CHAPTER 4

LINEARIZED STATE SPACE MODEL OF POWER SYSTEMS WITH HVDC LINKS

4.1 Introduction to State Space Analysis

In time domain, the system representation of differential equations can be difficult to analyze. The mathematics get more burdensome as the order of the equations increases. The combination of several differential equations into one single system could be quite difficult.

State space form provides a convenient time-domain representation that is useful for insight analysis. Furthermore, state variable descriptions need not assume zero initial conditions, and hence allowing the analysis and design of system characteristics that are not possible with frequency domain representations [16].

State equations are simply collections of first order differential equations that together represent exactly the same information as the original higher-order differential equations. Nevertheless, the set of variables used to write these n first-order equations is not unique. The state variables are normally chosen for convenience in the analysis, as one set of the state variables may result in mathematical expressions that make the solution or other characteristics of the system more apparent.

The collection of state variables at any given time is known as the *state* of the system, and the set of all values that can be taken on by the state is known as the *state space*. The state of the system represents complete information of the system such that if the state at time t_0 is known, it is possible to compute the state at all future times.

The behaviors of a dynamic system, such as a power system, may be described by a set of n first-order nonlinear ordinary differential equations of the following form [2] :

$$\dot{x} = f_i(x_1, x_2, ..., x_n; u_1, u_2, ..., u_r; t) \tag{4.1}$$

where n is the order of the system and r is the number of inputs. This can be written in the following concise form using vector matrix notation:

$$\dot{x} = f(x, u, t) \tag{4.2}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_r \end{bmatrix}, \qquad f = \begin{bmatrix} f_1 \\ f_2 \\ \dots \\ f_n \end{bmatrix}$$

The column vector $\boldsymbol{x}$ is referred to as the *state vector*, and its entries x_i as *state variables*. The column vector u is the input vector to the system. These are the external signals that influence the performance of the system. Time is denoted by t, and the derivative of a state variable x with respect to time is denoted by $\dot{x}$. If the derivatives of the state variables are not explicit functions of time, we can simplify equation (4.2) to

$$\dot{x} = f(x, u) \tag{4.3}$$

We are often interested in output variables which can be observed on the actual system. They can be expressed in terms of the state variables and the input variables in the following form:

$$y = g(x, u) \tag{4.4}$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_m \end{bmatrix} \qquad g = \begin{bmatrix} g_1 \\ g_2 \\ \dots \\ g_m \end{bmatrix}$$

The column vector y is the vector of output variables, and g is a vector of nonlinear functions relating state and input variables to the output variables.

4.1.1 Linearization

If equations (4.3) and (4.4) are linearized around the given operating states and inputs (x_0, y_0), then the linearized state and output equations can be written as follows :

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{4.5}$$

$$\Delta y = C\Delta x + D\Delta u \tag{4.6}$$

where

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{\partial f_n}{\partial x_1} & & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\partial f_1}{u} & \dots & \frac{\partial f_1}{\partial u_r} \\ \dots & \dots & \dots \\ \frac{\partial f_n}{\partial u_1} & & \frac{\partial f_n}{\partial u_r} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \dots & \frac{\partial g_1}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{\partial g_m}{\partial x_1} & & \frac{\partial g_m}{\partial x_n} \end{bmatrix} \qquad D = \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & \dots & \frac{\partial g_1}{\partial u_r} \\ \dots & \dots & \dots \\ \frac{\partial g_m}{\partial u_1} & & \frac{\partial g_m}{\partial u_r} \end{bmatrix} \tag{4.7}$$

where

Δx is the vector of state deviation from x_0

Δy is the vector of output deviation from y_0

Δu is the vector of input deviation from u_0

A is the state or plant matrix of size nxn

B is the control or input matrix of size nxr

C is the output matrix of size mxn

D is the feedforward matrix of size mxr

Modern control theory which is based on state space concepts is extremely useful not only for designing a specific optimal control system, but also for improving the principles on which the system will operate. By using the state space approach the control engineer will be able to design the systems with performance characteristics that cannot be achieved by the classical approach, such as the frequency response method or the root locus method [17].

A modern complex system may have many inputs and outputs, and they can be interrelated in a complicated manner. To analyze such a system, it is essential to reduce the complexity of the mathematical expressions. For small-signal analysis, the linearized state space approach to non-linear systems is best suited from this viewpoint.

While conventional control theory is based on the input-output relationship, or transfer function, modern control theory is based on the description of system equations in terms of n first order differential equations. The use of vector matrix notation greatly simplifies the mathematical representation of system equations. The increase in the number of state variables, the number of inputs, or the number of outputs does not significantly increase the complexity of the equations.

4.1.2 Advantages of State Space Analysis

Some of the advantages of the state space approach are as follows [18]:

1. It provides a convenient, compact notation, and allows the application of the powerful vector matrix theory.
2. The uniform notation for all systems, regardless of order, makes possible a uniform set of solution techniques and computer algorithm.
3. Able to define and explain more completely many system characteristics and attributes.

After the state space representation of the system is obtained, we can see the eigenvalues of the system by inspection of matrix A, which reveal the stability of system. Also we can check controllability and observability of the system to see the possibility of applying proper controller in the system.

4.1.3 Eigenvalue and Stability

The time dependent characteristic of a mode corresponding to an eigenvalues λ_i is given by an exponential term of $e^{\lambda_i t}$. Therefore, the stability of the system can be determined by the eigenvalues as follows :

a. *Real eigenvalues* correspond to a non-oscillatory mode. A negative real eigenvalue represent a decaying mode; the larger its magnitude, the faster the dacay. A positive real eigenvalue represent aperiodic instability.
b. *Complex eigenvalues* occur in conjugate pairs. Each pair corresponds to an oscillatory mode.

The real components of the eigenvalues give the damping, and the imaginary components the frequency of oscillation. A negative real part represents a damped oscillation whereas a positive real part represents oscillation of increasing amplitude. Thus, for a complex pair of eigenvalues $\lambda = \sigma \pm j\omega$,

the frequency of oscillation in Hz is given by :

$$f = \frac{\omega}{2\pi} \tag{4.8}$$

In addition, the damping ratio can be computed as :

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}} \tag{4.9}$$

The damping ratio ζ determines the rate of decay of the amplitude of the oscillation. The time constant of amplitude decay is $\frac{1}{|\sigma|}$. Positive damping ratio infers decaying mode, while the negative damping ratio infers instability mode.

4.1.4 Participation Factor

Participation factor analysis aids in the identification of how each state variable is reflected on a given mode or eigenvalue. Specifically, given a linear system of the form :

$$\dot{x} = Ax$$

a participation factor which is equivalent to a sensitivity measure of an eigenvalue with respect to a diagonal entry of the system A matrix, is defined as :

$$p_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}} \tag{4.10}$$

where λ_i is the i^{th} system eigenvalue

a_{kk} is a diagonal entry in the system A matrix

p_{ki} is the participation factor relating the k^{th} state variable to the i^{th} eigenvalue.

The participation factor may also be defined by :

$$p_{ki} = \frac{w_{ki} v_{ki}}{w_i' v_i} \tag{4.11}$$

where w_{ki} and v_{ki} are the k^{th} entries in the left and right eigenvector associated with the i^{th} eigenvalue. The right eigenvector, v_i and the left eigenvector w_i associated with the i^{th} eigenvalue λ_i satisfy :

$$Av_i = \lambda_i v_i \tag{4.12}$$

$$w_i' A = w_i' \lambda_i \tag{4.13}$$

An eigenvector may be scaled by any value resulting in a new vector, which is also an eigenvector. In any case, since $\sum_{k=1}^{n} w_{ki} v_{ki} = w_i' v_i$, it follow from (4.11) that the sum of all the participation factors associated with a given eigenvalues is equal to 1, i.e.,

$$\sum_{k=1}^{n} p_{ki} = 1$$

This property is useful since all participation factors lie on a scale from zero to one. To handle participation factors corresponding to complex eigenvalues, we introduce some modifications as follows. The eigenvectors corresponding to a complex eigenvalue will have complex elements. Hence, p_{ki} is defined as

$$p_{ki} = \frac{|v_{ki}||w_{ik}|}{\sum_{k=1}^{n} |v_{ki}||w_{ik}|} \tag{4.14}$$

A further normalization can be done by making the largest of the participation factors equal to unity.

4.1.5 Controllability and Observability Analysis

The state-space method is the modern approach for control system design and analysis. The controllability and observability are important structural properties of a control system. The controllability and observability analysis can be used to check whether the system is controllable or observable. This is the formulation to conduct those analysis [19] :

1. Controllability analysis is used to calculate the controllability matrix and check whether the system is controllable. The controllability matrix is :
   $$CM = \begin{bmatrix} B & AB & A^2B & \ldots & A^{(n-1)}B \end{bmatrix} \tag{4.15}$$
   If CM has full row rank, the system is controllable.
2. Observability is used to calculate the observability matrix and check whether the system is observable. If the observability matrix
   $$OM = [C \quad CA \quad CA^2 \quad \ldots \quad CA^{(n-1)}]^T \tag{4.16}$$
   If OM has full vector rank, the system is observable.

When the system is completely state controllable and available for feedback, then poles of the closed loop system may be placed at any desired location by means of state feedback through an appropriate state feedback gain matrix. It's call *pole placement* or *pole-assignment technique*. This job will be done by engineer in control area.

4.1.6 PBH Test

PBH (Popov-Belevich-Hautus) test provides a means to classify the modes of a system as controllable or uncontrollable and observable or unobservable. There are two kind of tests [19] :

1. PBH rank test for controllability

By duality, a pair {A,B} is controllable if and only if

$$\text{Rank }([sI - A \quad B]) = n\text{, } \forall s \in \lambda_i\text{, } i = 1,\ldots,n \tag{4.17}$$

where λ_i, $i = 1,\ldots,n$ are eigenvalues of A

2. PBH rank test for observability

By duality, a pair {A,C} is observable if and only if

$$\text{Rank}\left(\begin{bmatrix} C \\ sI - A \end{bmatrix}\right) = n\text{, } \forall s \in \lambda_i\text{, } i = 1,\ldots,n \tag{4.18}$$

where λ_i, $i = 1,\ldots,n$ are eigenvalues of A

4.2 Characteristics of Small Signal Stability Problems

In large power systems, small signal stability problems may be either local or global in nature.

4.2.1 Local Problems

Local problems involve a small part of the system. They may be associated with rotor angle oscillations of a single generator or a single plant against the rest of the power system. Such oscillations are called *local plant mode* oscillations.

Local problems may also be associated with oscillations between the rotor of few generators close to each other. Such oscillations are called *intermachine or interplant mode oscillations*. Usually, the local plant mode and interplant mode oscillations have frequencies in the range of 1 to 3 Hz.

Other possible local problems include instability of modes associated with controls of equipment such as generator excitation systems, HVDC converters, and static var compensators. The problems associated with *control modes* are due to inadequate tuning of the control systems.

Analysis of local small signal stability problems requires a detailed representation of a small portion of the complete interconnected power system.

4.2.2 Global Problems

Global small signal stability problems are caused by interactions among large groups of generators and have widespread effects.

They involve oscillations of a group of generators in one area swinging against a group of generators in another area. Such oscillations are called *interarea mode oscillations*.

Large interconnected system usually have two distinct forms of interarea oscillations :

a. A very low frequency mode involving all the generators in the system. The frequency of this mode of oscillation is on the order of 0.1 to 0.3 Hz.
b. Higher frequency modes involving subgroups of generators sswinging against each other. The frequency of these oscillations is typically in the range of 0.4 to 1 Hz.

4.3 Linearized State Space Model of Power Systems with HVDC Links

Dynamic stability of power system is analyzed by monitoring the eigenvalues of the linearized system as a power system is progressively loaded. Instability occurs when a pair of complex eigenvalues crosses the right half plane [14].

The formulation of the state equations involves the development of linearized equation about an operating point and elimination of all non state and non input variables. However, the need to allow for the representation of extensive transmission network, loads, excitation system, and HVDC links makes the process very complex [2]. Therefore, the formulation of the state equations requires a systematic procedure for treating the wide range of devices.

The objective of this thesis is to generate an algorithm to develop the state model of complex power system, which consists of generators, AC network and HVDC links. Firstly, we have to get the mathematical equations which represent the dynamic behaviors of all components. It means we need to get the differential equations of all components. Beside that, we have to interface each component, so we need equation which connected component to others. After that we need to linearize all equations and arrange them in the matrix form. Lastly, we need to eliminate the non-state and non-input variable by substitution. This algorithm is shown in Fig. 4.1.

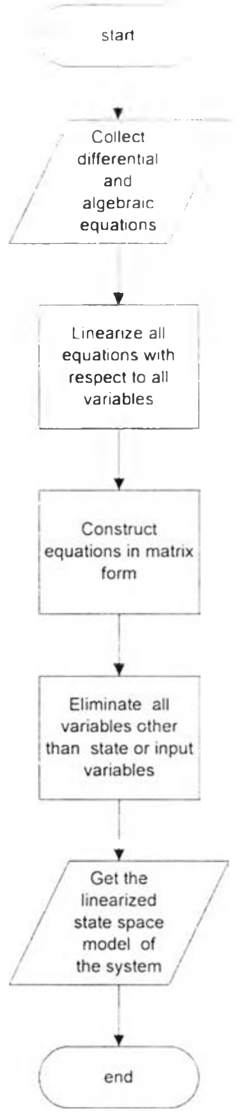

Figure 4.1 Main algorithm to generate linearized state space model.

4.3.1 Collections of the Equations

To develop state model, we need to collect equations of all components. We use both differential equations and algebraic equations to get the model.

From all components that explained above, thus we have:

a. Generating Unit

This part consists of synchronous generator and exciter system which are explained at Chapter 3, thus we have:

1. Four differential equations for each synchronous generator (3.4 (3.5), (3.6), (3.7) and three differential equations for each exciter (3.11), (3.12), (3.13) so that we have seven differential equations for each generating unit, *7m*.
2. Two real stator algebraic equations (3.2), (3.3) for each generating unit.

b. HVDC
   1. Three differential equations (2.12), (2.13), (2.15) for each HVDC link, *3h*.
   2. Two AC/DC interface equations, we have to choose whether the HVDC export or import power. When it exports we use equations (2.1), (2.7).When it imports we use equations (2.2), (2.10).
c. Network
   1. Two real network equations (3.14), (3.15) for each generator buses.
   2. Two real network equations for each HVDC buses, when it exports power, we use equations (3.16), (3.17) and when it imports power, we use equations (3.18), (3.19).
   3. Two real network equations (3.20), (3.20) for each load buses.

We assigned already buses of generator as $t = 1,2,...,m$, buses of HVDC as $h = m+1,...,m+d$, and the rest buses of the network is load buses as $l = m+d+1,...,m+d+p$. Note that m is the number of generator, d is the number of HVDC, p is the number of loads and n is the number of bus ($n = m+d+p$).

4.3.2 Linearized Model of Generating Units

In this thesis, each generating unit comprises of synchronous generator and exciter. To do linearization of generating unit, we have to collect differential equations of synchronous generator and exciter. Beside that, we have to collect stator algebraic equations also. We will recall those equations and rewrite in this part. After that we will linearize all equations respect to all variables.

In this case, we have multiple number of generator. Recall differential equations for generating unit in (3.4), (3.5), (3.6), (3.7), (3.11), (3.12), (3.13) :

$$\dot{\delta}_t = \omega_t - \omega_s$$

$$\dot{\omega}_t = \frac{T_{Mt} - \left[(E'_{qt} - X'_{dt} I_{dt}) I_{qt} + (E'_{dt} - X'_{qt} I_{qt}) I_{dt}\right] - D_t(\omega_t - \omega_s)}{M_t}$$

$$\dot{E}'_{qt} = \frac{\left(E_{fdt} - (X_{dt} - X'_{dt}) I_{dt} - E'_{qt}\right)}{\tau'_{d0t}}$$

$$\dot{E}'_{dt} = \frac{\left((X_{qt} - X'_{qt}) I_{qt} - E'_{dt}\right)}{\tau'_{q0t}}$$

$$\dot{E}_{fdt} = -\frac{K_{Et} + S_{Et}(E_{fdt})}{\tau_{Et}} E_{fdt} + \frac{V_{Rt}}{T_{Et}}$$

$$\dot{V}_{Rt} = -\frac{V_{Rt}}{\tau_{At}} + \frac{K_{At}}{\tau_{At}} R_{ft} - \frac{K_{At} K_{Ft}}{\tau_{At} \tau_{Ft}} E_{fdt} + \frac{K_{At}}{\tau_{At}} (V_{reft} - V_t)$$

$$\dot{R}_{ft} = -\frac{Rft}{\tau_{Ft}} + \frac{K_{Ft}}{(\tau_{Ft})^2} E_{fdt}$$

for $t = 1,2,...,m$

Recall stator algebraic equations (3.2), (3.3) :

$$E'_{dt} - V_t \sin(\delta_t - \theta_t) - R_{st} I_{dt} + X'_{qt} I_{qt} = 0$$

$$E'_{qt} - V_t \cos(\delta_t - \theta_t) - R_{st} I_{qt} + X'_{dt} I_{dt} = 0$$

for $i = 1,2,\ldots,m$

a. The linearization of the differential equations from generating unit yields

$$\Delta\dot{\delta}_i = \Delta\omega_i \tag{4.19}$$

$$\begin{aligned}\Delta\dot{\omega}_i =& \frac{1}{M_i}\Delta T_{Mi} - \frac{E'_{qi0}}{M_i}\Delta I_{qi} + \frac{X'_{di}I_{di0}}{M_i}\Delta I_{qi} + \frac{X'_{di}I_{qi0}}{M_i}\Delta I_{di} - \frac{I_{qi0}}{M_i}\Delta E'_{qi} \\ & - \frac{E'_{di0}}{M_i}\Delta I_{di} - \frac{I_{di0}}{M_i}\Delta E'_{di} - \frac{X'_{qi}I_{di0}}{M_i}\Delta I_{qi} - \frac{X'_{qi}I_{qi0}}{M_i}\Delta I_{di} - \frac{D_i}{M_i}\Delta\omega_i\end{aligned} \tag{4.20}$$

$$\Delta\dot{E}'_{qi} = \frac{1}{\tau'_{d0i}}\Delta E_{fdi} - \frac{1}{\tau'_{d0i}}\Delta E'_{qi} - \frac{(X_{di} - X'_{di})}{\tau'_{d0i}}\Delta I_{di} \tag{4.21}$$

$$\Delta\dot{E}'_{di} = \frac{(X_{qi} - X'_{qi})}{\tau'_{q0i}}\Delta I_{qi} - \frac{1}{\tau'_{q0i}}\Delta E'_{di} \tag{4.22}$$

$$\Delta\dot{E}_{fdi} = -f_{si}(E_{fdi0})\Delta E_{fdi} + \frac{1}{\tau_{Ei}}\Delta V_{Ri} \tag{4.23}$$

$$\Delta\dot{V}_{Ri} = -\frac{1}{\tau_{Ai}}\Delta V_{Ri} + \frac{K_{Ai}}{\tau_{Ai}}\Delta R_{fi} - \frac{K_{Ai}K_{Fi}}{\tau_{Ai}\tau_{Fi}}\Delta E_{fdi} + \frac{K_{Ai}}{\tau_{Ai}}\Delta V_{refi} - \frac{K_{Ai}}{\tau_{Ai}}\Delta V_i \tag{4.24}$$

$$\dot{R}_{fi} = -\frac{1}{\tau_{Fi}}R_{fi} + \frac{K_{Fi}}{(\tau_{Fi})^2}\Delta E_{fdi} \tag{4.25}$$

where $f_{si}(E_{fdi0}) = \dfrac{K_{Ei} + E_{fdi0}\partial S_E(E_{fdi0}) + S_E(E_{fdi0})}{\tau_{Ei}}$

for $i = 1,2,\ldots,m$

Writing those linearized equations in matrix notation, we obtain:

$$\begin{bmatrix} \Delta\dot{\delta}_i \\ \Delta\dot{\omega}_i \\ \Delta\dot{E}_{qi} \\ \Delta\dot{E}_{di} \\ \Delta\dot{E}_{fdi} \\ \Delta\dot{V}_{Ri} \\ \Delta\dot{R}_{Fi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{M_i} & \frac{I_{qi0}}{M_i} & \frac{I_{di0}}{M_i} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau'_{d0i}} & 0 & -\frac{1}{\tau'_{d0i}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau'_{q0i}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_{si}(E_{fdi0}) & -\frac{1}{\tau_{Ei}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{K_{Ai}K_{Fi}}{\tau_{Ai}\tau_{Fi}} & -\frac{1}{\tau_{Ai}} & \frac{K_{Ai}}{\tau_{Ai}} \\ 0 & 0 & 0 & 0 & \frac{K_{Fi}}{\tau_{Fi}} & 0 & -\frac{1}{\tau_{Fi}} \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E_{qi} \\ \Delta E_{di} \\ \Delta E_{fdi} \\ \Delta V_{Ri} \\ \Delta R_{Fi} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ \frac{I_{qi0}(X'_{di} - X'_{qi}) - E'_{di0}}{M_i} & \frac{I_{di0}(X'_{di} - X'_{qi}) - E'_{qi0}}{M_i} \\ -\frac{(X_{di} - X'_{di})}{\tau'_{d0i}} & 0 \\ 0 & \frac{X_{qi} - X'_{qi}}{\tau'_{q0i}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_{di} \\ \Delta I_{qi} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{Ai}}{\tau_{Ai}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta_i \\ \Delta V_i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{Ai}}{\tau_{Ai}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta T_{Mi} \\ \Delta V_{refi} \end{bmatrix} \tag{4.26}$$

Denoting $\begin{bmatrix} \Delta I_{di} \\ \Delta I_{qi} \end{bmatrix} = \Delta I_{gi}$, $\begin{bmatrix} \Delta\theta_i \\ \Delta V_i \end{bmatrix} = \Delta V_{gi}$, $\begin{bmatrix} \Delta T_{Mi} \\ \Delta V_{refi} \end{bmatrix} = \Delta u_{gi}$

Equation (4.26) can be written as

$$\Delta\dot{x}_{gi} = A_{g1i}\Delta x_{gi} + B_{1i}\Delta I_{gi} + C_{1i}\Delta V_{gi} + E_{gi}\Delta u_{gi} \tag{4.27}$$

For the m-machine system, (4.27) can be expressed in matrix form as

$$\Delta\dot{x}_g = A_{g1}\Delta x_g + B_1\Delta I_g + C_1\Delta V_g + E_g\Delta u_g \tag{4.28}$$

where A_{g1}, B_1, C_1 and E_g are block diagonal matrices

b. Linearize the stator algebraic equations from generating unit yields :

$$\Delta E'_{di} - \sin(\delta_{i0} - \theta_{i0})\Delta V_i - V_{i0}\cos(\delta_{i0} - \theta_{i0})\Delta\delta_i + V_{i0}\cos(\delta_{i0} - \theta_{i0})\Delta\theta_i$$
$$- R_{si}\Delta I_{di} + X'_{qi}\Delta I_{qi} = 0 \tag{4.29}$$

$$\Delta E'_{qi} - \cos(\delta_{i0} - \theta_{i0})\Delta V_i + V_{i0}\sin(\delta_{i0} - \theta_{i0})\Delta\delta_i - V_{i0}\sin(\delta_{i0} - \theta_{i0})\Delta\theta_i$$
$$- R_{si}\Delta I_{qi} - X'_{di}\Delta I_{di} = 0 \tag{4.30}$$

for $i = 1,2,\ldots,m$

Writing (4.29) and (4.30) in matrix form, we have :

$$\begin{bmatrix} -V_{i0}\cos(\delta_{i0} - \theta_{i0}) & 0 & 0 & 1 & 0 & 0 & 0 \\ V_{i0}\sin(\delta_{i0} - \theta_{i0}) & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E_{qi} \\ \Delta E_{di} \\ \Delta E_{fdi} \\ \Delta V_{Ri} \\ \Delta R_{Fi} \end{bmatrix} + \begin{bmatrix} -R_{si} & X'_{qi} \\ -X'_{di} & -R_{si} \end{bmatrix} \begin{bmatrix} \Delta I_{di} \\ \Delta I_{qi} \end{bmatrix} \tag{4.31}$$

$$+ \begin{bmatrix} V_{i0}\cos(\delta_{i0} - \theta_{i0}) & -\sin(\delta_{i0} - \theta_{i0}) \\ -V_{i0}\sin(\delta_{i0} - \theta_{i0}) & -\cos(\delta_{i0} - \theta_{i0}) \end{bmatrix} \begin{bmatrix} \Delta\theta_i \\ \Delta V_i \end{bmatrix} = 0$$

Rewriting (4.31) we obtain

$$0 = A_{g2i}\Delta x_{gi} + B_{2i}\Delta I_{gi} + C_{2i}\Delta V_{gi} \tag{4.32}$$

In matrix notation (4.32) can be written as

$$0 = A_{g2}\Delta x_g + B_2\Delta I_g + C_2\Delta V_g \tag{4.33}$$

Where A_{g2}, B_2 and C_2 are block diagonal matrices.

4.3.3 Linearized Model of HVDC

Recall differential equations for HVDC in eqs. (2.12), (2.13), (2.15):

$$\dot{I}_{dch} = \frac{\dfrac{3\sqrt{2}}{\pi}(V_{rh}\cos\alpha_h - V_{ih}\cos\gamma_h) - \left(\dfrac{3}{\pi}(X_{rh} - X_{ih}) + R_{dc}\right)I_{dch}}{\tau_{dch}R_{dch}}$$

$$\dot{\alpha}_h = \frac{K_{rh}(I_{dcrh} - I_{dch} + \Delta u) - \alpha_h}{\tau_{rh}}$$

$$\dot{\gamma}_h = \frac{K_{ih}(0.9I_{dcrh} - I_{dch}) - \gamma_h}{\tau_{ih}}$$

for $h = m+1,\ldots,m+d$

For AC/DC interface we need to know whether in the interface bus the power export or import through HVDC. When the power import to our system, its mean interface bus in our system acts as a rectifier, and when the power import to our system, it means interface bus in our system acts as an inverter.

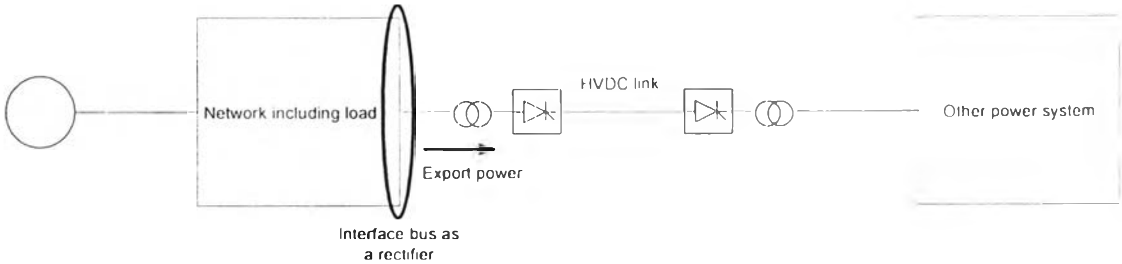

Figure 4.2 System configuration with exporting power.

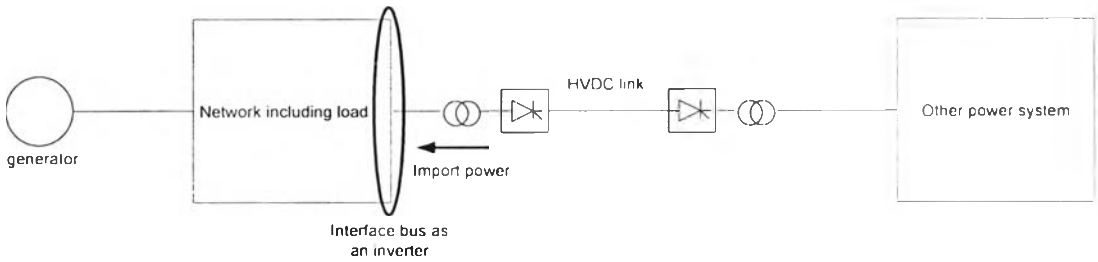

Figure 4.3 System configuration with importing power.

In Fig 4.2, when power export to other system so we use equation 2.1 and 2.7 :

$$E_{drh} = \frac{3\sqrt{2}}{\pi} V_{rh} \cos\alpha_h - \frac{3}{\pi} X_{rh} I_{dch}$$

$$\cos\phi_{rh} = \frac{E_{drh}}{E_{drh0}} = E_{drh} \frac{\pi}{3\sqrt{2}} V_{rh}^{-1}$$

for $h = m+1,\ldots,m+d$

In Fig 4.2, when power import to our system so we use equation 2.2 and 2.10 :

$$E_{dih} = \frac{3\sqrt{2}}{\pi} V_{ih} \cos\gamma_h - \frac{3}{\pi} X_{ih} I_{dch}$$

$$\cos\phi_{ih} = \frac{E_{dih}}{E_{dih0}} = E_{dih} \frac{\pi}{3\sqrt{2}} V_{ih}^{-1}$$

for $h = m+1,\ldots,m+d$

a. The linearization of the differential equations representing HVDC performance

When power export to other system, it means the interface bus acts as rectifier, we consider linearization respect to voltage at rectifier and we did not consider voltage at inverter because it is out of the system. Because of this reason, linearization of equation (2.12) yield :

$$\Delta \dot{I}_{dch} = \frac{\frac{3\sqrt{2}}{\pi}\cos\alpha_{h0}}{\tau_{dch}R_{dch}}\Delta V_{rh} - \frac{\frac{3\sqrt{2}}{\pi}V_{rh0}\sin\alpha_{h0}}{\tau_{dch}R_{dch}}\Delta\alpha_h + \frac{\frac{3\sqrt{2}}{\pi}V_{ih0}\sin\gamma_{h0}}{\tau_{dch}R_{dch}}\Delta\gamma_h + \frac{\frac{3}{\pi}(X_{rh}-X_{ih})+R_{dch}}{\tau_{dch}R_{dch}}\Delta I_{dch} \tag{4.34}$$

When power import to other system, it means the interface bus acts as inverter so linearization of equation (2.12) yield :

$$\Delta \dot{I}_{dch} = -\frac{\frac{3\sqrt{2}}{\pi}\cos\gamma_{h0}}{\tau_{dch}R_{dch}}\Delta V_{ih} - \frac{\frac{3\sqrt{2}}{\pi}V_{rh0}\sin\alpha_{h0}}{\tau_{dch}R_{dch}}\Delta\alpha_h + \frac{\frac{3\sqrt{2}}{\pi}V_{ih0}\sin\gamma_{h0}}{\tau_{dch}R_{dch}}\Delta\gamma_h + \frac{\frac{3}{\pi}(X_{rh}-X_{ih})+R_{dch}}{\tau_{dch}R_{dch}}\Delta I_{dch} \tag{4.35}$$

For equations (2.13), (2.15), even the system import or export power when we do linearization will yield :

$$\Delta\dot{\alpha}_h = -\frac{1}{\tau_{rh}}\Delta\alpha_h - \frac{K_{rh}}{\tau_{rh}}\Delta I_{dch} + \frac{K_{rh}}{\tau_{rh}}\Delta I_{dcrh} + \frac{K_{rh}}{\tau_{rh}}\Delta u_{dch} \tag{4.36}$$

$$\Delta\dot{\gamma}_h = -\frac{K_{ih}}{\tau_{ih}}\Delta I_{dch} - \frac{1}{\tau_{ih}}\Delta\gamma_h + \frac{0.9K_{ih}}{\tau_{ih}}\Delta I_{dcrh} \tag{4.37}$$

Writing those linearized equations in matrix notation, we obtain :
When power export to other system :

$$\begin{bmatrix}\Delta\dot{I}_{dch}\\ \Delta\dot{\alpha}_h\\ \Delta\dot{\gamma}_h\end{bmatrix} = \begin{bmatrix}\frac{\frac{3}{\pi}(X_{rh}-X_{ih})+R_{dch}}{\tau_{dch}R_{dch}} & -\frac{\frac{3\sqrt{2}}{\pi}V_{rh0}\sin\alpha_{h0}}{\tau_{dch}R_{dch}} & \frac{\frac{3\sqrt{2}}{\pi}V_{ih0}\sin\gamma_{h0}}{\tau_{dch}R_{dch}}\\ -\frac{K_{rh}}{\tau_{rh}} & -\frac{1}{\tau_{rh}} & 0\\ -\frac{K_{ih}}{\tau_{ih}} & 0 & -\frac{1}{\tau_{ri}}\end{bmatrix}\begin{bmatrix}\Delta I_{dch}\\ \Delta\alpha_h\\ \Delta\gamma_h\end{bmatrix} + \begin{bmatrix}0 & \frac{\frac{3\sqrt{2}}{\pi}\cos\alpha_{h0}}{\tau_{dch}R_{dch}}\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\theta_{rh}\\ \Delta V_{rh}\end{bmatrix} + \begin{bmatrix}0 & 0\\ \frac{K_{rh}}{\tau_{rh}} & \frac{K_{rh}}{\tau_{rh}}\\ \frac{0.9K_{ih}}{\tau_{ih}} & 0\end{bmatrix}\begin{bmatrix}\Delta I_{dcrh}\\ \Delta u_{dch}\end{bmatrix} \tag{4.38}$$

When power being imported to the system,

$$\begin{bmatrix}\Delta\dot{I}_{dch}\\ \Delta\dot{\alpha}_h\\ \Delta\dot{\gamma}_h\end{bmatrix}=\begin{bmatrix}\frac{\frac{3}{\pi}(X_{rh}-X_{ih})+R_{dch}}{\tau_{dch}R_{dch}} & -\frac{\frac{3\sqrt{2}}{\pi}V_{rh0}\sin\alpha_{h0}}{\tau_{dch}R_{dch}} & \frac{\frac{3\sqrt{2}}{\pi}V_{ih0}\sin\gamma_{h0}}{\tau_{dch}R_{dch}}\\ -\frac{K_{rh}}{\tau_{rh}} & -\frac{1}{\tau_{rh}} & 0\\ -\frac{K_{ih}}{\tau_{ih}} & 0 & -\frac{1}{\tau_{ii}}\end{bmatrix}\begin{bmatrix}\Delta I_{dch}\\ \Delta\alpha_h\\ \Delta\gamma_h\end{bmatrix} \tag{4.39}$$

$$+\begin{bmatrix}0 & -\frac{\frac{3\sqrt{2}}{\pi}\cos\gamma_{h0}}{\tau_{dch}R_{dch}}\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\theta_{ih}\\ \Delta V_{ih}\end{bmatrix}+\begin{bmatrix}0 & 0\\ \frac{K_{rh}}{\tau_{rh}} & \frac{K_{rh}}{\tau_{rh}}\\ \frac{0.9K_{ih}}{\tau_{ih}} & 0\end{bmatrix}\begin{bmatrix}\Delta I_{dcrh}\\ \Delta u_{dch}\end{bmatrix}$$

Denoting $\begin{bmatrix}\Delta\theta_{rh}\\ \Delta V_{rh}\end{bmatrix}=\begin{bmatrix}\Delta\theta_{ih}\\ \Delta V_{ih}\end{bmatrix}=\begin{bmatrix}\Delta\theta_h\\ \Delta V_h\end{bmatrix}=\Delta V_{ch}$, $\begin{bmatrix}\Delta I_{dcrh}\\ \Delta u_{ah}\end{bmatrix}=\Delta u_{ch}$

Equation (4.38) and (4.39) can be written as :

$$\Delta\dot{x}_{ch}=A_{c1h}\Delta x_{ch}+D_{1h}\Delta V_{ch}+E_{ch}\Delta u_{ch} \tag{4.40}$$

For the d-HVDC system, (4.40) can be expressed in matrix form as :

$$\Delta\dot{x}_c=A_{c1}\Delta x_c+D_1\Delta V_c+E_c\Delta u_c \tag{4.41}$$

Where A_{c1}, D_1, and E_c are block diagonal matrices.

b. The linearization of the AC/DC interfacing equations on HVDC side

There are two possibilities, whether the power export to another system, or import to our system.

Case 1 : The power export to another system, so we have these following equations :

$$\Delta E_{drh}-\frac{3\sqrt{2}}{\pi}\cos\alpha_{h0}\Delta V_{rh}+\frac{3\sqrt{2}}{\pi}V_{rh0}\sin\alpha_{h0}\Delta\alpha_h+\frac{3}{\pi}X_{rh}\Delta I_{dc}=0 \tag{4.42}$$

$$-\sin\phi_{rh0}\Delta\phi_{rh}-\frac{\pi}{3\sqrt{2}}V_{rh0}^{-1}\Delta E_{drh}+E_{drh0}\frac{\pi}{3\sqrt{2}}V_{rh0}^{-2}\Delta V_{rh}=0 \tag{4.43}$$

Writing those linearized equations in matrix notation, we obtain:

$$\begin{bmatrix}\frac{3}{\pi}X_{rh} & \frac{3\sqrt{2}}{\pi}V_{rh0}\sin\alpha_{h0} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta I_{dch}\\ \Delta\alpha_h\\ \Delta\gamma_h\end{bmatrix}+\begin{bmatrix}0 & -\frac{3\sqrt{2}}{\pi}\cos\alpha_{h0}\\ 0 & E_{drh0}\frac{\pi}{3\sqrt{2}}V_{rh0}^{-2}\end{bmatrix}\begin{bmatrix}\Delta\theta_{rh}\\ \Delta V_{rh}\end{bmatrix} \tag{4.44}$$

$$+\begin{bmatrix}0 & 1\\ -\sin\phi_{h0} & -\frac{\pi}{3\sqrt{2}}V_{rh0}^{-1}\end{bmatrix}\begin{bmatrix}\Delta\phi_{rh}\\ \Delta E_{drh}\end{bmatrix}=0$$

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Case 2 : The power import to put system, so we have these following equations :

$$\Delta E_{dh} - \frac{3\sqrt{2}}{\pi}\cos\gamma_{h0}\Delta V_{th} + \frac{3\sqrt{2}}{\pi}V_{th0}\sin\gamma_{h0}\Delta\gamma_h + \frac{3}{\pi}X_{th}\Delta I_{dh} = 0 \tag{4.45}$$

$$-\sin\phi_{th0}\Delta\phi_{th} - \frac{\pi}{3\sqrt{2}}V_{th0}^{-1}\Delta E_{dh} + E_{dh0}\frac{\pi}{3\sqrt{2}}V_{th0}^{-2}\Delta V_{th} = 0 \tag{4.46}$$

Writing those linearized equations in matrix notation, we obtain:

$$\begin{bmatrix} \frac{3}{\pi}X_{th} & 0 & \frac{3\sqrt{2}}{\pi}V_{th0}\sin\gamma_{h0} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}\Delta I_{dch} \\ \Delta\alpha_h \\ \Delta\gamma_h\end{bmatrix} + \begin{bmatrix} 0 & -\frac{3\sqrt{2}}{\pi}\cos\gamma_{h0} \\ 0 & E_{dh0}\frac{\pi}{3\sqrt{2}}V_{th0}^{-2}\end{bmatrix}\begin{bmatrix}\Delta\theta_{th} \\ \Delta V_{th}\end{bmatrix}$$
$$+\begin{bmatrix} 0 & 1 \\ -\sin\phi_{th0} & -\frac{\pi}{3\sqrt{2}}V_{th0}^{-1}\end{bmatrix}\begin{bmatrix}\Delta\phi_{th} \\ \Delta E_{dh}\end{bmatrix} = 0 \tag{4.47}$$

Equation (4.46) and (4.47) can be written as :

$$0 = A_{c2h}\Delta x_{ch} + D_{2h}\Delta V_{ch} + G_1\Delta V_{dh} \tag{4.48}$$

where $\begin{bmatrix}\Delta\phi_{rh} \\ \Delta E_{drh}\end{bmatrix} = \begin{bmatrix}\Delta\phi_{th} \\ \Delta E_{dth}\end{bmatrix} = \begin{bmatrix}\Delta\phi_h \\ \Delta E_{dh}\end{bmatrix} = \Delta V_{dh}$

For the d-HVDC system, (4.48) can be expressed in matrix form as :

$$0 = A_{c2}\Delta x_c + D_2\Delta V_c + G_1\Delta V_d \tag{4.49}$$

Where A_{c2}, D_2 and G_I are block diagonal matrices.

4.3.4 Linearized Model of Transmission Network

In the network we have generator bus, HVDC bus and load bus. We will do linearization of power balance equations for all those kind of bus.

a. Generator buses

Recall equations for generator bus (3.14), (3.15):

$$I_{di}V_i\sin(\delta_i - \theta_i) + I_{qi}V_i\cos(\delta_i - \theta_i) - \sum_{k=1}^{n} V_iV_kY_{ik}\cos(\theta_i - \theta_k - \alpha_{ik}) = 0$$

$$I_{di}V_i\cos(\delta_i - \theta_i) - I_{qi}V_i\sin(\delta_i - \theta_i) - \sum_{k=1}^{n} V_iV_kY_{ik}\sin(\theta_i - \theta_k - \alpha_{ik}) = 0$$

Linearized those equations, we obtain:

$$
\begin{aligned}
&V_{i0}\sin(\delta_{i0}-\theta_{i0})\Delta I_{di}+I_{di0}\sin(\delta_{i0}-\theta_{i0})\Delta V_i+I_{di0}V_{i0}\cos(\delta_{i0}-\theta_{i0})\Delta\delta_i\\
&-I_{di0}V_{i0}\cos(\delta_{i0}-\theta_{i0})\Delta\theta_i+V_{i0}\cos(\delta_{i0}-\theta_{i0})\Delta I_{qi}+I_{qi0}\cos(\delta_{i0}-\theta_{i0})\Delta V_i\\
&-I_{qi0}V_{i0}\sin(\delta_{i0}-\theta_{i0})\Delta\delta_i+I_{qi0}V_{i0}\sin(\delta_{i0}-\theta_{i0})\Delta\theta_i\\
&-\left[\sum_{k=1}^{n}V_{k0}Y_{ik}\cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta V_i-V_{i0}\sum_{k=1}^{n}\left[Y_{ik}\cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta V_k\\
&+\left[V_{i0}\sum_{\substack{k=1\\k\neq i}}^{n}V_{k0}Y_{ik}\sin(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta\theta_i\\
&-V_{i0}\sum_{\substack{k=1\\k\neq i}}^{n}\left[V_{k0}Y_{ik}\sin(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta\theta_k=0
\end{aligned}
\tag{4.50}
$$

$$
\begin{aligned}
&V_{i0}\cos(\delta_{i0}-\theta_{i0})\Delta I_{di}+I_{di0}\cos(\delta_{i0}-\theta_{i0})\Delta V_i-I_{di0}V_{i0}\sin(\delta_{i0}-\theta_{i0})\Delta\delta_i\\
&+I_{di0}V_{i0}\sin(\delta_{i0}-\theta_{i0})\Delta\theta_i-V_{i0}\sin(\delta_{i0}-\theta_{i0})\Delta I_{qi}-I_{qi0}\sin(\delta_{i0}-\theta_{i0})\Delta V_i\\
&-I_{qi0}V_{i0}\cos(\delta_{i0}-\theta_{i0})\Delta\delta_i+I_{qi0}V_{i0}\cos(\delta_{i0}-\theta_{i0})\Delta\theta_i\\
&-\left[\sum_{k=1}^{n}V_{k0}Y_{ik}\sin(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta V_i-V_{i0}\sum_{k=1}^{n}\left[Y_{ik}\sin(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta V_k\\
&-\left[V_{i0}\sum_{\substack{k=1\\k\neq i}}^{n}V_{k0}Y_{ik}\cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta\theta_i\\
&+V_{i0}\sum_{\substack{k=1\\k\neq i}}^{n}\left[V_{k0}Y_{ik}\cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})\right]\Delta\theta_k=0
\end{aligned}
\tag{4.51}
$$

for $i=1,2,\ldots,m$

Rewriting (4.50) and (4.51) in matrix form, we obtain

$$
\begin{aligned}
0=&\begin{bmatrix}A_{g31}&&\\&\cdots&\\&&A_{g3m}\end{bmatrix}\begin{bmatrix}\Delta x_1\\\cdots\\\Delta x_m\end{bmatrix}+\begin{bmatrix}B_{31}&&\\&\cdots&\\&&B_{3m}\end{bmatrix}\begin{bmatrix}\Delta I_{g1}\\\cdots\\\Delta I_{gm}\end{bmatrix}\\
&+\begin{bmatrix}C_{31,1}&&C_{31,m}\\&\cdots&\\C_{3m,1}&&C_{3m,m}\end{bmatrix}\begin{bmatrix}\Delta V_{g1}\\\cdots\\\Delta V_{gm}\end{bmatrix}+\begin{bmatrix}D_{31,m+1}&&D_{31,m+d}\\&\cdots&\\D_{3m,m+1}&&D_{3m,m+d}\end{bmatrix}\begin{bmatrix}\Delta V_{cm+1}\\\cdots\\\Delta V_{cm+d}\end{bmatrix}\\
&+\begin{bmatrix}F_{11,m+d+1}\cdots&&F_{51,n}\\&\cdots&\\F_{1m,m+d+1}&&F_{5m,n}\end{bmatrix}\begin{bmatrix}\Delta V_{b_{m+d+1}}\\\cdots\\\Delta V_{b_n}\end{bmatrix}
\end{aligned}
\tag{4.52}
$$

In matrix notation (4.52) is

$$0=A_{g3}\Delta x_g+B_3\Delta I_g+C_3\Delta V_g+D_3\Delta V_c+F_1\Delta V_b \tag{4.53}$$

where $\Delta V_{hl} = \begin{bmatrix} \Delta\theta_l \\ \Delta V_l \end{bmatrix}$

Note that A_{g3}, B_3 are block diagonal matrices, whereas C_3, D_3, F_1 are full matrices.

b. HVDC buses

When the power export to another system so we have these following equations :

$$E_{drh} I_{dch} - \sum_{k=1}^{n} V_h V_k Y_{hk} \cos(\theta_h - \theta_k - \alpha_{hk}) = 0$$

$$E_{drh} I_{dch} \tan(\phi_{rh}) - \sum_{k=1}^{n} V_h V_k Y_{hk} \sin(\theta_h - \theta_k - \alpha_{hk}) = 0$$

When the power import to out system so we have these following equations :

$$E_{dih} I_{dch} - \sum_{k=1}^{n} V_h V_k Y_{hk} \cos(\theta_h - \theta_k - \alpha_{hk}) = 0$$

$$E_{dih} I_{dch} \tan(\phi_{ih}) - \sum_{k=1}^{n} V_h V_k Y_{hk} \sin(\theta_h - \theta_k - \alpha_{hk}) = 0$$

Linearizing the first two equations, when the power exports to other system, we obtain:

$$\begin{aligned}
& I_{dch0} \Delta E_{drh} + E_{drh0} \Delta I_{dch} - \left[\sum_{k=1}^{n} V_{k0} Y_{hk} \cos(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta V_h \\
& + V_{h0} \sum_{k=1}^{n} \left[Y_{hk} \cos(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta V_k \\
& - \left[\sum_{\substack{k=1 \\ k \neq l}}^{n} V_{h0} V_{k0} Y_{hk} \sin(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta \theta_h \\
& - V_{h0} \sum_{\substack{k=1 \\ k \neq l}}^{n} \left[V_{k0} Y_{hk} \sin(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta \theta_k = 0
\end{aligned} \tag{4.54}$$

$$\begin{aligned}
& I_{dch0} \tan(\phi_{rh}) \Delta E_{drh} + E_{drh0} \tan(\phi_{rh}) \Delta I_{dch} + E_{drh0} I_{dc0} \sec^2 \phi_{rh0} \Delta \phi_{rh} \\
& - \left[\sum_{k=1}^{n} V_{k0} Y_{hk} \sin(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta V_h - V_{h0} \sum_{k=1}^{n} \left[Y_{hk} \sin(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta V_k \\
& - \left[V_{h0} \sum_{\substack{k=1 \\ k \neq l}}^{n} V_{k0} Y_{hk} \cos(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta \theta_h \\
& + V_{h0} \sum_{\substack{k=1 \\ k \neq l}}^{n} \left[V_{k0} Y_{hk} \cos(\theta_{h0} - \theta_{k0} - \alpha_{hk}) \right] \Delta \theta_k = 0
\end{aligned} \tag{4.55}$$

Linearizing the last two equations, when the power imports from other system, we obtain:

$$I_{dch0}\Delta E_{dh} + E_{dh0}\Delta I_{dch} - \left[\sum_{k=1}^{n} V_{k0}Y_{hk}\cos(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta V_h$$
$$+V_{h0}\sum_{k=1}^{n}\left[Y_{hk}\cos(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta V_k$$
$$-\left[\sum_{\substack{k=1\\k\neq l}}^{n} V_{h0}V_{k0}Y_{hk}\sin(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta\theta_h \tag{4.56}$$
$$-V_{h0}\sum_{\substack{k=1\\k\neq l}}^{n}\left[V_{k0}Y_{hk}\sin(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta\theta_k = 0$$

$$I_{dch0}\tan(\phi_{th})\Delta E_{dh} + E_{dh0}\tan(\phi_{th})\Delta I_{dch} + E_{dh0}I_{dc0}\sec^2\phi_{th0}\Delta\phi_{th}$$
$$-\left[\sum_{k=1}^{n} V_{k0}Y_{hk}\sin(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta V_h - V_{h0}\sum_{k=1}^{n}\left[Y_{hk}\sin(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta V_k$$
$$-\left[V_{h0}\sum_{\substack{k=1\\k\neq l}}^{n} V_{k0}Y_{hk}\cos(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta\theta_h \tag{4. 57}$$
$$+V_{h0}\sum_{\substack{k=1\\k\neq l}}^{n}\left[V_{k0}Y_{hk}\cos(\theta_{h0}-\theta_{k0}-\alpha_{hk})\right]\Delta\theta_k = 0$$

Rewriting (4.54 and 4.55) and (4.56 and 4.57) in matrix form, we obtain :

$$0=\begin{bmatrix} A_{c3m+1} & & \\ & \cdots & \\ & & A_{c3m+d}\end{bmatrix}\begin{bmatrix}\Delta x_{m+1}\\ \cdots \\ \Delta x_{m+d}\end{bmatrix}+\begin{bmatrix} C_{4m+1,1} & \cdots & C_{4m+1,m}\\ & & \\ C_{4m+d,1} & & C_{4m+d,m}\end{bmatrix}\begin{bmatrix}\Delta V_{g1}\\ \cdots \\ \Delta V_{gm}\end{bmatrix}+$$
$$\begin{bmatrix} D_{4m+1,m+1} & & D_{4m+1,m+d}\\ & \cdots & \\ D_{4M+d,m+1} & & D_{4m+d,m+d}\end{bmatrix}\begin{bmatrix}\Delta V_{cm+1}\\ \cdots \\ \Delta V_{cm+d}\end{bmatrix}+\begin{bmatrix} F_{2m+1,m+d+1} & & F_{2m+1,n}\\ & \cdots & \\ F_{2m+d,m+d+1} & & F_{2m+d,n}\end{bmatrix}\begin{bmatrix}\Delta V_{hm+d+1}\\ \cdots \\ \Delta V_{hn}\end{bmatrix}+$$
$$\begin{bmatrix} G_{2m+1} & & \\ & \cdots & \\ & & G_{2m+d}\end{bmatrix}\begin{bmatrix}\Delta V_{dm+1}\\ \cdots \\ \Delta V_{dm+d}\end{bmatrix} \tag{4.58}$$

In matrix notation (4.58) is

$$0 = A_{c3}\Delta x_c + C_4\Delta V_g + D_4\Delta V_c + F_2\Delta V_h + G_2\Delta V_d \tag{4.59}$$

Note that A_{c3}, G_2 are block diagonal matrices, whereas C_4, D_4, F_2 are full matrices.

c. Load buses

Recall equations for load bus (3.20), (3.21):

$$P_{Ll}(V_l)-\sum_{k=1}^{n}V_lV_kY_{lk}\cos(\theta_l-\theta_k-\alpha_{lk})=0$$

$$Q_{Ll}(V_l)-\sum_{k=1}^{n}V_lV_kY_{lk}\sin(\theta_l-\theta_k-\alpha_{lk})=0$$

Linearized those equations, we obtain:

$$\begin{aligned}&\frac{\partial P_{Ll}(V_l)}{\partial V_l}\Delta V_l-\left[\sum_{k=1}^{n}V_{k0}Y_{lk}\cos(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta V_l\\&-V_{l0}\sum_{k=1}^{n}\left[Y_{lk}\cos(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta V_k\\&+\left[V_{l0}\sum_{\substack{k=1\\k\neq l}}^{n}V_{k0}Y_{lk}\sin(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta\theta_l\\&-V_{l0}\sum_{\substack{k=1\\k\neq l}}^{n}\left[V_{k0}Y_{lk}\sin(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta\theta_k=0\end{aligned} \tag{4.60}$$

$$\begin{aligned}&\frac{\partial Q_{Ll}(V_l)}{\partial V_l}\Delta V-\left[\sum_{k=1}^{n}V_{k0}Y_{lk}\sin(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta V_l\\&-V_{l0}\sum_{k=1}^{n}\left[Y_{lk}\sin(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta V_k\\&-\left[\sum_{\substack{k=1\\k\neq l}}^{n}V_{l0}V_{k0}Y_{lk}\cos(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta\theta_l\\&+V_{l0}\sum_{\substack{k=1\\k\neq l}}^{n}\left[V_{k0}Y_{lk}\cos(\theta_{l0}-\theta_{k0}-\alpha_{lk})\right]\Delta\theta_k=0\end{aligned} \tag{4.61}$$

Rewriting (4.60) and (4.61) in matrix form, we obtain

$$0=\begin{bmatrix}C_{5m+d+1,1} & & C_{5m+d+1,m}\\ & \cdots & \\ C_{5n,1} & & C_{5n,m}\end{bmatrix}\begin{bmatrix}\Delta V_{g1}\\ \cdots\\ \Delta V_{gm}\end{bmatrix}+\begin{bmatrix}D_{5m+d+1,m+1} & & D_{5m+d+1,m+d}\\ & \cdots & \\ D_{5n,m+1} & & D_{5n,m+d}\end{bmatrix}\begin{bmatrix}\Delta V_{cm+1}\\ \cdots\\ \Delta V_{cm+d}\end{bmatrix}$$

$$+\begin{bmatrix}F_{3m+d+1,m+d+1} & & F_{3m+d+1,n}\\ & \cdots & \\ F_{3n,m+d+1} & & F_{3n,n}\end{bmatrix}\begin{bmatrix}\Delta V_{hm+d+1}\\ \cdots\\ \Delta V_{hn}\end{bmatrix} \tag{4.62}$$

In matrix notation (4.62) is

$$0=C_5\Delta V_g+D_5\Delta V_c+F_3\Delta V_h \tag{4.63}$$

where C_5, D_5, F_3 are full matrices.

4.3.5 Eliminations of Non State and Non Input Variables

Rewriting eqs. (4.28), (4.33), (4.41), (4.49), (4.53), (4.59), and (4.63), we have the following sets of equations :

$$\Delta\dot{x}_g = A_{g1}\Delta x_g + B_1\Delta I_g + C_1\Delta V_g + E_g\Delta u_g \tag{4.64}$$

$$0 = A_{g2}\Delta x_g + B_2\Delta I_g + C_2\Delta V_g \tag{4.65}$$

$$\Delta\dot{x}_c = A_{c1}\Delta x_c + D_1\Delta V_c + E_c\Delta u_c \tag{4.66}$$

$$0 = A_{c2}\Delta x_c + D_2\Delta V_c + G_1\Delta V_d \tag{4.67}$$

$$0 = A_{g3}\Delta x_g + B_3\Delta I_g + C_3\Delta V_g + D_3\Delta V_c + F_1\Delta V_h \tag{4.68}$$

$$0 = A_{c3}\Delta x_c + C_4\Delta V_g + D_4\Delta V_c + F_2\Delta V_h + G_2\Delta V_d \tag{4.69}$$

$$0 = C_5\Delta V_g + D_5\Delta V_c + F_3\Delta V_h \tag{4.70}$$

where $x_g = [x'_1 \quad \dots \quad x'_m]'$

$$x_i = [\delta_i, \omega_i, E'_{qi}, E'_{qi}, E_{fdi}, V_{Ri}, R_{fi}]'$$
$$u_g = [u'_1 \quad \dots \quad u'_m]'$$
$$u_i = [T_{Mi} \quad V_{refi}]'$$
$$x_c = [x'_{m+1} \quad \dots \quad x'_d]'$$
$$x_h = [I_{dch} \quad \alpha_h \quad \gamma_h]'$$
$$u_c = [u'_{m+1} \quad \dots \quad u'_d]'$$
$$u_h = [I_{dcrh} \quad u_{dcrh}]'$$
$$I_g = [I_{di} \quad I_{qi} \quad \dots \quad I_{dm} \quad I_{qm}]'$$
$$V_g = [\theta_i \quad V_i \quad \dots \quad \theta_m \quad V_m]'$$
$$V_c = [\theta_{m+1} \quad V_{m+1} \quad \dots \quad \theta_d \quad V_d]'$$
$$V_h = [\theta_{d+1} \quad V_{d+1} \quad \dots \quad \theta_n \quad V_n]'$$
$$V_d = [\phi_{m+1} \quad E_{m+1} \quad \dots \phi_d \quad E_d]'$$

To get the A_{sys} which is matrix A of the system including complex power system, we need to eliminate the non-state variable and non-input variable such that the linearization is respect to state and input variable. It means we need to eliminate I_g, V_g, V_c, V_h, V_d from those equations such that the linearized model is just respect to state and input variables as in eq. (4.3).

Firstly, we will eliminate I_g from eqs. (4.64) and (4.68) using eq. (4.65). Thus, from eq. (4.65):

$$\Delta I_g = -B_2^{-1}A_{g2}\Delta x_g - B_2^{-1}C_2\Delta V_g \tag{4.71}$$

Equation (4.71) substitute in (4.64) we get:

$$\Delta\dot{x}_g = (A_{g1} - B_1B_2^{-1}A_{g2})\Delta x_g + (C_2 - B_1B_2^{-1}C_2)\Delta V + E_g\Delta u_g \tag{4.72}$$

Let assign $K_1 = (A_{g1} - B_1 B_2^{-1} A_{g2})$ and $K_2 = (C_1 - B_1 B_2^{-1} C_2)$ thus (4.72) is now expressed as $\Delta\dot{x}_g = K_1 \Delta x_g + K_2 \Delta V_g + E_g \Delta u_g$ (4.73)

Equation (4.65) substitute in (4.68) we get:

$$0 = (A_{g3} - B_3 B_2^{-1} A_{g2})\Delta x_g + (C_3 - B_3 B_2^{-1} C_2)\Delta V_g + D_3 \Delta V_c + F_1 \Delta V_h \qquad (4.74)$$

Let assign $K_3 = (A_{g3} - B_3 B_2^{-1} A_{g2})$ and $K_4 = (C_2 - B_3 B_2^{-1} C_2)$ thus (4.74) is now expressed as $0 = K_3 \Delta x_g + K_4 \Delta V_g + D_3 \Delta V_c + F_1 \Delta V_h$ (4.75)

Secondly we have to eliminate V_d. From (4.67) we get :

$$\Delta V_d = -G_1^{-1} A_{c2} \Delta x_c - G_1^{-1} D_2 \Delta V_c \qquad (4.76)$$

Substitute in (4.69) we get:

$$0 = (A_{c3} - G_2 G_1^{-1} A_{c2})\Delta x_c + C_4 \Delta V_g + (D_4 - G_2 G_1^{-1} D_2)\Delta V_c + F_2 \Delta V_h \qquad (4.77)$$

Let assign $K_5 = (A_{c3} - G_2 G^{-1}{}_1 A_{c2})$ and $K_6 = (D_4 - G_2 G_1^{-1} D_2)$ thus (4.77) is now expressed as $0 = K_5 \Delta x_c + C_4 \Delta V_g + K_6 \Delta V_c + F_2 \Delta V_h$ (4.78)

Until this step, we already eliminate 2 equations, which are equation (4.65) and (4.67). It means we still have five equations. Recall those equations:

1. $\Delta\dot{x}_g = K_1 \Delta x_g + K_2 \Delta V_g + E_g \Delta u_g$
2. $\Delta\dot{x}_c = A_{c1} \Delta x_c + D_1 \Delta V_c + E_c \Delta u_c$
3. $0 = K_3 \Delta x_g + K_4 \Delta V_g + D_3 \Delta V_c + F_1 \Delta V_h$
4. $0 = K_5 \Delta x_c + C_4 \Delta V_g + K_6 \Delta V_c + F_2 \Delta V_h$
5. $0 = C_5 \Delta V_g + D_5 \Delta V_c + F_3 \Delta V_h$

Arrange those equations in matrix form:

$$\begin{bmatrix} \Delta\dot{x}_g \\ \Delta\dot{x}_c \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} K_1 & 0 & K_2 & 0 & 0 \\ 0 & A_{c1} & 0 & D_1 & 0 \\ K_3 & 0 & K_4 & D_3 & F_1 \\ 0 & K_5 & C_4 & K_6 & F_2 \\ 0 & 0 & C_5 & D_5 & F_3 \end{bmatrix} \begin{bmatrix} \Delta x_g \\ \Delta x_c \\ \Delta V_g \\ \Delta V_c \\ \Delta V_h \end{bmatrix} + \begin{bmatrix} E_g & 0 \\ 0 & E_c \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u_g \\ \Delta u_c \end{bmatrix} \qquad (4.79)$$

In more compact form, we can write (4.79) as:

$$\begin{bmatrix} \Delta\dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta V_N \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} \Delta u \qquad (4.80)$$

where $\Delta V_N = \begin{bmatrix} \Delta V_g \\ \Delta V_c \\ \Delta V_h \end{bmatrix}$

$$\Delta x = \begin{bmatrix} \Delta x_g \\ \Delta x_c \end{bmatrix}$$

$$\Delta u = \begin{bmatrix} \Delta u_g \\ \Delta u_c \end{bmatrix}$$

$$M_1 = \begin{bmatrix} K_1 & 0 \\ 0 & A_{c1} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} K_2 & 0 & 0 \\ 0 & D_1 & 0 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} K_3 & 0 \\ 0 & K_5 \\ 0 & 0 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} K_4 & D_3 & F_1 \\ C_4 & K_6 & F_2 \\ C_5 & D_5 & F_3 \end{bmatrix}$$

The system A_{sys} matrix is obtained as: $\Delta\dot{x} = A_{sys}\Delta x + E\Delta u$ (4.81)

where $A_{sys} = M_1 - M_2 M_4^{-1} M_3$

$B_{sys} = E$

Thus, we got already matrices A_{sys} and B_{sys}. For matrices C_{sys} and D_{sys} we can get also. In this case we want to observe δ and ω in the generator and also observe I_{dc} in the HVDC link. The output equation for generator side is :

$$\Delta y_{gi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E_{qi} \\ \Delta E_{di} \\ \Delta E_{fdi} \\ \Delta V_{Ri} \\ \Delta R_{Fi} \end{bmatrix} \tag{4.82}$$

For the m-machine system, equation (4.82) can be expressed in matrix form as

$$\Delta y_g = C_g \Delta x_g \tag{4.83}$$

where C_g is diagonal matrix.

The output equation for HVDC side is :

$$\Delta y_{ch} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_{dch} \\ \Delta\alpha_h \\ \Delta\gamma_h \end{bmatrix} \tag{4.84}$$

For the m-machine system, equation (4.84) can be expressed in matrix form as

$$\Delta y_c = C_c \Delta x_c \tag{4.85}$$

where C_c is diagonal matrix.

We can put equation (4.84) and (4.85) together

$$\begin{bmatrix} \Delta y_g \\ \Delta y_c \end{bmatrix} = \begin{bmatrix} C_g & 0 \\ 0 & C_c \end{bmatrix} \begin{bmatrix} \Delta x_g \\ \Delta x_c \end{bmatrix} \tag{4.86}$$

The system C_{sys} matrix is obtained as:

$$\Delta y = C_{sys} \Delta x \tag{4.87}$$

where $\Delta y = \begin{bmatrix} \Delta y_g \\ \Delta y_c \end{bmatrix}$

$$C_{sys} = \begin{bmatrix} C_g & 0 \\ 0 & C_c \end{bmatrix}$$

This completes the calculation to get linearized state space model of power systems with HVDC links. After that, we will develop the program based on this calculation which will be discussed in Chapter 5.