Chapter 6
Conclusion

6.1 Derivation of the Clocked SE Using the Ordinary SE

In this thesis, we have derived the clocked SE by using the ordinary SE. We have derived the clocked SE in two ways : (i) straight forward substitution and (ii) reduction of the composite system. We have applied the quantum-classical boundary to derive the ket state in the Schroedinger picture from the entangled wave function of a composite system. Specifically, we have considered the Jaynes-Cummings model, which is a two-level atom coupling to a quantized field.

By straightforward substitution, we have considered the boundary condition of the traversal wave function at $x = a$ and $x = b$ for $\tau = 0$ and t and have used a theorem of differential calculus to obtain the clocked SE from the ordinary SE. This is in contrast to what has been claimed by Sokolovski [19,20].

To reduce the composite system, we have shown that the effect of observation, acting on the object system, is analogous to the constraint given by Sokolovski, which is the Dirac-delta function. We have started with using the operator $\Theta_{ab}[\mathrm{x}]$ to compare with the classical variable $\Lambda(t)$ and have considered the coupling interaction between the apparatus system and object system to be in the form of a weak measurement. When the coupling interaction is turned on, the initial state becomes an entangled state Eq.(4.23). After we have eliminated the apparatus system by using the identity Eq.(4.28). Then the SE for the whole system have been reduced to the effective SE for the observed particle. Then the phase factor or the variable $\Lambda(t)$ appears as a constraint for the evolution of the observed state. Those paths of the observed particle which give the value of $\Theta_{ab}[x(t)]$ in the time period t, synchronizing with $\Lambda(t)$. are the same as the constrained paths of the observed system which have spent a duration τ in the region $a \leq x \leq b$. In this way the measurement system is used to constrain the evolution of the observed system. just as the Dirac delta function in the derivation of Sokolovski. We have obtained the traversal wave function in another way from the effective propagator. We have used Eq.(4.34) analogous to the probability theorem. The effective propagator in the form of Eq.(4.38) corresponds to the propagator of Sokolovski [19]. By defining the initial traversal wave function Eq.(4.39), we have obtained the traversal wave function Eq.(4.40). Thus we have obtained the clocked SE and the traversal wave function by starting from the ordinary SE. This work has been published in Physical Review A **73**. 012108 (2006).

6.2 Applications

We have use the quantum-classical boundary of Bohm and Aharonov to show that the ket state in the Schroedinger picture can be derived from the entangled wave function of the composite system when the environment tends to be the classical limit. We have started by considering the composite system in the time independent SE. Eq.(5.11) and have expanded the total wave function. This definition it is different from the total wave function in the Born-Oppenhimer expansion [45] used by Briggs and Rost [46] to derive the time dependent Schroedinger equation. We have considered the nature of time in quantum mechanics by deriving the variable determining the time of measurement. We have assumed that the environment has great mass and large energy compared to the quantum system energy. In this way the environment wave function will be narrow enough in $\overrightarrow{R}$-space and it will move in an essentially classical way which will determine the orbit $\overrightarrow{R}(t)$ of the environment motion by the Hamilton-Jacobi equation in Eq.(5.22). So we have replaced $\overrightarrow{R}$ by the parameter t, which means to be "*time*", defined through a trajectory $\overrightarrow{R}(t)$. The full quantum equation, Eq.(5.21), mixed by the back-coupling from the system, becomes a fundamental differential equation for characterizing the time evolution of the quantum system and leads to the ket state in the Schroedinger picture, Eq.(5.31).

As an example, we have considered the Jaynes-Cummings model. By using the model in Sec. 5.2 we assume that the large number of photons allows the wave function $\kappa(Q)$ to be written in the WKB form. The variable Q can be replaced by the new parameter t, which is time. In the limit of large number of photons, the TDSE of the Jaynes-Cummings model is reduced to the TISE for the two-level atom coupling to a classical field. In addition, the total state in Eq.(5.33) can be expanded in the form in Eq.(5.46), which is the a state in the Schroedinger picture.