Chapter 1
Introduction

1.1 The Tunneling Time

Quantum tunneling is a microscopic phenomenon in which a particle can penetrate and in most cases pass through a potential barrier. This barrier is assumed to be higher than the energy of the particle, therefore such a motion is not allowed by the law of classical dynamics. In this thesis we consider the problem in one dimension which is the simplest problem. However, extension of one-dimensional tunneling to higher dimensions is not straightforward. In addition there are certain characteristics that appear in two-or three-dimensional tunneling which do not show up in the one-dimensional motion, as such as electron-electron scattering rate. Furthermore, recent advances concerning time delay and tunneling time and some of the problems associated with their measurement are also discussed.

The issue of how much time a particle takes to tunnel, a problem first addressed in 1932 [1], was solved by using time analogous to the classical time. In 1982 Büttiker and Landauer [2] pointed out that an incoming peak or centroid does not, in any obvious physically causative sense, turn into an outgoing peak or centroid. Quantum tunneling processes and the definition of quantum tunneling time have been investigated extensively by using numerical [3,4], experimental [5-10], and analytic methods in a number of works [11]. By using the Wigner function of the propagating wave packet a quantum tunneling time was obtained [11,12]. A way which use the measurement process for discussing the tunneling time is the local Larmor time [13-17]. The local Larmor time was defined by using an averaged spin component $\langle s_y \rangle$ of the particles due to the Larmor precession arising from a homogeneous magnetic field confined to the barrier region. Another approach to the traversal time problem, which uses trajectories, follows naturally from the Feynman path integral. In 1987, Sokolovski and Baskin [18] applied this formulation to calculate a traversal time using the functional

$$t_{ab}^{cl}[x(t)] = \int_0^t \Theta_{ab}[x(t')]dt' \tag{1.1}$$

where $\Theta_{ab}[x] = 1$ for $a \leq x \leq b$ and 0 otherwise. The operator $\boldsymbol{\Theta}_{ab}[\mathbf{x}]$ measures whether a particle is in the barrier region or not. It is a Hermitian projection operator with eigenvalues 0 and 1 by having the eigenfunction $(1 - \Theta_{ab}[x])g(x)$ and $\Theta_{ab}[x]g(x)$ ($g(x)$ is an arbitrary function), respectively.

In 1995, Sokolovski [19] introduced a generalized Schroedinger equation for solving the traversal time wave function, called the clocked Schroedinger equation (the clocked SE). The Feynman path integral with constrained paths was used to derive the clocked SE and Sokolovski intimated that the clocked SE can be derived only from the path integral, i.e. it cannot be derived from the Schroedinger equation (the SE). Thus, Feynman quantum mechanics appears to be capable of analyzing physical quantities and aspects of quantum motion that cannot be discussed by a theory based exclusively on the SE. In 1997, Sokolovski [20] solved the clocked SE for the cases of bound motion and wave packet scattering. Later [21] he analyzed the time average of a physical quantity F by using the path integral with paths constructed, according to the value f of a certain classical functional $F[x(t)]$. In 2001, Sokolovski and Liu [22] defined a quantum measurement as destruction of coherence between certain components of the Schroedinger wave function, related to the particle's possible paths. Steinberg [23] presented an approach to tunneling time by considering the outcome of a weak measurement in the sense of Aharonov, Albert and Vaidman [24]. However, Steinberg did not derive the clocked SE and the traversal time wave function form the ordinary SE. In this thesis we will derive the clocked SE by using directly the ordinary SE.

1.2 Outline of Thesis

This thesis is organized as follows. In Chapter 2, we consider the order of magnitude of the tunneling time [12] and present some other possible ways of formulating the tunneling time following Sokolovski [21], Sokolovski and Liu [22] and Sokolovski and Baskin [37].

In Chapter 3, we consider the quantum-classical boundary by following Aharonov and Bohm [26]. This method was discussed by Aharonov and Bohm in connection with the process of time measurement by considering the time-measuring variable.

In Chapter 4. we derive the clocked SE from the ordinary SE and discuss its physical meaning [49]. We derive the clocked SE form the ordinary SE using two methods. (1) We substitute the restored wave function into the ordinary SE and use a theorem of differential calculus. By using the meaning of the traversal wave function. we obtain the clocked SE. (2) We reduce the composite system composed of the observed system and the apparatus system. We start with the effective SE by eliminating the apparatus system. Then we obtain the clocked SE and the effective wave function which is equivalent to Sokolovski's definition of the traversal wave function. If we reduce the total propagator by defining an effective propagator using the meaning of the conditional probability then we obtain an effective propagator which is analogous to the propagator of Sokolovski [21].

In Chapter 5. we use the quantum-classical boundary to derive the wave function of the quantum system in the Schroedinger picture at time t from the entangled wave function and the differential equation which characterizes the time evolution of the quantum system. As an example. we consider the time independent Jaynes-Cummings model to derive the time dependent Schroedinger of the two-level atom coupling to a classical field. We start with the entangled state in a different form of the entangled state in the Born-Oppenhimer expansion [45]. Braun, Strunz and Briggs [50] had shown that the interaction between a two-level atom and the quantized electromagnetic field in the Jaynes-Cummings model in the limit of large number of photons approaches the classical limit. Our results are the same as of Braun et al.

Finally, we summarize the results in Chapter 6.