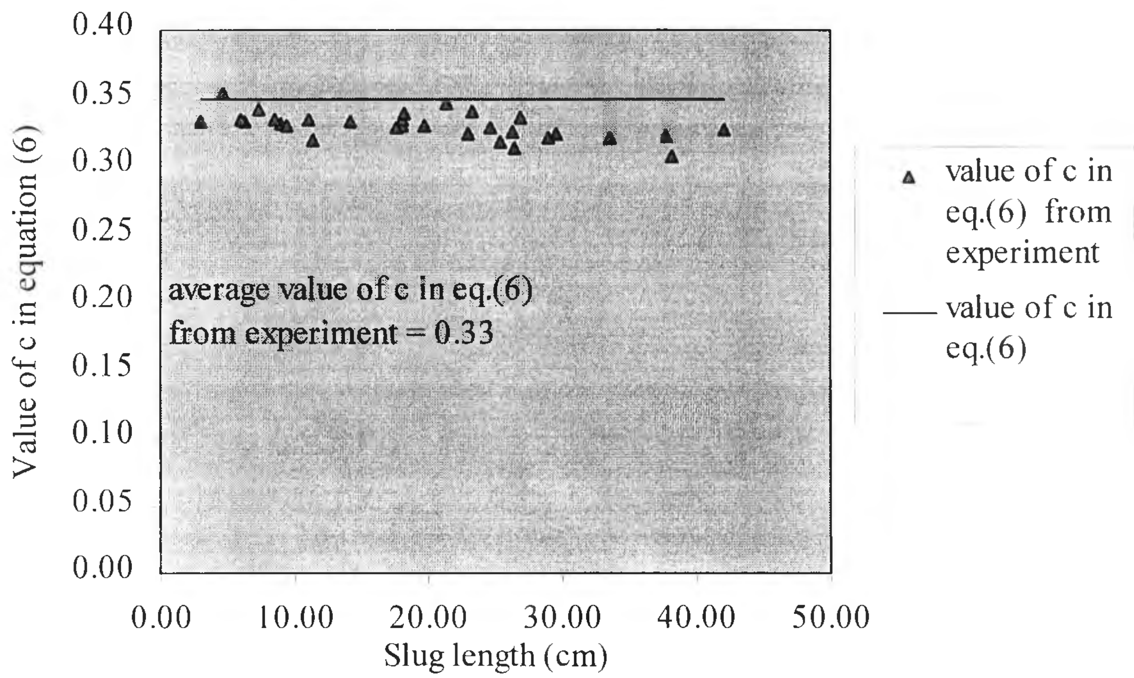

Figure 4.4 Experimental values of c in Eqn. (6) versus slug length.

4.3 Determination Void Fractions at a Variety of Air and Water Flow Rate Within Slug Flow

In Figures 4.5-4.11, the void fractions are plotted as a function of the superficial air velocities for constant superficial water velocities and an inside diameter. These data cover the superficial air and water velocities in the range of 2.93 to 70.42 cm/s and 0 to 14.70 cm/s, respectively. Increasing the superficial air velocity while the keeping a superficial water velocity and an inside diameter of tube constant made void fractions increase. The void fractions from experiments were close to those predicted by equation (9) and the trend was the same one predicted by the model. This model was satisfactory for predicting void fractions within an error 15%.

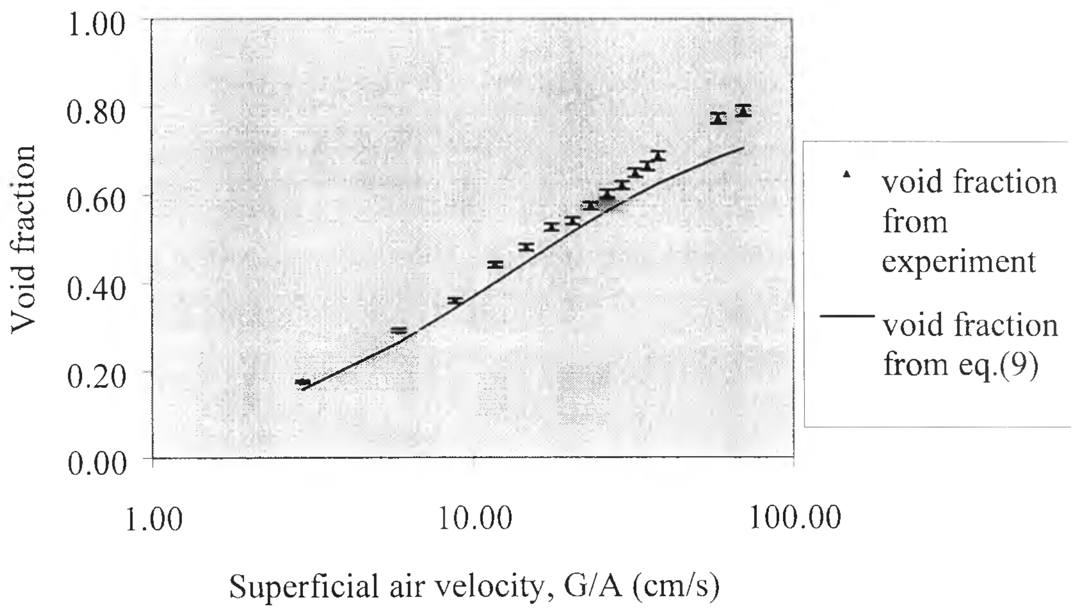

Figure 4.5 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at no net water flow rate.

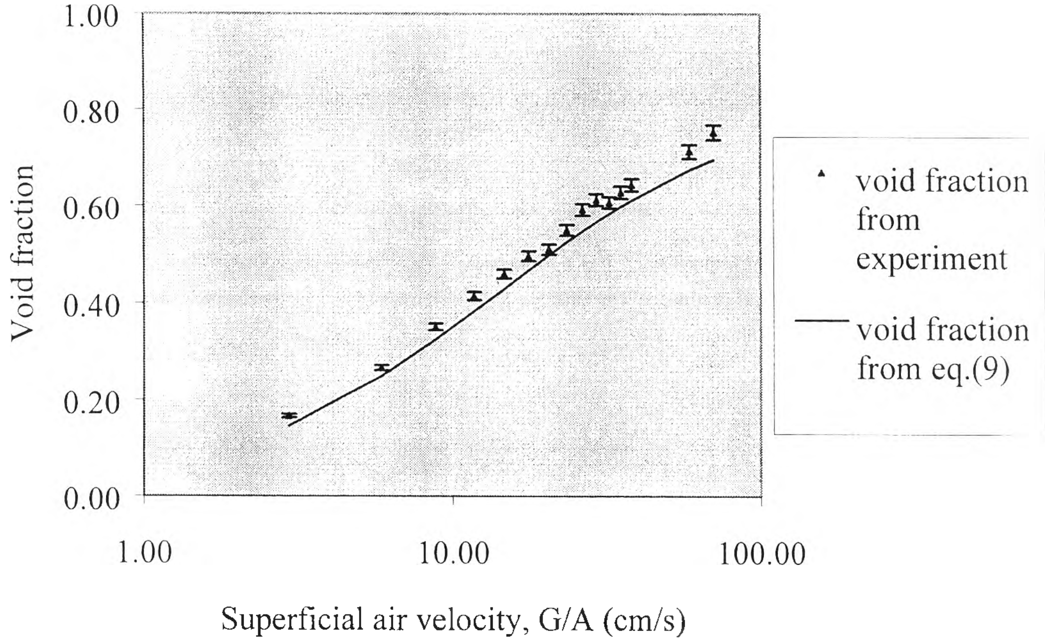

Figure 4.6 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at 200 ml/min (1.17 cm/s) water flow rate.

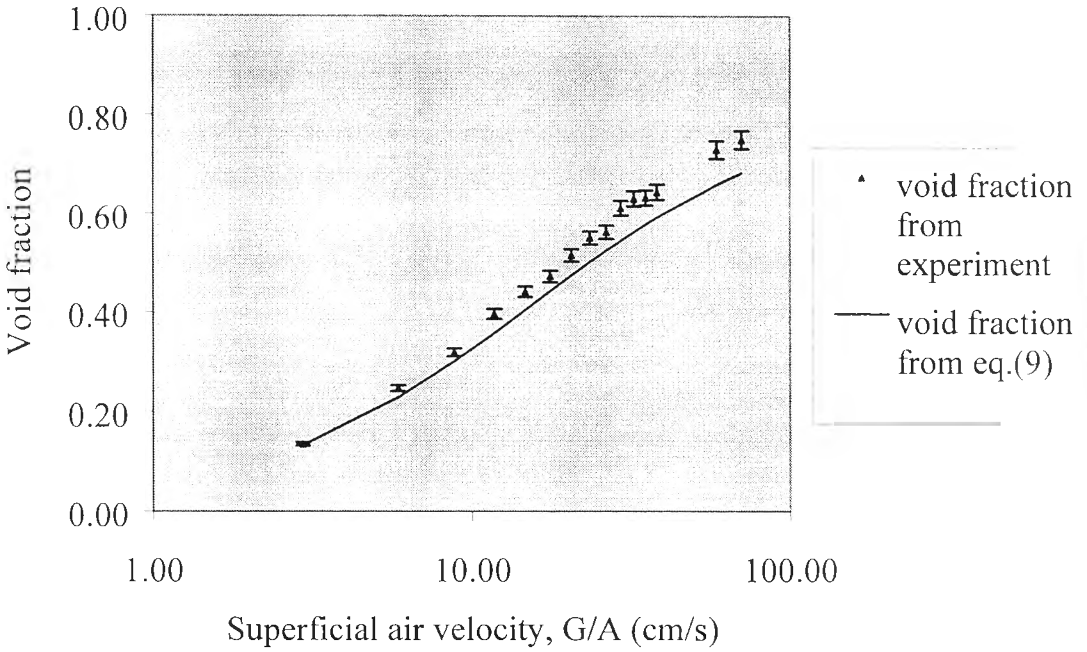

Figure 4.7 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at 500 ml/min (2.93 cm/s) water flow rate.

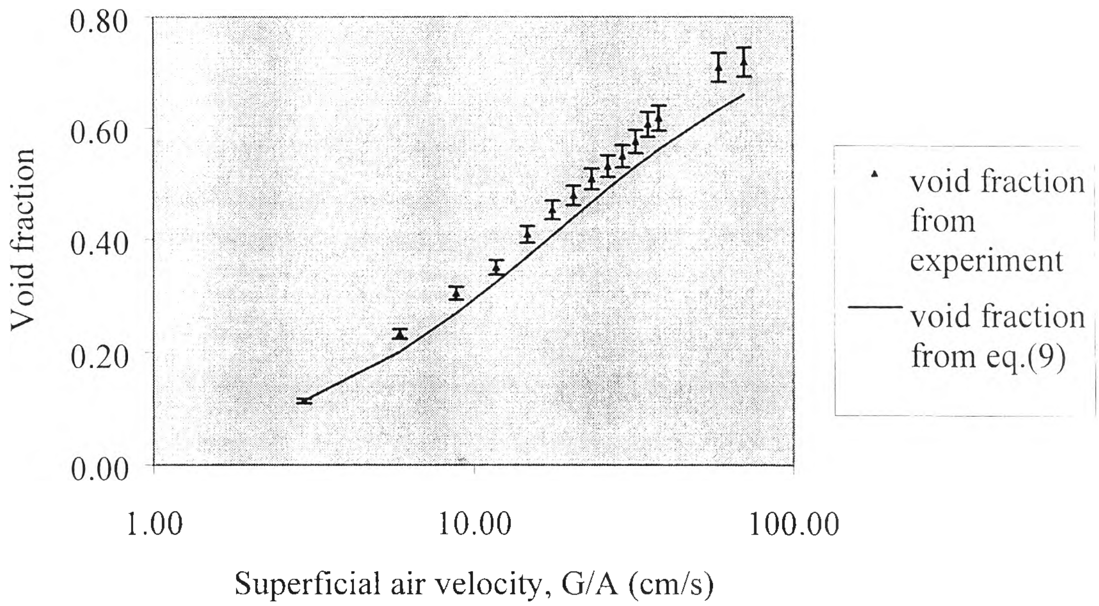

Figure 4.8 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at 1000 ml/min (5.88 cm/s) water flow rate.

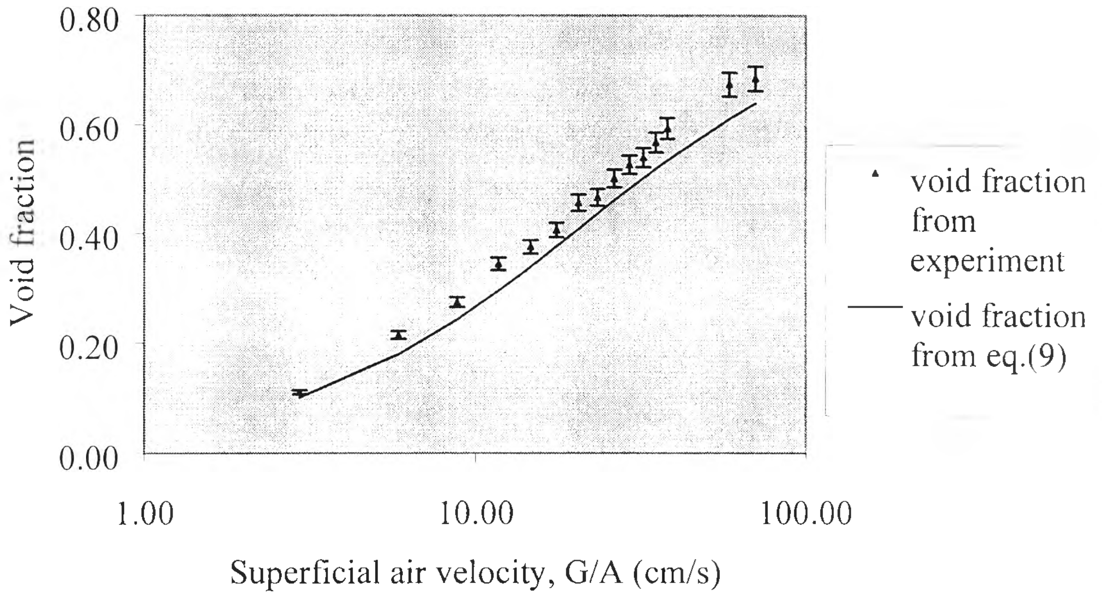

Figure 4.9 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at 1500 ml/min (8.80 cm/s) water flow rate.

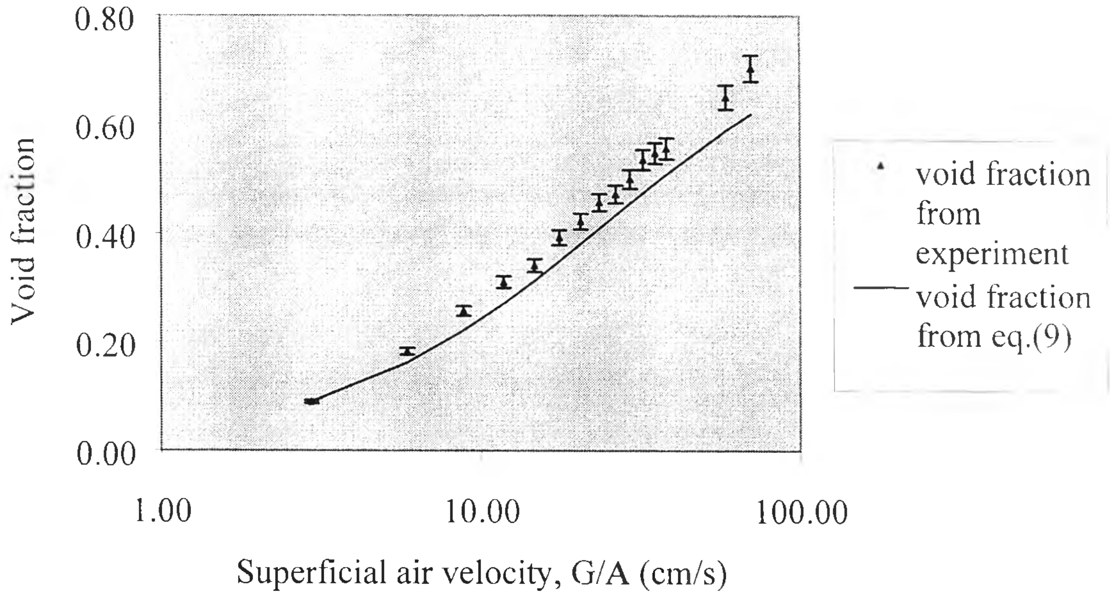

Figure 4.10 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at 2000 ml/min (11.74 cm/s) water flow rate.

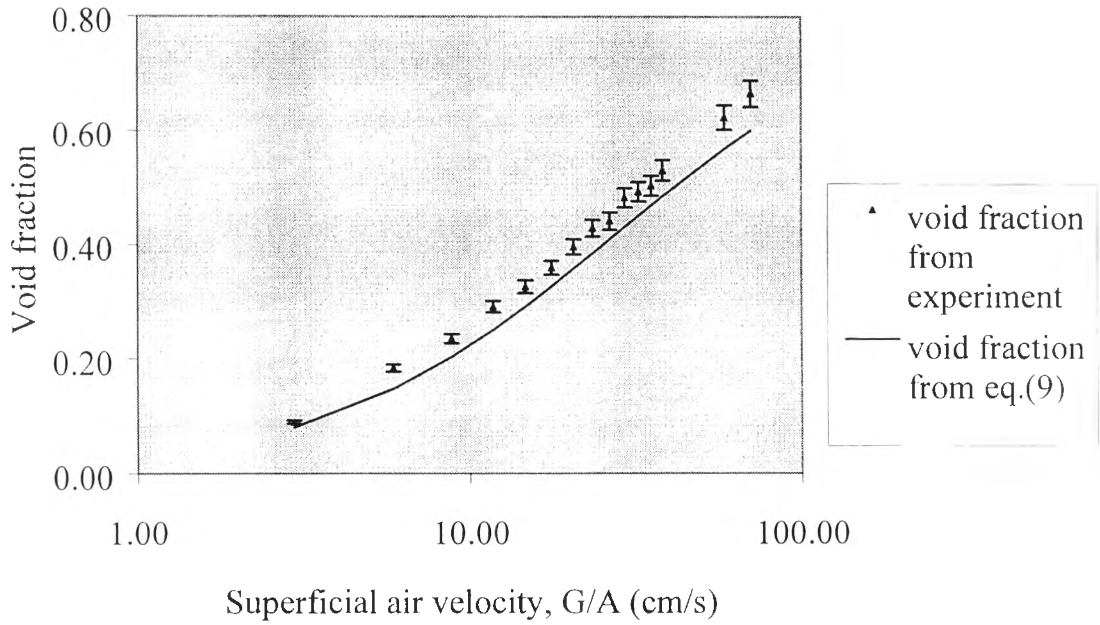

Figure 4.11 Comparison of void fraction calculated from Eqn. (9) with that determined experimentally with a 19 mm inside diameter tube at 2500 ml/min (14.70 cm/s) water flow rate.

4.4 Determination Rise Velocities of Continuously Generated Slugs (u_s)

In Figures 4.12-4.18, the rise velocities of continuously generated slugs are plotted as a function of the superficial air velocities at a variety of water flow rates as same as flow rates of void fraction experiments. Increasing the superficial air velocity for a variety of water flow rates linearly increased the rise velocity of slugs. The data of this experiment showed excellent agreement (within ±7%) with those predicted by Eqn. (7) for various air and water flow rates.

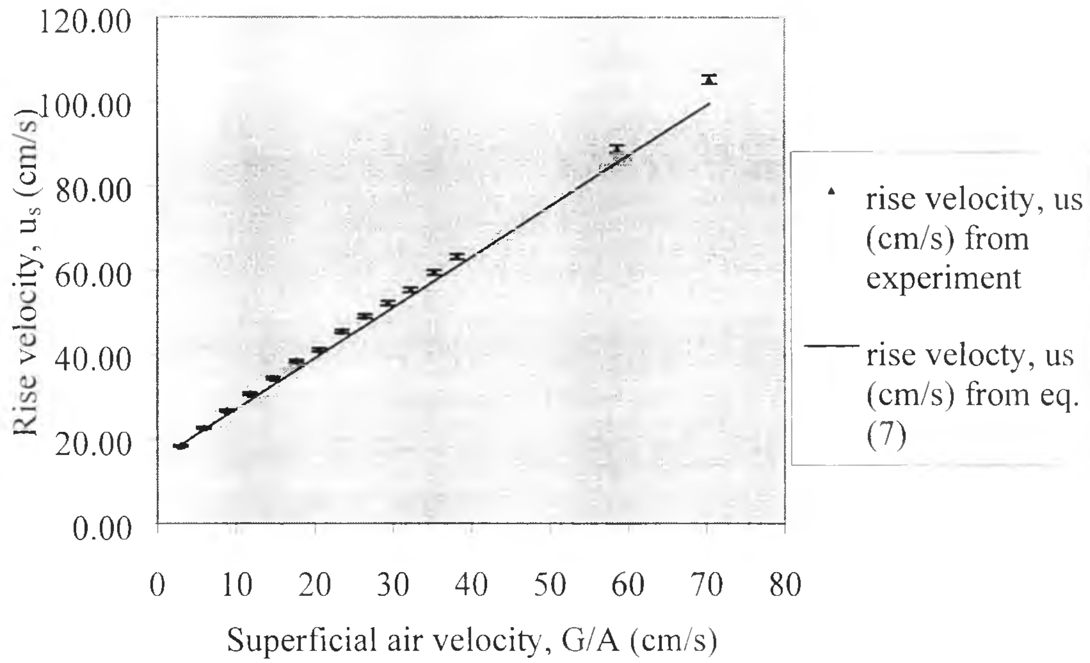

Figure 4.12 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at no net water flow rate.

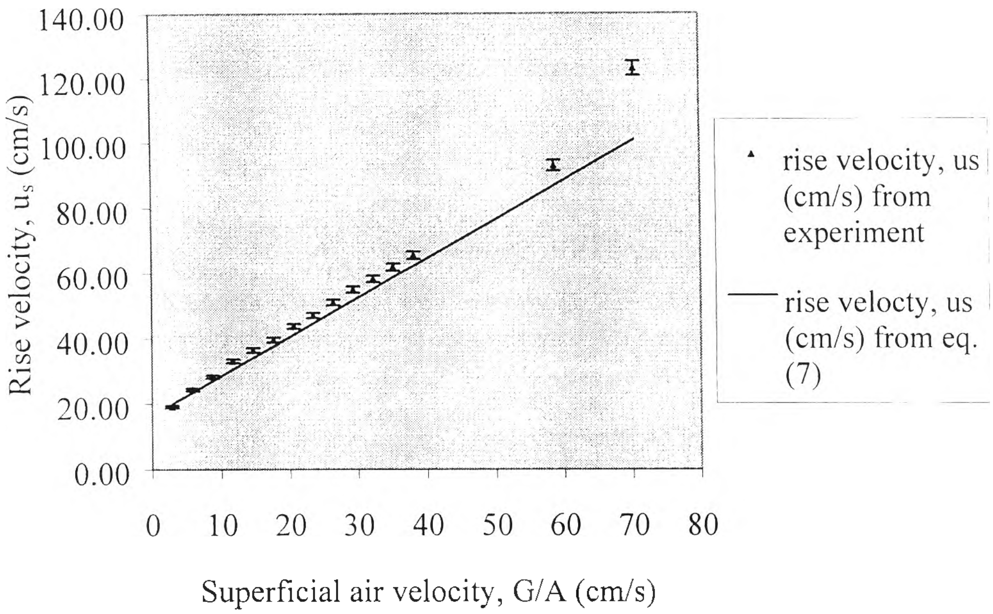

Figure 4.13 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at 200 ml/min (1.17 cm/s) water flow rate.

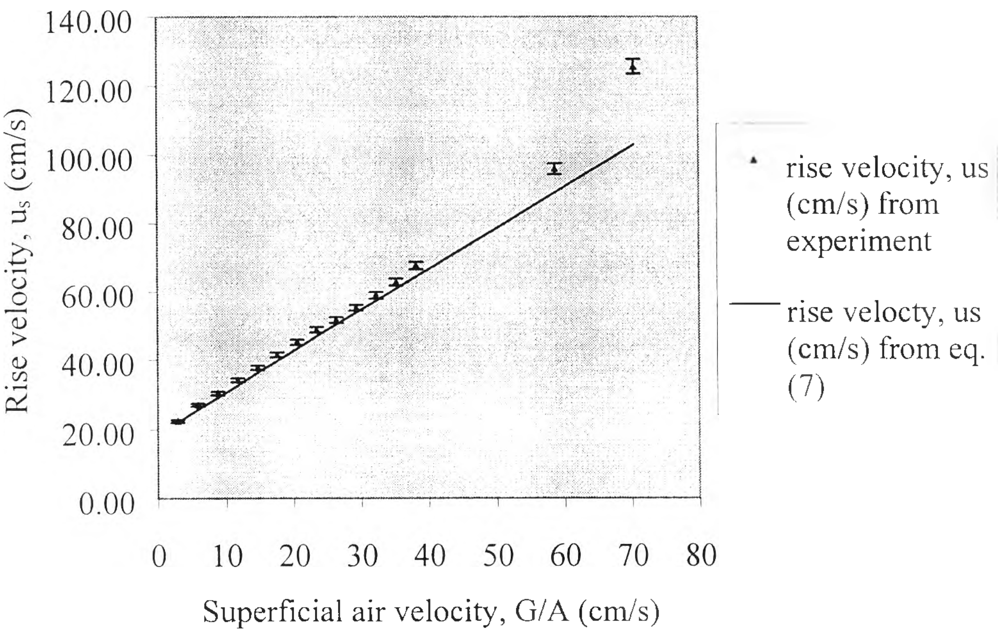

Figure 4.14 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at 500 ml/min (2.93 cm/s) water flow rate.

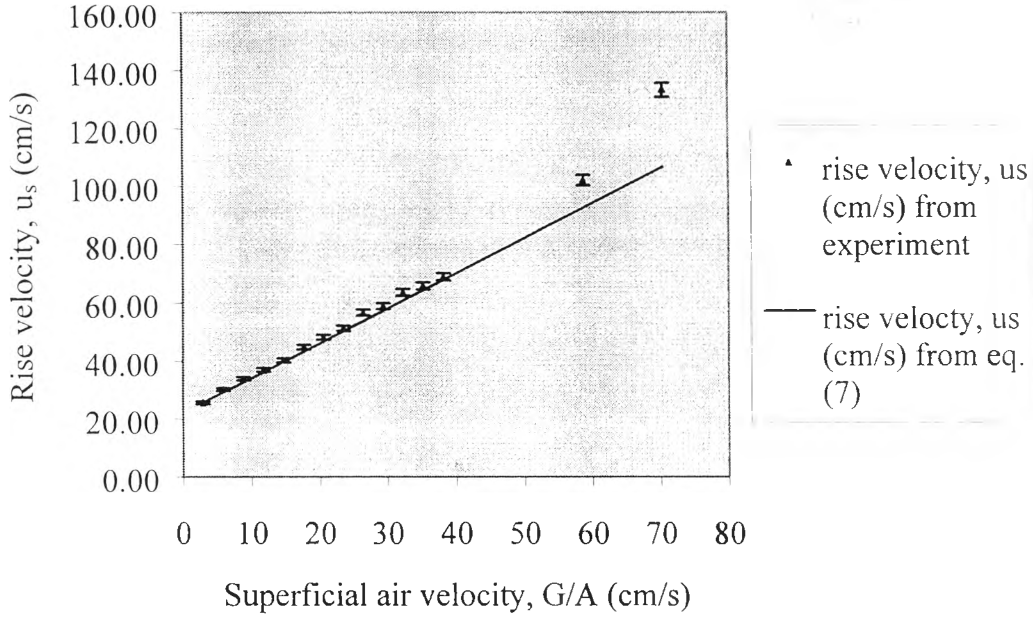

Figure 4.15 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at 1000 ml/min (5.88 cm/s) water flow rate.

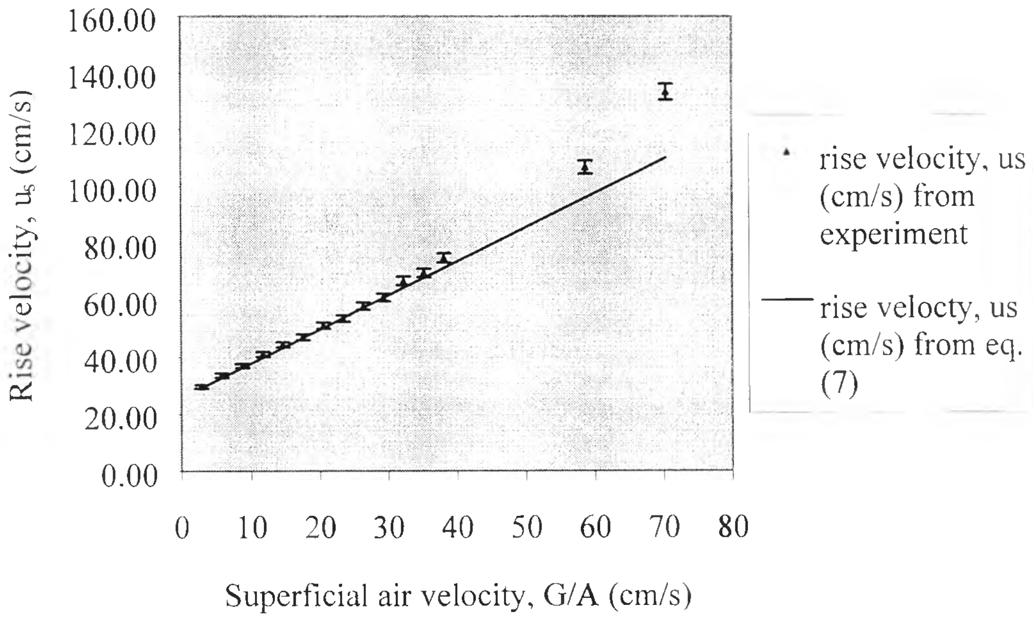

Figure 4.16 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at 1500 ml/min (8.80 cm/s) water flow rate.

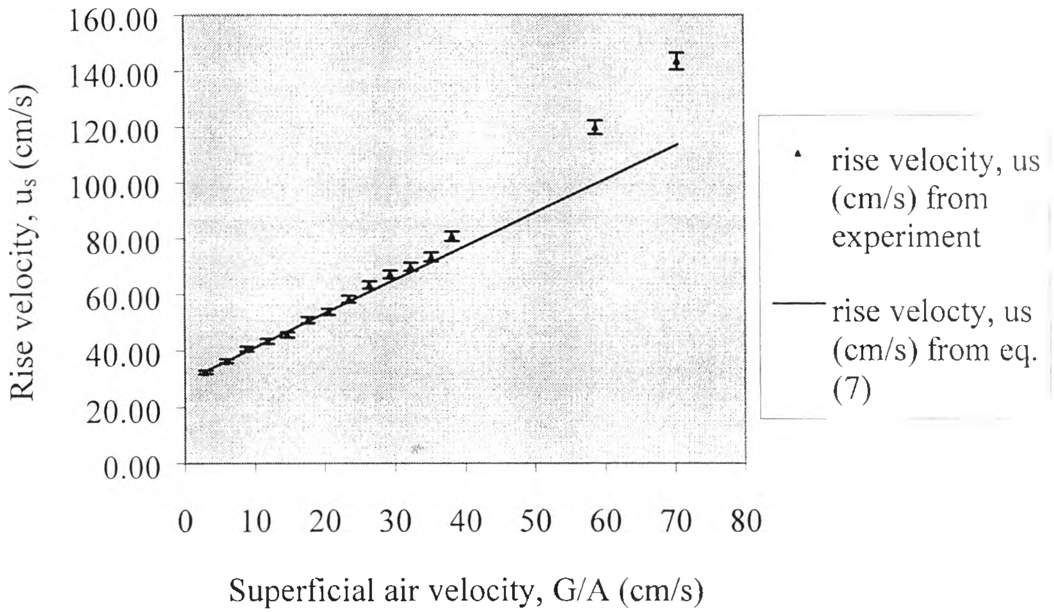

Figure 4.17 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at 2000 ml/min (11.74 cm/s) water flow rate.

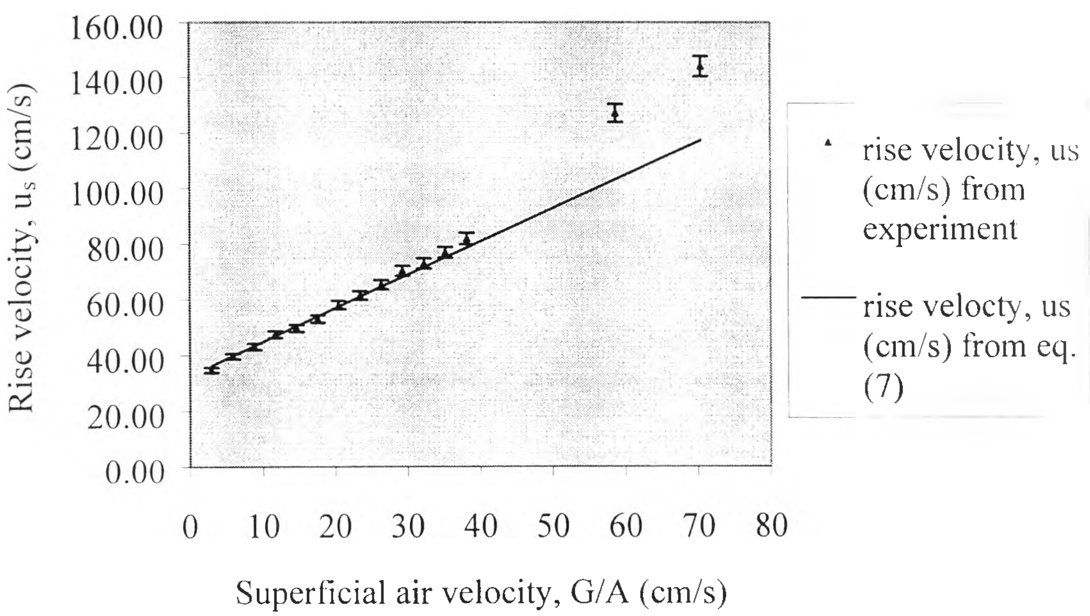

Figure 4.18 Comparison of rise velocity of continuously generated slugs from Eqn. (7) with that determined experimentally at 2500 ml/min (14.70 cm/s) water flow rate.

4.5 Determination Air- Lift Pump Operation

In Figure 4.19, the superficial air velocities for incipient air-lift operation are plotted as a function of initial height of water in the main and reservoir columns. Decreasing the initial height of water in the main and reservoir columns increased the required a superficial air velocity for incipient air-lift pump operation. Average data were within ±23.17% of values predicted by Eqn. (12) and the spread of individual values was within –56.12 to 1.44%. It was acceptable within ±30% error. The predicted values for higher heights of water in the columns or lower required superficial air velocities for incipient air-lift pump operation showed better agreement with the data from an experiment.

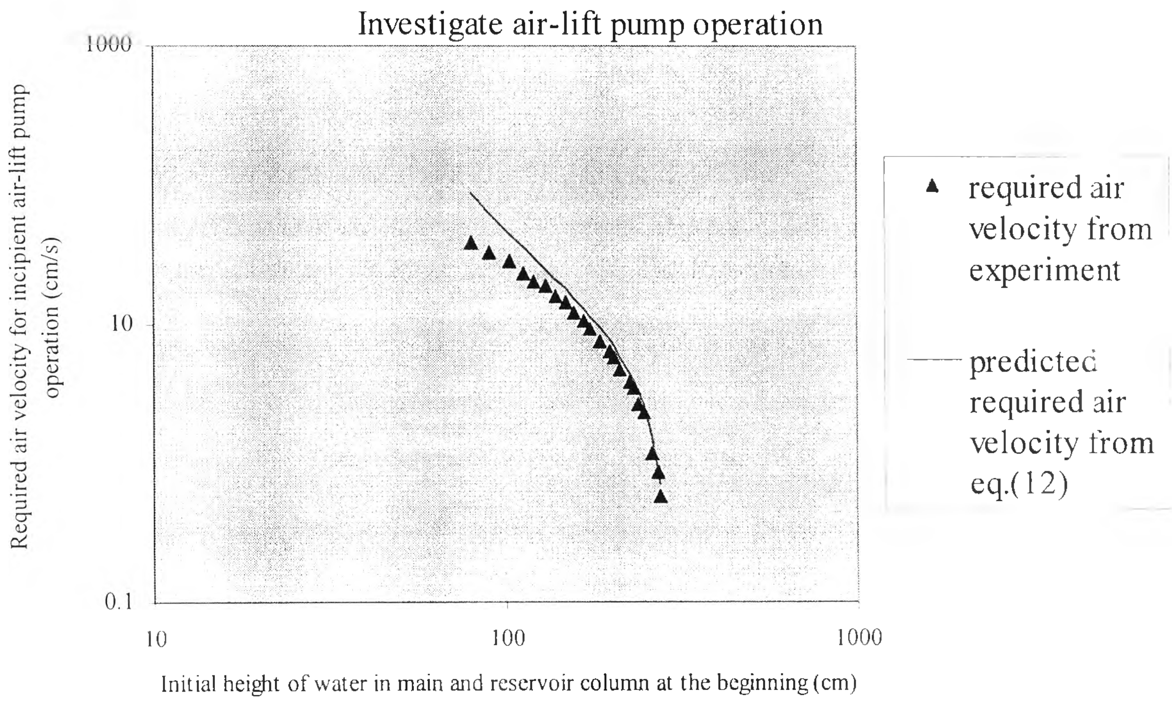

Figure 4.19 Investigation of the air-lift pump operation.

4.6 Determine u_b from Potential-Flow Theory

This development paralleled that of Davies *et al.* (1950) for determining the rise velocity U_b of gas slug or bubble ascending in an otherwise stagnant inviscid liquid in a vertical tube of radius a. By Superimposing a downwards velocity U_b on the system, the analysis was performed for liquid streaming downwards past a stationary bubble, as shown in Figure 4.20.

The following relationships may be assumed for the velocities potential function, and stream function in the coordinate system shown, with symmetry about the z-axis:

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial z} \tag{13}$$

$$v_z = \frac{\partial \phi}{\partial z} = -\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{14}$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{15}$$

$$\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = 0 \tag{16}$$

The following forms are assumed for the potential and stream function:

$$\phi = U_b z - Ae^{kz/a} f(r) \tag{17}$$

$$\psi = -\frac{1}{2}U_b r^2 + Are^{kz/a} g(r) \tag{18}$$

Substitute (17) in (15),

Also note that:

$$\frac{\partial \phi}{\partial r} = -Ae^{kz/a} f'(r) \tag{19}$$

$$\frac{\partial^2 \phi}{\partial r^2} = -Ae^{kz/a} f''(r) \tag{20}$$

$$\frac{\partial \phi}{\partial z} = U_b - A\left(\frac{k}{a}\right)e^{kz/a}f(r) \tag{21}$$

$$\frac{\partial \phi}{\partial z} = -A\left(\frac{k}{a}\right)^2 e^{kz/a}f(r) \tag{22}$$

Thus Eqn. (15) becomes

$$-Ae^{kz/a}f''(r) - \frac{1}{r}Ae^{kz/a}f'(r) - A\left(\frac{k}{a}\right)^2 e^{kz/a}f(r) = 0 \tag{23}$$

Deviding both sides by $-Ae^{kz/a}$

$$f''(r) + \frac{1}{r}f'(r) + \left(\frac{k}{a}\right)^2 f(r) = 0 \tag{24}$$

and multiplying both sides r^2

$$r^2 f''(r) + rf'(r) + \left(\frac{kr}{a}\right)^2 f(r) = 0 \tag{25}$$

Also note that for Bessel's equation:

$$x^2 f''(x) + xf'(x) + \left(\lambda x^2 - n^2\right)f(x) = 0 \tag{26}$$

$$\lambda = \alpha^2 \tag{27}$$

The solution is:

$$f(x) = J_n(\alpha x) \tag{28}$$

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(1+n+k)}\left(\frac{x}{2}\right)^{2k+n} \tag{29}$$

And the following relations will be needed:

$$\frac{d}{dx}J_0(x) = -J_1(x) \tag{30}$$

$$\frac{d}{dx}[xJ_1(x)] = xJ_0(x) \tag{31}$$

$$J_0(0) = 1 \tag{32}$$

$$J_1(0) = 0 \tag{33}$$

Therefore Eqn. (25) becomes

$$r^2 f''(r) + rf'(r) + \left(\left(\frac{k}{a}\right)^2 r^2 - 0\right) f(r) = 0 \tag{34}$$

So

$$\lambda = \frac{k^2}{a^2} = \alpha^2 \tag{35}$$

$$\alpha = \frac{k}{a} \tag{36}$$

$$n = 0 \tag{37}$$

Therefore the solution of Eqn. (25) is:

$$f(r) = J_0\left(\frac{k}{a} r\right) \tag{38}$$

In a similar way, substitute (18) in (16):

$$r^2 g''(r) + rg'(r) + \left(\left(\frac{k}{a}\right)^2 r^2 - 1\right) g(r) = 0 \tag{39}$$

The solution of Eqn. (39) is:

$$g(r) = J_1\left(\frac{k}{a} r\right) \tag{40}$$

Therefore, the potential and stream functions are:

$$\phi = U_b z - Ae^{kz/a} J_0\left(\frac{k}{a} r\right) \tag{41}$$

$$\psi = -\frac{1}{2} U_b r^2 + Are^{kz/a} J_1\left(\frac{k}{a} r\right) \tag{42}$$

Therefore, Eqn. (13) becomes:

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial z} = A\left(\frac{k}{a}\right) e^{kz/a} J_1\left(\frac{k}{a} r\right) \tag{43}$$

and Eqn. (14) becomes:

$$v_z = \frac{\partial \phi}{\partial z} = -\frac{1}{r} \frac{\partial \psi}{\partial r} = U_b - A\left(\frac{k}{a}\right) e^{kz/a} J_0\left(\frac{k}{a} r\right) \tag{44}$$

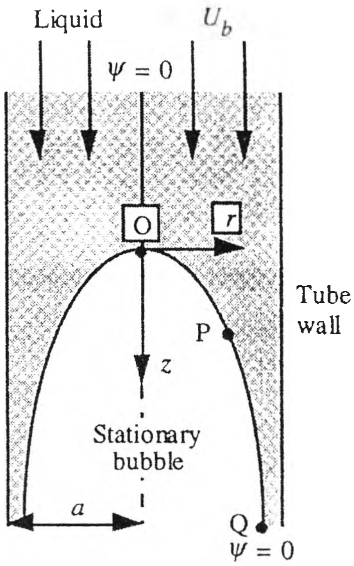

Figure 4.20 Gas slug in a tube.

Far above the nose of the bubble (z is large negative.), check at r =0:

$$v_r = A\left(\frac{k}{a}\right)e^{kz/a}J_1(0) = 0 \tag{45}$$

$$v_z = U_b - A\left(\frac{k}{a}\right)e^{kz/a}J_0(0) = U_b - A\left(\frac{k}{a}\right)e^{kz/a} \tag{46}$$

Because z is large negative, so that $e^{kz/a} \approx 0$.

$$v_z = U_b \tag{47}$$

And

$$v = \sqrt{v_z^2 + v_r^2} = v_z \tag{48}$$

$$v = U_b \tag{49}$$

It shows that the velocity is uniformly U_b downwards at far above the nose of the bubble. Since there is no radial velocity component at the tube wall (r=a).

$$v_r = A\left(\frac{k}{a}\right)e^{kz/a}J_1\left(\frac{k}{a}r\right) = 0 \tag{50}$$

$$A\left(\frac{k}{a}\right)e^{kz/a}J_1(k) = 0 \tag{51}$$

$$A \neq 0 \tag{52}$$

$$k \neq 0 \tag{53}$$

$$\therefore J_1(k) = 0 \tag{54}$$

The lowest root of that equation is k=3.832.

Along r=0, the centerline above the nose of the bubble at the origin O:

$$\psi = -\frac{1}{2}U_b r^2 + Are^{kz/a} J_1\left(\frac{k}{a}r\right) \tag{55}$$

$$\psi = 0 \tag{56}$$

At the wall (r=a) is also streamline, whose value of ψ corresponds to a flow rate through the tube of $\pi a^2 U_b$. The stream function at a point is ψ if the flow rate in the negative z direction, through a circle formed by rotating the point about the axis of symmetry, is $2\pi\psi$.

$$-\pi a^2 U_b = 2\pi\psi \tag{57}$$

$$\psi = -\frac{1}{2}U_b a^2 \tag{58}$$

At the stagnant point O ,z=0, r=0

$$v_r = A\left(\frac{k}{a}\right)e^{kz/a} J_1\left(\frac{k}{a}r\right) = A\left(\frac{k}{a}\right)J_1(0) = 0 \tag{59}$$

$$v_z = U_b - A\left(\frac{k}{a}\right)e^{kz/a} J_0\left(\frac{k}{a}r\right) = U_b - A\left(\frac{k}{a}\right)J_0(0) = U_b - A\left(\frac{k}{a}\right) \tag{60}$$

The requirement that v_z is zero gives

$$v_z = U_b - A\left(\frac{k}{a}\right) = 0 \tag{61}$$

$$A = \frac{aU_b}{k} \tag{62}$$

Note that because of the gas in the bubble, the pressure is uniform along OPQ, so that the Bernoulli condition along the bubble surface is:

$$v_r^2 + v_z^2 = 2gz \tag{63}$$

With only a single term in the approximation for ϕ, this condition can only be observed at a single point, which we shall take at r= a/2. By considering the streamline ψ=0, and assuming that $J_1(3.832/2)=0.580$ and also assuming that $J_0(3.832/2)=0.273$.

Along the streamline ψ=0:

$$\psi = -\frac{1}{2}U_b r^2 + Are^{kz/a}J_1\left(\frac{k}{a}r\right) \quad (64)$$

$$0 = -\frac{1}{2}U_b\frac{a^2}{4} + \frac{aU_b}{k}\left(\frac{a}{2}\right)e^{kz/a}J_1\left(\frac{k}{2}\right) \quad (65)$$

$$0 = -\frac{1}{8}U_b a^2 + \frac{a^2U_b}{2k}e^{kz/a}J_1\left(\frac{k}{2}\right) \quad (66)$$

$$\therefore \frac{1}{8}U_b a^2 = \frac{a^2U_b}{2k}e^{kz/a}J_1\left(\frac{k}{2}\right) \quad (67)$$

$$\frac{1}{8}U_b a^2 = \frac{a^2U_b}{2(3.832)}e^{3.832z/a}J_1\left(\frac{3.832}{2}\right) \quad (68)$$

$$\frac{1}{8}U_b a^2 = \frac{a^2U_b}{7.664}e^{3.832z/a}(0.580) \quad (69)$$

$$e^{3.832z/a} = 1.651 \quad (70)$$

$$3.832z/a = 0.501 \quad (71)$$

$$z/a = 0.131 \quad (72)$$

From Eqn. (63)

$$v_r^2 + v_z^2 = 2gz \quad (63)$$

$$\left[A\left(\frac{k}{a}\right)e^{kz/a}J_1\left(\frac{k}{a}r\right)\right]^2 + \left[U_b - A\left(\frac{k}{a}\right)e^{kz/a}J_0\left(\frac{k}{a}r\right)\right]^2 = 2gz \quad (73)$$

$$\left[\frac{aU_b}{k}\left(\frac{k}{a}\right)e^{kz/a}J_1\left(\frac{k}{2}\right)\right]^2 + \left[U_b - \frac{aU_b}{k}\left(\frac{k}{a}\right)e^{kz/a}J_0\left(\frac{k}{2}\right)\right]^2 = 2gz \quad (74)$$

$$\left[U_b e^{3.832(0.131)}J_1\left(\frac{3.832}{2}\right)\right]^2 + \left[U_b - U_b e^{3.832(0.131)}J_0\left(\frac{3.832}{2}\right)\right]^2 = 2gz \quad (75)$$

$$[U_b(1.652)(0.580)]^2 + [U_b - U_b(1.652)(0.273)]^2 = 2gz \quad (76)$$

$$0.918U_b^2 + 0.301U_b^2 = 2gz \quad (77)$$

$$1.219U_b^2 = 2gz \tag{78}$$

$$U_b^2 = 1.641\,gz \tag{79}$$

$$z = 0.131a \tag{80}$$

$$U_b^2 = 0.215\,ga \tag{81}$$

$$U_b = 0.464\sqrt{ga} \tag{82}$$

and a =D/2

$$U_b = 0.464\sqrt{gD / 2} \tag{83}$$

$$U_b = 0.328\sqrt{gD} \tag{84}$$

$$\therefore U_b = c\sqrt{gD} \tag{85}$$

$$c \approx 0.33 \tag{86}$$

$$U_b = 0.33\sqrt{gD} \tag{87}$$