Chapter 4

Simulation of Superconductor-Insulator Transition in Two-Dimensional Disordered Josephson Junction Arrays

In this chapter we will discuss the disorder effect on the Josephson junction arrays. For a disordered system, it is of necessity to perform a numerical calculation.

A model that we use is the 2D XY model, neglecting the charging energy effect. We check our program with the XY model with the clean system studied

numerically by Gawiec and Grempel (Gawiec and Grempel, 1991). In Section 4.2 we describe the procedure of finding the quasiground state and how to get the order parameter which signifies the phase transition, as we tune the impurity concentration to a certain critical value.

4.1 Limit of Neglecting Charging Effect

Since the charging energy associated with each grain is inversely proportional to the grain size, when the grain size of the superconductor is large, the charging energy is so small that it is enough to take into account only the Josephson coupling energy term in our Hamiltonian, that is,

$$H = -\sum_{\langle ij \rangle} (E_J)_{ij} \cos(\phi_i - \phi_j) \tag{4.1}$$

where $(E_J)_{ij}$ is the Josephson coupling energy between site i and site j, ϕ_i is the phase associated with each grain size, $\sum_{\langle ij \rangle}$ denotes the sum over nearest neighbors ij, and $(E_J)_{ij}$ take a random value for any pair ij. The disorder in our system comes from the randomness of $(E_J)_{ij}$. Eq.(4.1) can be mapped into the spin-glass XY model if we think of ϕ_i as the angle between the planar spin $\mathbf{S}_i$, $|\mathbf{S}_i| = 1$, at the x-axis, so that

$$H = -\sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \tag{4.2}$$

where for simplicity we have let $(E_J)_{ij}$ equal to J_{ij}. As we mentioned in the Chapter 2, the presence of disorder can be interpreted as the existence the impu-

rity in the lattice. For a finite concentration x of impurities, we use the x value to indicate the probability associated with the value of J_{ij} by

$$J_{ij} = \begin{cases} J & \text{with probability } 1-x, \\ -J & \text{with probability } x. \end{cases} \tag{4.3}$$

4.2 Model Approach

We want to find the configuration which give ground states energy. The configurations that minimize the energy (4.2) obey the equations

$$\mathbf{S}_i = \frac{\mathbf{H}_i}{|\mathbf{H}_i|} \tag{4.4}$$

$$\mathbf{H}_i = \sum_j J_{ij}\mathbf{S}_j. \tag{4.5}$$

These equations express that, in equilibrium, each spin must lie in the direction of the internal magnetic field at its site. To solve eq.(4.4) and eq.(4.5), we start from some initial random configurations and sequentially rotates the spins into the direction of their local field. Since, as a result of each move, the local fields themselves change, the procedure is repeated a large number of times until the value of H converges.

From the fact that eq.(4.4) and eq.(4.5) are necessary but not sufficient conditions for a minimum energy configuration the iterative method described above may not be an efficient way of locating ground states. That method can only take us from an initial configuration to a stationary one along a path composed of a sequence of straight segments along each of which the orientation of

one spin varies while the rest are held fixed. This path is far from being the optimal one for our problem.

We illustrate the procedure with data from a 20×20 system with $x = 0.2$. Fig. 4.1 shows the plot of energy versus magnetization. It is found that by starting from 100 different random initial conditions, each of them evolves into a different minimum, thereby exhibiting Fig. 4.1 exhibits the existence of sets of states that lie very close in energy but have widely different magnetizations. This reflects the fact that important morphological differences may exist between states that are essentially degenerate. It is clear that, to appropiately describe the $T = 0$ properties of the model, we need to average physical quantities over those sets of quasidegenerate states whose energies lie near the (unknown) absolute minimum. However, as illustrated in Fig. 4.1, the energies of most of the configurations that one can reach by random generation lie far above the ground state and, to have enough low-energy data, one would need to generate huge numbers of them.

Gawiece and Grempel (Gawiece and Grempel, 1991) have found a more efficient way to reach the low-energy part of the spectum based on the morphological properties of the stationary states. Fig. 4.2 shows the configuration that corresponds to the lowest points in Fig. 4.1 at $E = -1.5747J$.

When we use the spin-glass XY model to deal with the magnetic material, thus the structure is characterized by the existence of ferromagnetically ordered domains in regions that are either relatively free of impurities or where impurities are isolated, surrounded by other regions where the density of frus-

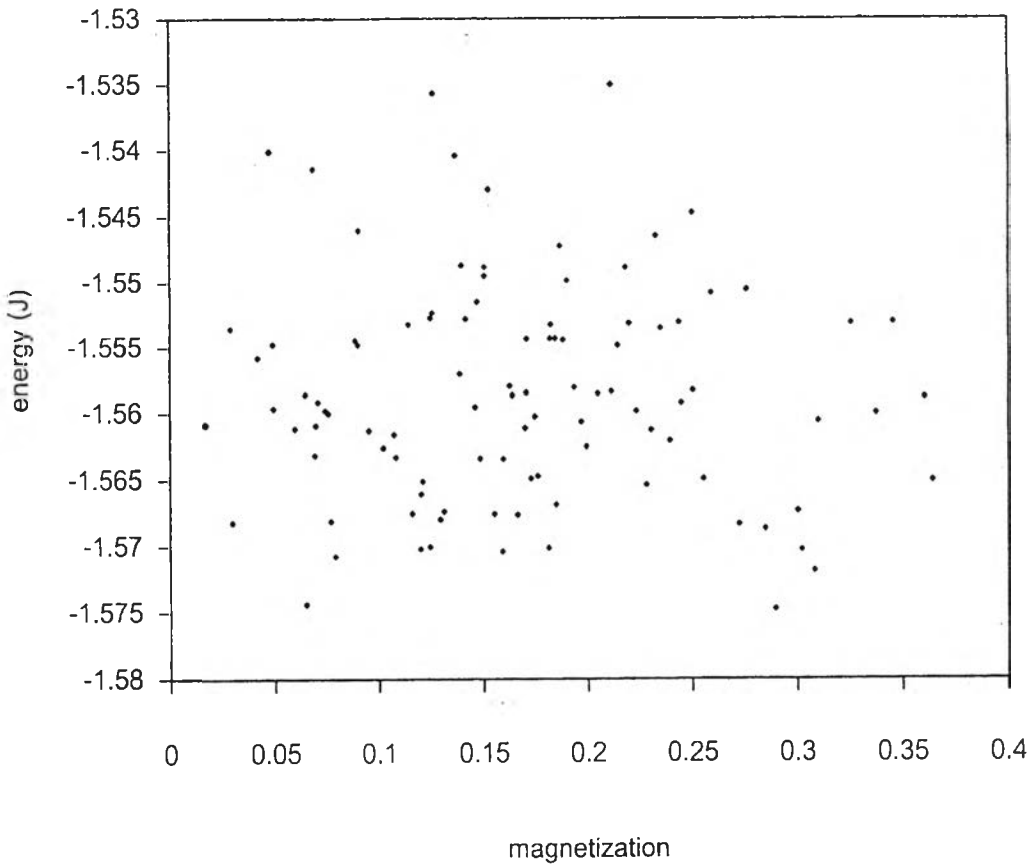

Figure 4.1: The energies of 100 stationary states as a function of their magnetization. These states were obtained by starting the minimization of the energy from 50 different random initial conditions. Data for a 20×20 system with $x = 0.2$.

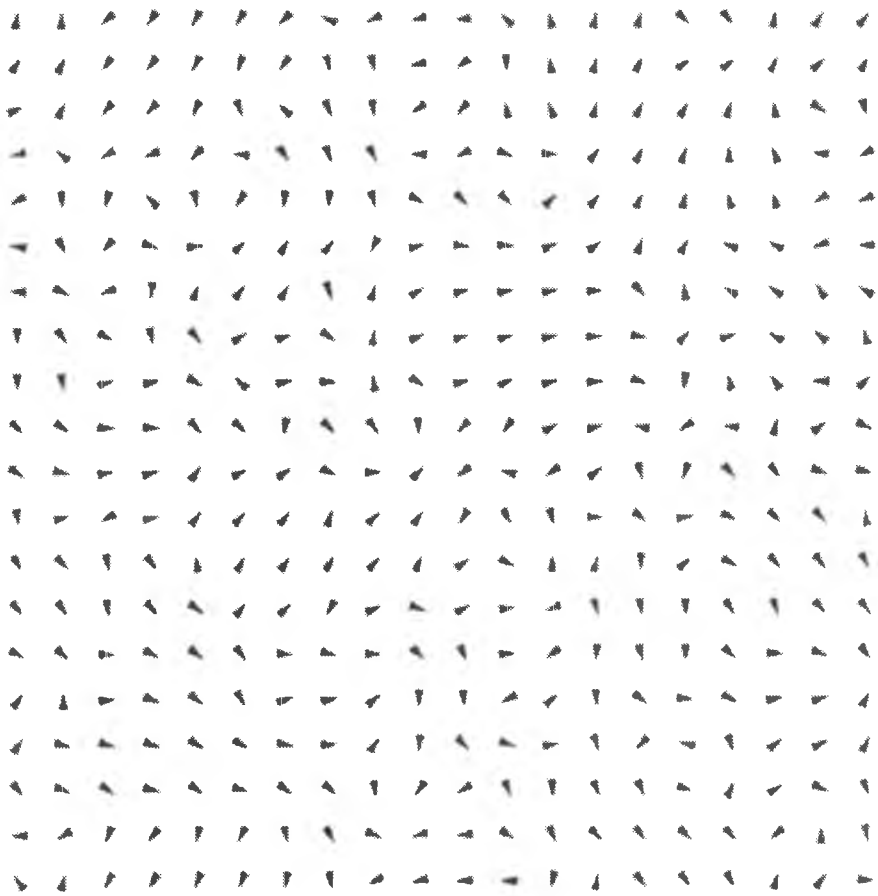

Figure 4.2: Spin configuration for the state of lowest energy in Fig. 4.1.

tated plaquettes is high. We can deform this configuration, denoted henceforth as the parent configuation, by rotating independently but rigidly the domains and giving arbitrary angles to the spins that belong to the boundaries between domains or, more generally, to a highly frustrated environment. If this configuration is used as the starting point of a new series of minimizations we may expect to generate states with energies lower than that of the parent state. The reason is that, by preparing the new initial state in this manner, the environment of those spins that were already in a high local field stays unchanged (except for a global rotation) whereas the spins that were in a weak local field have a chance to increase it.

We implemented this idea in practice by using the value of the internal field H acting on a spin as a criterion to decide whether and by how much the spin should be rotated. We first choose a value of local field H_L and set a threshold H^*. We then reinitialize the spins of the parent configurations, assign new but arbitrary angles to those spins whose local field H_L is less than a threshold H^* and impose small random deviations with respect to their previous equilibrium configuration to all the others. Then we let the system find a minimum, starting from this new initial condition. The procedure is repeated for different values of H^*, each of which leads to a new minimum. Fig. 4.3 shows the energies of the sequence of stationary states thus obtained as a function of H^*. Each point in the figure represents the energy of the stationary state reached by minimization of energy in which the starting configuration is derived from that of Fig. 4.2 by

randomly initializing all the spins whose molecular field is less than H^*.

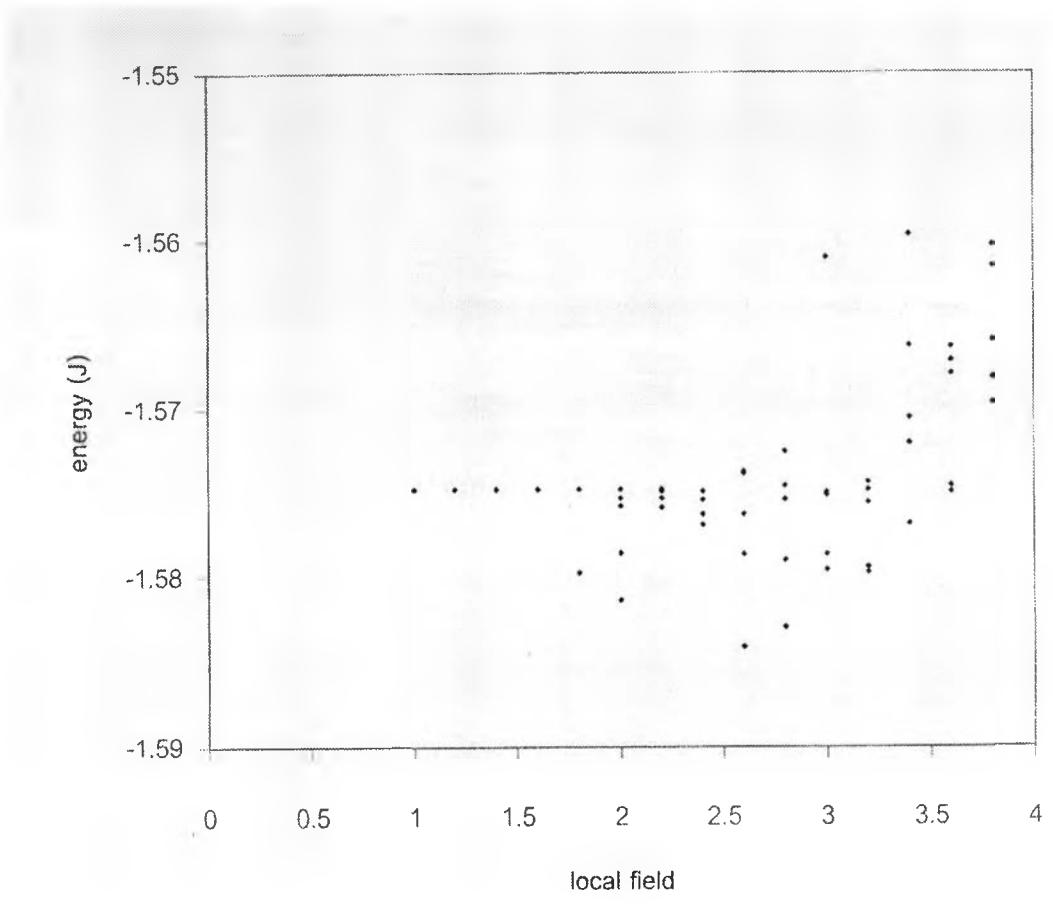

Figure 4.3: Energy as a function of threshold field H^*.

Notice that, as expected, most of the new states have an energy lower than that of the parent state. The spin distribution of the state at $E = -1.5879J$, the lowest of the series, is shown in Fig. 4.4.

Comparison between Fig. 4.2 and Fig. 4.4 shows that the domain structures of the parent and of the daughter states are basically the same but that the two configurations differ by large-scale collective rotations of the spins in the domains. It would have been extremely hard to generate one of these states starting

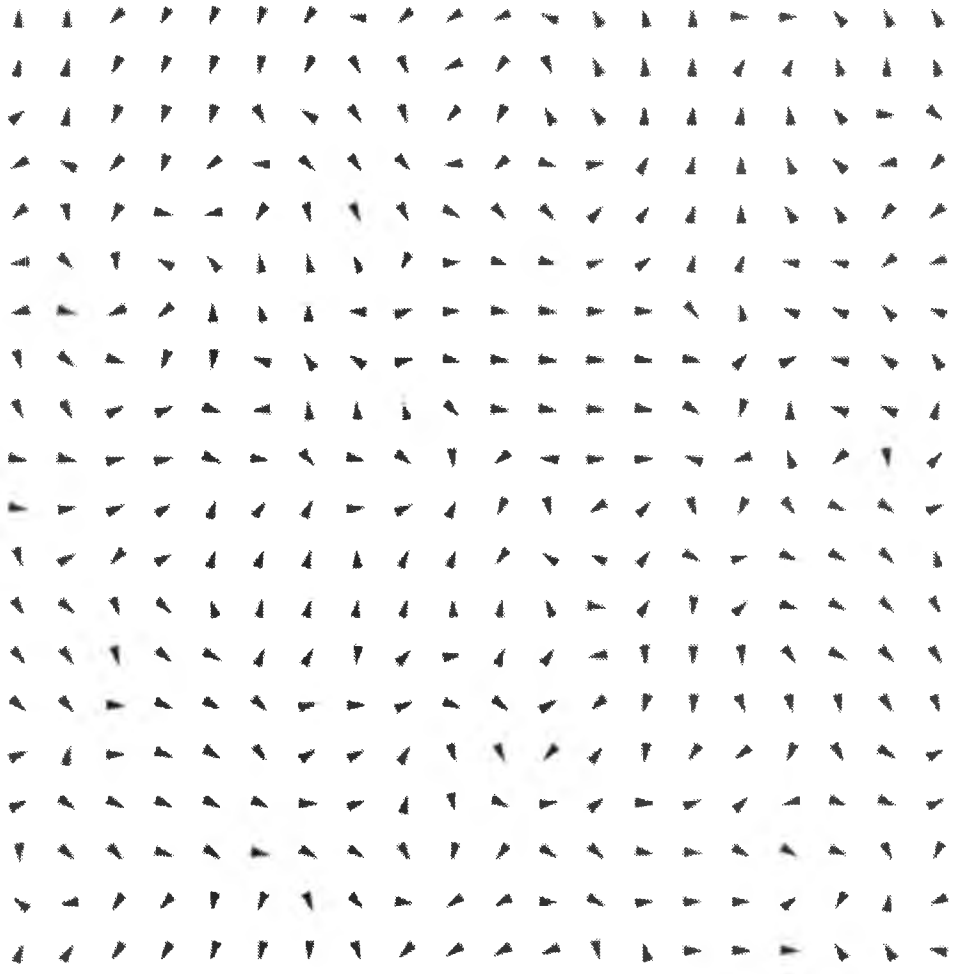

Figure 4.4: Spin configuration for the state of lowest energy in Fig. 4.3.

from the other algorithm.

To find states with even lower energy, we may use this configuration as a parent and generate a new set of minima by repeating the steps just described, and so on. This generates an arborescent structure that stops when we reach the point where every new state produced has an energy higher than that of its parent. At this point we estimate that we have reached the ground state.

To take degeneracy into account, we compute the observable quantities as appropriate statistical averages over sets of states that lie near the ground state. The latter has been found by going backward and collecting the information about all the states in the hierarchy whose energy. In our work we get the ground states by choosing the states which have the energy within a band of width $\Delta E \approx 10^{-5}$ J. This width corresponds to about ten times our uncertainty in the energies per spin.

We have applied the method in the aforementioned to study several systems corresponding to a wide range of concentrations, x. In the following we describe in details the results obtained for the 20×20 size. All along we have used periodic boundary conditions. The points in the curves represent averages over the sets of quasidegenerate states and five random configurations of bonds. We present the results for the order parameter, which we have used the spontaneous magnetization as an order parameter, that is

$$M = \left[\left\langle \frac{1}{N} \left| \sum_{i=1}^{N} \mathbf{S}_i \right| \right\rangle \right]_{dis} . \tag{4.6}$$

In this expression the angular brackets stand for an average over the set of quasidegenerate ground states, and the rectangular ones stand for an average over the set of J_{ij} which have the same value x

The finite but nonzero value of M in this spin glass corresponds to the existance of the ferromagnetic state, but our system was mapped from eq.(4.1), thus the existance of ferromagnetic state implies the existence of the superconducting phase of the two-dimensional disordered Josephson junction arrays. Therefore, we can look for the phase transition by looking through the value of M.

We can see from Fig. 4.5 that M begins to break down at x=0.1. This means that at this value of impurity, the superconductivity of Josephson junction arrays was destroyed.

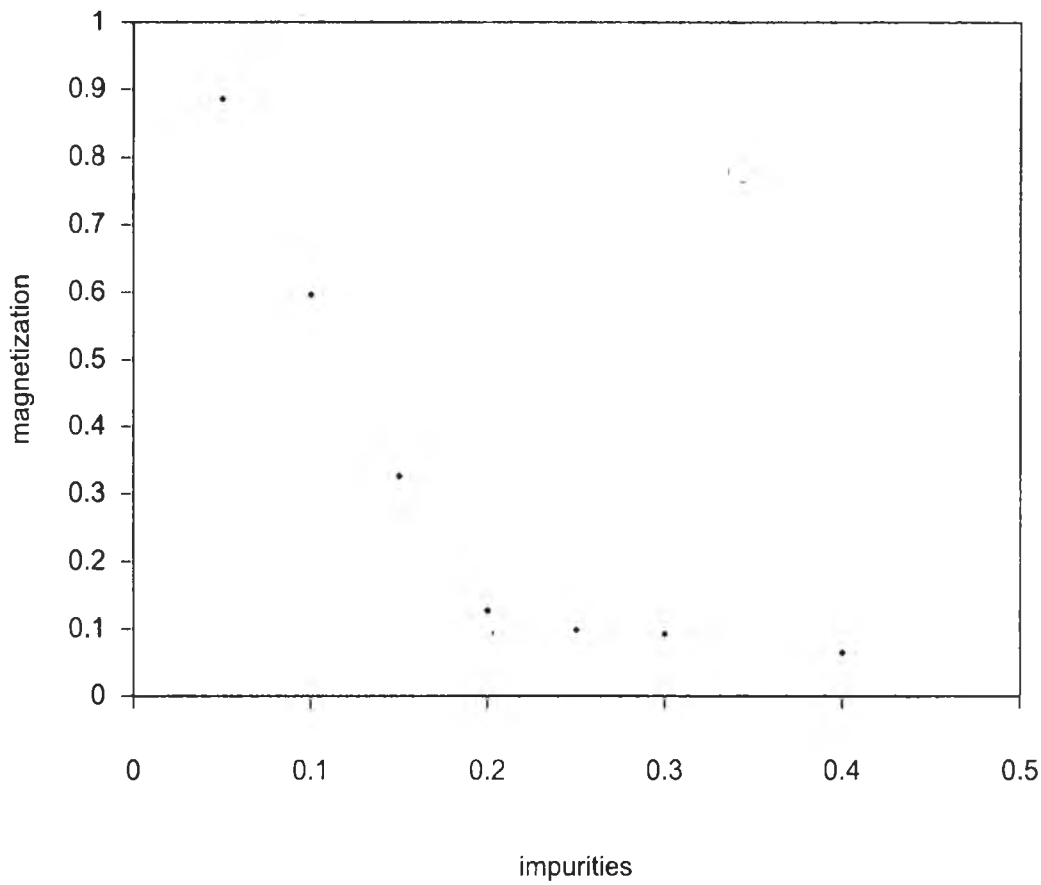

Figure 4.5: The magnetization per spin M versus the concentration of impurities x.