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Appendices

Appendix A: Feynman's Average

The mean value of the functional $t^{cl}_{\Omega}[x] = \int_0^t dt'\Theta_{\Omega}[x(t')]$, where $\Theta_{\Omega}[x(t')] = 1$ for $a \leq x \leq b$ and 0 otherwise, is given by

$$\tau_D = \langle \Psi_f | \, t^{cl}_{\Omega} \, | \Psi_i \rangle \tag{1}$$

or

$$\tau_D = \frac{\int dx_2 \int dx_1 \Psi_f^*(x_2) \int D[x(t)] t^{cl}_{\Omega}[x] \exp\left[\frac{i}{\hbar} S[x(t)]\right] \Psi_i(x_1)}{\int dx_2 \int dx_1 \Psi_f^*(x_2) \int D[x(t)] \exp\left[\frac{i}{\hbar} S[x(t)]\right] \Psi_i(x_1)} . \tag{2}$$

In particular case, the final state Ψ_f is obtained from Ψ_i by the evolution:

$$\begin{aligned} \Psi_f(x_2, t_2) &= \int dx_1 \langle x_2 | \mathrm{U}(t,0) | x_1 \rangle \langle x_1 | \Psi_i \rangle \\ &= \int dx_1 \int D[x(t)] \, \exp\left\{\frac{i}{\hbar} S[x(t)]\right\} \Psi_i(x_1, t_1 = 0), \end{aligned} \tag{3}$$

so that we have

$$\int dx_2 \int dx_1 \Psi_f^*(x_2) \int D[x(t)] \exp\left[\frac{i}{\hbar} S[x(t)]\right] \Psi_i(x_1) = |\Psi_i|^2 = 1.$$

We have assumed that the initial state is normalized.

Now we consider

$$\begin{aligned} \int D[x(t)] t^{cl}_{\Omega} \exp\left[\frac{i}{\hbar} S[x(t)]\right] &= \int_0^t dt' \int D[x(t)] \Theta_{\Omega}[x(t')] \exp\left[\frac{i}{\hbar} S[x(t')]\right] \\ &= \int_0^t dt' \int_{\Omega} dx K(x_2, x; t, t') K(x, x_1; t', 0). \end{aligned} \tag{4}$$

where $K(x, x_1; t', 0)$ is the propagator for the particle moving from x_1 at $t = 0$ to x_2 at $t'$and $K(x_2, x; t, t')$ is the propagator for the particle moving from x at t' to x_2 at t. Thus Eq.(1) can be written as

$$\begin{aligned} \tau_D &= \langle \Psi_f | \, t^{cl}_{\Omega}[x] \, | \Psi_i \rangle \qquad (5) \\ &= \int_0^t dt' \int_{\Omega} dx \left[\int dx_2 \Psi_f^*(x_2) K(x_2, x; t, t')\right] \times \left[\int dx_1 \Psi_i(x_1) K(x, x_1; t', 0)\right] . \end{aligned}$$

By using the relation

$$\begin{aligned} \int dx_2 \Psi_f^*(x_2) K(x_2, x; t, t') &= \int dx_2 \langle \Psi_i | \mathbf{U}^\dagger(t, 0) | x_2 \rangle \langle x_2 | \mathbf{U}(t, t') | x \rangle \\ &= \langle \Psi_i | \mathbf{U}^\dagger(t, 0) \mathbf{U}(t, t') | x \rangle \\ &= \langle \Psi_i | \mathbf{U}^\dagger(t', 0) | x \rangle \\ &= \Psi^*(x, t') \end{aligned} \tag{6}$$

and

$$\Psi(x, t') = \int dx_1 \Psi_i(x_1) K(x, x_1; t', 0), \tag{7}$$

we obtain the Feynman average of the functional $t_\Omega^{cl}[x]$ in the form

$$\overline{\tau}_D = \int_0^t dt' \int_\Omega dx \, |\Psi(x, t')|^2 . \tag{8}$$

Appendix B:

A Theorem of Differential Calculus

A theorem of differential calculus is used. Let $\Phi(\lambda) = \int\limits_{v(\lambda)}^{u(\lambda)} dx f(x,\lambda)$ where $u(\lambda)$ and $v(\lambda)$ are differentiable functions in a closed interval $[\lambda_0, \lambda_1]$; $f(x,\lambda)$ and $f'(x,\lambda)$ are continuous in the region $\lambda_0 \leq \lambda \leq \lambda_1$, then

$$\frac{\partial}{\partial \lambda}\Phi(\lambda) = \frac{\partial u}{\partial \lambda} f(x,u) - \frac{\partial v}{\partial \lambda} f(x,v) + \int\limits_{v(\lambda)}^{u(\lambda)} dx \frac{\partial}{\partial \lambda} f(x,\lambda). \tag{1}$$

We prove the above theorem of differential calculus by considering the integral

$$\Phi(\lambda) \equiv \Phi(u(\lambda), v(\lambda), \lambda) = \int\limits_{v(\lambda)}^{u(\lambda)} dx f(x.\lambda) = F(x,\lambda)|_{v(\lambda)}^{u(\lambda)} = F(u(\lambda),\lambda) - F(v(\lambda).\lambda). \tag{2}$$

The partial differentives of $F(u, v. \lambda)$ with respect to u and v are

$$\frac{\partial}{\partial u}\Phi(u,v,\lambda) = \frac{\partial}{\partial u} F(u(\lambda),\lambda) = f(x,\lambda)_{u=x}, \tag{3}$$

$$\frac{\partial}{\partial v}\Phi(u,v,\lambda) = \frac{\partial}{\partial v} F(v(\lambda),\lambda) = f(x,\lambda)_{v=x}. \tag{4}$$

We consider the partial differential $\frac{\partial}{\partial \lambda}\Phi(\lambda)$:

$$\frac{\partial}{\partial \lambda}\Phi(\lambda) = \frac{\partial}{\partial \lambda} F(u(\lambda),\lambda) - \frac{\partial}{\partial \lambda} F(v(\lambda).\lambda) + \frac{\partial}{\partial \lambda}\left(F(u(\lambda),\lambda) - F(v(\lambda).\lambda)\right)_{u,v} \tag{5}$$

and

$$\frac{\partial}{\partial \lambda} F(u(\lambda).\lambda) = \frac{\partial u(\lambda)}{\partial \lambda}\frac{\partial}{\partial u} F(u(\lambda),\lambda). \tag{6}$$

$$\frac{\partial}{\partial \lambda} F(v(\lambda).\lambda) = \frac{\partial v(\lambda)}{\partial \lambda}\frac{\partial}{\partial v} F(u(\lambda),\lambda). \tag{7}$$

We substitute Eq.(6) and Eq.(7) into Eq.(5) to obtain

$$\frac{\partial}{\partial \lambda}\Phi(\lambda) = \frac{\partial u(\lambda)}{\partial \lambda} f(x.\lambda)_{u=x} - \frac{\partial v(\lambda)}{\partial \lambda} f(x.\lambda)_{v=x} + \frac{\partial}{\partial \lambda}\left(F(u(\lambda).\lambda) - F(v(\lambda).\lambda)\right)_{u,v}. \tag{8}$$

By using the relation

$$\frac{\partial}{\partial\lambda}\left(F(u(\lambda),\lambda) - F(v(\lambda),\lambda)\right)_{u,v} = \frac{\partial}{\partial\lambda}\Phi(u,v,\lambda) = \int\limits_{v(\lambda)}^{u(\lambda)} dx \frac{\partial}{\partial\lambda} f(x,\lambda). \tag{9}$$

Eq.(8) becomes

$$\frac{\partial}{\partial\lambda}\Phi(\lambda) = \frac{\partial u(\lambda)}{\partial\lambda} f(x,\lambda)_{u=x} - \frac{\partial v(\lambda)}{\partial\lambda} f(x,\lambda)_{v=x} + \int\limits_{v(\lambda)}^{u(\lambda)} dx \frac{\partial}{\partial\lambda} f(x.\lambda). \tag{10}$$

This is the theorem of differential calculus.

For an example, we want to use the theorem of differential calculus to find the partial derivative of a function $\Phi(\lambda)$, which is defined as

$$\Phi(\lambda) = \int\limits_{\lambda}^{\lambda^2} dx\,(ax + \lambda)\,. \tag{11}$$

Let us consider the function $\Phi(\lambda)$.

$$\Phi(\lambda) = \int\limits_{\lambda}^{\lambda^2} dx\,(ax + \lambda) = \left(\frac{a}{2}\left(\lambda^2\right)^2 + \lambda\left(\lambda^2\right)\right) - \left(\frac{a}{2}(\lambda)^2 + \lambda(\lambda)\right). \tag{12}$$

or

$$\Phi(\lambda) = \frac{a}{2}\lambda^4 + \lambda^3 - (\frac{a}{2} - 1)\lambda^2.$$

By straightforward substitution, we find the partial differential $\frac{\partial}{\partial\lambda}\Phi(\lambda)$ and we can obtain

$$\frac{\partial}{\partial\lambda}\Phi(\lambda) = 2a\lambda^3 + 3\lambda^2 - (a-2)\lambda. \tag{13}$$

By using the theorem of differential calculus, we have

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\Phi(\lambda) &= \frac{\partial}{\partial\lambda}(\lambda^2)\,(ax + \lambda)_{x=\lambda^2} - \frac{\partial}{\partial\lambda}(\lambda)\,(ax + \lambda)_{x=\lambda} + \int\limits_{\lambda}^{\lambda^2} dx \frac{\partial}{\partial\lambda}(ax + \lambda) \\
&= 2\lambda(a\lambda^2 + \lambda) - (a\lambda + \lambda) + \int\limits_{\lambda}^{\lambda^2} dx \\
&= 2a\lambda^3 + 3\lambda^2 - (a-2)\lambda. \qquad (14)
\end{aligned}$$

The result in Eq.(14) is equal to Eq. (13) which uses directly the partial differentiation of $\Phi(\lambda)$ with respect to λ.

Appendix C:
The Continuous Condition

In quantum mechanics, the wave function $\psi(x,t)$ means the probability amplitude of a particle located at x at time t. To interpret $\psi(x,t\,|\tau)$ as the probability amplitude for the particle at x to spend in $\Omega \equiv [a,b]$ prior to time t, a net time τ, we must require that $\psi(x,t\,|\tau)$ is square integrable both the x and τ,

$$N(t) = \int dx \left| \int_0^t d\tau \psi(x,t\,|\tau)\,\right|^2. \qquad (1)$$

The traversal wave function $\psi(x,t\,|\tau)$ means the probability amplitude of the particle located at x of having been in the region $\Omega \equiv [a,b]$ with net duration τ, prior to time t. We are to interpret $|\psi(x,t\,|\tau)\,|^2$ as a probability density for position measurement. So it implies strong constraints on $\psi(x,t\,|\tau)$:

1) $|\psi(x,t\,|\tau)\,|$ must tend to zero sufficiently and rapidly as $x \to \pm\infty$, so that the integral of $|\psi(x,t\,|\tau)\,|^2$ converges.

2) For a probability interpretation to be valid, we must also require that $\psi(x,t\,|\tau)$ is continuous in x and t, as a discontinuity of $\psi(x,t\,|\tau)$ would lead to ambiguous predictions for probabilities near the discontinuity.

3) $\psi(x,t\,|\tau)$ is a differentiable function in x and t. $\psi(x,t\,|\tau)$ satisfies the clocked SE

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) - i\hbar\Theta_{ab}[x]\frac{\partial}{\partial \tau} - i\hbar\frac{\partial}{\partial t}\right)\psi(x,t|\tau) = 0. \qquad (2)$$

4) In the time-independent potential case, we may assume that $\frac{\partial}{\partial t}\psi(x,t|\tau)$ is continuous everywhere. If the potential energy is a smooth function , we may assume that $\frac{\partial}{\partial x}\psi(x,t|\tau)$ is continuous everywhere.

Appendix D: Eliminating the Apparatus Degree of Freedom

We define the SE for an observed particle, which contains the effects of the measurement, by eliminating the apparatus system using the identity

$$\frac{g}{2\pi\hbar}\int dp \frac{\langle\phi(p),t|\,\mathbf{H}_{total}-i\hbar\frac{\partial}{\partial t}\,|\Psi(t)\rangle}{\langle\phi(p),t|\,\phi(p),t\rangle}=0. \tag{1}$$

We substitute the total state in Eq.(4.19) into Eq.(1)

$$\begin{aligned}&\frac{g}{2\pi\hbar}\int dp \frac{\langle\phi(p),t|\,\mathbf{H}_{total}-i\hbar\frac{\partial}{\partial t}\,|\Psi(t)\rangle}{\langle\phi(p),t|\,\phi(p),t\rangle}\\ =\;& \frac{g}{2\pi\hbar}\int dp\int dp' \frac{\langle\phi(p),t|\,\mathbf{H}_0+\frac{\mathbf{P}^2}{2M}+g(t)\mathbf{P}(\mathbf{\Theta_{ab}}[\mathbf{x}]-\mathbf{\Lambda}(t))-i\hbar\frac{\partial}{\partial t}\,|\phi(p'),t\rangle\otimes|\psi_{p'},\tau,t\rangle}{\langle\phi(p),t|\,\phi(p),t\rangle}\end{aligned} \tag{2}$$

where $|\psi_{p'},\tau,t\rangle$ is given by

$$|\psi_{p'},\tau,t\rangle = e^{\frac{i}{\hbar}gp'\tau}e^{-(\frac{i}{\hbar})\int_0^t(\mathbf{H}_0+gp'\mathbf{\Theta}_{ab}[\mathbf{x}])dt'}|\psi_0\rangle = e^{\frac{i}{\hbar}gp'\tau}|\psi_{p'},t\rangle \tag{3}$$

and the variable τ is defined as

$$\tau=\int_0^t \Lambda(t')dt'. \tag{4}$$

We multiply $e^{\frac{i}{\hbar}gp'\tau}$ with the time evolution of the observed system, so the state of the apparatus in time is of the form

$$|\phi(p'),t\rangle = e^{-\frac{i}{\hbar}\frac{p'^2}{2M}t}\,|p'\rangle\,\langle p'|\,\phi_0\rangle. \tag{5}$$

By differentiating on t,

$$\frac{\partial}{\partial t}F(t,y(t))=\frac{\partial}{\partial t}F(t,y)+\frac{\partial y(t)}{\partial t}\frac{\partial}{\partial y}F(t,y). \tag{6}$$

and

$$\frac{\partial \tau}{\partial t} = \Lambda(t) \tag{7}$$

we obtain

$$\begin{aligned} -i\hbar\frac{\partial}{\partial t}\left|\phi(p'),t\right\rangle \otimes \left|\psi_{p'},\tau,t\right\rangle &= \frac{p'^2}{2M}\left|\phi(p'),t\right\rangle \otimes \left|\psi_{p'},\tau,t\right\rangle - \left|\phi(p'),t\right\rangle \otimes i\hbar\frac{\partial}{\partial t}\left|\psi_{p'},\tau,t\right\rangle \\ &= \frac{p'^2}{2M}\left|\phi(p'),t\right\rangle \otimes \left|\psi_{p'},\tau,t\right\rangle \\ &\quad + gp'\Lambda(t)\left|\phi(p'),t\right\rangle \otimes \left|\psi_{p'},\tau,t\right\rangle - \left|\phi(p'),t\right\rangle \otimes i\hbar\left(\frac{\partial}{\partial t}\left|\psi_{p'},\tau,t\right\rangle\right)_\tau \end{aligned} \tag{8}$$

So Eq.(2) becomes

$$\begin{aligned} \frac{g}{2\pi\hbar}\int dp \frac{\langle\phi_0|\,\phi_0\rangle\left[\mathbf{H}_0 + gp'(\Theta_{ab}[x]) - i\hbar(\frac{\partial}{\partial t})_\tau\right]\left|\psi_{p'},\tau,t\right\rangle}{\langle\phi_0|\,\phi_0\rangle} &= 0 \\ \frac{g}{2\pi\hbar}\int dp\left[\mathbf{H}_0 + gp'\Theta_{ab}[x] - (i\hbar\frac{\partial}{\partial t})_\tau\right]\left|\psi_{p'},\tau,t\right\rangle &= 0 \\ \left\{\mathbf{H}_0 - i\hbar\frac{\partial}{\partial\tau}\boldsymbol{\Theta}_{ab}[\mathbf{x}] - (i\hbar\frac{\partial}{\partial t})_\tau\right\}\left|\psi,t\right|\tau\rangle &= 0. \end{aligned} \tag{9}$$

where $|\psi,t|\,\tau\rangle = \frac{g}{2\pi\hbar}\int dp e^{\frac{i}{\hbar}gp\tau} e^{-(\frac{i}{\hbar})\int_0^t(\mathbf{H}_0 + gp\boldsymbol{\Theta}_{ab}[\mathbf{x}])dt'} |\psi_0\rangle$ and the effective wave function can be written as

$$\begin{aligned} \psi(x,t\,|\tau) &= \frac{g}{2\pi\hbar}\int dp\langle x\,|\psi_p,\tau,t\rangle \\ &= \frac{g}{2\pi\hbar}\int dp\,\langle x|\,e^{\frac{i}{\hbar}gp\tau} e^{-(\frac{i}{\hbar})\int_0^t(\mathbf{H}_0 + gp\boldsymbol{\Theta}_{ab}[\mathbf{x}])dt'}|\psi_0\rangle\,. \end{aligned} \tag{10}$$

Appendix E:
To Reduce the Total Propagator

The effective propagator which uses the relation

$$K_{eff}(x, x_0; t) = \frac{\int dy \int dy_0 K_{total} K_a^*}{\int dy \int dy_0 K_a K_a^*} \tag{1}$$

where $K_a(y, y_0; t) = \langle y| \exp\{-\frac{i}{\hbar} \int_0^t \mathbf{H}_A dt'\} |y_0\rangle$ is the propagator of the pointer particle, in the form of a free particle propagator, $K_{total}(x, x_0; y, y_0; t) = \langle y| \otimes \langle x| U(t, t_0 = 0) |y_0\rangle \otimes |x_0\rangle$ is the propagator of the whole system. Straightforward substitution of the total propagator of Eq.(4.29) into Eq.(4.30) leads to

$$\begin{aligned} K_{eff}(x, x_0; t) &= \int dy \int dy_0 \langle y| \otimes \langle x| e^{[-\frac{i}{\hbar} \int_0^t (\mathbf{H}_0 + \mathbf{H}_A + g\mathbf{P}(\Theta_{ab}[\mathbf{x}] - \Lambda(t')))dt']} |y_0\rangle \\ &\times \langle y_0| e^{\frac{i}{\hbar} \int_0^t \mathbf{H}_A dt'} |y\rangle \otimes |x_0\rangle \\ &\times \left[\int dy \int dy_0 \langle y| \exp\{-\frac{i}{\hbar} \int_0^t \mathbf{H}_A dt'\} |y_0\rangle \langle y_0| \exp\{\frac{i}{\hbar} \int_0^t \mathbf{H}_A dt'\} |y\rangle \right]^{-1} . \end{aligned} \tag{2}$$

Using the completeness relation $I = \int dy_0 |y_0\rangle \langle y_0|$, Eq.(4.32) becomes

$$K_{eff}(x, x_0; t) = \int dy \langle y| \otimes \langle x| e^{[-\frac{i}{\hbar} \int_0^t (\mathbf{H}_0 + g\mathbf{P}(\Theta_{ab}[\mathbf{x}] - \Lambda(t')))dt']} |y\rangle \otimes |x_0\rangle \times \left[\int dy \langle y| y\rangle \right]^{-1} . \tag{3}$$

and inserting the completeness relation $I = \int dp |p\rangle \langle p|$ between $\langle y|$ and $|y\rangle$, leads to

$$\begin{aligned} K_{eff}(x, x_0; t) &= \int dy \int dp \langle y| p\rangle \langle p| \otimes \langle x| e^{[-\frac{i}{\hbar} \int_0^t (\mathbf{H}_0 + g\mathbf{P}(\Theta_{ab}[\mathbf{x}] - \Lambda(t')))dt']} |y\rangle \otimes |x_0\rangle \\ &\times \left[\int dy \int dp \langle y |p\rangle \langle p| y\rangle \right]^{-1} \end{aligned}$$

$$
\begin{aligned}
&= \int dy \int dp\, \langle p| \otimes \langle x|\, e^{[-\frac{i}{\hbar}\int_0^t (\mathbf{H}_0 + gp(\Theta_{ab}[\mathbf{x}] - \Lambda(t')))dt']} \otimes |x_0\rangle \\
&\quad \times \left[\int dy \int dp\right]^{-1} \\
&= \left(\frac{g}{2\pi\hbar}\right) \int dp\, \langle x|\, e^{[-\frac{i}{\hbar}\int_0^t (\mathbf{H}_0 + gp(\Theta_{ab}[\mathbf{x}] - \Lambda(t')))dt']} \otimes |x_0\rangle \times \left[\left(\frac{g}{2\pi\hbar}\right)\int dp\right]^{-1} \\
&= \int D[x]\delta(\tau - t_{ab}^{cl}[x])e^{iS[x(t)]/\hbar}/\delta(0), \qquad (4)
\end{aligned}
$$

where $\delta(0) = \left(\frac{g}{2\pi\hbar}\right)\int dp$ is the normalization.

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